



Important Neutrosophic Rules for Decision-Making in the Case of Uncertain Data

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Abstract

An urgent need to make a rational decision has emerged in the world of rapid changes based on quantitative methods that reduce the proportion of risk, especially if the decisions are fateful and the decision issues are huge and complex, noting that the decision-making process and the selection of the optimal alternative depends on the quality of the data that describes the issue that the decision is intended to be taken. Because the theory of administrative decision-making depends on this data and on the type of this data if it is confirmed data or unconfirmed data and not specified with sufficient accuracy, or random data that is repeated according to a certain probability distribution law, after which the decision maker uses the methods used to obtain on the optimal decision. In this research we will study the theory of administrative decisions in the case of uncertain data, which is the situation that the decision maker faces, and he does not know anything certain about the state that nature (market or management --) will take, nor even about the possibilities of any of them, then it is assumed that the cases the possible ones are equal and they enter the analysis at the same opportunity and make a trade-off between the alternatives available to him in all circumstances. In the classical logic, a set of rules was used to help the decision maker to make the ideal decision, and since the ideal decision depends on specific classical values that do not take into account the changes that may occur in the work environment, which is represented by high prices or unavailability of materials or others, it was necessary to search for a better method that helps us to avoid dealing with specific values and gives us a margin of freedom. Therefore, in this research, we will study the theory of decision-making in the case of uncertain data using the Neutrosophic Logic, the logic that helps us to face fluctuations and changes that we may encounter during work, through uncertainty that the Neutrosophical values have, which we will take in the elements of the profit (or loss) matrix and rely on them in the decision-making process, as we will take these values in the form of fields representing the minimum field of profit (or loss) that we can get in the worst cases of nature, and represents the upper limit of the field of profit (or loss) that we can get in the best cases of nature, and we will show the most important rules used in the case of uncertain data with an applied example of each rule.

Keywords: Decision-Making Theory; Neutrosophic Logic; Decision-Making in Case of Uncertainty

1. Introduction

To avoid economic inflation and the sensitivity of political and military situations, the need to know the results of any decision appeared before taking it, which prompted mathematicians to develop mathematical models aimed at searching for optimal strategies in conditions of doubt and uncertainty. From the most prominent scientists who had a great impact in this field Von Neumann - Morgan Strain - Nash and the scientist Herbert Simon, who is considered the father of the theory of decision-making, and this is due to the great importance of the theories he developed, as it opened the way for new methods called today artificial intelligence [1,2,3]. The decision, in all fields, is increasing day by day. It was necessary for a new logic to keep pace with the great scientific development, and to limit risks and losses. The Neutrosophic logic, the logic whose emersion was a revolution in all fields of science, was founded by the American mathematical philosopher Florentin Smarandache [4,5,6,7,8], where he presented it in 1995 as a generalization of Fuzzy Logic, and as an extension of the theory of fuzzy categories, presented by Lotfi Zadeh [9] in 1965, where it came to replace the binary logic that recognizes right and wrong by entering a third neutral case, which can be interpreted as undetermined or uncertain, Neutrosophic logic has grown and developed significantly in

recent years through its application in measurement, sets, graphs, and many scientific and practical fields, [10,11,12,13,14,15,16, 17,18,19,20,21,22,23].

2. Discussion: [1, 2]

When the data is uncertain and the decision maker does not know anything about the state that nature will take, not even about the possibilities of any of them occurring, he will deal with them on the basis that they are equivalent, and enter into the analysis at the same opportunity, then he will make a trade-off between the alternatives available to him in all circumstances by using specific rules that were developed to study the state of uncertainty [1,2], we will study in this research some of the rules that were used in the classical logic to determine the optimal decision in the event of uncertainty, according to the Neutrosophic Logic, where we will take in the elements of the profit (or loss) Neutrosophic values of the matrix .

The Problem of Decision-making:

Let us have a set of alternatives, which we will symbolize with $a_1, a_2, a_3, \dots, a_m$, and a group of states of nature, which we will symbolize with $\theta_1, \theta_2, \theta_3, \dots, \theta_n$, and we have the profit (or loss) resulting from the alternative i , since $i = 1, 2, 3, \dots, m$, and the state of nature j , since $j = 1, 2, 3, \dots, n$, we will represent the above in the following table:

Table 1: Profit Matrix for Uncertainty Case

alternatives \ states of nature	states of nature				
	θ_1	θ_2	θ_3	-----	θ_n
a_1	Nx_{11}	Nx_{12}	Nx_{13}	-----	Nx_{1n}
a_2	Nx_{21}	Nx_{22}	Nx_{23}	-----	Nx_{2n}
a_3	Nx_{31}	Nx_{32}	Nx_{33}	-----	Nx_{3n}
...	-----	-----	-----	-----	-----
...	-----	-----	-----	-----	-----
a_m	Nx_{m1}	Nx_{m2}	Nx_{m3}	-----	Nx_{mn}

Required: Determining the appropriate alternative so that the decision maker achieves the greatest profit (i.e. the least loss).

To determine the optimal alternative, and since the situation is a state of uncertainty, we will use one of the following rules:

1- Neutrosophic Laplace's Rule (Average Gain or Loss):

In this rule we calculate the average profit (or loss) corresponding to each of the available alternatives, then we choose the largest (smallest) average, and the alternative corresponding to this value is the appropriate decision.

Table2: Neutrosophic Laplace's Rule (Average Gain or Loss):

alternatives \ states of nature	states of nature					average of profit or loss
	θ_1	θ_2	θ_3	-----	θ_n	
a_1	Nx_{11}	Nx_{12}	Nx_{13}	-----	Nx_{1n}	$\overline{NX}_1 = \frac{\sum_{j=1}^n Nx_{1j}}{n}$

a_2	Nx_{21}	Nx_{22}	Nx_{23}	-----	Nx_{2n}	$\overline{NX}_2 = \frac{\sum_{j=1}^n Nx_{2j}}{n}$
a_3	Nx_{31}	Nx_{32}	Nx_{33}	-----	Nx_{3n}	$\overline{NX}_3 = \frac{\sum_{j=1}^n Nx_{3j}}{n}$
---	-----	-----	-----	-----	-----	
a_m	Nx_{m1}	Nx_{m2}	Nx_{m3}	-----	Nx_{mn}	$\overline{NX}_m = \frac{\sum_{j=1}^n Nx_{mj}}{n}$

In the case of profit, we have:

$$\overline{NX}^* = \text{Max}\{\overline{NX}_1, \overline{NX}_2, \overline{NX}_3, \dots, \overline{NX}_m\}$$

In case of loss, we have:

$$\overline{NX}^* = \text{Min}\{\overline{NX}_1, \overline{NX}_2, \overline{NX}_3, \dots, \overline{NX}_m\}$$

In addition, the appropriate alternative is the alternative corresponding to \overline{NX}^*

We illustrate the above with the following example:

Practical example:

An investor wants an amount of millions of dollars in the available investment areas, which are:

- a_1 Cars trading.
- a_2 Real estate trading.
- a_3 Purchasing shares of a service company.
- a_4 Establishing an industrial project.

It is required to determine the appropriate alternative for the investor according to the following economic conditions:

- θ_1 Declining of the economic situation.
- θ_2 Stabilization of the economic situation.
- θ_3 Improving of the economic situation.

The annual profits of these investments, according to the economic conditions, are shown in the following table: (One thousand dollars annually).

Table 3: profit matrix for the example

alternatives \ states of nature	θ_1	θ_2	θ_3
	a_1	[100, 190]	[200, 260]

a ₂	[- 400 , - 220]	[160 , 170]	[600 , 650]
a ₃	[100 , 170]	[290 , 370]	[300 , 360]
a ₄	[133 , 180]	[160 , 280]	[290 , 400]

Required: find the appropriate alternative using the Neutrosophic Laplace Rule:

We calculate the average profit corresponding to each alternative, and thus we get the following table:

Table 4: Applies Neutrosophic Laplace's Rule

states of nature alternatives	θ ₁	θ ₂	θ ₃	average of profit
a ₁	[100 , 190]	[200 , 260]	[300 , 390]	$\frac{[100 , 190] + [200 , 260] + [300 , 390]}{3} = [200 , 280]$
a ₂	[- 400 , - 220]	[160 , 170]	[600 , 650]	$\frac{[- 400 , - 200] + [160 , 170] + [600 , 650]}{3} = [120 , 206.7]$
a ₃	[100 , 170]	[290 , 370]	[300 , 360]	$\frac{[100 , 170] + [290 , 370] + [300 , 360]}{3} = [230 , 300]$
a ₄	[133 , 180]	[160 , 280]	[290 , 400]	$\frac{[133 , 180] + [160 , 280] + [290 , 400]}{3} = [194.3 , 286.7]$

By comparing the elements of the last column, we notice that the largest value of the average profit is [230 , 300] thousand dollars corresponding to the Cars trading (a₃), so we consider the alternative a₃ to be the appropriate decision for the investor according to Laplace's rule.

2- The Neutrosophic Maxi Max rule:

This rule is based on calculating the largest value of profit for each alternative (in each line), then finding the largest of these, and considering the value that corresponds to it as the appropriate decision.

Therefore, if we denote NY for the Maxi Max Nx_{ij} for the profit from variant i versus the state of nature j, we get the following relationship:

$$NY = \text{Max}_i \left[\text{Max}_j Nx_{ij} \right] = \text{Max}_i NY_i$$

We explain this rule through the following table:

Table 5: The Neutrosophic Maxi Max rule

states of nature alternatives	θ ₁	θ ₂	θ ₃	-----	θ _n	NY _i = Max _j Nx _{ij}
a ₁	Nx ₁₁	Nx ₁₂	Nx ₁₃	-----	Nx _{1n}	NY ₁ = Max _j Nx _{1j}
a ₂	Nx ₂₁	Nx ₂₂	Nx ₂₃	-----	Nx _{2n}	NY ₂ = Max _j Nx _{2j}

a_3	Nx_{31}	Nx_{32}	Nx_{33}	-----	Nx_{3n}	$NY_3 = \text{Max}_j Nx_{3j}$
---	-----	-----	-----	-----	-----	-----
a_m	Nx_{m1}	Nx_{m2}	Nx_{m3}	-----	Nx_{mn}	$NY_m = \text{Max}_j Nx_{mj}$

We illustrate the above with the following example:

Practical example: We will take the same example given in the previous rule:

Table 6: profit matrix for the example

alternatives \ states of nature	θ_1	θ_2	θ_3
a_1	[100 , 190]	[200 , 260]	[300 , 390]
a_2	[- 400 , - 220]	[160 , 170]	[600 , 650]
a_3	[100 , 170]	[290 , 370]	[300 , 360]
a_4	[133 , 180]	[160 , 280]	[290 , 400]

It is required to determine the appropriate alternative using the Maxi Max rule.

Table 7: Applies Neutrosophic Maxi Max rule

alternatives \ states of nature	θ_1	θ_2	θ_3	$NY_i = \text{Max}_j Nx_{ij}$
a_1	[100 , 190]	[200 , 260]	[300 , 390]	[300 , 390]
a_2	[- 400 , - 220]	[160 , 170]	[600 , 650]	[600 , 650]
a_3	[100 , 170]	[290 , 370]	[300 , 360]	[300 , 360]
a_4	[133 , 180]	[160 , 280]	[290 , 400]	[290 , 400]

By comparing the elements of the last column, we get: $NY = \text{Max}_i \left[\text{Max}_j x_{ij} \right] = [600 , 650]$

That is, the largest values are the value [600 , 650] corresponding to alternative a_2 . Thus, alternative a_2 is the appropriate alternative according to the Maxi Max rule, that is, the investor must invest the amount in real estate trading.

3- The Neutrosophic Maxi Min Rule:

This rule is based on calculating the smallest value of gain for each alternative (in each line), if the conditions of nature are bad, and then searching for the largest of these small ones, and considering the value corresponding to it as the appropriate decision.

Therefore, if we denote NZ for the largest of the smallest and Nx_{ij} for the profit from variant i versus the state of nature j , we get the following relation:

$$NZ = \text{Max}_i \left[\text{Min}_j Nx_{ij} \right] = \text{Max}_i NZ_i$$

We explain this rule through the following table:

Table 8: The Neutrosophic Maxi Min Rule

states of nature alternatives	θ_1	θ_2	θ_3	-----	θ_n	$NZ_i = \text{Min}_j Nx_{ij}$
a1	Nx_{11}	Nx_{12}	Nx_{13}	-----	Nx_{1n}	$NZ_1 = \text{Min}_j Nx_{1j}$
a2	Nx_{21}	Nx_{22}	Nx_{23}	-----	Nx_{2n}	$NZ_2 = \text{Min}_j Nx_{2j}$
a3	Nx_{31}	Nx_{32}	Nx_{33}	-----	Nx_{3n}	$NZ_3 = \text{Min}_j Nx_{3j}$
---	-----	-----	-----	-----	-----	-----
---	-----	-----	-----	-----	-----	-----
a _m	Nx_{m1}	Nx_{m2}	Nx_{m3}	-----	Nx_{mn}	$NZ_m = \text{Min}_j Nx_{mj}$

We illustrate the above with the following example:

Practical example: We will take the same example given in the previous rule:

Table 9: profit matrix for the example

states of nature alternatives	θ_1	θ_2	θ_3
a1	[100 , 190]	[200 , 260]	[300 , 390]
a2	[- 400 , - 220]	[160 , 170]	[600 , 650]
a3	[100 , 170]	[290 , 370]	[300 , 360]
a4	[133 , 180]	[160 , 280]	[290 , 400]

It is required to determine the appropriate alternative using the Maxi Min Rule.

Table 10: Applies Neutrosophic Maxi Min Rule

states of nature alternatives	θ_1	θ_2	θ_3	$NZ_i = \text{Max}_j Nx_{ij}$
a1	[100 , 190]	[200 , 260]	[300 , 390]	[100 , 190]
a2	[- 400 , - 220]	[160 , 170]	[600 , 650]	[- 400 , - 220]
a3	[100 , 170]	[290 , 370]	[300 , 360]	[100 , 170]
a4	[133 , 180]	[160 , 280]	[290 , 400]	[133 , 180]

Comparing the elements of the last column, we get:

$$NZ = \text{Max}_i \left[\text{Min}_j x_{ij} \right] = [133 , 180]$$

That is, the largest values are the value [133 , 180] corresponding to the alternative a4. Thus, the alternative a4 is the appropriate alternative according to Maxi Min Rule, that is, the investor must invest the amount in establishing an industrial project.

4- The Neutrosophic Horwes Rule:

Since most of the decision-makers do not look at the future with a completely optimistic or completely pessimistic view, but rather they expect the future to take a middle state between the states of optimism and pessimism, so (Horwes) put a composite rule of the two rules given by the relationship:

$$NH = \text{Max}_i[\alpha NY_i + (1 - \alpha)NZ_i]$$

Since this rule $0 \leq \alpha \leq 1$ gives us a certain value confined between NZ, NY, that is $NZ \leq NH \leq NY$, noting that NH is related to the value we give to α , and this makes it tend towards the value NY if $\alpha > 0.5$ and towards NZ if $\alpha < 0.5$, and if we put $\alpha = 0$ we get NZ and if we put $\alpha = 1$ we get NY.

We explain this rule through the following table:

Table 11: The Neutrosophic Horwes Rule

states of nature alternatives	θ_1	θ_2	θ_3	-----	θ_n	$NH_i = \alpha NY_i + (1 - \alpha)NZ_i$
a ₁	Nx_{11}	Nx_{12}	Nx_{13}	-----	Nx_{1n}	$NH_1 = \alpha NY_1 + (1 - \alpha)NZ_1$
a ₂	Nx_{21}	Nx_{22}	Nx_{23}	-----	Nx_{2n}	$NH_2 = \alpha NY_2 + (1 - \alpha)NZ_2$
a ₃	Nx_{31}	Nx_{32}	Nx_{33}	-----	Nx_{3n}	$NH_3 = \alpha NY_3 + (1 - \alpha)NZ_3$
---	-----	-----	-----	-----	-----	-----
a _m	Nx_{m1}	Nx_{m2}	Nx_{m3}	-----	Nx_{mn}	$NH_m = \alpha NY_m + (1 - \alpha)NZ_m$

Where NY_i and NZ_i are calculated as in the previous two rules, and we choose the appropriate alternative according to this rule.

$$NH = \text{Max}_i[\alpha NY_i + (1 - \alpha)NZ_i]$$

We illustrate this rule through the example given in the previous rules and consider $\alpha = 0.2$.

Table 12: Applies Neutrosophic Horwes Rule

states of nature alternatives	θ_1	θ_2	θ_3	NY_i	NZ_i	NH_i
a ₁	[100 , 190]	[200 , 260]	[300 , 390]	[300 , 390]	[100 , 190]	[140 , 230]
a ₂	[- 400 , - 22]	[160 , 170]	[600 , 650]	[600 , 650]	[- 400 , - 22]	[- 200 , - 4]
a ₃	[100 , 170]	[290 , 370]	[300 , 360]	[300 , 360]	[100 , 170]	[140 , 208]
a ₄	[133 , 180]	[160 , 280]	[290 , 400]	[290 , 400]	[133 , 180]	[164.4 , 224]

We apply the rule $NH = \text{Max}_i[\alpha NY_i + (1 - \alpha)NZ_i]$ to the elements of the last column. We note that the largest value is [164.4 , 224] and it corresponds to the a₄ alternative, and therefore the a₄ alternative is the appropriate decision according to the (Horwes) rule, that is, the investor must invest the amount in establishing an industrial project.

5. The Neutrosophic Min Max Rule (Savage Rule):

This rule depends on new expressions called Savage Expressions that we symbolize with NS_{ij} and get them from the relationship:

$$NS_{ij} = \left[\text{Max}_i Nx_{ij} \right] - Nx_{ij}$$

Table 13: The Neutrosophic Min Max Rule (Savage Rule)

							Savage expressions				
states of nature alternatives	θ_1	θ_2	θ_3	---	θ_n	θ'_1	θ'_2	θ'_3	--	θ'_n	
	a1	Nx_{11}	Nx_{12}	Nx_{13}	---	Nx_{1n}	NS_{11}	NS_{12}	NS_{13}	-----	NS_{1n}
a2	Nx_{21}	Nx_{22}	Nx_{23}	---	Nx_{2n}	NS_{21}	NS_{22}	NS_{23}	-----	NS_{2n}	
a3	Nx_{31}	Nx_{32}	Nx_{33}	---	Nx_{3n}	NS_{31}	NS_{32}	NS_{33}	-----	NS_{3n}	
...	-----	-----	-----	---	-----	-----	-----	-----	-----		
a _m	Nx_{m1}	Nx_{m2}	Nx_{m3}	---	Nx_{mn}	NS_{m1}	NS_{m2}	NS_{m3}	-----	NS_{mn}	

That is, we take the largest value in each column and then subtract the values in that column from it, we get a new column, and from the new columns we get a new matrix that we call the Savage Expression Matrix, then we apply to the elements of this matrix the following rule $\text{Max}_j NS_{ij}$, that is, we choose from each line the largest value, and thus we get on a new column in the matrix of Savage amounts, as in the following table:

Table 14: Savage Expressions

Savage expressions					$\text{Max}_j NS_{ij}$
θ'_1	θ'_2	θ'_3	--	θ'_n	
NS_{11}	NS_{12}	NS_{13}	-----	NS_{1n}	$\text{Max}_j NS_{1j}$
NS_{21}	NS_{22}	NS_{23}	-----	NS_{2n}	$\text{Max}_j NS_{2j}$
NS_{31}	NS_{32}	NS_{33}	-----	NS_{3n}	$\text{Max}_j NS_{3j}$
-----	-----	-----	-----		
NS_{m1}	NS_{m2}	NS_{m3}	-----	NS_{mn}	$\text{Max}_j NS_{mj}$

We compare the elements of the new column and choose the smallest value for which the corresponding alternative is the appropriate decision, i.e. we apply the relationship:

$$\text{Min}_i \left(\text{Max}_j NS_{ij} \right)$$

We illustrate this rule through the example given in the previous rules:

Table 15: Applies Neutrosophic Min Max Rule (Savage Rule)

states of nature alternatives	θ_1	θ_2	θ_3	Savage expressions		
				θ'_1	θ'_2	θ'_3
a1	[100, 190]	[200, 260]	[300, 390]	[10, 33]	[90, 110]	[260, 300]
a2	[-400, -220]	[160, 170]	[600, 650]	[-267, -40]	[130, 200]	[0, 0]
a3	[100, 170]	[290, 370]	[300, 360]	[10, 33]	[0, 0]	[290, 300]
a4	[133, 180]	[160, 280]	[290, 400]	[0, 0]	[90, 130]	[250, 310]

We have taken the largest value in each column and then subtracted from it the values in that column, and thus we got a new matrix that we call the Savage Amounts Matrix.

We applied the following rule $Max_j NS_{ij}$ to the elements of this matrix, and thus we got a new column in the Savage Amounts Matrix as shown in the following table:

Table 16: Savage Expressions

Savage Expressions			$Max_j NS_{ij}$
θ'_1	θ'_2	θ'_3	
[10, 33]	[90, 110]	[260, 300]	[260, 300]
[-267, -40]	[130, 200]	[0, 0]	[130, 200]
[10, 33]	[0, 0]	[290, 300]	[290, 300]
[0, 0]	[90, 130]	[250, 310]	[250, 310]

We compare the elements of the new column and choose the smallest value for which the corresponding alternative is the appropriate decision, i.e. we apply the relationship:

$$Min_i \left(Max_j NS_{ij} \right) = [130, 200]$$

In addition, the value [130, 200] is the appropriate value according to the rule (Savage), and it is corresponding to the alternative a2, that is, the appropriate decision is the alternative a2 according to this rule (Savage), the investor must invest the amount in the real estate trade.

6. Conclusion and results:

From the previous study, we get the following table

Table 17: Comparison results

The decision	Alternative	The profit	The rule
Cars trading	a3	[230, 300]	Laplace
Real estate trade	a2	[600, 650]	Maxi Max
Establishing an industrial project	a4	[133, 180]	Maxi Min

Establishing an industrial project	a_4	[164.4 , 224]	Horwes
Real estate trade	a_2	[130 , 200]	Savage

We note from the table that:

The alternative a_2 was chosen according to two rules, the Maxi Max rule and Min Max Rule (Savage Rule), but with a different estimate of the profit, and the reason for this is due to the optimist's view and his choice of the highest profit.

The alternative a_4 was chosen according to two rules, the Maxi Min rule and Horwes rule. The reason for this is that the rule of Horwes is related to the value we give to α , and this makes it tend towards NZ if $\alpha < 0.5$ and in this issue $\alpha = 0,2$ and $0,2 < 0,5$ were chosen, and if it was chosen is greater than $0,5$ we would have found a convergence between the base of the Horwes and the base of Maxi Max.

Alternative a_3 was chosen according to Laplace's rule only.

Alternative a_1 was not selected according to any of the rules.

The decision remains with the investor because he is the decision maker.

We also note that the Neutrosophical values of profits give us for each alternative the lowest possible profit in the event that was chosen and the greatest profit, according to the state of nature.

We are looking forward in the near future to prepare research in decision-making theory that includes other cases and decision trees.

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