$N_{nc}\gamma$ Maps in N_{nc}-Topological Spaces

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Abstract

In this article, the concept of N-neutrosophic crisp γ-open and N-neutrosophic crisp γ-closed mappings in N-neutrosophic crisp topological spaces are introduced and studied some of their related properties. Also, N-neutrosophic crisp γ irresolute mapping is introduced in N-neutrosophic crisp topological spaces. Further, it is extended to N-neutrosophic crisp γ-homeomorphism, N-neutrosophic crisp γ-Completely homeomorphism and N-neutrosophic crisp $\gamma T_{\frac{1}{2}}$-space in N-neutrosophic crisp topological spaces and establishes some of their related properties. Finally, Strongly N-neutrosophic crisp γ continuous and Perfectly N-neutrosophic crisp γ continuous functions is also discuss.

Keywords: $N_{nc}\gamma O$ map, $N_{nc}\gamma C$ map, $N_{nc}\gamma T_{\frac{1}{2}}$-space, $N_{nc}\gamma$-homeomorphism, $N_{nc}\gamma$-Completely homeomorphism, $StN_{nc}\gamma Cts$, $PeN_{nc}\gamma Cts$
AMS (2000) subject classification: 54A10, 54A40, 54C05.

1 Introduction

In our daily routine, we have used the crisp sets in most of our life. The concepts of neutrosophy and neutrosophic set are the recent tools in a topological space. It was first introduced by Smarandache[7,8] in the beginning of 20^{th} century. In 2014, Salama, Smarandache and Kroumov[5] has provided the basic concept of neutrosophic crisp set in a topological space. After that Al-Omeri[2] also investigated some fundamental properties of neutrosophic crisp topological Spaces. Al-Hamido[1] explore the concept of N-neutrosophic crisp topological spaces and investigate some of their basic properties in N-terms. By using N-terms of topological spaces, we can defined $1_{nc}ts, 2_{nc}ts, \cdots, N_{nc}ts$.
In 1996, Andrijevic[3] introduced b-open sets and develop some of their works in general topology. The notion of γ-open set in topological spaces was introduced by Min[4] and worked in the field of general topology. Vadivel et al.[10] presented γ-open sets in neutrosophic crisp topological spaces via N-terms of topology. Also, he[11,12] introduced γ-continuous and γ-contra continuous functions in N-neutrosophic crisp topological spaces and almost γ-Continuous Functions in N-Neutrosophic Crisp Topological Spaces.

2 Preliminaries

Definition 2.1. [6] For any non-empty fixed set Y, a neutrosophic crisp set (briefly, ncs) K, is an object having the form $K = \langle K_1, K_2, K_3\rangle$ where K_1, K_2 & K_3 are subsets of Y satisfying any one of the types

(T1) $K_\eta \cap K_\xi = \varphi, \eta \neq \xi \;\&\; \bigcup_{\eta=1}^{3} K_\eta \subset Y, \forall\, \eta, \xi = 1, 2, 3.$

(T2) $K_\eta \cap K_\xi = \varphi, \eta \neq \xi \;\&\; \bigcup_{\eta=1}^{3} K_\eta = Y, \forall\, \eta, \xi = 1, 2, 3.$

(T3) $\bigcap_{\eta=1}^{3} K_\eta = \varphi \;\&\; \bigcup_{\eta=1}^{3} K_\eta = Y, \forall\, \eta = 1, 2, 3.$

Definition 2.2. [6] Types of ncs's $\emptyset_N$ and Y_N in Y are as

https://doi.org/10.54216/IJNS.180302
Received: January 13, 2022 Accepted: April 07, 2022

(i) $\emptyset_N = \langle \emptyset, \emptyset, Y \rangle$ or $\langle \emptyset, Y, Y \rangle$ or $\langle \emptyset, Y, \emptyset \rangle$ or $\langle \emptyset, \emptyset, \emptyset \rangle$.

(ii) $Y_N = \langle Y, \emptyset, \emptyset \rangle$ or $\langle Y, Y, \emptyset \rangle$ or $\langle Y, \emptyset, Y \rangle$ or $\langle Y, Y, Y \rangle$.

Definition 2.3. [6] Let Y be a non-empty set & the ncs's K & M in the form $K = \langle K_{11}, K_{22}, K_{33} \rangle$, $M = \langle M_{11}, M_{22}, M_{33} \rangle$, then

(i) $K \subseteq M \Leftrightarrow K_{11} \subseteq M_{11}, K_{22} \subseteq M_{22}$ & $K_{33} \supseteq M_{33}$ or $K_{11} \subseteq M_{11}, K_{22} \supseteq M_{22}$ & $K_{33} \supseteq M_{33}$.

(ii) $K \cap M = \langle K_{11} \cap M_{11}, K_{22} \cap M_{22}, K_{33} \cup M_{33} \rangle$ or $\langle K_{11} \cap M_{11}, K_{22} \cup M_{22}, K_{33} \cup M_{33} \rangle$

(iii) $K \cup M = \langle K_{11} \cup M_{11}, K_{22} \cup M_{22}, K_{33} \cap M_{33} \rangle$ or $\langle K_{11} \cup M_{11}, K_{22} \cap M_{22}, K_{33} \cap M_{33} \rangle$

Definition 2.4. [6] Let $K = \langle K_1, K_2, K_3 \rangle$ a ncs on Y, then the complement of K (briefly, K^c) may be defined in three different ways:

(C1) $K^c = \langle {K_1}^c, {K_2}^c, {K_3}^c \rangle$, or

(C2) $K^c = \langle K_3, K_2, K_1 \rangle$, or

(C3) $K^c = \langle K_3, {K_2}^c, K_1 \rangle$.

Definition 2.5. [1] Let Y be a non-empty set. Then ${}_{nc}\Gamma_1$, ${}_{nc}\Gamma_2$, $\cdots$, ${}_{nc}\Gamma_N$ are N-arbitrary crisp topologies defined on Y and the collection $N_{nc}\Gamma$ is called N_{nc}-topology on Y is

$N_{nc}\Gamma = \{A \subseteq Y : A = (\bigcup_{\eta j=1}^{N} E_{\eta j}) \cup (\bigcap_{\eta j=1}^{N} F_{\eta j}), E_{\eta j}, F_{\eta j} \in {}_{nc}\Gamma_j\}$

and it satisfies the following axioms:

(i) $\emptyset_N, Y_N \in N_{nc}\Gamma$.

(ii) $\bigcup_{j=1}^{\infty} K_\eta \in N_{nc}\Gamma \; \forall \; \{K_\eta\}_{\eta=1}^{\infty} \in N_{nc}\Gamma$.

(iii) $\bigcap_{\eta=1}^{n} K_\eta \in N_{nc}\Gamma \; \forall \; \{K_\eta\}_{\eta=1}^{n} \in N_{nc}\Gamma$.

Then $(Y, N_{nc}\Gamma)$ is called a N_{nc}-topological space (briefly, $N_{nc}ts$) on Y. The N_{nc}-open sets ($N_{nc}os$) are the elements of $N_{nc}\Gamma$ in Y and the complement of $N_{nc}os$ is called N_{nc}-closed sets ($N_{nc}cs$) in Y. The elements of Y are known as N_{nc}-sets ($N_{nc}s$) on Y.

Definition 2.6. [1] Let $(Y, N_{nc}\Gamma)$ be $N_{nc}ts$ on Y and K be an $N_{nc}s$ on Y, then the N_{nc} interior of K (briefly, $N_{nc}int(K)$) and N_{nc} closure of K (briefly, $N_{nc}cl(K)$) are defined as

$$N_{nc}int(K) = \cup\{A : A \subseteq K \text{ \& } G \text{ is a } N_{nc}os\}$$

$$N_{nc}cl(K) = \cap\{D : K \subseteq D \text{ \& } D \text{ is a } N_{nc}cs\}.$$

Definition 2.7. [1] Let $(Y, N_{nc}\Gamma)$ be any $N_{nc}ts$. Let K be an $N_{nc}s$ in $(Y, N_{nc}\Gamma)$. Then K is said to be a N-neutrosophic crisp

(i) regular open[10] set (briefly, $N_{nc}ros$) if $K = N_{nc}int(N_{nc}cl(K))$.

(ii) pre open set (briefly, $N_{nc}\mathcal{P}os$) if $K \subseteq N_{nc}int(N_{nc}cl(K))$.

(iii) semi open set (briefly, $N_{nc}\mathcal{S}os$) if $K \subseteq N_{nc}cl(N_{nc}int(K))$.

(iv) α-open set (briefly, $N_{nc}\alpha os$) if $K \subseteq N_{nc}int(N_{nc}cl(N_{nc}int(K)))$.

(v) γ-open[10] set (briefly, $N_{nc}\gamma os$) set if $K \subseteq N_{nc}cl(N_{nc}int(K)) \cup N_{nc}int(N_{nc}cl(K))$.

The complement of an $N_{nc}\mathcal{P}os$ (resp. $N_{nc}\mathcal{S}os$, $N_{nc}\alpha os$, $N_{nc}ros$ & $N_{nc}\gamma os$) is called an N_{nc}-pre (resp. N_{nc}-semi, N_{nc}-α, N_{nc}-regular & N_{nc}-γ) closed set (briefly, $N_{nc}\mathcal{P}cs$ (resp. $N_{nc}\mathcal{S}cs$, $N_{nc}\alpha cs$, $N_{nc}rcs$ & $N_{nc}\gamma c$)) in Y.

The family of all $N_{nc}\mathcal{P}os$ (resp. $N_{nc}\mathcal{P}cs$, $N_{nc}\mathcal{S}os$, $N_{nc}\mathcal{S}cs$, $N_{nc}\alpha os$, $N_{nc}\alpha cs$, $N_{nc}\gamma os$ & $N_{nc}\gamma cs$) of Y is denoted by $N_{nc}\mathcal{P}OS(Y)$ (resp. $N_{nc}\mathcal{P}CS(Y)$, $N_{nc}\mathcal{S}OS(Y)$, $N_{nc}\mathcal{S}CS(Y)$, $N_{nc}\alpha OS(Y)$, $N_{nc}\alpha CS(Y)$, $N_{nc}\gamma OS(Y)$ & $N_{nc}\gamma CS(Y)$).

https://doi.org/10.54216/IJNS.180302
Received: January 13, 2022 Accepted: April 07, 2022

Definition 2.8. [10] Let $(Y, N_{nc}\Gamma)$ be a $N_{nc}ts$ on Y and K be an $N_{nc}s$ on Y then

(i) $N_{nc}\gamma int(K)$(resp. $N_{nc}rint(K)) = \cup\{D : D \subseteq K$ and D is a $N_{nc}\gamma o$ (resp. $N_{nc}ro)\}$.

(ii) $N_{nc}\gamma cl(K)$(resp. $N_{nc}rcl(K)) = \cap\{D : K \subseteq D$ and D is a $N_{nc}\gamma c$ (resp. $N_{nc}rc)\}$.

Definition 2.9. Let $(X_1, N_{nc}\Gamma)$ and $(X_2, N_{nc}\Psi)$ be any two $N_{nc}ts$'s. A map $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is said to be

(i) N_{nc} (resp. $N_{nc}\gamma$)-continuous (briefly, $N_{nc}Cts$[9] (resp. $N_{nc}\gamma Cts$[11])) if the inverse image of every $N_{nc}os$ in $(X_2, N_{nc}\Psi)$ is a $N_{nc}os$ (resp. $N_{nc}\gamma os$) in $(X_1, N_{nc}\Gamma)$.

(ii) strongly N_{nc} continuous (briefly, $StN_{nc}Cts$[13]) function if the inverse image of every subset in $(X_2, N_{nc}\Psi)$ is N-neutrosophic crisp clopen (i.e both $N_{nc}o$ and $N_{nc}c$) (briefly, $N_{nc}clo$) in $(X_1, N_{nc}\Gamma)$.

3 N-Neutrosophic Crisp γ-Irresolute Functions

Definition 3.1. Let $(X_1, N_{nc}\Gamma)$ and $(X_2, N_{nc}\Psi)$ be any two $N_{nc}ts$'s. A map $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is said to be $N_{nc}\gamma$-irresolute function (briefly, $N_{nc}\gamma Irr$), if for the inverse image of every $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$ is a $N_{nc}\gamma cs$ in $(X_1, N_{nc}\Gamma)$.

Theorem 3.2. Let $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a mapping, if $N_{nc}\gamma Irr$, then ρ is $N_{nc}\gamma Cts$.

Proof. Let C be $N_{nc}cs$ in X_2, then C is $N_{nc}\gamma cs$ in X_2, since every $N_{nc}cs$ is $N_{nc}\gamma cs$. By hypothesis, $\rho^{-1}(C)$ is $N_{nc}\gamma cs$. Therefore ρ is $N_{nc}\gamma Cts$. □

Remark 3.3. The converse of the above theorem need not be true as shown in the following example.

Example 3.4. Let $X = \{l_1, m_1, n_1, o_1, p_1\} = Y$, ${}_{nc}\Gamma_1 = \{\varphi_N, X_N, L, M, N\}$, ${}_{nc}\Gamma_2 = \{\varphi_N, X_N\}$. $L = \langle\{n_1\}, \{\varphi\}, \{l_1, m_1, o_1, p_1\}\rangle$, $M = \langle\{l_1, m_1\}, \{\varphi\}, \{n_1, o_1, p_1\}\rangle$, $N = \langle\{l_1, m_1, n_1\}, \{\varphi\}, \{o_1, p_1\}\rangle$, then we have $2_{nc}\Gamma = \{\varphi_N, X_N, L, M, N\}$. ${}_{nc}\Psi_1 = \{\varphi_N, Y_N, O, P, Q\}$, ${}_{nc}\Psi_2 = \{\varphi_N, Y_N\}$. $O = \langle\{l_1, m_1\}, \{\varphi\}, \{n_1, o_1, p_1\}\rangle$, $P = \langle\{n_1, o_1\}, \{\varphi\}, \{l_1, m_1, p_1\}\rangle$, $Q = \langle\{l_1, m_1, n_1, o_1\}, \{\varphi\}, \{p_1\}\rangle$, then we have $2_{nc}\Psi = \{\varphi_N, Y_N, O, P, Q\}$.
Define $\rho : (X, 2_{nc}\Gamma) \to (Y, 2_{nc}\Psi)$ as $\rho(l_1) = l_1$, $\rho(m_1) = m_1$, $\rho(n_1) = n_1$, $\rho(o_1) = p_1$ & $\rho(p_1) = p_1$, then $2_{nc}\gamma Cts$ mapping but not $2_{nc}\gamma Irr$ mapping, the set $\rho^{-1}(\langle\{m_1, o_1, p_1\}, \{\varphi\}, \{l_1, n_1\}\rangle) = \langle\{m_1, o_1, p_1\}, \{\varphi\}, \{l_1, n_1\}\rangle$ is a $2_{nc}\gamma os$ in Y but not $2_{nc}\gamma os$ in X.

Theorem 3.5. A function $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is $N_{nc}\gamma Irr$ if and only if for every $N_{nc}\gamma os$ K in X_2, $\rho^{-1}(K)$ is $N_{nc}\gamma os$ in X_1.

Proof. Follows from the fact that the complement of $N_{nc}\gamma os$ is $N_{nc}\gamma cs$ and vice versa. □

Theorem 3.6. If $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ are both $N_{nc}\gamma Irr$, then $\rho_2 \circ \rho_1 : (X_1, N_{nc}\Gamma) \to (X_3, N_{nc}\Phi)$ is $N_{nc}\gamma Irr$.

Proof. Let K be $N_{nc}\gamma os$ in X_3. Then $\rho_2^{-1}(K)$ is $N_{nc}\gamma os$ in X_2, since ρ_2 is $N_{nc}\gamma Irr$ and $\rho_1^{-1}(\rho_2^{-1}(K)) = (\rho_2 \circ \rho_1)^{-1}(K)$ is $N_{nc}\gamma os$ in X_1 since ρ_1 is $N_{nc}\gamma Irr$. Hence $\rho_2 \circ \rho_1$ is $N_{nc}\gamma Irr$. □

Theorem 3.7. (i) If $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is $N_{nc}\gamma Irr$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ is $N_{nc}\gamma Cts$, then $\rho_2 \circ \rho_1 : (X_1, N_{nc}\Gamma) \to (X_3, N_{nc}\Phi)$ is $N_{nc}\gamma Cts$.

(ii) If $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is $N_{nc}\gamma Cts$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ is $N_{nc}Cts$, then $\rho_2 \circ \rho_1 : (X_1, N_{nc}\Gamma) \to (X_3, N_{nc}\Phi)$ is $N_{nc}\gamma Cts$.

Proof. (i) Let K be $N_{nc}os$ in X_3. Then, $\rho_2^{-1}(K)$ is $N_{nc}\gamma os$ in X_2, since ρ_2 is $N_{nc}\gamma Cts$ & $\rho_1^{-1}(\rho_2^{-1}(K)) = (\rho_2 \circ \rho_1)^{-1}(K)$ is $N_{nc}\gamma os$ in X_1, since ρ_1 is $N_{nc}\gamma Irr$. Hence $\rho_2 \circ \rho_1$ is $N_{nc}\gamma Cts$.

(ii) Let K be $N_{nc}os$ in X_3. Then, $\rho_2^{-1}(K)$ is $N_{nc}os$ in X_2, since ρ_2 is $N_{nc}Cts$ & $\rho_1^{-1}(\rho_2^{-1}(K)) = (\rho_2 \circ \rho_1)^{-1}(K)$ is $N_{nc}\gamma os$ in X_1, since ρ_1 is $N_{nc}\gamma Cts$. Hence $\rho_2 \circ \rho_1$ is $N_{nc}\gamma Cts$.

□

https://doi.org/10.54216/IJNS.180302
Received: January 13, 2022 Accepted: April 07, 2022

4 N-Neutrosophic crisp γ-open mapping

Definition 4.1. A mapping $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is N_{nc} (resp. N_{nc} regular, $N_{nc}\alpha$, N_{nc} semi, N_{nc} pre & $N_{nc}\gamma$)-open mapping (briefly, $N_{nc}O$ (resp. $N_{nc}rO$, $N_{nc}\alpha O$, $N_{nc}\mathcal{S}O$, $N_{nc}\mathcal{P}O$ & $N_{nc}\gamma O$)) if the image of every $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$ is a $N_{nc}os$ (resp. $N_{nc}ros$, $N_{nc}\alpha os$, $N_{nc}\mathcal{S}os$, $N_{nc}\mathcal{P}os$ & $N_{nc}\gamma os$) in $(X_2, N_{nc}\Psi)$.

Theorem 4.2. Let $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a mapping. The statements are true for a map ρ but not converse. Every

(i) $N_{nc}rO$ is a $N_{nc}O$.

(ii) $N_{nc}O$ is a $N_{nc}\alpha O$.

(iii) $N_{nc}\alpha O$ is a $N_{nc}\mathcal{S}O$.

(iv) $N_{nc}\alpha O$ is a $N_{nc}\mathcal{P}O$.

(v) $N_{nc}\mathcal{S}O$ is a $N_{nc}\gamma O$.

(vi) $N_{nc}\mathcal{P}O$ is a $N_{nc}\gamma O$.

Proof. (ii) Let $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a $N_{nc}O$ and K is a $N_{nc}os$ in X_1. Then $\rho(K)$ is $N_{nc}\alpha os$ in X_2. Since every $N_{nc}o$ is $N_{nc}\alpha o$, $\rho(K)$ is $N_{nc}\alpha os$ in X_2. Therefore ρ is $N_{nc}\alpha O$.
The other cases are similar. □

Example 4.3. Let $X = \{l_1, m_1, n_1, o_1, p_1\}$, ${}_{nc}\Gamma_1 = \{\varphi_N, X_N, L, M, N\}$, ${}_{nc}\Gamma_2 = \{\varphi_N, X_N\}$. $L = \langle\{n_1\}, \{\varphi\}, \{l_1, m_1, o_1, p_1\}\rangle$, $M = \langle\{l_1, m_1\}, \{\varphi\}, \{n_1, o_1, p_1\}\rangle$, $N = \langle\{l_1, m_1, n_1\}, \{\varphi\}, \{o_1, p_1\}\rangle$, then we have $2_{nc}\Gamma = \{\varphi_N, X_N, L, M, N\}$.
Let $\rho : (X, 2_{nc}\Gamma) \to (X, 2_{nc}\Gamma)$ be an identity function. Then ρ is $2_{nc}O$ but not $2_{nc}rO$, the set $\rho(\langle\{l_1, m_1, n_1\}, \{\varphi\}, \{o_1, p_1\}\rangle) = \langle\{l_1, m_1, n_1\}, \{\varphi\}, \{o_1, p_1\}\rangle$ is a $2_{nc}os$ but not a $2_{nc}ros$.

Example 4.4. Let $X = \{l_1, m_1, n_1, o_1\}$, ${}_{nc}\Gamma_1 = \{\varphi_N, X_N, L, M, N\}$, ${}_{nc}\Gamma_2 = \{\varphi_N, X_N\}$. $L = \langle\{l_1\}, \{\varphi\}, \{m_1, n_1, o_1\}\rangle$, $M = \langle\{m_1, o_1\}, \{\varphi\}, \{l_1, n_1\}\rangle$, $N = \langle\{l_1, m_1, o_1\}, \{\varphi\}, \{n_1\}\rangle$, then we have $2_{nc}\Gamma = \{\varphi_N, X_N, L, M, N\}$. Let $Y = \{w_1, x_1, y_1, z_1\}$, ${}_{nc}\Psi_1 = \{\varphi_N, Y_N, \langle\{w_1, z_1\}, \{\varphi\}, \{x_1, y_1\}\rangle\}$, ${}_{nc}\Psi_2 = \{\varphi_N, Y_N\}$, then we have $2_{nc}\Psi = \{\varphi_N, Y_N, \langle\{w_1, z_1\}, \{\varphi\}, \{x_1, y_1\}\rangle\}$.
Define $\rho : (X, 2_{nc}\Gamma) \to (Y, 2_{nc}\Psi)$ as $\rho(l_1) = w_1$, $\rho(m_1) = x_1$, $\rho(n_1) = y_1$ & $\rho(o_1) = z_1$, then $2_{nc}\alpha O$ but not $2_{nc}O$, the set $\rho(\langle\{l_1\}, \{\varphi\}, \{m_1, n_1, o_1\}\rangle) = \langle\{w_1\}, \{\varphi\}, \{x_1, y_1, z_1\}\rangle$ is a $2_{nc}\alpha os$ but not $2_{nc}os$.

Example 4.5. Let $X = \{l_1, m_1, n_1, o_1\}$, ${}_{nc}\Gamma_1 = \{\varphi_N, X_N, L, M, N\}$, ${}_{nc}\Gamma_2 = \{\varphi_N, X_N\}$. $L = \langle\{l_1\}, \{\varphi\}, \{m_1, n_1, o_1\}\rangle$, $M = \langle\{m_1, o_1\}, \{\varphi\}, \{l_1, n_1\}\rangle$, $N = \langle\{l_1, m_1, o_1\}, \{\varphi\}, \{n_1\}\rangle$, then we have $2_{nc}\Gamma = \{\varphi_N, X_N, L, M, N\}$. Let $Y = \{w_1, x_1, y_1, z_1\}$, ${}_{nc}\Psi_1 = \{\varphi_N, Y_N, P, Q, R\}$, ${}_{nc}\Psi_2 = \{\varphi_N, Y_N\}$. $P = \langle\{z_1\}, \{\varphi\}, \{w_1, x_1, y_1\}\rangle$, $Q = \langle\{x_1, y_1\}, \{\varphi\}, \{w_1, z_1\}\rangle$, $R = \langle\{x_1, y_1, z_1\}, \{\varphi\}, \{w_1\}\rangle$, then we have $2_{nc}\Psi = \{\varphi_N, Y_N, P, Q, R\}$.
Define $\rho : (X, 2_{nc}\Gamma) \to (Y, 2_{nc}\Psi)$ as $\rho(l_1) = x_1$, $\rho(m_1) = y_1$, $\rho(n_1) = z_1$ & $\rho(o_1) = w_1$, then

(i) $2_{nc}\mathcal{S}O$ (resp. $2_{nc}\gamma O$) but not $2_{nc}\alpha O$ (resp. $2_{nc}\mathcal{P}O$), the set $\rho(\langle\{m_1, o_1\}, \{\varphi\}, \{l_1, n_1\}\rangle) = \langle\{y_1, w_1\}, \{\varphi\}, \{x_1, z_1\}\rangle$ is a $2_{nc}\mathcal{S}os$ (resp. $2_{nc}\gamma os$) but not $2_{nc}\alpha os$ (resp. $2_{nc}\mathcal{P}os$).

(ii) $2_{nc}\mathcal{P}O$ (resp. $2_{nc}\gamma O$) but not $2_{nc}\alpha O$ (resp. $2_{nc}\mathcal{S}O$), the set $\rho(\langle\{l_1\}, \{\varphi\}, \{m_1, n_1, o_1\}\rangle) = \langle\{x_1\}, \{\varphi\}, \{y_1, z_1, w_1\}\rangle$ is a $2_{nc}\mathcal{P}os$ (resp. $2_{nc}\gamma os$) but not $2_{nc}\alpha os$ (resp. $2_{nc}\mathcal{S}os$).

[arrows] $N_{nc}rO$ map [d]
$N_{nc}O$ map [d]
$N_{nc}\alpha O$ map [dl] [dr]
$N_{nc}\mathcal{S}O$ map [dr] $N_{nc}\mathcal{P}O$ map [dl]
$N_{nc}\gamma O$ map

Figure 1: $N_{nc}\gamma O$ mapping function in $N_{nc}ts$.

https://doi.org/10.54216/IJNS.180302
Received: January 13, 2022 Accepted: April 07, 2022

Theorem 4.6. A mapping $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is $N_{nc}\gamma O$ iff for every $N_{nc}s$ K of $(X_1, N_{nc}\Gamma)$, $\rho(N_{nc}int(K)) \subseteq N_{nc}\gamma int(\rho(K))$.

Proof. Necessity: Let ρ be a $N_{nc}\gamma O$ and K be a $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$. Now, $N_{nc}int(K) \subseteq K$ implies $\rho(N_{nc}int(K)) \subseteq \rho(K)$. Since ρ is a $N_{nc}\gamma O$, $\rho(N_{nc}int(K))$ is $N_{nc}\gamma os$ in $(X_2, N_{nc}\Psi)$ such that $\rho(N_{nc}int(K)) \subseteq \rho(K)$ therefore $\rho(N_{nc}\ int(K)) \subseteq N_{nc}\gamma int(\rho(K))$.
Sufficiency: Assume K is a $N_{nc}os$ of $(X_1, N_{nc}\Gamma)$. Then $\rho(K) = \rho(N_{nc}int(K)) \subseteq N_{nc}\gamma int(\rho(K))$. But $N_{nc}\gamma int(\rho(K)) \subseteq \rho(K)$. So $\rho(K) = N_{nc}\gamma int(K)$ which implies $\rho(K)$ is a $N_{nc}\gamma os$ of $(X_2, N_{nc}\Psi)$ and hence ρ is a $N_{nc}\gamma O$. □

Theorem 4.7. If $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is a $N_{nc}\gamma O$ mapping then $N_{nc}int(\rho^{-1}(K)) \subseteq \rho^{-1}(N_{nc}\ \gamma int(K))$ for every $N_{nc}s$ K of $(X_2, N_{nc}\Psi)$.

Proof. Let K be a $N_{nc}s$ of $(X_2, N_{nc}\Psi)$. Then $N_{nc}int(\rho^{-1}(K))$ is a $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$. Since ρ is $N_{nc}\gamma O$, $\rho(N_{nc}int(\rho^{-1}(K)))$ is $N_{nc}\gamma os$ in $(X_2, N_{nc}\Psi)$ and hence $\rho(N_{nc}int(\rho^{-1}(K))) \subseteq N_{nc}\gamma int(\rho(\rho^{-1}(K))) \subseteq N_{nc}\gamma int(K)$. Thus $N_{nc}int(\rho^{-1}(K)) \subseteq \rho^{-1}(N_{nc}\gamma int(K))$. □

Theorem 4.8. A mapping $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is $N_{nc}\gamma O$ iff for each $N_{nc}s$ K of $(X_2, N_{nc}\Psi)$ and for each $N_{nc}cs$ N of $(X_1, N_{nc}\Gamma)$ containing $\rho^{-1}(K)$ there is a $N_{nc}\gamma cs$ K of $(X_2, N_{nc}\Psi)$ such that $K \subseteq N$ and $\rho^{-1}(K) \subseteq N$.

Proof. Necessity: Assume ρ is a $N_{nc}\gamma O$. Let K be the $N_{nc}cs$ of $(X_2, N_{nc}\Psi)$ and N is a $N_{nc}cs$ of $(X_1, N_{nc}\Gamma)$ such that $\rho^{-1}(K) \subseteq N$. Then $K = (\rho^{-1}(N^c))^c$ is $N_{nc}\gamma cs$ of $(X_2, N_{nc}\Psi)$ such that $\rho^{-1}(K) \subseteq N$.
Sufficiency: Assume M is a $N_{nc}os$ of $(X_1, N_{nc}\Gamma)$. Then $\rho^{-1}(\rho(M))^c \subseteq M^c$ & M^c is $N_{nc}cs$ in $(X_1, N_{nc}\Gamma)$. By hypothesis there is a $N_{nc}\gamma cs$ K of $(X_2, N_{nc}\Psi)$ such that $(\rho(M))^c \subseteq K$ and $\rho^{-1}(K) \subseteq M^c$. Therefore $M \subseteq (\rho^{-1}(K))^c$. Hence $K^c \subseteq \rho(M) \subseteq \rho((\rho^{-1}(K))^c) \subseteq K^c$ which implies $\rho(M) = K^c$. Since K^c is $N_{nc}\gamma os$ of $(X_2, N_{nc}\Psi)$. Hence $\rho(M)$ is $N_{nc}\gamma o$ in $(X_2, N_{nc}\Psi)$ and thus ρ is $N_{nc}\gamma O$. □

Theorem 4.9. A mapping $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is $N_{nc}\gamma O$ iff $\rho^{-1}(N_{nc}\gamma cl(M)) \subseteq N_{nc}cl(\rho^{-1}(M))$ for every $N_{nc}s$ M of $(X_2, N_{nc}\Psi)$.

Proof. Necessity: Assume ρ is a $N_{nc}\gamma O$. For any $N_{nc}s$ M of $(X_2, N_{nc}\Psi)$, $\rho^{-1}(M) \subseteq N_{nc}cl(\rho^{-1}(M))$. Therefore by Theorem 4.8 there exists a $N_{nc}\gamma cs$ K in $(X_2, N_{nc}\Psi)$ such that $M \subseteq K$ & $\rho^{-1}(K) \subseteq N_{nc}cl (\rho^{-1}(M))$. Therefore we obtain that $\rho^{-1}(N_{nc}\gamma cl(M)) \subseteq \rho^{-1}(K) \subseteq N_{nc}cl(\rho^{-1}(M))$.
Sufficiency: Assume M is a $N_{nc}s$ of $(X_2, N_{nc}\Psi)$ and K is a $N_{nc}cs$ of $(X_1, N_{nc}\Gamma)$ containing $\rho^{-1}(M)$. Put $\varphi = N_{nc}cl(M)$, then $M \subseteq \varphi$ and φ is $N_{nc}\gamma c$ and $\rho^{-1}(\varphi) \subsetneq N_{nc}cl(\rho^{-1}(M)) \subseteq K$. Then by Theorem 4.8, ρ is $N_{nc}\gamma O$. □

Theorem 4.10. If $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ be $N_{nc}ts$ and $\rho_2 \circ \rho_1 : (X_1, N_{nc}\Gamma) \to (X_3, N_{nc}\Phi)$ is $N_{nc}\gamma O$. If $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ is $N_{nc}\gamma Irr$ then $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is $N_{nc}\gamma O$.

Proof. Let K be a $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$. Then $\rho_2 \circ \rho_1(K)$ is $N_{nc}\gamma os$ of $(X_3, N_{nc}\Phi)$ because $\rho_2 \circ \rho_1$ is $N_{nc}\gamma O$. Since ρ_2 is $N_{nc}\gamma Irr$ and $\rho_2 \circ \rho_1(K)$ is $N_{nc}\gamma os$ of $(X_3, N_{nc}\Phi)$ therefore $\rho_2^{-1}(\rho_2 \circ \rho_1(K)) = \rho_1(K)$ is $N_{nc}\gamma os$ in $(X_2, N_{nc}\Psi)$. Hence ρ_1 is $N_{nc}\gamma O$. □

Theorem 4.11. If $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is $N_{nc}O$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ is $N_{nc}\gamma O$ then $\rho_2 \circ \rho_1 : (X_1, N_{nc}\Gamma) \to (X_3, N_{nc}\Phi)$ is $N_{nc}\gamma O$.

Proof. Let K be a $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$. Then $\rho_1(K)$ is a $N_{nc}os$ of $(X_2, N_{nc}\Psi)$ because ρ_1 is a $N_{nc}O$. Since ρ_2 is $N_{nc}\gamma O$, $\rho_2(\rho_1(K)) = (\rho_2 \circ \rho_1)(K)$ is $N_{nc}\gamma os$ of $(X_3, N_{nc}\Phi)$. Hence $\rho_2 \circ \rho_1$ is $N_{nc}\gamma O$. □

5 N-Neutrosophic crisp γ-closed mapping

Definition 5.1. A mapping $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is N_{nc} (resp. N_{nc} regular, $N_{nc}\alpha$, N_{nc} semi, N_{nc} pre & $N_{nc}\gamma$)-closed mapping (briefly, $N_{nc}C$ (resp. $N_{nc}rC$, $N_{nc}\alpha C$, $N_{nc}\mathcal{S}C$, $N_{nc}\mathcal{P}C$ & $N_{nc}\gamma C$)) if the image of every $N_{nc}cs$ in $(X_1, N_{nc}\Gamma)$ is a $N_{nc}cs$ (resp. $N_{nc}rcs$, $N_{nc}\alpha cs$, $N_{nc}\mathcal{S}cs$, $N_{nc}\mathcal{P}cs$ & $N_{nc}\gamma cs$) in $(X_2, N_{nc}\Psi)$.

Theorem 5.2. Let $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a mapping. The statements are true for a map ρ but not converse. Every

(i) $N_{nc}rC$ is a $N_{nc}C$.

Received: January 13, 2022 Accepted: April 07, 2022

(ii) $N_{nc}C$ is a $N_{nc}\alpha C$.

(iii) $N_{nc}\alpha C$ is a $N_{nc}\mathcal{S}C$.

(iv) $N_{nc}\alpha C$ is a $N_{nc}\mathcal{P}C$.

(v) $N_{nc}\mathcal{S}C$ is a $N_{nc}\gamma C$.

(vi) $N_{nc}\mathcal{P}C$ is a $N_{nc}\gamma C$.

Proof. (ii) Let $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a $N_{nc}C$ and K is a $N_{nc}cs$ in X_1. Then $\rho(K)$ is $N_{nc}\alpha cs$ in X_2. Since every $N_{nc}cs$ is $N_{nc}\alpha cs$, $\rho(K)$ is $N_{nc}\alpha cs$ in X_2. Therefore ρ is $N_{nc}\alpha C$.
The other cases are similar. □

Example 5.3. In Example 4.3, then ρ is $2_{nc}C$ mapping but not $2_{nc}rC$ mapping, the set $\rho(\langle\{o_1, p_1\}, \{\varphi\}, \{l_1, m_1, n_1\}\rangle) = \langle\{o_1, p_1\}, \{\varphi\}, \{l_1, m_1, n_1\}\rangle$ is a $2_{nc}cs$ but not a $2_{nc}rcs$.

Example 5.4. In Example 4.4, then ρ is $2_{nc}\alpha C$ but not $2_{nc}C$, the set $\rho(\langle\{m_1, n_1, o_1\}, \{\varphi\}, \{l_1\}\rangle) = \langle\{x_1, y_1, z_1\}, \{\varphi\}, \{w_1\}\rangle$ is a $2_{nc}\alpha cs$ but not $2_{nc}cs$.

Example 5.5. In Example 4.5, then ρ is

(i) $2_{nc}\mathcal{S}C$ (resp. $2_{nc}\gamma C$) but not $2_{nc}\alpha C$ (resp. $2_{nc}\mathcal{P}C$), the set $\rho(\langle\{l_1, n_1\}, \{\varphi\}, \{m_1, o_1\}\rangle) = \langle\{x_1, z_1\}, \{\varphi\}, \{y_1, w_1\}\rangle$ is a $2_{nc}\mathcal{S}cs$ (resp. $2_{nc}\gamma cs$) but not $2_{nc}\alpha cs$ (resp. $2_{nc}\mathcal{P}cs$).

(ii) $2_{nc}\mathcal{P}C$ (resp. $2_{nc}\gamma C$) but not $2_{nc}\alpha C$ (resp. $2_{nc}\mathcal{S}C$), the set $\rho(\langle\{m_1, n_1, o_1\}, \{\varphi\}, \{l_1\}\rangle) = \langle\{y_1, z_1, w_1\}, \{\varphi\}, \{x_1\}\rangle$ is a $2_{nc}\mathcal{P}cs$ (resp. $2_{nc}\gamma cs$) but not $2_{nc}\alpha cs$ (resp. $2_{nc}\mathcal{S}cs$).

[arrows] $N_{nc}rC$ map [d]
$N_{nc}C$ map [d]
$N_{nc}\alpha C$ map [dl] [dr]
$N_{nc}\mathcal{S}C$ map [dr] $N_{nc}\mathcal{P}C$ map [dl]
$N_{nc}\gamma C$ map

Figure 2: $N_{nc}\gamma C$ mapping function in $N_{nc}ts$.

Theorem 5.6. A mapping $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is $N_{nc}\gamma C$ iff for each $N_{nc}s$ K of $(X_2, N_{nc}\Psi)$ and for each $N_{nc}os$ M of $(X_1, N_{nc}\Gamma)$ containing $\rho^{-1}(K)$ there is a $N_{nc}\gamma os$ N of $(X_2, N_{nc}\Psi)$ such that $K \subseteq N$ and $\rho^{-1}(N) \subseteq M$.

Proof. Necessity: Assume ρ is a $N_{nc}\gamma C$. Let K be the $N_{nc}cs$ of $(X_2, N_{nc}\Psi)$ and M is a $N_{nc}os$ of $(X_1, N_{nc}\Gamma)$ such that $\rho^{-1}(K) \subseteq M$. Then $\rho = X_2 - \rho^{-1}(M^c)$ is $N_{nc}\gamma os$ of $(X_2, N_{nc}\Psi)$ such that $\rho^{-1}(N) \subseteq M$.
Sufficiency: Assume L is a $N_{nc}cs$ of $(X_1, N_{nc}\Gamma)$. Then $(\rho(L))^c$ is a $N_{nc}s$ of $(X_2, N_{nc}\Psi)$ and L^c is $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$ such that $\rho^{-1}((\rho(L))^c) \subseteq L^c$. By hypothesis there is a $N_{nc}\gamma os$ N of $(X_2, N_{nc}\Psi)$ such that $(\rho(L))^c \subseteq N$ and $\rho^{-1}(N) \subseteq L^c$. Therefore $L \subseteq (\rho^{-1}(N))^c$. Hence $N^c \subseteq \rho(N) \subseteq \rho((\rho^{-1}(N))^c) \subseteq N^c$ which implies $\rho(L) = N^c$. Since N^c is $N_{nc}\gamma cs$ of $(X_2, N_{nc}\Psi)$. Hence $\rho(L)$ is $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$ and thus ρ is $N_{nc}\gamma C$. □

Theorem 5.7. If $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is $N_{nc}C$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ is $N_{nc}\gamma C$. Then $\rho_2 \circ \rho_1 : (X_1, N_{nc}\Gamma) \to (X_3, N_{nc}\Phi)$ is $N_{nc}\gamma C$.

Proof. Let K be a $N_{nc}cs$ in $(X_1, N_{nc}\Gamma)$. Then $\rho_1(K)$ is $N_{nc}cs$ of $(X_2, N_{nc}\Psi)$ because ρ_1 is $N_{nc}C$. Now $(\rho_2 \circ \rho_1)(K) = \rho_2(\rho_1(K))$ is $N_{nc}\gamma cs$ in $(X_3, N_{nc}\Phi)$ because ρ_2 is $N_{nc}\gamma C$. Thus $\rho_2 \circ \rho_1$ is $N_{nc}\gamma C$. □

Theorem 5.8. If $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is $N_{nc}\gamma C$, then $N_{nc}\gamma cl(\rho(N)) \subsetneq \rho(N_{nc}cl(N))$.

Proof. Obvious. □

Theorem 5.9. Let $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ are $N_{nc}\gamma C$. If every $N_{nc}\gamma cs$ of $(X_2, N_{nc}\Psi)$ is $N_{nc}c$ then, $\rho_2 \circ \rho_1 : (X_1, N_{nc}\Gamma) \to (X_3, N_{nc}\Phi)$ is $N_{nc}\gamma C$.

Received: January 13, 2022 Accepted: April 07, 2022

Proof. Let K be a $N_{nc}cs$ in $(X_1, N_{nc}\Gamma)$. Then $\rho_1(K)$ is $N_{nc}\gamma cs$ of $(X_2, N_{nc}\Psi)$ because ρ_1 is $N_{nc}\gamma C$. By hypothesis $\rho_1(K)$ is $N_{nc}cs$ of $(X_2, N_{nc}\Psi)$. Now $\rho_2(\rho_1(K)) = (\rho_2 \circ \rho_1)(K)$ is $N_{nc}\gamma cs$ in $(X_3, N_{nc}\Phi)$ because ρ_2 is $N_{nc}\gamma C$. Thus $\rho_2 \circ \rho_1$ is $N_{nc}\gamma C$. □

Theorem 5.10. Let $\rho : (X_1, N_{nc}\Gamma) \rightarrow (X_2, N_{nc}\Psi)$ be a objective mapping, then the following statements are equivalent:

(i) ρ is a $N_{nc}\gamma O$.

(ii) ρ is a $N_{nc}\gamma C$.

(iii) ρ^{-1} is $N_{nc}\gamma Cts$.

Proof. (i) ⇒ (ii): Let us assume that ρ is a $N_{nc}\gamma O$. By definition, K is a $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$, then $\rho(K)$ is a $N_{nc}\gamma os$ in $(X_2, N_{nc}\Psi)$. Here, K is $N_{nc}cs$ in $(X_1, N_{nc}\Gamma)$, then $X_1 - K$ is a $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$. By assumption, $\rho(X_1 - K)$ is a $N_{nc}\gamma os$ in $(X_2, N_{nc}\Psi)$. Hence, $X_2 - \rho(X_1 - K)$ is a $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$. Therefore, ρ is a $N_{nc}\gamma C$.
(ii) ⇒ (iii): Let K be a $N_{nc}cs$ in $(X_1, N_{nc}\Gamma)$ By (ii), $\rho(K)$ is a $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$. Hence, $\rho(K) = (\rho^{-1})^{-1}(K)$, so ρ^{-1} is a $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$. Hence, ρ^{-1} is $N_{nc}\gamma Cts$.
(iii) ⇒ (i): Let K be a $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$ By (iii), $(\rho^{-1})^{-1}(K) = \rho(K)$ is a $N_{nc}\gamma O$. □

6 N-Neutrosophic crisp γ-homeomorphism

Definition 6.1. A bijection $\rho : (X_1, N_{nc}\Gamma) \rightarrow (X_2, N_{nc}\Psi)$ is called a N_{nc}-homeomorphism (briefly N_{nc} Hom) if ρ and ρ^{-1} are $N_{nc}Cts$.

Definition 6.2. A bijection $\rho : (X_1, N_{nc}\Gamma) \rightarrow (X_2, N_{nc}\Psi)$ is called a $N_{nc}\gamma$-homeomorphism (briefly $N_{nc}\gamma Hom$) if ρ and ρ^{-1} are $N_{nc}\gamma Cts$.

Theorem 6.3. Each $N_{nc}Hom$ is a $N_{nc}\gamma Hom$.

Proof. Let ρ be $N_{nc}Hom$, then ρ and ρ^{-1} are $N_{nc}Cts$. But every $N_{nc}Cts$ is $N_{nc}\gamma Cts$. Hence, ρ and ρ^{-1} is $N_{nc}\gamma Cts$. Therefore, ρ is a $N_{nc}\gamma Hom$. □

Example 6.4. Let $X = \{l_1, m_1, n_1, o_1\}$, ${}_{nc}\Gamma_1 = \{\varphi_N, X_N, L, M, N\}$, ${}_{nc}\Gamma_2 = \{\varphi_N, X_N\}$. $L = \langle\{n_1\}, \{\varphi\}, \{l_1, m_1, o_1\}\rangle$, $M = \langle\{l_1, m_1\}, \{\varphi\}, \{n_1, o_1\}\rangle$, $N = \langle\{l_1, m_1, n_1\}, \{\varphi\}, \{o_1\}\rangle$, then we have $2_{nc}\Gamma = \{\varphi_N, X_N, L, M, N\}$. Let $Y = \{w_1, x_1, y_1, z_1\}$, ${}_{nc}\Psi_1 = \{\varphi_N, Y_N, W, X, Y\}$, ${}_{nc}\Psi_2 = \{\varphi_N, Y_N\}$. $W = \langle\{x_1\}, \{\varphi\}, \{w_1, y_1, z_1\}\rangle$, $X = \langle\{w_1, y_1\}, \{\varphi\}, \{x_1, z_1\}\rangle$, $Y = \langle\{w_1, x_1, y_1\}, \{\varphi\}, \{z_1\}\rangle$, then we have $2_{nc}\Psi = \{\varphi_N, Y_N, W, X, Y\}$.
Define $\rho : (X, 2_{nc}\Gamma) \rightarrow (Y, 2_{nc}\Psi)$ as $\rho(l_1) = z_1$, $\rho(m_1) = y_1$, $\rho(n_1) = y_1$ & $\rho(o_1) = w_1$, then $2_{nc}\gamma Hom$ but not $2_{nc}Hom$, the set $\rho^{-1}(\langle\{w_1, y_1\}, \{\varphi\}, \{x_1, z_1\}\rangle) = \langle\{m_1, n_1, o_1\}, \{\varphi\}, \{l_1\}\rangle$ is a $2_{nc}\gamma os$ but not $2_{nc}os$.

Theorem 6.5. Let $\rho : (X_1, N_{nc}\Gamma) \rightarrow (X_2, N_{nc}\Psi)$ be a bijective mapping. If ρ is $N_{nc}\gamma Cts$, then the following statements are equivalent:

(i) ρ is a $N_{nc}\gamma C$.

(ii) ρ is a $N_{nc}\gamma O$.

(iii) ρ^{-1} is a $N_{nc}\gamma Hom$.

Proof. (i) ⇒ (ii) : Assume that ρ is a bijective mapping and a $N_{nc}\gamma C$. Hence, ρ^{-1} is a $N_{nc}\gamma Cts$. We know that each $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$ is a $N_{nc}\gamma os$ in $(X_2, N_{nc}\Psi)$. Hence, ρ is a $N_{nc}\gamma O$.
(ii) ⇒ (iii) : Let ρ be a bijective and $N_{nc}O$. Further, ρ^{-1} is a $N_{nc}\gamma Cts$. Hence, ρ & ρ^{-1} are $N_{nc}\gamma Cts$. $\Longrightarrow$ ρ is a $N_{nc}\gamma Hom$.
(iii) ⇒ (i): Let ρ be a $N_{nc}\gamma Hom$, then ρ & ρ^{-1} are $N_{nc}\gamma Cts$. Since each $N_{nc}cs$ in $(X_1, N_{nc}\Gamma)$ is a $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$, then ρ is a $N_{nc}\gamma C$. □

Definition 6.6. A $N_{nc}ts$ $(X, N_{nc}\Gamma)$ is said to be a N-neutrosophic crisp $\gamma T_{\frac{1}{2}}$ (briefly, $N_{nc}\gamma T_{\frac{1}{2}}$)-space if every $N_{nc}\gamma cs$ is $N_{nc}c$ in $(X, N_{nc}\Gamma)$.

Theorem 6.7. Let $\rho : (X_1, N_{nc}\Gamma) \rightarrow (X_2, N_{nc}\Psi)$ be a $N_{nc}\gamma Hom$, then ρ is a $N_{nc}Hom$ if $(X_1, N_{nc}\Gamma)$ and $(X_2, N_{nc}\Psi)$ are $N_{nc}\gamma T_{\frac{1}{2}}$-space.

https://doi.org/10.54216/IJNS.180302
Received: January 13, 2022 Accepted: April 07, 2022

Proof. Assume that K is a $N_{nc}cs$ in $(X_2, N_{nc}\Psi)$, then $\rho^{-1}(K)$ is a $N_{nc}\gamma cs$ in $(X_1, N_{nc}\Gamma)$. Since $(X_1, N_{nc}\Gamma)$ is an $N_{nc}\gamma T_{\frac{1}{2}}$-space, $\rho^{-1}(K)$ is a $N_{nc}cs$ in $(X_1, N_{nc}\Gamma)$. Therefore, ρ is $N_{nc}Cts$. By hypothesis, ρ^{-1} is $N_{nc}\gamma Cts$. Let L be a $N_{nc}cs$ in $(X_1, N_{nc}\Gamma)$. Then, $(\rho^{-1})^{-1}(L) = \rho(L)$ is a $N_{nc}cs$ in $(X_2, N_{nc}\Psi)$, by presumption. Since $(X_2, N_{nc}\Psi)$ is a $N_{nc}\gamma T_{\frac{1}{2}}$-space, $\rho(L)$ is a $N_{nc}cs$ in $(X_2, N_{nc}\Psi)$. Hence, ρ^{-1} is $N_{nc}Cts$. Hence, ρ is a $N_{nc}Hom$. □

Theorem 6.8. Let $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a $N_{nc}ts$, then the following are equivalent if $(X_2, N_{nc}\Psi)$ is a $N_{nc}\gamma T_{\frac{1}{2}}$-space:

(i) ρ is $N_{nc}\gamma C$.

(ii) If K is a $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$, then $\rho(K)$ is $N_{nc}\gamma os$ in $(X_2, N_{nc}\Psi)$.

(iii) $\rho(N_{nc}int(K)) \subseteq N_{nc}cl(N_{nc}int(\rho(K)))$ for every $N_{nc}s$ K in $(X_1, N_{nc}\Gamma)$.

Proof. (i) ⇒ (ii): Obvious.
(ii) ⇒ (iii): Let K be a $N_{nc}s$ in $(X_1, N_{nc}\Gamma)$. Then, $N_{nc}int(K)$ is a $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$. Then, $\rho(N_{nc}int(K))$ is a $N_{nc}\gamma os$ in $(X_2, N_{nc}\Psi)$. Since $(X_2, N_{nc}\Psi)$ is a $N_{nc}\gamma T_{\frac{1}{2}}$-space, so $\rho(N_{nc}int(K))$ is a $N_{nc}os$ in $(X_2, N_{nc}\Psi)$. Therefore, $\rho(N_{nc}int(K)) = N_{nc}int(\rho(N_{nc}int(K))) \subseteq N_{nc}cl(N_{nc}int(\rho(K)))$.
(iii) ⇒ (i): Let K be a $N_{nc}cs$ in $(X_1, N_{nc}\Gamma)$. Then, K^c is a $N_{nc}os$ in $(X_1, N_{nc}\Gamma)$. From, $\rho(N_{nc}int(K^c)) \subseteq N_{nc}cl(N_{nc}int(\rho(K^c)))$. Hence, $\rho(K^c) \subseteq N_{nc}cl(N_{nc}int(\rho(K^c)))$. Therefore, $\rho(K^c)$ is $N_{nc}\gamma os$ in $(X_2, N_{nc}\Psi)$. Therefore, $\rho(K)$ is a $N_{nc}\gamma cs$ in $(X_1, N_{nc}\Gamma)$. Hence, ρ is a $N_{nc}\gamma C$. □

Theorem 6.9. Let $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ be $N_{nc}\gamma C$, where $(X_1, N_{nc}\Gamma)$ and $(X_3, N_{nc}\Phi)$ are two $N_{nc}ts$'s and $(X_2, N_{nc}\Psi)$ a $N_{nc}\gamma T_{\frac{1}{2}}$-space, then the composition $\rho_2 \circ \rho_1$ is $N_{nc}\gamma C$.

Proof. Let K be a $N_{nc}cs$ in $(X_1, N_{nc}\Gamma)$. Since ρ_1 is $N_{nc}\gamma c$ and $\rho_1(K)$ is a $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$, by assumption, $\rho_1(K)$ is a $N_{nc}cs$ in $(X_2, N_{nc}\Psi)$. Since ρ_2 is $N_{nc}\gamma c$, then $\rho_2(\rho_1(K))$ is $N_{nc}\gamma c$ in $(X_3, N_{nc}\Phi)$ and $\rho_2(\rho_1(K)) = (\rho_2 \circ \rho_1)(K)$. Therefore, $\rho_2 \circ \rho_1$ is $N_{nc}\gamma C$. □

Theorem 6.10. Let $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ be two $N_{nc}ts$'s, then the following hold:

(i) If $\rho_2 \circ \rho_1$ is $N_{nc}\gamma O$ and ρ_1 is $N_{nc}Cts$, then ρ_2 is $N_{nc}\gamma O$.

(ii) If $\rho_2 \circ \rho_1$ is $N_{nc}O$ and ρ_2 is $N_{nc}\gamma Cts$, then ρ_1 is $N_{nc}\gamma O$.

Proof. Obvious. □

7 N-Neutrosophic crisp γ-Completely Homeomorphism

Definition 7.1. A bijection $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is called a $N_{nc}\gamma$-Completely homeomorphism (briefly, $N_{nc}\gamma CHom$) if ρ and ρ^{-1} are $N_{nc}\gamma Irr$.

Theorem 7.2. Each $N_{nc}\gamma CHom$ is a $N_{nc}\gamma Hom$. But not conversely.

Proof. We take that K is a $N_{nc}cs$ in $(X_2, N_{nc}\Psi)$. This shows that K is a $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$. By assumption, $\rho^{-1}(K)$ is a $N_{nc}\gamma cs$ in $(X_1, N_{nc}\Gamma)$. Hence, ρ is a $N_{nc}\gamma Cts$. Hence, ρ and ρ^{-1} are $N_{nc}\gamma Cts$. Hence ρ is a $N_{nc}\gamma Hom$. □

Example 7.3. In Example 4.5, ρ is $2_{nc}\gamma Hom$ but not $2_{nc}Hom$, the set $\rho^{-1}(\langle\{y_1\}, \{\varphi\}, \{x_1, z_1, w_1\}\rangle) = \langle\{n_1\}, \{\varphi\}, \{l_1, m_1, o_1\}\rangle$ is a $2_{nc}\gamma os$ in Y but not $2_{nc}\gamma os$ in X.

Theorem 7.4. If $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is a $N_{nc}\gamma CHom$, then $N_{nc}\gamma cl(\rho^{-1}(L)) \subseteq \rho^{-1}(N_{nc}cl\ (L))$ for each $N_{nc}ts$ L in $(X_2, N_{nc}\Psi)$.

Proof. Let L be a $N_{nc}ts$ in $(X_2, N_{nc}\Psi)$. Then, $N_{nc}cl(L)$ is a $N_{nc}cs$ in $(X_2, N_{nc}\Psi)$, and every $N_{nc}cs$ is a $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$. Assume ρ is $N_{nc}\gamma Irr$, $\rho^{-1}(N_{nc}cl(L))$ is a $N_{nc}\gamma cs$ in $(X_1, N_{nc}\Gamma)$, then $N_{nc}cl(\rho^{-1}(N_{nc}cl(L))) = \rho^{-1}(N_{nc}cl(L))$. Here, $N_{nc}\gamma cl\ (\rho^{-1}(L)) \subseteq N_{nc}\gamma cl(\rho^{-1}(N_{nc}cl(L))) = \rho^{-1}(N_{nc}cl(L))$. Therefore, $N_{nc}\gamma cl(\rho^{-1}(L)) \subseteq \rho^{-1}(N_{nc}cl(L))$ for every $N_{nc}s$ L in $(X_2, N_{nc}\Psi)$. □

Theorem 7.5. Let $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a $N_{nc}\gamma CHom$, then $N_{nc}\gamma cl(\rho^{-1}(L)) = \rho^{-1}(N_{nc}\gamma cl(L))$ for each $N_{nc}s$ L in $(X_2, N_{nc}\Psi)$.

https://doi.org/10.54216/IJNS.180302

Received: January 13, 2022 Accepted: April 07, 2022

Proof. Since ρ is a $N_{nc}\gamma CHom$, then ρ is a $N_{nc}\gamma Irr$. Let L be a $N_{nc}s$ in $(X_2, N_{nc}\Psi)$. Clearly, $N_{nc}\gamma cl(L)$ is a $N_{nc}\gamma cs$ in $(X_1, N_{nc}\Gamma)$. Then $N_{nc}\gamma cl(L)$ is a $N_{nc}\gamma cs$ in $(X_1, N_{nc}\Gamma)$. Since $\rho^{-1}(L) \subseteq \rho^{-1}(N_{nc}\gamma cl(L))$, then $N_{nc}\gamma cl(\rho^{-1}(L)) \subseteq N_{nc}\gamma cl(\rho^{-1}(N_{nc}\gamma cl(L))) = \rho^{-1}(N_{nc}\gamma cl(L))$. Therefore, $N_{nc}\gamma cl(\rho^{-1}(L)) \subseteq \rho^{-1}(N_{nc}\gamma cl(L))$. Let ρ be a $N_{nc}\gamma CHom$. ρ^{-1} is a $N_{nc}\gamma Irr$. Let us consider $N_{nc}s$ $\rho^{-1}(L)$ in $(X_1, N_{nc}\Gamma)$, which implies $N_{nc}\gamma cl(\rho^{-1}(L))$ is a $N_{nc}\gamma cs$ in $(X_1, N_{nc}\Gamma)$. Hence, $N_{nc}\gamma cl(\rho^{-1}(L))$ is a $N_{nc}\gamma cs$ in $(X_1, N_{nc}\Gamma)$. This implies that $(\rho^{-1})^{-1}(N_{nc}\gamma cl(\rho^{-1}(L))) = \rho(N_{nc}\gamma cl(\rho^{-1}(L)))$ is a $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$. This proves $L = (\rho^{-1})^{-1}(\rho^{-1}(L)) \subseteq (\rho^{-1})^{-1}(N_{nc}\gamma cl(\rho^{-1}(L))) = \rho(N_{nc}\gamma cl(\rho^{-1}(L)))$. Therefore, $N_{nc}\gamma cl(L) \subseteq N_{nc}\gamma cl(\rho(N_{nc}\gamma cl(\rho^{-1}(L)))) = \rho(N_{nc}\gamma cl(\rho^{-1}(L)))$, since ρ^{-1} is a $N_{nc}\gamma Irr$. Hence, $\rho^{-1}(N_{nc}\gamma cl(L)) \subseteq \rho^{-1}(\rho(N_{nc}\gamma cl(\rho^{-1}(L)))) = N_{nc}\gamma cl(\rho^{-1}(L))$. That is, $\rho^{-1}(N_{nc}\gamma cl(L)) \subseteq N_{nc}\gamma cl(\rho^{-1}(L))$. Hence, $N_{nc}\gamma cl(\rho^{-1}(L)) = \rho^{-1}$ $(N_{nc}\gamma cl(L))$. □

Theorem 7.6. If $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ are $N_{nc}\gamma CHom$'s, then $\rho_2 \circ \rho_1$ is a $N_{nc}\gamma CHom$.

Proof. Let ρ_1 and ρ_2 to be two $N_{nc}\gamma CHom$'s. Assume K is a $N_{nc}\gamma cs$ in $(X_3, N_{nc}\Phi)$. Then, $\rho_2^{-1}(K)$ is a $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$. Then, by hypothesis, $\rho_1^{-1}(\rho_2^{-1}(K))$ is a $N_{nc}\gamma cs$ in $(X_1, N_{nc}\Gamma)$. Hence,$\rho_2 \circ \rho_1$ is a $N_{nc}\gamma Irr$. Now, let L be a $N_{nc}\gamma cs$ in $(X_1, N_{nc}\Gamma)$. Then, by presumption, $\rho_1(\rho_2)$ is a $N_{nc}\gamma cs$ in $(X_2, N_{nc}\Psi)$. Then, by hypothesis, $\rho_2(\rho_1(L))$ is a $N_{nc}\gamma cs$ in $(X_3, N_{nc}\Phi)$. $\implies$ $\rho_2 \circ \rho_1$ is a $N_{nc}\gamma Irr$. Hence, $\rho_2 \circ \rho_1$ is a $N_{nc}\gamma CHom$. □

8 Strongly and Perfectly N-neutrosophic crisp γ continuous

Definition 8.1. Let $(X_1, N_{nc}\Gamma)$ and $(X_2, N_{nc}\Psi)$ be two $N_{nc}ts$'s. A function $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is called strongly N-neutrosophic crisp γ continuous (briefly, $StN_{nc}\gamma Cts$) function if the inverse image of every $N_{nc}\gamma o$ set in X_2 is $N_{nc}o$ in X_1.

Definition 8.2. Let $(X_1, N_{nc}\Gamma)$ and $(X_2, N_{nc}\Psi)$ be two $N_{nc}ts$'s. A function $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ is called a perfectly N-neutrosophic crisp (resp. γ) continuous (briefly, $PeN_{nc}Cts$ (resp. $PeN_{nc}\gamma Cts$)) function if the inverse image of every $N_{nc}o$ (resp. $N_{nc}\gamma o$) set in X_2 is $N_{nc}clo$ in X_1.

Theorem 8.3. Let $(X_1, N_{nc}\Gamma)$ and $(X_2, N_{nc}\Psi)$ be two $N_{nc}ts$'s and $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a function. Then

(i) If ρ is $PeN_{nc}\gamma Cts$, then ρ is $PeN_{nc}Cts$.

(ii) If ρ is $StN_{nc}\gamma Cts$, then ρ is $N_{nc}Cts$.

Proof. (i) Let $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be $PeN_{nc}\gamma Cts$. Let K be a $N_{nc}o$ set in X_2. Since ρ is $PeN_{nc}\gamma Cts$, $\rho^{-1}(K)$ is $N_{nc}clo$ in X_1. Therefore ρ is $PeN_{nc}Cts$.
(ii) Let $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be $StN_{nc}\gamma Cts$. Let G be a $N_{nc}o$ set in X_2. Since ρ is $StN_{nc}\gamma Cts$, $\rho^{-1}(G)$ is $N_{nc}o$ in X_1. Therefore ρ is $N_{nc}Cts$. □

Theorem 8.4. Let $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be $StN_{nc}\gamma Cts$ and A be $N_{nc}o$ in X_1. Then the restriction, $\rho_A : A \to X_2$ is $StN_{nc}\gamma Cts$.

Proof. Let K be any $N_{nc}\gamma o$ set in X_2. Since ρ is $StN_{nc}\gamma Cts$, $\rho^{-1}(K)$ is $N_{nc}o$ in X_1. But $\rho_A^{-1}(K) = A \cap \rho^{-1}(K)$. Since A and $\rho^{-1}(K)$ are $N_{nc}o$, $\rho_A^{-1}(K)$ is $N_{nc}o$ in A. Hence ρ_A is $StN_{nc}\gamma Cts$. □

Theorem 8.5. Every $PeN_{nc}\gamma Cts$ is $StN_{nc}\gamma Cts$.

Proof. Let $\rho : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be $PeN_{nc}\gamma Cts$ and K be $N_{nc}\gamma o$ in X_2. Since ρ is $PeN_{nc}\gamma Cts$, $\rho^{-1}(K)$ is $N_{nc}clo$ in X_1. That is, $\rho^{-1}(K)$ is both $N_{nc}o$ and $N_{nc}c$ in X_1. Hence ρ is $StN_{nc}\gamma Cts$. □

Theorem 8.6. If $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ are $StN_{nc}\gamma Cts$, then their composition $\rho_2 \circ \rho_1 : (X_1, N_{nc}\Gamma) \to (X_3, N_{nc}\Phi)$ is also $StN_{nc}\gamma Cts$.

Proof. Let K be a $N_{nc}\gamma o$ set in X_3. Since ρ_2 is a $StN_{nc}\gamma Cts$ function, $\rho_2^{-1}(K)$ is $N_{nc}o$ in X_2. Since ρ_1 is a $StN_{nc}\gamma Cts$ function, $\rho_1^{-1}(\rho_2^{-1}(K)) = (\rho_2 \circ \rho_1)^{-1}(K)$ is $N_{nc}o$ in X_1. Therefore $\rho_2 \circ \rho_1$ is $StN_{nc}\gamma Cts$. □

Theorem 8.7. If $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ are $PeN_{nc}\gamma Cts$, then their composition $\rho_2 \circ \rho_1 : (X_1, N_{nc}\Gamma) \to (X_3, N_{nc}\Phi)$ is also $PeN_{nc}\gamma Cts$.

https://doi.org/10.54216/IJNS.180302

Received: January 13, 2022 Accepted: April 07, 2022

Proof. Let K be a $N_{nc}\gamma o$ set in X_3. Since ρ_2 is a $PeN_{nc}\gamma Cts$ function, $\rho_2^{-1}(K)$ is $N_{nc}clo$ in X_2. That is $\rho_2^{-1}(K)$ is both $N_{nc}o$ and $N_{nc}c$. Since ρ_1 is a $PeN_{nc}\gamma Cts$ function, $\rho_1^{-1}(\rho_2^{-1}(K)) = (\rho_2 \circ \rho_1)^{-1}(K)$ is $N_{nc}clo$ in X_1. Therefore $\rho_2 \circ \rho_1$ is $PeN_{nc}\gamma Cts$. □

Theorem 8.8. Let $\rho_1 : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ and $\rho_2 : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ be functions. Then,

(i) If ρ_2 is $StN_{nc}\gamma Cts$ and ρ_1 is $N_{nc}\gamma Cts$, then $\rho_2 \circ \rho_1$ is $N_{nc}\gamma Irr$.

(ii) If ρ_2 is $PeN_{nc}\gamma Cts$ and ρ_1 is $N_{nc}Cts$, then $\rho_2 \circ \rho_1$ is $StN_{nc}\gamma Cts$.

(iii) If ρ_2 is $StN_{nc}\gamma Cts$ and ρ_1 is $PeN_{nc}\gamma Cts$, then $\rho_2 \circ \rho_1$ is $PeN_{nc}\gamma Cts$.

(iv) If ρ_2 is $N_{nc}\gamma Cts$ and ρ_1 is $StN_{nc}\gamma Cts$, then $\rho_2 \circ \rho_1$ is $N_{nc}Cts$.

Proof. (i) Let K be a $N_{nc}\gamma o$ set in X_3. Since ρ_2 is a $StN_{nc}\gamma Cts$ function, $\rho_2^{-1}(K)$ is $N_{nc}o$ in X_2. Since ρ_1 is a $N_{nc}\gamma Cts$ function, $\rho_1^{-1}(\rho_2^{-1}(K)) = (\rho_2 \circ \rho_1)^{-1}(K)$ is $N_{nc}\gamma o$ in X_1. Hence $\rho_2 \circ \rho_1$ is $N_{nc}\gamma Irr$.
(ii) Let K be a $N_{nc}\gamma o$ set in X_3. Since ρ_2 is a $PeN_{nc}\gamma Cts$ function, $\rho_2^{-1}(K)$ is $N_{nc}clo$ in X_2. That is, $\rho_2^{-1}(K)$ is both $N_{nc}o$ and $N_{nc}c$. Since ρ_1 is a $N_{nc}Cts$, $\rho_1^{-1}(\rho_2^{-1}(K)) = (\rho_2 \circ \rho_1)^{-1}(K)$ is $N_{nc}o$ in X_1. Therefore $\rho_2 \circ \rho_1$ is $StN_{nc}\gamma Cts$.
(iii) Let K be a $N_{nc}\gamma o$ set in X_3. Since ρ_2 is a $StN_{nc}\gamma Cts$ function, $\rho_2^{-1}(K)$ is $N_{nc}o$ in X_2. Since ρ_1 is a $PeN_{nc}\gamma Cts$ function, $\rho_1^{-1}(\rho_2^{-1}(K)) = (\rho_2 \circ \rho_1)^{-1}(K)$ is $N_{nc}clo$ in X_1. Hence $\rho_2 \circ \rho_1$ is $PeN_{nc}\gamma Cts$.
(iv) Let K be a $N_{nc}o$ set in X_3. Since ρ_2 is a $N_{nc}\gamma Cts$ function, $\rho_2^{-1}(K)$ is $N_{nc}\gamma o$ in X_2. Since ρ_1 is a $StN_{nc}\gamma Cts$ function, $\rho_1^{-1}(\rho_2^{-1}(K)) = (\rho_2 \circ \rho_1)^{-1}(K)$ is $N_{nc}o$ in X_1. Therefore $\rho_2 \circ \rho_1$ is $N_{nc}Cts$. □

9 Conclusions

In this paper, the new concept of a $N_{nc}\gamma$ irresolute, $N_{nc}\gamma O$ and $N_{nc}\gamma C$, $N_{nc}Hom$ and a $N_{nc}\gamma Hom$ in $N_{nc}ts$ was discussed. Also, we studied their properties and theorems with examples in $N_{nc}ts$. Also, we demonstrated $N_{nc}\gamma CHom$'s, $N_{nc}\gamma T_{\frac{1}{2}}$-space, $StN_{nc}\gamma Cts$ and $PeN_{nc}\gamma Cts$ functions with some of their properties. We can carry out the further research on N-neutrosophic crisp γ-compactness, N-neutrosophic crisp γ-connectedness and N-neutrosophic locally γ-connectedness in $N_{nc}ts$.
Funding: This research received no external funding.
Conflicts of Interest: The authors declare no conflict of interest.

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https://doi.org/10.54216/IJNS.180302

Received: January 13, 2022 Accepted: April 07, 2022