Neutrosophic Infi-Semi-Open Set via Neutrosophic Infi-Topological Spaces

Suman Das [1], Rakhal Das [2], Surapati Pramanik [3*], and Binod Chandra Tripathy [4]

[1] Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India; email:sumandas18842@gmail.com
[2] Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India; email:rakhaldas95@gmail.com
[3*] Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, Narayanpur, 743126, West Bengal, India; email: surapati.math@gmail.com
[4] Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India; email:tripathybc@gmail.com

*** Correspondence:** surapati.math@gmail.com

Abstract

In this article an attempt is made to introduce the notion of neutrosophic infi-topological space as an extension of infi-topological space and fuzzy infi-topological space. Besides, we define some open sets, namely, neutrosophic infi-open set, neutrosophic infi-semi-open set, neutrosophic infi-pre-open set, neutrosophic infi-b-open set. Then, we define some continuous functions namely, neutrosophic infi-continuous function, neutrosophic infi-semi-continuous function, neutrosophic infi-pre-continuous function, neutrosophic infi-b-continuous function via neutrosophic infi-topological space. Further, we formulate several interesting results on them via neutrosophic infi-topological spaces.

Keywords: Neutrosophic Set**,** Neutrosophic Infi-Topology; Neutrosophic Infi-Open Set; Neutrosophic Infi-Continuous Function.

1. Introduction

Johann Listing was first introducing the term topology in the 19th century. In 1955, Kelly [27] studied different basic concepts of topological spaces namely neighborhood, open set, closed set, interior, closure, compactness, continuity. Afterwards, Levine [28] introduced the concept of semi-open set and semi-continuous functions via general topological space. Masshour et al. [31] grounded the notion of supra topological space in 1983. In 2002, Csaszar [4] presented the notion of generalized topology and generalized continuous functions on it. Csaszar [5] also studied the separation axioms for generalized topologies. Later on, the generalized open sets via generalized topological spaces was presented by Csaszar [6] in 2005. In 2009, Csaszar [7] also introduced the notion of products of generalized topologies. The concept of infi-topological spaces was grounded by Das et al. [16]. In 1965, Zadeh [49] grounded the notion of Fuzzy Set (FS) theory to deal with the situation involving the uncertainty. Afterwards, Chang [3] presented the notion of fuzzy topological spaces in 1968. Hutton [25] studied the normality via fuzzy topological spaces. In 1975, Gantner et al. [24] presented the idea of compactness via fuzzy topological spaces in 1978. Later on, Saha and Bhattacharya [44] introduced and studied the concept of countable fuzzy topological spaces and countable fuzzy vector spaces. In 2021, Das et al. [8] introduced the notion of fuzzy infi topological spaces by extending the notion of FS and infi-topological space. Later on, Atanassov [2] introduced the notion of Intuitionistic Fuzzy Set (IFS) theory in 1986. Afterwards, Smarandache [38] grounded the notion of Neutrosophic

DOI: https://doi.org/10.54216/IJNS.180204

Received: November 15, 2021 Accepted: March 09, 2022

Set (in short NS) theory in 1998 by generalizing the idea of FS and IFS theory. In an NS, every element has three independent memberships namely truth membership, indeterminacy membership and false membership. Later on, Wang et al. [40] grounded the concept of single-valued NS. Till now, many researchers around the globe used NS, SVNS and their extensions in the theoretical [10, 15, 20, 22, 33,34, 35, 36, 37, 38, 39, 40, 41, 47] area as well as practical area of research [11, 17-18, 29]. In 2020, Das and Tripathy [19] introduced the neutrosophic multiset topological spaces. Thereafter, the idea of Neutrosophic Topological Space (NTS) was grounded by Salama and Alblowi [48] in 2012. Later on, Arokiarani et al. [1] defined the notion of semi-open functions via NTS. In 2016, Iswaraya and Bageerathi [26] presented the concept of semi-open set and semi-closed set in NTSs. Afterwards, Rao and Srinivasa [43] introduced the idea of neutrosophic pre-open set and neutrosophic pre-closed set via NTSs. The notion of neutrosophic generalized closed sets via NTSs was studied by Pushpalatha and Nandhini [42]. The idea of neutrosophic *b*-open sets via NTSs was introduced by Ebenanjar et al. [23]. Maheswari et al. [30] studied the idea of neutrosophic generalized *b*-closed sets in NTSs. In the year 2019, the concept of generalized neutrosophic *b*-open set via NTSs was studied by Das and Pramanik [12]. Later on, Das and Pramanik [13] also grounded the notion of neutrosophic Φ-open sets and neutrosophic Φ-continuous mappings via NTSs. Afterwards, Das and Pramanik [14] presented the notions of neutrosophic simply soft open set via neutrosophic soft topological spaces. Mohammed Ali Jaffer and Ramesh [32] presented the notion of neutrosophic generalized pre regular closed set via NTSs. In 2021, Das [9] introduced the notion of neutrosophic supra simply open set and neutrosophic supra simply compactness via neutrosophic supra topological spaces. Recently, Das and Tripathy [21] presented the idea of neutrosophic simply *b*-open set via NTSs.

In this article, we procure the notion of neutrosophic infi-topological space, and studied some basic properties of neutrosophic infi-topological spaces like neutrosophic infi-interior, neutrosophic infi-closure, neutrosophic infi-continuous mapping and neutrosophic infi-open mapping, etc. via neutrosophic infi-topological spaces.

Research Gap: No investigation on neutrosophic infi-topological space has been reported in the literature.

Motivation: To fill the research gap, we procure the notion of neutrosophic infi-topological space, and study its different properties.

The rest of this paper has been designed as follows:

In section-2, we recall some relevant definitions and results on FS, NS, NTS, Infi-Topological Space, etc. In section-3, we procure the notion of neutrosophic infi-topological space, and formulate several interesting results on it. Finally, section-4 represents the concluding remarks of the work done in this article.

2. Some Relevant Results

In this section, we provide some definitions and results those are very relevant and useful for the preparation of the main results of this article.

Definition 2.1.[16] Assume that W is a fixed set. Then, a collection $\Im$ of subsets of W is said to be an infi-topology on W if the following two conditions hold:

(i) $\emptyset, W \in \Im$;

(ii) $Y_1, Y_2 \in \Im \Rightarrow Y_1 \cap Y_2 \in \Im$;

Then, the structure (W, $\Im$) is called an infi-topological space. Every member of $\Im$ is called an infi-open set. If $Y \in \Im$, then Y^c is said to be an infi-closed set.

Definition 2.2.[49] Suppose that W is a fixed set. Then P, a FS over W is defined by:

$$P=\{(q, T_P(q)): q \in W\},$$

where $T_P(q)$ ($\in[0, 1]$) denotes the truth membership values of $q \in W$.

Definition 2.3.[49] The absolute FS (1_F) and null FS (0_F) over W are defined as follows:

(i) $1_N=\{(q, 1): q \in W\}$;

(ii) $0_N=\{(q, 0): q \in W\}$.

Definition 2.4.[8] Suppose that W is a fixed set. Then, a collection $\Im$ of FSs over W is called a fuzzy infi-topology on W if the following two conditions hold:

(i) $0_N, 1_N \in \Im$;

Received: November 15, 2021 Accepted: March 09, 2022

(ii) $Y_1, Y_2 \in \Im \Rightarrow Y_1 \cap Y_2 \in \Im$.

Then, the structure (W, $\Im$) is said to be a fuzzy infi-topological space. Every member of $\Im$ is called a fuzzy infi-open set. If $Y \in \Im$, then Y^c is said to be a fuzzy infi-closed set in (W, $\Im$).

Definition 2.5.[46] Suppose that W is a universe of discourse. Then P, an NS over W is defined by:

$$P=\{(q, T_P(q), I_P(q), F_P(q)): q \in W\},$$

where $T_P(q)$, $I_P(q)$, $F_P(q)$ ($\in [0, 1]$) are the truth membership, indeterminacy membership and falsity membership values of $q \in W$. So, $0 \le T_P(q) + I_P(q) + F_P(q) \le 3$, for all $q \in W$.

Definition 2.6.[46] The absolute NS (1_N) and null NS (0_N) over a fixed set W are defined as follows:

(i) $1_N = \{(q,1,0,0): q \in W\}$;

(ii) $0_N = \{(q,0,1,1): q \in W\}$.

Definition 2.3.[46] Assume that $X=\{(q,T_X(q),I_X(q),F_X(q)): q \in W\}$ and $Y = \{(q,T_Y(q),I_Y(q), F_Y(q)): q \in W\}$ are any two NSs over a fixed set W. Then, $X \subseteq Y$ if and only if $T_X(q) \le T_Y(q)$, $I_X(q) \ge I_Y(q)$, $F_X(q) \ge F_Y(q)$, for all $q \in W$.

Definition 2.7.[46] Suppose that $X=\{(q,T_X(q),I_X(q),F_X(q)): q \in W\}$ and $Y = \{(q,T_Y(q),I_Y(q), F_Y(q)): q \in W\}$ are any two NSs over a fixed set W. Then, the intersection of X and Y is defined by

$$X \cap Y=\{(q, \min\{T_X(q),T_Y(q)\}, \max\{I_X(q),I_X(q)\}, \max\{F_X(q),F_X(q)\}): q \in W\}.$$

Definition 2.8.[46] Assume that $X=\{(q,T_X(q),I_X(q),F_X(q)): q \in W\}$ and $Y = \{(q,T_Y(q),I_Y(q), F_Y(q)): q \in W\}$ are any two NSs over a fixed set W. Then, the union of X and Y is defined by

$$X \cup Y=\{(q, \max\{T_X(q),T_Y(q)\}, \min\{I_X(q),I_X(q)\}, \min\{F_X(q),F_X(q)\}): q \in W\}.$$

Definition 2.9.[46] Suppose that $X=\{(q,T_X(q),I_X(q),F_X(q)): q \in W\}$ is an NS over a fixed set W. Then, the complement of X is defined by $X^c=\{(q,1-T_X(q),1-I_X(q),1-F_X(q)): q \in W\}$.

3. Neutrosophic Infi-Topological space

In this section, an attempt is made to introduce the notion of neutrosophic infi-topological space (NITS) as an extension of infi-topological space and fuzzy infi-topological space, and study some of its basic properties. Further, we formulate several interesting results on NITSs.

Definition 3.1. Suppose that W is a fixed set. Then, a family $\Im$ of NSs over W is said to be an neutrosophic Infi-Topology (NIT) on W if the following two conditions holds:

(i) $0_N, 1_N \in \Im$;

(ii) $Y_1, Y_2 \in \Im \Rightarrow Y_1 \cap Y_2 \in \Im$.

Then, the structure (W, $\Im$) is called an NITS. Every member of $\Im$ is called an neutrosophic infi-open set (NIOS). If $Y \in \Im$, then Y^c is said to be an neutrosophic infi-closed set (NICS).

Clearly, every NTS is an NITS.

Example 3.1. Suppose that W={a, b} is a fixed set. Let $\Im=\{0_N, 1_N, P, Q\}$ be a collection of NSs such that P={(a,0.5,0.3,0.6), (b,0.9,0.8,0.2)} and Q={(a,0.8,0.2,0.5), (b,1.0,0.5,0.2)}. Then, $\Im$ is an NIT on W. Therefore, (W, $\Im$) is an NITS.

Remark 3.1. In an NITS (W, $\Im$), the null NS (0_N) and the absolute NS (1_N) are both NIOS and NICS.

The notion of neutrosophic infi-interior i.e., $N_{i\text{-}int}$ and neutrosophic infi-closure i.e., $N_{i\text{-}cl}$ of an NS are defined as follows:

Definition 3.2. Assume that (W, $\Im$) is an NITS. Suppose that X is an NS over W. Then, the neutrosophic infi-interior of X i.e., $N_{i\text{-}int}(X)$ is the union of all NIOSs contained in X and the neutrosophic infi-closure of X i.e., $N_{i\text{-}cl}(X)$ is the intersection of all NICSs containing X.

Therefore, $N_{i\text{-}int}(X) = \cup\{Y: Y \subseteq X$ and Y is an NIOS in (W, $\Im$)$\}$ and $N_{i\text{-}cl}(X) = \cap\{Z: X \subseteq Z$ and Z is an NICS in (W,$\Im$)$\}$.

Remark 3.2. Clearly, $N_{i\text{-}int}(X)$ is the largest NIOS in (W, $\Im$) which is contained in X and $N_{i\text{-}cl}(X)$ is the smallest NICS in (W, $\Im$) that contains X.

Theorem 3.1. Suppose that (W, $\Im$) is an NITS. Assume that Q and R are any two NSs over W. Then, the following properties hold:

(i) $N_{i\text{-}int}(Q) \subseteq Q \subseteq P\text{-}N_{i\text{-}cl}(Q)$;

(ii) $Q \subseteq R \Rightarrow P\text{-}N_{i\text{-}cl}(Q) \subseteq P\text{-}N_{i\text{-}cl}(R)$;

(iii) $Q \subseteq R \Rightarrow P\text{-}N_{i\text{-}int}(Q) \subseteq P\text{-}N_{i\text{-}int}(R)$;

DOI: https://doi.org/10.54216/IJNS.180204

Received: November 15, 2021 Accepted: March 09, 2022

(iv) $P\text{-}N_{i\text{-}cl}(Q\cup R) = P\text{-}N_{i\text{-}cl}(Q) \cup P\text{-}N_{i\text{-}cl}(R)$;
(v) $P\text{-}N_{i\text{-}cl}(Q\cap R) \subseteq P\text{-}N_{i\text{-}cl}(Q) \cap P\text{-}N_{i\text{-}cl}(R)$;
(vi) $P\text{-}N_{i\text{-}int}(Q\cup R) \supseteq P\text{-}N_{i\text{-}int}(Q) \cup P\text{-}N_{i\text{-}int}(R)$;
(vii) $P\text{-}N_{i\text{-}int}(Q\cap R) \subseteq P\text{-}N_{i\text{-}int}(Q) \cap P\text{-}N_{i\text{-}int}(R)$.

Proof. (i) It is known that $N_{i\text{-}int}(Q) = \cup\{R: R$ is an NIOS in $(W, \mathfrak{I})$ and $R\subseteq Q\}$. Since, each $R\subseteq Q$, so $\cup\{R: R$ is an NIOS in $(W, \mathfrak{I})$ and $R\subseteq Q\} \subseteq Q$, i.e., $N_{i\text{-}int}(Q)\subseteq Q$.
Again, $N_{i\text{-}cl}(Q) = \cap\{Z: Z$ is an NICS in $(W, \mathfrak{I})$ and $Q\subseteq Z\}$. Since, each $Z\supseteq Q$, so $\cap\{Z: Z$ is an NICS in $(W, \mathfrak{I})$ and $Q\subseteq Z\} \supseteq Q$, i.e., $N_{i\text{-}cl}(Q)\supseteq Q$**.**
Therefore, $N_{i\text{-}int}(Q)\subseteq Q\subseteq N_{i\text{-}cl}(Q)$.
(ii) Suppose that $(W, \mathfrak{I})$ be an NITS. Assume that Q and R are two NSs over W such that $Q\subseteq R$.
Now, $N_{cl}(Q) = \cap\{Z: Z$ is an NICS in $(W, \mathfrak{I})$ and $Q\subseteq Z\}$
$\subseteq \cap\{Z: Z$ is an NICS in $(W, \mathfrak{I})$ and $R\subseteq Z\}$ [Since $Q\subseteq R$]
$= N_{i\text{-}cl}(R)$
$\Rightarrow N_{i\text{-}cl}(Q) \subseteq N_{i\text{-}cl}(R)$.
Therefore, $Q\subseteq R \Rightarrow N_{i\text{-}cl}(Q)\subseteq N_{i\text{-}cl}(R)$.
(iii) Assume that $(W, \mathfrak{I})$ is an NITS. Suppose that Q and R are two NSs over W such that $Q\subseteq R$.
Now, $N_{i\text{-}int}(Q) = \cup\{Z: Z$ is an NIOS in $(W, \mathfrak{I})$ and $Z\subseteq Q\}$
$\subseteq \cup\{Z: Z$ is an NIOS in $(W, \mathfrak{I})$ and $Z\subseteq R\}$ [Since $Q\subseteq R$]
$= N_{i\text{-}int}(R)$
$\Rightarrow N_{i\text{-}int}(Q) \subseteq N_{i\text{-}int}(R)$.
Therefore, $Q\subseteq R \Rightarrow N_{i\text{-}int}(Q)\subseteq N_{i\text{-}int}(R)$.
(iv) Suppose that Q and R are any two neutrosophic subsets of an NITS $(W, \mathfrak{I})$. It is known that $Q\subseteq Q\cup R$ and $R\subseteq Q\cup R$.
Now, $Q \subseteq Q\cup R$
$\Rightarrow N_{i\text{-}cl}(Q) \subseteq N_{i\text{-}cl}(Q\cup R)$;
and $R \subseteq Q\cup R$
$\Rightarrow N_{i\text{-}cl}(R) \subseteq N_{i\text{-}cl}(Q\cup R)$.
Therefore, $N_{i\text{-}cl}(Q)\cup N_{i\text{-}cl}(R) \subseteq N_{i\text{-}cl}(Q\cup R)$ (1)
We have, $Q\subseteq N_{i\text{-}cl}(Q)$, $R\subseteq N_{i\text{-}cl}(R)$. Therefore, $Q\cup R \subseteq N_{i\text{-}cl}(Q)\cup N_{i\text{-}cl}(R)$.
Further, it is known that $N_{i\text{-}cl}(Q)\cup N_{i\text{-}cl}(R)$ is an NICS in $(W, \mathfrak{I})$. It is clear that, $N_{i\text{-}cl}(Q)\cup N_{i\text{-}cl}(R)$ is an NICS in $(W, \mathfrak{I})$, which contains $Q\cup R$. But it is known that $N_{i\text{-}cl}(Q\cup R)$ is the smallest NICS in $(W, \mathfrak{I})$, which contains $Q\cup R$. Therefore, $N_{i\text{-}cl}(Q\cup R) \subseteq N_{i\text{-}cl}(Q)\cup N_{i\text{-}cl}(R)$ (2)
From eq. (1) and eq. (2), we have $N_{i\text{-}cl}(Q\cup R) = N_{i\text{-}cl}(Q)\cup N_{i\text{-}cl}(R)$.
(v) Suppose that Q and R are any two neutrosophic subsets of an NITS $(W, \mathfrak{I})$. It is known that $Q\cap R\subseteq Q$, $Q\cap R\subseteq R$.
Now, $Q\cap R\subseteq Q$
$\Rightarrow N_{i\text{-}cl}(Q\cap R) \subseteq N_{i\text{-}cl}(Q)$;
and $Q\cap R\subseteq R$
$\Rightarrow N_{i\text{-}cl}(Q\cap R) \subseteq N_{i\text{-}cl}(R)$.
Therefore, $N_{i\text{-}cl}(Q\cap R) \subseteq N_{i\text{-}cl}(Q)\cap N_{i\text{-}cl}(R)$.
(vi) Assume that Q and R are two neutrosophic subsets of an NITS $(W, \mathfrak{I})$. It is known that $Q\subseteq Q\cup R$ and $R\subseteq Q\cup R$. Thus, we get
$Q\subseteq Q\cup R$
$\Rightarrow N_{i\text{-}int}(Q) \subseteq N_{i\text{-}int}(Q\cup R)$;
and $R\subseteq Q\cup R$
$\Rightarrow N_{i\text{-}int}(R) \subseteq N_{i\text{-}int}(Q\cup R)$.
Therefore, $N_{i\text{-}int}(Q)\cup N_{i\text{-}int}(R) \subseteq N_{i\text{-}int}(Q\cup R)$.
(vii) Suppose that Q and R are two neutrosophic subsets of an NITS $(W, \mathfrak{I})$. It is known that $Q\cap R\subseteq Q$, $Q\cap R\subseteq R$.
Now, $Q\cap R\subseteq Q$
$\Rightarrow N_{i\text{-}int}(Q\cap R) \subseteq N_{i\text{-}int}(Q)$;
and $Q\cap R\subseteq R$

DOI: https://doi.org/10.54216/IJNS.180204

Received: November 15, 2021 Accepted: March 09, 2022

$\Rightarrow N_{i\text{-}int}(Q\cap R) \subseteq N_{i\text{-}int}(R)$.
Therefore, $N_{i\text{-}int}(Q\cap R) \subseteq N_{i\text{-}int}(Q)\cap N_{i\text{-}int}(R)$.
Theorem 3.2. Let Q be an neutrosophic subset of an NITS (W, $\Im$). Then, the following properties hold:
(i) $(N_{i\text{-}int}(Q))^c = N_{i\text{-}cl}(Q^c)$;
(ii) $(N_{i\text{-}cl}(Q))^c = N_{i\text{-}int}(Q^c)$.
Proof. (i) Suppose that (W, $\Im$) is an NITS, and Q={(w, T_Q(w), I_Q(w), F_Q(w)): w∈W} is an neutrosophic subset of W.
Therefore, P-N_{int}(Q)
= $\cup\{Z_i : i\in\Delta$ and Z_i is an NIOS in (W, $\Im$) such that $Z_i \subseteq Q\}$
= {(w, $\vee T_{Z_i}(w)$, $\wedge I_{Z_i}(w)$, $\wedge F_{Z_i}(w)$) : w ∈W}, where for all i∈Δ and Z_i is an NIOS in (W, $\Im$) such that $Z_i\subseteq Q$.
This implies, $(N_{i\text{-}int}(Q))^c$ = {(w, $\wedge T_{Z_i}(w)$, $\vee I_{Z_i}(w)$, $\vee F_{Z_i}(w)$): w∈W}.
Since, $\wedge T_{Z_i}(w) \le T_E(w)$, $\vee I_{Z_i}(w) \ge I_E(w)$, $\vee F_{Z_i}(w) \ge F_E(w)$, for each i∈Δ and w∈W, so $N_{i\text{-}cl}(Q^c)$ = {(w, $\wedge T_{Z_i}(w)$, $\vee I_{Z_i}(w)$, $\vee F_{Z_i}(w)$): w∈W}} = $\cap\{Z_i$: i∈Δ and Z_i is an NICS in (W, $\Im$) such that $Q^c\subseteq Z_i\}$. Therefore, $(N_{i\text{-}int}(Q))^c = N_{i\text{-}cl}(Q^c)$.

(ii) Let (W, $\Im$) be an NITS, and Q = {(w, T_Q(w), I_Q(w), F_Q(w)): w∈W} be an neutrosophic subset of W.
Therefore, P-N_{cl}(Q)
= $\cap\{Z_i$: i∈Δ and Z_i is an NICS in (W, $\Im$) such that $Z_i \supseteq Q\}$
= {(w, $\wedge T_{Z_i}(w)$, $\vee I_{Z_i}(w)$, $\vee F_{Z_i}(w)$): w∈W}, where Z_i is an NICS in (W, $\Im$) such that $Z_i \supseteq Q$, for all i∈Δ.
This implies, $(N_{i\text{-}cl}(Q))^c$ = {(w, $\vee T_{Z_i}(w)$, $\wedge I_{Z_i}(w)$, $\wedge F_{Z_i}(w)$): w∈W}.
Since $\vee T_{Z_i}(w) \ge T_E(w)$, $\wedge I_{Z_i}(w) \le I_E(w)$, $\wedge F_{Z_i}(w) \le F_E(w)$, for each i∈Δ and w∈W, so $N_{i\text{-}int}(Q^c)$ = {(w, $\vee T_{Z_i}(w)$, $\wedge I_{Z_i}(w)$, $\wedge F_{Z_i}(w)$): w∈W} = $\cup\{Z_i$: i∈Δ and Z_i is an NIOS in (W, $\Im$) such that $Z_i \subseteq Q^c\}$. Therefore, $(N_{i\text{-}cl}(Q))^c = N_{i\text{-}int}(Q^c)$.
Theorem 3.3. Let X be an neutrosophic subset of an NITS (W, $\Im$). Then, the following properties hold:
(i) Q is an NIOS if and only if $N_{i\text{-}int}(Q) = Q$;
(ii)Q is a NIOS if and only if $N_{i\text{-}cl}(Q) = Q$.
Proof. (i) Let Q be an NIOS in an NITS (W, $\Im$). Now, $N_{i\text{-}int}(Q)$ = ∪{Z: Z is an NIOS in (W, $\Im$) and Z ⊆Q}. Since, Q is an NIOS in (W, $\Im$), so Q is the largest NIOS, which is contained in Q. This implies, ∪{Z: Z is an NIOS in (W, $\Im$) and Z ⊆Q} = Q. Therefore, $N_{i\text{-}int}(Q) = Q$.
(ii) Let Q be an NICS in an NITS (W, $\Im$). Now, $N_{i\text{-}cl}(Q)$ = ∩{Z : Z is an NICS in (W, $\Im$) and Q⊆Z}. Since, Q is an NICS in (W, $\Im$), so Q is the smallest NICS, which contains Q. This implies, ∩{Z: Z is an NICS in (W, $\Im$) and Q⊆Z} = Q. Therefore, $N_{i\text{-}cl}(Q) = Q$.
Definition 3.3. Let (W, $\Im$) be an NITS. Then X, an NS over W is called as
(i) neutrosophic infi-semi-open (in short NISO) set if and only if $X \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X))$;
(ii) neutrosophic infi-pre-open (in short NIPO) set if and only if $X \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X))$.
Remark 3.3. The complement of NISO set and NIPO set in an NITS (W, $\Im$) are called neutrosophic infi-semi-closed (in short NISC) set and neutrosophic infi-pre-closed (in short NIPC) set respectively.
Theorem 3.4. Suppose that (W, $\Im$) is an NITS. Then,
(i) every NIOS is an NISO set.
(ii) every NIOS is an NIPO set.
Proof. (i) Let (W, $\Im$) be an NITS. Let X be an NIOS. Therefore, $X=N_{i\text{-}int}(X)$. It is known that $X\subseteq N_{i\text{-}cl}(X)$. This implies, $X\subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X))$. Therefore, X is an NISO set in (W, $\Im$).
(ii) Let (W, $\Im$) be an NITS. Let X be an NIOS. Therefore, $X=N_{i\text{-}int}(X)$. It is known that, $X\subseteq N_{i\text{-}cl}(X)$. This implies, $N_{i\text{-}int}(X)\subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X))$ i.e. $X=N_{i\text{-}int}(X) \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X))$. Therefore, $X\subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X))$. Hence, X is an NIPO set in (W, $\Im$).
Remark 3.4. The converse of the theorem 3.4 may not be true in general, which follows from the following example.
Example 3.2. Assume that (W, $\Im$) is an NITS, where $\Im$={0_N, 1_N, {(a,0.4,0.4,0.3), (b,0.3,0.3,0.4)}, {(a,0.6,0.4,0.1), (b,0.4,0.1,0.3)}}. Then,

DOI: https://doi.org/10.54216/IJNS.180204

Received: November 15, 2021 Accepted: March 09, 2022

(i) Q={(a,0.6,0.4,0.1), (b,0.8,0.1,0.2)} is an NISO set but it is not an NIOS in (W, $\Im$).
(ii) P={(a,0.7,0.9,0.2), (b,0.7,0.4,0.3)} is an NIPO set but it is not an NIOS in (W, $\Im$).

Theorem 3.5. In an NITS (W, $\Im$), the union of any two NISO sets is also an NISO set.

Proof. Suppose that X and Y be any two NISO sets in an NITS (W, $\Im$). Therefore,

$$X \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X)) \tag{3}$$

and
$$Y \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(Y)) \tag{4}$$

From eq. (3) and eq. (4), we have

$$X \cup Y \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X)) \cup N_{i\text{-}cl}(N_{i\text{-}int}(Y))$$
$$= N_{i\text{-}cl}(N_{i\text{-}int}(X) \cup N_{i\text{-}int}(Y))$$
$$\subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X \cup Y)).$$

Therefore, $X \cup Y \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X \cup Y))$. Hence, $X \cup Y$ is an NISO set in (W, $\Im$).

Theorem 3.6. In an NITS (W, $\Im$), the union of any two NIPO sets is also an NIPO set.

Proof. Suppose that X and Y are any two NIPO sets in an NITS (W, $\Im$). Therefore,

$$X \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X)) \tag{5}$$

and
$$Y \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(Y)) \tag{6}$$

From eq. (5) and eq. (6), we have,

$$X \cup Y \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X)) \cup N_{i\text{-}int}(N_{i\text{-}cl}(Y))$$
$$\subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X) \cup N_{i\text{-}cl}(Y))$$
$$= N_{i\text{-}int}(N_{i\text{-}cl}(X \cup Y)).$$

Therefore, $X \cup Y \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X \cup Y))$. Hence, $X \cup Y$ is an NIPO set in (W, $\Im$).

Definition 3.4. Let (W, $\Im$) is an NITS. Then, an NS X over W is called a neutrosophic infi-α-open (in short NI-α-O) set if and only if $X \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(N_{i\text{-}int}(X)))$. The complement of an NI-α-O set is called an neutrosophic infi-α-closed (in short NI-α-C) set.

Corollaries 3.1. In an NITS (W, $\Im$), every NIOS is an NI-α-O set.

Remark 3.5. The converse of the above proposition may not be true in general, which follows from the following example.

Example 3.3. Let us consider an NITS (W, $\Im$) as shown in Example 3.2. Clearly, the NS Q = {(a,0.6,0.4,0.1), (b,0.8,0.1,0.2)} is an NI-α-O set but it is not an NIOS in (W, $\Im$).

Theorem 3.7. In an NITS (W, $\Im$), every NI-α-O set is an NISO set.

Proof. Assume that X be an NI-α-O set in (W, $\Im$). Therefore, $X \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(N_{i\text{-}int}(X)))$. It is known that $N_{i\text{-}int}(N_{i\text{-}cl}(N_{i\text{-}int}(X))) \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X))$. Thus we have, $X \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X))$. Hence, X is a NISO set. Therefore, every NI-α-O set is an NISO set.

Remark 3.6. The converse of the above example may not be true in general, which follows from the following example.

Example 3.4. Suppose that (W, $\Im$) be an NITS, where $\Im$ ={0_N, 1_N, {(a,0.6,0.7,0.8), (b,0.5,0.5, 0.6)}, {(a,0.4,0.8,0.8), (b,0.5,0.8,0.8)}}. Then, it can be easily verified that A={(a,0.6,0.3,0.3), (b,0.5,0.4,0.4)} is an NISO set in (W, $\Im$), but it is not an NI-α-O set in (W, $\Im$).

Theorem 3.8. In an NITS (W, $\Im$), every NI-α-O set is an NIPO set.

Proof. Let (W, $\Im$) is an NITS. Assume that X be an NI-α-O set in (W, $\Im$). Therefore, $X \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(N_{i\text{-}int}(X)))$. It is known that, $N_{i\text{-}int}(X) \subseteq X$. This implies, $N_{i\text{-}cl}(N_{i\text{-}int}(X)) \subseteq N_{i\text{-}cl}(X)$. Which implies $N_{i\text{-}int}(N_{i\text{-}cl}(N_{i\text{-}int}(X))) \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X)$. Therefore, $X \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X)$. Hence, X is an NIPO set. Therefore, every NI-α-O set is an NIPO set in (W, $\Im$).

Remark 3.7. The converse of the above example may not be true in general, which follows from the following example.

Example 3.5. Suppose that (W, $\Im$) is an NITS as shown in Example 3.2. Then, the NS P={(a, 0.7,0.9,0.2), (b,0.7,0.4,0.3)} is an NIPO set in (W, $\Im$) but it is not an NI-α-O set in (W, $\Im$).

Definition 3.5. Assume that (W, $\Im$) is an NITS. Then, an NS X over W is called an neutrosophic infi-b-open (in short NI-b-O) set if and only if $X \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X)) \cup N_{i\text{-}cl}(N_{i\text{-}int}(X))$.

Remark 3.8. An NS X is called an neutrosophic infi-b-closed (in short NI-b-C) set iff X^c is an NI-b-O set i.e., if $N_{i\text{-}int}(N_{i\text{-}cl}(X)) \cap N_{i\text{-}cl}(N_{i\text{-}int}(X)) \subseteq X$.

Theorem 3.9. In an NITS (W, $\Im$), every NIPO (NISO) set is an NI-b-O set.

DOI: https://doi.org/10.54216/IJNS.180204

Received: November 15, 2021 Accepted: March 09, 2022

Proof. Let X be an NIPO set in an NITS (W, $\mathfrak{I}$). Therefore, $X \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X))$. This implies, $X \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X)) \cup N_{i\text{-}cl}(N_{i\text{-}int}(X))$. Hence, X is an NI-b-O set. Therefore, every NIPO set is an NI-b-O set.

Similarly, it can be easily shown that every NISO set is an NI-b-O set.

Theorem 3.10. The union of any two NI-b-O sets in an NITS (W, $\mathfrak{I}$) is also an NI-b-O set.

Proof. Suppose that X and Y be any two NI-b-O sets in an NITS (W, $\mathfrak{I}$).

Therefore, $X \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X)) \cup N_{i\text{-}cl}(N_{i\text{-}int}(X))$ (7)

and $Y \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(Y)) \cup N_{i\text{-}cl}(N_{i\text{-}int}(Y))$ (8)

It is known that, $X \subseteq X\cup Y$ and $Y \subseteq X\cup Y$.

Now, $X \subseteq X\cup Y$

$\Rightarrow N_{i\text{-}int}(X) \subseteq N_{i\text{-}int}(A\cup B)$

$\Rightarrow N_{i\text{-}cl}(N_{i\text{-}int}(X)) \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X\cup Y))$ (9)

and $X \subseteq X\cup Y$

$\Rightarrow N_{i\text{-}cl}(X) \subseteq N_{i\text{-}cl}(A\cup B)$

$\Rightarrow N_{i\text{-}int}(N_{i\text{-}cl}(X)) \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X\cup Y))$ (10)

Similarly, it can be shown that

$N_{i\text{-}cl}(N_{i\text{-}int}(Y)) \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X\cup Y))$ (11)

$N_{i\text{-}int}(N_{i\text{-}cl}(Y)) \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(X\cup Y))$ (12)

From eq. (7) and eq. (8) we have,

$X\cup Y \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X)) \cup N_{i\text{-}int}(N_{i\text{-}cl}(X)) \cup N_{i\text{-}cl}(N_{i\text{-}int}(Y)) \cup N_{i\text{-}int}(N_{i\text{-}cl}(Y))$

$\subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X\cup Y)) \cup N_{i\text{-}int}(N_{i\text{-}cl}(X\cup Y)) \cup N_{i\text{-}cl}(N_{i\text{-}int}(X\cup Y)) \cup N_{i\text{-}int}(N_{i\text{-}cl}(X\cup Y))$

[By eqs. (9), (10), (11), & (12)]

$= N_{i\text{-}cl}(N_{i\text{-}int}(X\cup Y)) \cup N_{i\text{-}int}(N_{i\text{-}cl}(X\cup Y))$

$\Rightarrow X\cup Y \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X\cup Y)) \cup N_{i\text{-}int}(N_{i\text{-}cl}(X\cup Y))$.

Therefore, $X\cup Y$ is an NI-b-O set.

Hence, the union of two NI-b-O sets is also an NI-b-O set.

Theorem 3.11. In an NITS (W, $\mathfrak{I}$), the intersection of any two NI-b-C sets is also an NI-b-C set.

Proof. Assume that (W, $\mathfrak{I}$) is an NITS. Suppose that X and Y are any two NI-b-C sets in (W, $\mathfrak{I}$). Therefore,

$N_{i\text{-}int}(N_{i\text{-}cl}(X)) \cap N_{i\text{-}cl}(N_{i\text{-}int}(X)) \subseteq X$ (13)

and $N_{i\text{-}int}(N_{i\text{-}cl}(Y)) \cap N_{i\text{-}cl}(N_{i\text{-}int}(Y)) \subseteq Y$ (14)

Since, $X\cap Y \subseteq X$ and $X\cap Y \subseteq Y$, so we get

$N_{i\text{-}int}(X\cap Y) \subseteq N_{i\text{-}int}(X) \Rightarrow N_{i\text{-}cl}(N_{i\text{-}int}(X\cap Y)) \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(X))$ (15)

$N_{i\text{-}cl}(X\cap Y) \subseteq N_{i\text{-}cl}(X) \Rightarrow N_{i\text{-}int}(N_{i\text{-}cl}(X\cap Y)) \subseteq N_{i\text{-}int}(P\text{-}N_{cl}(X))$ (16)

$N_{i\text{-}int}(X\cap Y) \subseteq N_{i\text{-}int}(Y) \Rightarrow N_{i\text{-}cl}(N_{i\text{-}int}(X\cap Y)) \subseteq N_{i\text{-}cl}(N_{i\text{-}int}(Y))$ (17)

and $N_{i\text{-}cl}(X\cap Y) \subseteq N_{i\text{-}cl}(Y) \Rightarrow N_{i\text{-}int}(N_{i\text{-}cl}(X\cap Y)) \subseteq N_{i\text{-}int}(N_{i\text{-}cl}(Y))$ (18)

From eq. (13) & (14) we get,

$X\cap Y \supseteq N_{i\text{-}int}(N_{i\text{-}cl}(X)) \cap N_{i\text{-}cl}(N_{i\text{-}int}(X)) \cap N_{i\text{-}int}(N_{i\text{-}cl}(Y)) \cap N_{i\text{-}cl}(N_{i\text{-}int}(Y))$

$\supseteq N_{i\text{-}int}(N_{i\text{-}cl}(X\cap Y)) \cap N_{i\text{-}cl}(N_{i\text{-}int}(X\cap Y)) \cap N_{i\text{-}int}(N_{i\text{-}cl}(X\cap Y)) \cap N_{i\text{-}cl}(N_{i\text{-}int}(X\cap Y))$

[By eqs. (15), (16), (17) & (18)]

$= N_{i\text{-}int}(N_{i\text{-}cl}(X\cap Y)) \cap N_{i\text{-}cl}(N_{i\text{-}int}(X\cap Y))$

$\Rightarrow X\cap Y \supseteq N_{i\text{-}cl}(N_{i\text{-}int}(X\cap Y)) \cap N_{i\text{-}int}(N_{i\text{-}cl}(X\cap Y))$.

Hence, $X\cap Y$ is an NI-b-C set in (W, $\mathfrak{I}$).

Therefore, the intersection of any two NI-b-C sets is also an NI-b-C set.

Definition 3.6. Suppose that (W, $\mathfrak{I}_1$) and (M, $\mathfrak{I}_2$) are any two NITSs. Then, a one to one and onto mapping ξ:(W, $\mathfrak{I}_1$)$\rightarrow$ (M, $\mathfrak{I}_2$) is called as

(i) neutrosophic infi-continuous mapping (NI-C-mapping) if and only if $\xi^{-1}(L)$ is an NIOS in W, whenever L is an NIOS in M;

(ii) neutrosophic infi-semi-continuous mapping (NIS-C-mapping) if and only if $\xi^{-1}(L)$ is an NISO set in W, whenever L is an NIOS in M;

(iii) neutrosophic infi-pre-continuous mapping (NIP-C-mapping) if and only if $\xi^{-1}(L)$ is an NIPO set in W, whenever L is an NIOS in M;

DOI: https://doi.org/10.54216/IJNS.180204

Received: November 15, 2021 Accepted: March 09, 2022

(iv) neutrosophic infi-b-continuous mapping (NIP-C-mapping) if and only if $\xi^{-1}(L)$ is an NI-b-O set in W, whenever L is an NIOS in M;
(v) neutrosophic infi-α-continuous mapping (NIP-C-mapping) if and only if $\xi^{-1}(L)$ is an NI-α-O set in W, whenever L is an NIOS in M;

Theorem 3.12. Suppose that $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ and $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ are any two NI-C-mappings. Then, the composition mapping $\zeta\circ\xi:(W, \Im_1)\rightarrow(V, \Im_3)$ is also an NI-C-mapping.

Proof. Let $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ and $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ be any two NI-C-mappings. Assume that Q is an NIOS in $(V, \Im_3)$. Since, $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ is an NI-C-mapping, so $\zeta^{-1}(Q)$ is an NIOS in $(M, \Im_2)$. Further, since $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NI-C-mapping, so $\xi^{-1}(\zeta^{-1}(Q))=(\zeta\circ\xi)^{-1}(Q)$ is an NIOS in (Ψ, τ_1, τ_2). Hence, $(\zeta\circ\xi)^{-1}(Q)$ is an NIOS in $(W, \Im_1)$ whenever Q is an NIOS in $(V, \Im_3)$. Therefore, the composition mapping $\zeta\circ\xi:(W, \Im_1)\rightarrow(V, \Im_3)$ is also an NI-C-mapping.

Theorem 3.13. Let $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ be an NIS-C-mapping, and $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ be an NI-C-mapping. Then, the composition mapping $\zeta\circ\xi:(W, \Im_1)\rightarrow(V, \Im_3)$ is an NIS-C-mapping.

Proof. Let $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ be an NIS-C-mapping, and $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ be an NI-C-mapping. Assume that Q is an NIOS in $(V, \Im_3)$. Since, $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ is an NI-C-mapping, so $\zeta^{-1}(Q)$ is an NIOS in $(M, \Im_2)$. Further, since $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NIS-C-mapping, so $\xi^{-1}(\zeta^{-1}(Q))=(\zeta\circ\xi)^{-1}(Q)$ is an NISO set in (Ψ, τ_1, τ_2). Hence, $(\zeta\circ\xi)^{-1}(Q)$ is an NISO set in $(W, \Im_1)$ whenever Q is an NIOS in $(V, \Im_3)$. Therefore, the composition mapping $\zeta\circ\xi:(W, \Im_1)\rightarrow(V, \Im_3)$ is an NIS-C-mapping.

Theorem 3.14. Let $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ be an NIP-C-mapping, and $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ be an NI-C-mapping. Then, the composition mapping $\zeta\circ\xi:(W, \Im_1)\rightarrow(V, \Im_3)$ is an NIP-C-mapping.

Proof. Let $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ be an NIP-C-mapping, and $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ be an NI-C-mapping. Assume that Q is an NIOS in $(V, \Im_3)$. Since, $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ is an NI-C-mapping, so $\zeta^{-1}(Q)$ is an NIOS in $(M, \Im_2)$. Further, since $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NIP-C-mapping, so $\xi^{-1}(\zeta^{-1}(Q))=(\zeta\circ\xi)^{-1}(Q)$ is an NIPO set in (Ψ, τ_1, τ_2). Hence, $(\zeta\circ\xi)^{-1}(Q)$ is an NIPO set in $(W, \Im_1)$ whenever Q is an NIOS in $(V, \Im_3)$. Therefore, the composition mapping $\zeta\circ\xi:(W, \Im_1)\rightarrow(V, \Im_3)$ is an NIP-C-mapping.

Theorem 3.15. Let $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ be an NI-b-C-mapping, and $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ be an NI-C-mapping. Then, the composition mapping $\zeta\circ\xi:(W, \Im_1)\rightarrow(V, \Im_3)$ is an NI-b-C-mapping.

Proof. Let $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ be an NI-b-C-mapping, and $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ be an NI-C-mapping. Assume that Q is an NIOS in $(V, \Im_3)$. Since, $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ is an NI-C-mapping, so $\zeta^{-1}(Q)$ is an NIOS in $(M, \Im_2)$. Further, since $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NI-b-C-mapping, so $\xi^{-1}(\zeta^{-1}(Q))=(\zeta\circ\xi)^{-1}(Q)$ is an NI-b-O set in (Ψ, τ_1, τ_2). Hence, $(\zeta\circ\xi)^{-1}(Q)$ is an NI-b-O set in $(W, \Im_1)$ whenever Q is an NIOS in $(V, \Im_3)$. Therefore, the composition mapping $\zeta\circ\xi:(W, \Im_1)\rightarrow(V, \Im_3)$ is an NI-b-C-mapping.

Theorem 3.16. Let $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ be an NI-α-C-mapping, and $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ be an NI-C-mapping. Then, the composition mapping $\zeta\circ\xi:(W, \Im_1)\rightarrow(V, \Im_3)$ is an NI-α-C-mapping.

Proof. Let $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ be an NI-α-C-mapping, and $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ be an NI-C-mapping. Assume that Q is an NIOS in $(V, \Im_3)$. Since, $\zeta:(M, \Im_2)\rightarrow(V, \Im_3)$ is an NI-C-mapping, so $\zeta^{-1}(Q)$ is an NIOS in $(M, \Im_2)$. Further, since $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NI-α-C-mapping, so $\xi^{-1}(\zeta^{-1}(Q))=(\zeta\circ\xi)^{-1}(Q)$ is an NI-α-O set in (Ψ, τ_1, τ_2). Hence, $(\zeta\circ\xi)^{-1}(Q)$ is an NI-α-O set in $(W, \Im_1)$ whenever Q is an NIOS in $(V, \Im_3)$. Therefore, the composition mapping $\zeta\circ\xi:(W, \Im_1)\rightarrow(V, \Im_3)$ is an NI-α-C-mapping.

Theorem 3.17.
(i) Every NI-C-mapping is an NIP-C-mapping;
(ii) Every NI-C-mapping is an NIS-C-mapping;
(iii) Every NIP-C-mapping is an NI-b-C-mapping;
(iv Every NIS-C-mapping is an NI-b-C-mapping;
(v) Every NI-C-mapping is an NI-b-C-mapping.

Proof. (i) Let $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ be an NI-C-mapping, and L be an NIOS in M. Since ξ is an NI-C-mapping, so $\xi^{-1}(L)$ is an NIOS in W. Further, since every NIOS is again an NIPO set, so $\xi^{-1}(L)$ is an NIPO set in $(W, \Im_1)$. Therefore, $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NIP-C-mapping.

DOI: https://doi.org/10.54216/IJNS.180204

Received: November 15, 2021 Accepted: March 09, 2022

(ii) Suppose that $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NI-C-mapping, and L be an NIOS in M. Since ξ is an NI-C-mapping, so $\xi^{-1}(L)$ is an NIOS in W. Further, since every NIOS is again an NISO set, so $\xi^{-1}(L)$ is an NISO set in $(W, \Im_1)$. Therefore, $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NIS-C-mapping.

(iii) Suppose that $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NIP-C-mapping, and L is an NIOS in M. Since ξ is an NIP-C-mapping, so $\xi^{-1}(L)$ is an NIPO set in W. Further, since every NIPO set is an NI-b-O set, so $\xi^{-1}(L)$ is an NI-b-O set in W. Therefore, $\xi^{-1}(L)$ is an NI-b-O set in W whenever L is an NIOS in M. Hence, $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NI-b-C-mapping.

(iv) Suppose that $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NIS-C-mapping, and L is an NIOS in M. Since ξ is an NIS-C-mapping, so $\xi^{-1}(L)$ is an NISO set in W. Further, since every NISO set is an NI-b-O set, so $\xi^{-1}(L)$ is an NI-b-O set in W. Therefore, $\xi^{-1}(L)$ is an NI-b-O set in W whenever L is an NIOS in M. Hence, $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NI-b-C-mapping.

(v) Suppose that $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NI-C-mapping, and L is an NIOS in M. Since ξ is an NI-C-mapping, so $\xi^{-1}(L)$ is an NIOS in W. Further, since every NIOS is again an NI-b-O set, so $\xi^{-1}(L)$ is an NI-b-O set in $(W, \Im_1)$. Therefore, $\xi:(W, \Im_1)\rightarrow (M, \Im_2)$ is an NI-b-C-mapping.

5. Conclusions

In this article, we introduce the notion of neutrosophic infi-topological space, and study different types of open sets, namely, NIOS, NIPO set, NISO set, NI-b-O set and NI-α-O set. By defining NIOS, NIPO set, NISO set, NI-b-O set and NI-α-O set, we formulate some interesting results on NITSs in the form of theorems, propositions, etc. It is hoped that, in the future, based on these notions and various open sets on NITS, many new investigations can be easily done.

Funding: "This research received no external funding."

Conflicts of Interest: "The authors declare that they have no conflict of interest."

References

[1] I. Arokiarani, R. Dhavaseelan, S. Jafari, & M. Parimala, On some new notations and functions in neutrosophic topological spaces. *Neutrosophic Sets and Systems*, 2017, v. 16, 16-19.

[2] K. Atanassov, Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 1986, v. 20, 87-96.

[3] C.L. Chang, Fuzzy topological spaces. *Journal of Mathematical Analysis and Applications*, 1968, v. 24, no. 1, 182-190.

[4] A. Csaszar, Generalized topology, generalized continuity. *Acta Math. Hungar.*, 2002, v. 96, 351-357.

[5] A. Csaszar, Separation axioms for generalized topologies. *Acta Math. Hungar.*, 2004, v. 104, 63-69.

[6] A. Csaszar, Generalized open sets in generalized topologies. *Acta Math. Hungar.*, 2005, v. 106, no. (1-2), 53-66.

[7] A. Csaszar, Products of generalized topologies. *Acta Math. Hungar.*, 2009, v. 123, 127-132.

[8] B. Das, B. Bhattacharya, & A.K. Saha, Some remarks on fuzzy infi topological spaces. *Proyecciones Journal of Mathematics*, 2021, v. 40, no. 2, 399-415.

[9] S. Das, Neutrosophic supra simply open set and neutrosophic supra simply compact space. *Neutrosophic Sets and Systems*, 2021, v. 43, 105-113.

[10] S. Das, R. Das, C. Granados, & A. Mukherjee, Pentapartitioned neutrosophic Q-ideals of Q-algebra. *Neutrosophic Sets and Systems*, 2021, v. 41, 52-63.

[11] S. Das, R. Das, & B.C. Tripathy, Multi-criteria group decision making model using single-valued neutrosophic set. *LogForum*, 2020, v. 16, no. 3, 421-429.

[12] S. Das, & S. Pramanik, Generalized neutrosophic b-open sets in neutrosophic topological space. *Neutrosophic Sets and Systems*, 2020, v. 35, 522-530.

Received: November 15, 2021 Accepted: March 09, 2022

[13] S. Das, & S. Pramanik, Neutrosophic Φ-open sets and neutrosophic Φ-continuous functions. *Neutrosophic Sets and Systems*, **2020, v.** 38, 355-367.

[14] S. Das, & S. Pramanik, Neutrosophic simply soft open set in neutrosophic soft topological space. *Neutrosophic Sets and Systems*, 2020, v. 38, 235-243.

[15] S. Das, & S. Pramanik, Neutrosophic Tri-Topological Space. *Neutrosophic Sets and Systems*, 2021, v. 45, 366-377.

[16] B. Das, A.K. Saha, & B. Bhattacharya, On infi-topoogical spaces. *The Journal of Fuzzy Mathematics*, 2017, v. 25, no. 2, 437-448.

[17] S. Das, B. Shil, & S. Pramanik, SVPNS-MADM strategy based on GRA in SVPNS Environment. *Neutrosophic Sets and Systems*, 2021, v. 47, 50-65.

[18] S. Das, B. Shil, & B.C. Tripathy, Tangent similarity measure based MADM-strategy under SVPNS-environment. *Neutrosophic Sets and Systems*, 2021, v. 43, 93-104.

[19] R. Das, & B.C. Tripathy, Neutrosophic multiset topological space. *Neutrosophic Sets and Systems*, 2020, v. 35, 142-152.

[20] S. Das, & B.C. Tripathy, Pairwise neutrosophic-b-open set in neutrosophic bitopological spaces. *Neutrosophic Sets and Systems*, 2020, v. 38, 135-144.

[21] S. Das, & B.C. Tripathy, Neutrosophic simply *b*-open set in neutrosophic topological spaces. *Iraqi Journal of Science*, 2021, v. 62, no. 12, 4830-4838.

[22] S. Das, & B.C. Tripathy, Pentapartitioned neutrosophic topological space. *Neutrosophic Sets and Systems*, 2021, v. 45, 121-132.

[23] E. Ebenanjar, J. Immaculate, & C.B. Wilfred, On neutrosophic *b*-open sets in neutrosophic topological space. *Journal of Physics Conference Series*, 2018, v. 1139, no. 1, 012062.

[24] T.E. Gantner, R.C. Steinlage, & R.H. Warren, Compactness in fuzzy topological spaces. *Journal of Mathematical Analysis and Applications*, 1978, v. 62, no. 3, 547-562.

[25] B. Hutton, Normality in fuzzy topological spaces. *Journal of Mathematical Analysis and Applications*, 1975, v. 50, 74-79.

[26] P. Iswarya, & K. Bageerathi, On neutrosophic semi-open sets in neutrosophic topological spaces. *International Journal of Mathematical Trends and Technology*, 2016, v. 37, no. 3, 214-223.

[27] J.C. Kelly, General topology. *Van Nostrand*, *Princeton*, *New Jersy*, 1955.

[28] N. Levine, Semi-open sets and semi-continuity in topological spaces. *Am. Math. Monthly*, 1963, v. 70, 36-41.

[29] P. Majumder, S. Das, R. Das, & B.C. Tripathy, identification of the most significant risk factor of COVID-19 in economy using cosine similarity measure under SVPNS-environment. *Neutrosophic Sets and Systems*, 2021, v. 46, 112-127.

[30] C. Maheswari, M. Sathyabama, & S. Chandrasekar, Neutrosophic generalized *b*-closed sets in neutrosophic topological spaces. *Journal of Physics Conference Series*, 2018, v. 1139, no. 1, 012065.

[31] A.S. Masshour, A.A. Allam, F.S. Mahmud, & F.H. Khedr, On supra topological spaces. *Indian J. Pure appl. Math.*, 1983, v. 14, no. 4, 502-510.

[32] I. Mohammed Ali Jaffer, & K. Ramesh, Neutrosophic generalized pre regular closed sets. *Neutrosophic Sets and Systems*, 2019, v. 30, 171-181.

[33] A. Mukherjee, & R. Das, Neutrosophic bipolar vague soft set and its application to decision making problems. *Neutrosophic Sets and Systems*, 2020, v. 32, 410-424.

[34] S. Pramanik, A critical review of Vivekanada's educational thoughts for women education based on neutrosophic logic. MS Academic, 2013, v.3, no. 1, 191-198.

[35] S. Pramanik, P.P. Dey, B.C. Giri, B. C., & F. Smarandache, An extended TOPSIS for multi-attribute decision making problems with neutrosophic cubic information. Neutrosophic Sets and Systems, 2017, v.17, 20-28.

[36] S. Pramanik, S. Dalapati, S., Alam, F.., Smarandache, S., & Roy, T.K. (2018). NS-cross entropy based MAGDM under single valued neutrosophic set environment. Information, 2018, v. 9, no.2, 37.

[37] S. Pramanik, R. Mallick, & A. Dasgupta Contributions of selected Indian researchers to multi-attribute decision making in neutrosophic environment. Neutrosophic Sets and Systems, 2018, v.20, 108-131.

[38] S. Pramanik, & R. Mallick, VIKOR based MAGDM strategy with trapezoidal neutrosophic numbers. Neutrosophic Sets and Systems, 2018, v. 22, 118-130.

Received: November 15, 2021 Accepted: March 09, 2022

[39] S. Pramanik, & R. Mallick, TODIM strategy for multi-attribute group decision making in trapezoidal neutrosophic number environment. Complex & Intelligent Systems, 2019, v. 5, n.4, 379–389.

[40] S. Pramanik, Rough neutrosophic set: an overview. In F. Smarandache, & S. Broumi, Eds.), Neutrosophic theories in communication, management and information technology (pp.275-311). New York. Nova Science Publishers, 2020.

[41] S. Pramanik, & R. Mallick, MULTIMOORA strategy for solving multi-attribute group decision making (MAGDM) in trapezoidal neutrosophic number environment. CAAI Transactions on Intelligence Technology, 2020, v. 5, no. 3, 150-156.

[42] A. Pushpalatha, & T. Nandhini, Generalized closed sets via neutrosophic topological spaces. *Malaya Journal of Matematik*, 2019, v. 7, no. 1, 50-54.

[43] V.V. Rao, & R. Srinivasa, Neutrosophic pre-open sets and pre-closed sets in neutrosophic topology. *International Journal of ChemTech Research*, 2017, v. 10, no. 10, 449-458.

[44] A.K. Saha, & D. Bhattacharya, Countable fuzzy topological space and Countable fuzzy topological vector space. *J. Math. Fund. Sci.*, 2015, v. 47, no. 2, 154-166.

[45] A.A. Salama, & S.A. Alblowi, Neutrosophic set and neutrosophic topological space. *ISOR Journal of Mathematics*, 2012, v. 3, no. 4, 31-35.

[46] F. Smarandache, A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. *Rehoboth: American Research Press*, 1998.

[47] B.C. Tripathy, & S. Das, Pairwise neutrosophic *b*-continuous function in neutrosophic bitopological spaces. *Neutrosophic Sets and Systems*, 2021, v. 43, 82-92.

[48] H. Wang, F. Smarandache, Y.Q. Zhang, & R. Sunderraman, Single valued neutrosophic sets. *Multispace and Multistructure*, 2010, v. 4, 410-413.

[49] L.A. Zadeh, Fuzzy sets. *Information and Control*, 1965, v. 8, no. 3, 338-353.

DOI: https://doi.org/10.54216/IJNS.180204

Received: November 15, 2021 Accepted: March 09, 2022