which is the truth membership of a NPD.

Using Eqn (2) indeterminacy of NPD can be determined as follows:

$$Ind_x = \lambda(I)^z \sum_{k=0}^{th} \frac{n(n-1)(n-2)..........(n-k-z-1)}{x!k!} \left(\frac{\lambda(S)}{n}\right)^k \left(1-\frac{\lambda(F)}{n}\right)^{n-z-k}$$

$$\text{Since } \left[n \to \infty \;\&\; \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^{n-z-k} = e^{-\lambda} \right]$$

$$Ind_x = \sum_{z=th+1}^{n} \lambda(I)^z \sum_{k=0}^{n-z} \frac{e^{-\lambda(F)k} \lambda(S)^k}{k!} \quad ....(5)$$

The falsehood membership function of NPD can be defined as follows using Eqn (3).

$$Fal_x = \lambda(S)^y \sum_{k=0}^{th} n^k \frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)................\left(1-\frac{(k+y-1)}{n}\right)\right]}{x!k!} \left(\frac{\lambda(F)}{n}\right)^k \left(1-\frac{\lambda(I)}{n}\right)^{n-y-k}$$

$$\text{Since } \left[n \to \infty \;\&\; \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^{n-y-k} = e^{-\lambda} \right]$$

$$Fal_x = \sum_{\substack{y=0 \\ y\neq x}}^{n} \lambda(S)^y \sum_{k=0}^{th} \frac{e^{-\lambda(I)k} \lambda(F)^k}{k!} \quad ....(6)$$

Therefore the neutrosophic poisson distribution can be determined as

$$Tr_x = \lambda(S)^x \sum_{k=0}^{th} \frac{e^{-\lambda(I)k} \lambda(F)^k}{k!}, \quad I_x = \sum_{z=th+1}^{n} \lambda(I)^z \sum_{k=0}^{n-z} \frac{e^{-\lambda(F)k} \lambda(S)^k}{k!}, \quad F_x = \sum_{\substack{y=0 \\ y\neq x}}^{n} \lambda(S)^y \sum_{k=0}^{th} \frac{e^{-\lambda(I)k} \lambda(F)^k}{k!}$$

4. Application of the neutrosophic poisson distribution:

In this section, we used [23] to apply the proposed notion to a real-world problem.

The atoms of a radioactive element disintegrate at random. Every gramme of this element emits between [0.4-0.9] alpha particles per second on average. When the indeterminacy threshold is reached during the next second, the probability of the number of alpha particles emitted from one gramme is exactly two, as shown below using the proposed NPD.

Solution:

Given data: $\begin{aligned}\lambda(S)&=0.4\\ \lambda(I)&=0.6\ ;\\ \lambda(F)&=0.9\end{aligned}$

$$x=2;k=2$$

From Equ(4) the formula for Neutrosophic truth membership function is given by

$$Tr_{x}=\lambda(S)^{x}\sum_{k=0}^{th}\frac{e^{-\lambda(I)k}\lambda(F)^{k}}{k!}$$

$$=(0.4)^{2}\sum_{k=0}^{2}\frac{e^{-0.6k}(0.9)^{k}}{k!}$$

$$Tr_{x}=0.75$$

From Equ(5) the formula for Neutrosophic truth membership function is given by

$$Fal_{x}=\sum_{\substack{y=0\\ y\neq x}}^{n}\lambda(S)^{y}\sum_{k=0}^{th}\frac{e^{-\lambda(I)k}\lambda(F)^{k}}{k!}$$

$$=(0.4)^{2}\sum_{k=0}^{2}\frac{e^{-0.6k}(0.9)^{k}}{k!}$$

$$Fal_{x}=0.52$$

From Equ(4) the formula for Neutrosophic truth membership function is given by

$$Ind_{x}=\sum_{z=th+1}^{n}\lambda(I)^{z}\sum_{k=0}^{n-z}\frac{e^{-\lambda(F)k}\lambda(S)^{k}}{k!}$$

$$=\sum_{z=th+1}^{n}(0.6)^{2}\sum_{k=0}^{z}\frac{e^{-0.9k}(0.4)^{k}}{k!}$$

$$Ind_{x}=0.25$$

$$(Tr_{x},Fal_{x},Ind_{x})=(0.75,0.52,0.25)\ \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Therefore from Eqn.(7) we applied all the values in [11] we get,

$$S(b) = \frac{1+0.75-0.52}{2}$$

$$S(b) = 0.615$$

As a result, the probability that 1 gm will produce 0.615 alpha particles in the following second is 0.615.

5. The Neutrosophic Exponential Distribution (NED):

In this section, we proposed NED's density function for Truth membership function, Indeteminacy and also for Falsity. Also proposed mean and variance of NED, and their Distribution function.

Definition:5.1

The Neutrosophic Exponential Distribution (NED) is a generalisation of the traditional exponential distribution. It can process any type of data, including non-specific data. The density function of NED is written as follows:

Table 1:

Density function for Neutrosophic Exponential Distribution(NED)		
Truth Membership	Indeteminacy	Falsity
$f_{Neu}\left(T_x(x)\right) = \lambda_{Neu}\left(T_x(x)\right)e^{-\left(\lambda_{Neu}\left(T_x(x)\right)\right)x}$	$f_{Neu}\left(I_x(x)\right) = \lambda_{Neu}\left(I_x(x)\right)e^{-\left(\lambda_{Neu}\left(I_x(x)\right)\right)x}$	$f_{Neu}\left(F_x(x)\right) = \lambda_{Neu}\left(F_x(x)\right)e^{-\left(\lambda_{Neu}\left(F_x(x)\right)\right)x}$

Where 'x' is a Neutrosophic random variable

$\lambda_{Neu}\left(T_x(x)\right)$ - Truth membership function's distribution parameter.

$\lambda_{Neu}\left(I_x(x)\right)$ - Indeterminacy membership function's distribution parameter.

$\lambda_{Neu}\left(F_x(x)\right)$ - Falsity membership function's distribution parameter.

Properties of NED:

Table 2:

	Truth Membership Function	Indeterminacy	Falsity
Expected Value for NED: $E(x)$	$E(T_x) = \frac{1}{\lambda_{Neu}\left(T_x(x)\right)}$	$E(I_x) = \frac{1}{\lambda_{Neu}\left(I_x(x)\right)}$	$E(F_x) = \frac{1}{\lambda_{Neu}\left(F_x(x)\right)}$
Variance for NED: $Var(x)$	$Var(T_x) = \frac{1}{\left(\lambda_{Neu}\left(T_x(x)\right)\right)^2}$	$Var(I_x) = \frac{1}{\left(\lambda_{Neu}\left(I_x(x)\right)\right)^2}$	$Var(F_x) = \frac{1}{\left(\lambda_{Neu}\left(F_x(x)\right)\right)^2}$

Table 3:

	Truth Membership Function	Indeterminacy	Falsity
Distribution Function for NED: $N\left(F\left(T_x, I_x, F_x\right)\right) = NP\left(X \le x\right)$	$\left(1-e^{-\left(\lambda_{Neu}\left(T_x(x)\right)\right)x}\right)$	$\left(1-e^{-\left(\lambda_{Neu}\left(I_x(x)\right)\right)x}\right)$	$\left(1-e^{-\left(\lambda_{Neu}\left(F_x(x)\right)\right)x}\right)$

6. Application of the neutrosophic exponential distribution:

We used the proposed notion in a case study problem from [21] with different intervals in this part.

Consider what amount of time it requires for a bank to end a client assistance's, which follows a dramatic dispersion with a normal of one moment. Compose a thickness work, mean, change, and dissemination work for the time it takes to end a customer's administration, and afterward propose the likelihood of ending a customer's administration in the stretch [1,2] minute.

Assume that x: indicates the time necessary per minute to terminate the client's service.

The average $\frac{1}{\lambda\left(T_x(x)\right)} = 1$; $\frac{1}{\lambda\left(I_x(x)\right)} = \frac{1}{1.5} = 0.6667$; $\frac{1}{\lambda\left(F_x(x)\right)} = \frac{1}{2} = 0.5$

The probability density function NED:

From table 1:

$$f_{Neu}\left(T_x(x)\right) = \lambda_{Neu}\left(T_x(x)\right)e^{-\left(\lambda_{Neu}\left(T_x(x)\right)\right)x} = 0.3679$$

$$f_{Neu}\left(I_x(x)\right) = \lambda_{Neu}\left(I_x(x)\right)e^{-\left(\lambda_{Neu}\left(I_x(x)\right)\right)x} = 0.3347$$

$$f_{Neu}\left(F_x(x)\right) = \lambda_{Neu}\left(F_x(x)\right)e^{-\left(\lambda_{Neu}\left(F_x(x)\right)\right)x} = 0.2707$$

From Table 2:

$$E\left(T_x\right) = \frac{1}{\lambda_{Neu}\left(T_x(x)\right)} = 1; Var\left(T_x\right) = \frac{1}{\left(\lambda_{Neu}\left(T_x(x)\right)\right)^2} = 1$$

$$E\left(I_x\right) = \frac{1}{\lambda_{Neu}\left(I_x(x)\right)} = 0.6667\,; Var\left(I_x\right) = \frac{1}{\left(\lambda_{Neu}\left(I_x(x)\right)\right)^2} = 0.4444$$

$$E\left(F_x\right) = \frac{1}{\lambda_{Neu}\left(F_x(x)\right)} = 0.5\,; Var\left(F_x\right) = \frac{1}{\left(\lambda_{Neu}\left(F_x(x)\right)\right)^2} = 0.2500$$

There's a chance that the client's service will be terminated in less than 1,0.67,0.5 minute. :

From Table 3

$$Neu\left(F\left(T_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=Neu\left(P\left(X\le 1\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(T_x(x)\right)\right)x}\right)=0.63$$

$$Neu\left(F\left(I_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=Neu\left(P\left(X\le 1\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(I_x(x)\right)\right)x}\right)=0.4883$$

$$Neu\left(F\left(F_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=Neu\left(P\left(X\le 1\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(F_x(x)\right)\right)x}\right)=0.39$$

Probability that the client's service will be terminated in less than a minute:

$$Neu\left(F\left(T_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(T_x(x)\right)\right)x}\right)=\left(1-e^{-[0.5,1.5]1}\right)$$

That is, the likelihood of terminating a client's service in less than a minute ranges between 0.085 and 0.085 when we use the numbers in [11].

Probability that the client's service will be terminated in less than 1.5 minutes:

$$Neu\left(F\left(I_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(I_x(x)\right)\right)x}\right)=\left(1-e^{-[0.5,1.5]1.5}\right)$$

When we utilise the numbers in [11], the probability of terminating a client's service in less than 1.5 minutes ranges between 0.2111 and 0.2111.

Probability of terminating a client's service in under 2 minutes:

$$Neu\left(F\left(F_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(F_x(x)\right)\right)x}\right)=\left(1-e^{-[0.5,1.5]2}\right)$$

When we apply the figures in [11], the probability of terminating a client's service in less than 2 minutes ranges between 0.435 and 0.435.

7. Neutrosophic Uniform Distribution (NUD):

In this section, we proposed NUD's density function for Truth membership function, Indeteminacy and also for Falsity. Also proposed mean and variance of NUD.

Definition:7.1

The numeric value of a continuous variable is its numeric value. Although X is a conventional Uniform distribution, the distribution parameters an or b, or both, are untrustworthy. For example, 'a' or 'b,' or both,' are sets of two or more components having a< b, and we can use NUD to stand for Truth Membership, Indeterminacy, and Falsity. The following is a NUD definition that has been offered.: $f\left(T_x\right)=\begin{cases}K, & a\left(T_x\right)<b\left(T_x\right)\\ 0 & otherwise\end{cases}$

Since the total probability always unity

$$\int_{a(T_x)}^{b(T_x)} f(T_x)dx = 1$$

$$\Rightarrow \int_{a(T_x)}^{b(T_x)} K dx = 1$$

$$[K]_{a(T_x)}^{b(T_x)} = 1$$

$$K = \frac{1}{b(T_x) - a(T_x)}$$

Density function of NUD for Truth Membership function is given by

$$f_{Neu}(T_x) = \frac{1}{b(T_x) - a(T_x)} \text{ for } a(T_x) < b(T_x) \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Similiarly we propose NUD for Indeterminacy

$$f_{Neu}(I_x) = \frac{1}{b(I_x) - a(I_x)} \text{ for } a(I_x) < b(I_x) \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Also we propose NUD for Falsity function

$$f_{Neu}(F_x) = \frac{1}{b(F_x) - a(F_x)} \text{ for } a(F_x) < b(F_x) \ldots\ldots\ldots\ldots(10)$$

Mean of a NUD:

Mean for NUD for Truth Membership function is given by

$$E(T_x) = \int_{a(T_x)}^{b(T_x)} f(T_x)dx = \frac{b(T_x) - a(T_x)}{2} \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Mean for NUD for Indeterminacy is given by

$$E(I_x) = \int_{a(T_x)}^{b(T_x)} f(I_x)dx = \frac{b(I_x) - a(I_x)}{2} \ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Mean for NUD for Falsity is given by

$$E(F_x) = \int_{a(T_x)}^{b(T_x)} f(F_x)dx = \frac{b(F_x) - a(F_x)}{2} \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Variance of NUD:

Variance of NUD for Truth Membership function is given by

$$Var(T_x) = \frac{\left(b(T_x) - a(T_x)\right)^2}{12} \text{ ....(14)}$$

Variance of NUD for Indeterminacy is given by

$$Var(I_x) = \frac{\left(b(I_x) - a(I_x)\right)^2}{12} \text{ ....(15)}$$

Variance of NUD for Falsity function is given by

$$Var(F_x) = \frac{\left(b(F_x) - a(F_x)\right)^2}{12} \text{ ....(16)}$$

8. Application of the neutrosophic Uniform distribution:

We used the proposed notion in a case study problem from [21] with different intervals in this part.

The station official explained that assuming 'x; is a variable that denotes a person's waiting time for a passenger's bus (in minutes), the bus arrival time is not mentioned.:

1-the bus arrival time is: either now or in 5 minutes [0,5] or in 15-20 minutes [15,20], then

2- the bus arrival time is: either now or in 5 minutes [0,5], or in 20-25 minutes [20,25], then

3- the bus will arrive in either 5 minutes [0,5] or 25-30 minutes [25,30], depending on when you arrive.

Here $a(T_x) = a(I_x) = a(F_x) = [0,5]$

$b(T_x) = [15,20]$;
$b(I_x) = [20,25]$;
$b(F_x) = [25,30]$;

Then the density

Density function of NUD for Truth Membership function is given by

From Eqn(8)⇒ $f_{Neu}(T_x) = \frac{1}{b(T_x) - a(T_x)} = \frac{1}{[15,20]-[0,5]} = \frac{1}{[10,15]} = [0.067, 0.1]$

Density function of NUD for Indeterminacy is given by

From Eqn(9)⇒ $f_{Neu}(I_x) = \frac{1}{b(I_x) - a(I_x)} = \frac{1}{[20,25]-[0,5]} = \frac{1}{[15,20]} = \frac{[0.05, 0.067]}{2} = [0.025, 0.033]$

Density function of NUD for Falsity function is given by

From Eqn(10)⇒ $f_{Neu}(F_x) = \frac{1}{b(F_x) - a(F_x)} = \frac{1}{[25,30]-[0,5]} = \frac{1}{[20,25]} = [0.04, 0.05]$

Mean of the bus arrival time 15-20 minutes is given by

$$E(T_x)=\frac{b(T_x)-a(T_x)}{2}=\frac{[15,20]-[0,5]}{2}=\frac{[10,15]}{2}=\frac{[0.067,0.1]}{2}=[0.0335,0.05]$$

Mean of the bus arrival time 20-25 minutes is provided by

$$E(I_x)=\frac{b(I_x)-a(I_x)}{2}=\frac{[20,25]-[0,5]}{2}=\frac{[15,20]}{2}=\frac{[0.05,0.067]}{2}=[0.025,0.0335]$$

Mean of the bus arrival time 25-30 minutes is provided by

$$E(F_x)=\frac{b(F_x)-a(F_x)}{2}=\frac{[25,30]-[0,5]}{2}=\frac{[20,25]}{2}=\frac{[0.04,0.05]}{2}=[0.02,0.025]$$

Variance of the bus arrival time 15-20 minutes is given by

From Eqn(14)⇒ $Var(T_x)=\frac{(b(T_x)-a(T_x))^2}{12}=\frac{[0.0335,0.5]^2}{12}=[0.000093,0.0208]$

Variance of the bus arrival time 20-25 minutes is given by

From Eqn(15)⇒ $Var(I_x)=\frac{(b(I_x)-a(I_x))^2}{12}=\frac{[0.05,0.067]^2}{12}=[0.000208,0.000374]$

Variance of the bus arrival time 25-30 minutes is given by

From Eqn(16)⇒ $Var(F_x)=\frac{(b(F_x)-a(F_x))^2}{12}=\frac{[0.04,0.05]^2}{12}=[0.000133,0.000208]$

9. Conclusion

Classical probability solely considers determinate data, but neutrosophic probability considers indeterminate data with varying degrees of indeterminacy. Hence in this paper, we proposed many of the standard distribution called Poisson distribution as a limiting case of Binomial distribution,NED,NUD under neutrosophic environment. Also, using the proposed concept probability value has been obtained for a real world problem. In future, probability distributions may be proposed under different neutrosophic environment.

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