MBJ-neutrosophic T-ideal on B-algebra

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Abstract

In this paper we define and study the MBJ-neutrosophic T-ideal through diferent concept like union, intersection. further we use the important properties to investigate the MBJ-neutrosophic T-ideal under cartesian product and homomorphic results.

Keywords: B-algebra, MBJ-neutrosophic set, MBJ-neutrosophic T-ideal, Cartesian product, Homomorphism.

1.Introduction

[1] presented the idea of fuzzy set. [2] gave the idea of BCK-algebra. [3] introduced the BCI-algebra and it is obvius that BCK-algebra is a proper sub class of BCI-algebra. [4] wrote on intuitionistic fuzzy G-algebra. [5] introduced the intuitionistic fuzzy sets. [6] provided the idea of interval-valued intuitionistic fuzzy sets. [7] studied the neutrosophic soft subalgebra and investigted many results. [8] initiated the intuitionistic fuzzy structure of B-algebra. [9] discussed the fuzzy dot subalgebra and fuzzy dot ideal of B-algebras. [10] introduced the B-algebra in 2002. [11,12] worked on t-intuitionistic fuzzy sets in fuzzy subgroups and subrings. Barbhuiya [13] studied the t-intuitionistic fuzzy BG-subalgebra. [14] studied ideals with its level subsets. [15] introduced the T-neutrosophic cubic set and deeply investigated this set with significant characterisitics. [16] defined and studied the MBJ-neutrosophic set through subalgebra and S-extension. [17,18] The concept of Neutrosophic set is given by Smarandache with truth, indeterminate and false membership function.

This paper is presented to studied the idea of MBJ-neutrosophic set throught the concept of T-ideal, homomorphic characteristics and cartesian product.

2. Preliminaries

First we cite some definitions which are used to present this paper. [3] An algebra $(Y,*,0)$ of type (2,0) is called a BCI-algebra if it satisfies the following conditions:

i) $(t_1 * t_2) * (t_1 * t_3) \leq (t_3 * t_2)$,

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ii) $t_1 * (t_1 * t_2) \leq t_2$,

iii) $t_1 \leq t_1$,

iv) $t_1 \leq t_2$ and $t_2 \leq t_1 \Rightarrow t_1 = t_2$,

v) $t_1 \leq 0 \Rightarrow t_1 = 0$, where $t_1 \leq t_2$ is defined by $t_1 * t_2 = 0$, for all $t_1, t_2, t_3 \in Y$.

[1] An algebra $(Y,* ,0)$ of type (2,0) is called a BCK-algebra if it satisfies the following conditions:

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iv) $t_1 \leq t_2$ and $t_2 \leq t_1 \Rightarrow t_1 = t_2$,

v) $0 \leq t_1 \Rightarrow t_1 = 0$, where $t_1 \leq t_2$ is defined by $t_1 * t_2 = 0$, for all $t_1, t_2, t_3 \in Y$.

[10] A nonempty set Y with a constant 0 and having binary operation $*$ is called B-algebra if it satisfies the following conditions:

i) $t_1 * t_1 = 0$

ii) $t_1 * 0 = x$

iii) $(t_1 * t_2) * t_3 = (t_1 * (t_3 * (0 * t_2))$ for all $t_1, t_2 \in Y$.

[10] Let K be a nonempty subset of B-algebra Y, then K is called a subalgebra of Y if $t_1 * t_2 \in K$, for all $t_1, t_2 \in K$.

[14] Let Y be a PS-algebra. A fuzzy set B of Y is called a fuzzy PS ideal of Y if it satisfies the following conditions:

i) $\vartheta(0) \geq \vartheta(t_1)$,

ii) $\vartheta(t_1) \geq min\{\vartheta(t_2 * t_1), \vartheta(t_2)\}$, for all $t_1, t_2 \in Y$.

Let Y be a group of objects denoted generally by t_1. Then a fuzzy set B of Y is defined as $B = \{< t_1, \vartheta_B(t_1) > |t_1 \in Y\}$, where $\vartheta_B(t_1)$ is called the existence ship value of t_1 in B and $\vartheta_B(t_1) \in [0,1]$.

A fuzzy set B [14] of PS-algebra Y is called a fuzzy PS subalgebra of Y if $\vartheta(t_1 * t_2) \geq min\{\vartheta(t_1), \vartheta(t_2)\}$, for all $t_1, t_2 \in Y$.

An intuitionistic fuzzy set **(IFS)** [5] B over Y is an object having the form $B = \{\langle t_1, \vartheta_B(t_1), \phi_B(t_1)\rangle | t_1 \in Y\}$, where $\vartheta_B(t_1)|Y \rightarrow [0,1]$ and $\phi_B(t_1)|Y \rightarrow [0,1]$, with the condition $0 \leq \vartheta_B(t_1) + \phi_B(t_1) \leq 1$, for all $t_1 \in Y$. $\vartheta_B(t_1)$ and $\phi_B(t_1)$ represent the degree of existence and the degree of non-existence of the element t_1 in the set B respectively.

An **IFS** $B = \{\langle t_1, \vartheta_B(t_1), \phi_B(t_1)\rangle | t_1 \in Y\}$ of Y is said to be an **IFID** of Y if it satisfies these three conditions:

(i) $\vartheta_B(0) \geq \vartheta_B(t_1), \phi_B(0) \leq \phi_B(t_1)$,

(ii) $\vartheta_B(t_1) \geq min\{\vartheta_B(t_1 * t_2), \vartheta_B(t_2)\}$,

(iii) $\phi_B(t_1) \leq max\{\phi_B(t_1 * t_2), \phi_B(t_2)\}$, for all $t_1, t_2 \in Y$.

An IVFS [1] B over Y is of the form $B = \{\langle t_1, \hat{\vartheta}_B(t_1)\rangle | t_1 \in Y\}$, where $\hat{\vartheta}_B(t_1)|Y \to D[0.1]$, where D[0,1] is the set of all sub intervals of [0,1]. $\hat{\vartheta}_B(t_1)$ is the interval of the existence value of the element t_1 in B, where $\hat{\vartheta}_B(t_1) = [\hat{\vartheta}_{B\,L}(t_1), \hat{\vartheta}_{B\,U}(t_1)]$ $\forall$ $t_1 \in Y$.

An IVIFS [6] B over Y is an object of the form $B = \{\langle t_1, \hat{\vartheta}_B(t_1), \hat{\phi}_B(t_1)\rangle | t_1 \in Y\}$ where $\hat{\vartheta}_B(t_1)|Y \to [0,1]$ and $\hat{\phi}_B(t_1)|Y \to [0,1]$, D[0,1] is the set of all subintervals of [0,1]. $\hat{\vartheta}_B(t_1)$ and $\hat{\phi}_B(t_1)$ are the intervals of existence value and non-existence value of element t_1 to B, $\hat{\vartheta}_B(t_1) = [\hat{\vartheta}_{B\,L}(t_1), \hat{\vartheta}_{B\,U}(t_1)]$ and $\hat{\phi}_B(t_1) = [\hat{\phi}_{B\,L}(t_1), \hat{\phi}_{B\,U}(t_1)]$, $\forall$ $t_1 \in Y$, with the condition $0 \leq \hat{\vartheta}_{B\,U}(t_1) + \hat{\phi}_{B\,U}(t_1) \leq 1$.

An IVIFS $\hat{B} = (\hat{\vartheta}_B, \hat{\phi}_B)$ in Y is called an IVIF-ideal [9] of Y when it fulfills these axioms.

i) $\hat{\vartheta}_B(0) \geq \hat{\vartheta}_B(t_1)$ and $\hat{\phi}_B(0) \leq \hat{\phi}_B(t_1)$,

ii) $\hat{\vartheta}_B(t_1) \geq rmin\{\hat{\vartheta}_B(t_1 * t_2), \hat{\vartheta}_B(t_2)\}$,

iii) $\hat{\phi}_B(t_1) \leq rmax\{\hat{\phi}_B(t_1 * t_2), \hat{\phi}_B(t_2)\}$.

Let $B = (\vartheta_B, \phi_B)$ be an IFS of Y. Let $t \in [0,1]$, then the IFS B^t is called the t-intuitionistic fuzzy subset [11, 12] of Y w.r.t B and is defined as $B^t = \{< t_1, \vartheta_{B^t}(t_1), \phi_{B^t}(t_1) > | t_1 \in Y\} =< \vartheta_{B^t}, \phi_{B^t} >$, where $\vartheta_{B^t}(t_1) = min\{\vartheta_B(t_1), t\}$ and $\vartheta_{B^t}(t_1) = max\{\phi_B(t_1), 1 - t\}$ for all $t_1 \in Y$.

Let **IFS** $B = \{\langle t_1, \vartheta_B(t_1), \phi_B(t_1)\rangle | t_1 \in Y\}$ of Y is said to be an **IFTID** of Y if it satisfies these three conditions:

(i) $\vartheta_B(0) \geq \vartheta_B(t_1), \phi_B(0) \leq \phi_B(t_1)$,

(ii) $\vartheta_B(t_1 * t_3) \geq min\{\vartheta_B((t_1 * t_2) * t_3), \vartheta_B(t_2)\}$,

(iii) $\phi_B(t_1 * t_3) \leq max\{\phi_B((t_1 * t_2) * t_3), \phi_B(t_2)\}$, for all $t_1, t_2 \in Y$.

An Neutrosophic fuzzy set [17, 18] on Y is defined by $B = \langle B_T, B_I, B_F\rangle | x \in Y$, where $B_T \to [0,1]$ is a truth membership function, $B_I \to [0,1]$, is an indeterminate membership function and $B_F \to [0,1]$ is a false membership function.

Let Y be a non empty set. MBJ-neutrosophic set [16] in Y, is a structure of the form $C = \{\langle M_C t_1, \hat{B}_C t_1, J_C t_1\rangle | t_1 \in Y\}$ where M_C and J_C are fuzzy sets in Y and M_C is a truth membership function, J_C is a false membership function and $\hat{B}$ is interval valued fuzzy set in Y and is an Indeterminate Interval Valued membership function.

3. MBJ-neutrosophic T-ideal of B-algebra

Definition 3.1. Let $C = (M_C, \hat{B}_C, J_C)$ is a MBJ-neutrosphic set of B-algebra Y. Let $t \in [0,1]$ then C is called MBJ-neutrosphic T-ideal (MBJNTID) of Y if it fulfills these assertions:

i) $M_C(0) \geq M_C(t_1)$, $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ and $J_C(0) \leq J_C(t_1)$.

ii) $M_C(t_1 * t_3) \geq min\{M_C((t_1 * t_2) * t_3), M_C(t_2)\}$.

iii) $\hat{B}_C(t_1 * t_3) \geq rmin\{\hat{B}_C((t_1 * t_2) * t_3), \hat{B}_C(t_2)\}$.

iv) $J_C(t_1 * t_3) \leq max\{J_C((t_1 * t_2) * t_3), J_C(t_2)\}$.

Theorem 3.1. Let $C = (M_C, \hat{B}_C, J_C)$ be a MBJNID of a B-algebra Y. If $t_1 * t_2 \leq t_3$, then $M_C(t_1 * x) \geq \min\{M_C(t_3), \hat{B}_C(t_2)\}$, $\hat{B}_C(t_1 * x) \geq rmin\{\hat{B}_C(t_3), \hat{B}_C(t_2)\}$ and $J_C(t_1 * x) \leq \max\{J_C(t_3), J_C(t_2\ )\}$ for all $t_1, t_2, t_3 \in Y$.

Proof. Let $t_1, t_2, t_3 \in Y$ such that $t_1 * t_2 = t_3$, now

$$M_C(t_1 * x) \geq \{min\{M_C((t_1 * t_2) * x), M_C(t_2)\}$$

$$\geq min\{min\{M_C(((t_1 * t_2) * t_3) * x), M_C(t_3)\}, M_C(t_2)\}$$

$$\geq min\{min\{M_C(0), M_C(t_3)\}, M_C(t_2)\}$$

$$\geq min\{M_C(t_3), M_C(t_2)\}$$

$$M_C(t_1 * x) = \min\{M_C(t_3), M_C(t_2)\}$$

and

$$\hat{B}_C(t_1 * x) \geq \{rmin\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}$$

$$\geq rmin\{rmin\{\hat{B}_C(((t_1 * t_2) * t_3) * x), \hat{B}_C(t_3)\}, \hat{B}_C(t_2)\}$$

$$\geq rmin\{rmin\{\hat{B}_C(0), \hat{B}_C(t_3)\}, \hat{B}_C(t_2)\}$$

$$\geq rmin\{\hat{B}_C(t_3), \hat{B}_C(t_2)\}$$

$$\hat{B}_C(t_1 * x) = rmin\{\hat{B}_C(t_3), \hat{B}_C(t_2)\}$$

and

$$J_C(t_1 * x) \leq \{max\{J_C((t_1 * t_2) * x), J_C(t_2)\}$$

$$\leq max\{max\{J_C(((t_1 * t_2) * t_3) * x)), J_C(t_3)\}, J_C(t_2)\}$$

$$\leq max\{max\{J_C(0), J_C(t_3)\}, J_C(t_2)\}$$

$$\leq max\{J_C(t_3), J_C(t_2)\}$$

$$J_C(t_1 * x) = max\{J_C(t_3), J_C(t_2)\}.$$

Theorem 3.2. Let $C = (M_C, J_C)$ be a MBJNTID of a B-algebra Y. If $t_1 \leq t_2$ then $M_C(t_1 * x) \geq M_C(t_2)$, $\hat{B}_C(t_1 * x) \geq \hat{B}_C(t_2)$ and $J_C(t_1 * x) \leq J_C(t_2)$ for all $t_1, t_2, t_3 \in Y$.

Proof. Let $t_1, t_2 \in Y$ such that $t_1 \leq t_2$. Then $t_1 * t_2 = 0$, now

$$M_C(t_1 * x) \geq \{min\{M_C((t_1 * t_2) * x), M_C(t_2)\}$$

$$\geq \{min\{M_C(0), M_C(t_2)\}$$

$$M_C(t_1 * x) = M_C(t_2)$$

and

$$\hat{B}_C(t_1 * x) \geq \{rmin\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}$$

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$\geq \{rmin\{\hat{B}_C(0), \hat{B}_C(t_2)\}$

$\hat{B}_C(t_1 * x) = \hat{B}_C(t_2)$

and

$J_C(t_1 * x) \leq \{max\{J_C((t_1 * t_2) * x), J_C(t_2)\}$

$\leq \{max\{J_C(0), J_C(t_2)\}$

$J_C(t_1 * x) = J_C(t_2)$.

Hence $M_C(t_1 * x) \geq M_C(t_2)$, $\hat{B}_C(t_1 * x) \geq \hat{B}_C(t_2)$ and $J_C(t_1 * x) \leq J_C(t_2)$ for all $t_1, t_2, t_3 \in Y$.

Theorem 3.3. If C is a MBJNTID, then it fulfills the condition $(M_C((t_1 * (t_2 * t_1)) * x) \geq (M_C(t_2)$, $(\hat{B}_C((t_1 * (t_2 * t_1)) * x) \geq (\hat{B}_C(t_2)$ and $(J_C((t_1 * (t_2 * t_1)) * x) \leq (J_C(t_2)$.

Proof. Let C is a MBJNTID, so

$M_C((t_1 * (t_2 * t_1)) * x) \geq min\{M_C(((t_1 * (t_2 * t_1)) * t_2) * x), M_C(t_2)\}$

$= min\{M_C(0), M_C(t_2)\}$

$M_C((t_1 * (t_2 * t_1)) * x) = M_C(t_2)$

and

$\hat{B}_C((t_1 * (t_2 * t_1)) * x) \geq rmin\{\hat{B}_C(((t_1 * (t_2 * t_1)) * t_2) * x), \hat{B}_C(t_2)\}$

$= rmin\{\hat{B}_C(0), \hat{B}_C(t_2)\}$

$\hat{B}_C((t_1 * (t_2 * t_1)) * x) = \hat{B}_C(t_2)$

and

$J_C((t_1 * (t_2 * t_1)) * x) \leq max\{J_C(((t_1 * (t_2 * t_1)) * t_2) * x), J_C(t_2)\}$

$= max\{J_C(0), J_C(t_2)\}$

$J_C((t_1 * (t_2 * t_1)) * x) = J_C(t_2)$.

Hence $(M_C((t_1 * (t_2 * t_1)) * x) \geq (M_C(t_2)$, $(\hat{B}_C((t_1 * (t_2 * t_1)) * x) \geq (\hat{B}_C(t_2)$ and $(J_C((t_1 * (t_2 * t_1)) * x) \leq (J_C(t_2)$.

Theorem 3.4. Let $A = (M_{\mathrm{A}}, \hat{B}_{\mathrm{A}}, J_{\mathrm{A}})$ and $C = (M_C, \hat{B}_{\mathrm{C}}, J_C)$ are two MBJNTIDs of a B-algebra Y. Then the intersection A ∩ C is also MBJNTID of Y.

Proof. Let $t_1, t_2 \in \mathrm{A} \cap C$, then $t_1, t_2 \in \mathrm{A}$ and $t_1, t_2 \in C$.

$M_{\mathrm{A}\cap C}(0) = M_{\mathrm{A}\cap C}(t_1 * t_1)$

$= min\{M_{\mathrm{A}}(t_1 * t_1), M_C(t_1 * t_1)\}$

$\geq min\{min\{M_{\mathrm{A}}(t_1), M_{\mathrm{A}}(t_1)\}, min\{M_C(t_1), M_C(t_1)\}\}$

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$= min\{M_{\mathrm{A}}(t_1), M_C(t_1)\}$

$M_{\mathrm{A}\cap C}(0) = M_{\mathrm{A}\cap C}(t_1)$

and

$\hat{B}_{\mathrm{A}\cap C}(0) = \hat{B}_{\mathrm{A}\cap C}(t_1 * t_1)$

$= rmin\{\hat{B}_{\mathrm{A}}(t_1 * t_1), \hat{B}_C(t_1 * t_1)\}$

$\geq rmin\{rmin\{\hat{B}_{\mathrm{A}}(t_1), \hat{B}_{\mathrm{A}}(t_1)\}, rmin\{\hat{B}_C(t_1), \hat{B}_C(t_1)\}\}$

$= rmin\{\hat{B}_{\mathrm{A}}(t_1), \hat{B}_C(t_1)\}$

$\hat{B}_{\mathrm{A}\cap C}(0) = \hat{B}_{\mathrm{A}\cap C}(t_1)$

and

$J_{\mathrm{A}\cap C}(0) = J_{\mathrm{A}\cap C}(t_1 * t_1)$

$= max\{J_C(t_1 * t_1), J_C(t_1 * t_1)\}$

$\leq max\{max\{J_C(t_1), J_C(t_1)\}, max\{J_C(t_1), J_C(t_1)\}\}$

$= max\{J_C(t_1), J_C(t_1)\}$

$J_{\mathrm{A}\cap C}(0) = J_{\mathrm{A}\cap C}(t_1)$,

now

$M_{\mathrm{A}\cap C}(t_1 * x) = min\{M_{\mathrm{A}}(t_1 * x), M_C(t_1 * x)\}$

$\geq min\{min\{M_{\mathrm{A}}((t_1 * t_2) * x), M_{\mathrm{A}}(t_2)\}, min\{M_C((t_1 * t_2) * x), M_C(t_2)\}\}$

$= min\{min\{M_{\mathrm{A}}((t_1 * t_2) * x), M_C((t_1 * t_2) * x)\}, min\{M_{\mathrm{A}}(t_2), M_C(t_2)\}\}$

$= \{min\{M_{\mathrm{A}\cap C}((t_1 * t_2) * x), M_{\mathrm{A}\cap C}(t_2)\}$

and

$\hat{B}_{\mathrm{A}\cap C}(t_1 * x) = rmin\{\hat{B}_{\mathrm{A}}(t_1 * x), \hat{B}_C(t_1 * x)\}$

$\geq rmin\{rmin\{\hat{B}_{\mathrm{A}}((t_1 * t_2) * x), \hat{B}_{\mathrm{A}}(t_2)\}, rmin\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}\}$

$= rmin\{rmin\{\hat{B}_{\mathrm{A}}((t_1 * t_2) * x), \hat{B}_C((t_1 * t_2) * x)\}, rmin\{\hat{B}_{\mathrm{A}}(t_2), \hat{B}_C(t_2)\}\}$

$= \{rmin\{\hat{B}_{\mathrm{A}\cap C}((t_1 * t_2) * x), \hat{B}_{\mathrm{A}\cap C}(t_2)\}$

and

$J_{\mathrm{A}\cap C}(t_1 * x) = max\{J_{\mathrm{A}}(t_1 * x), J_C(t_1 * x)\}$

$\leq max\{max\{J_{\mathrm{A}}((t_1 * t_2) * x), J_{\mathrm{A}}(t_2)\}, max\{J_C((t_1 * t_2) * x), J_C(t_2)\}\}$

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$= max\{max\{J_{A}((t_1 * t_2) * x), J_C((t_1 * t_2) * x)\}, max\{J_{A}(t_2), J_C(t_2)\}\}$

$= max\{J_{A\cap C}((t_1 * t_2) * x), J_{A\cap C}(t_2)\}.$

Hence the intersection $A \cap C$ is MBJNTID of Y.

Theorem 3.5. Let $A = (M_A, \hat{B}_A, J_A)$ and $C = (M_C, \hat{B}_C, J_C)$ are two MBJNTIDs of a B-algebra Y. Then the union $A \cup C$ is also MBJNTID of Y.

Proof. Let $t_1, t_2 \in A \cup C$, then $t_1, t_2 \in A$ and $t_1, t_2 \in C$.

$M_{A\cup C}(0) = M_{A\cup C}(t_1 * t_1)$

$= max\{M_A(t_1 * t_1), M_C(t_1 * t_1)\}$

$\geq max\{max\{M_A(t_1), M_A(t_1)\}, max\{M_C(t_1), M_C(t_1)\}\}$

$= max\{M_A(t_1), M_C(t_1)\}$

$M_{A\cup C}(0) = M_{A\cup C}(t_1)$

and

$\hat{B}_{A\cup C}(0) = \hat{B}_{A\cup C}(t_1 * t_1)$

$= rmax\{\hat{B}_A(t_1 * t_1), \hat{B}_C(t_1 * t_1)\}$

$\geq rmax\{rmax\{\hat{B}_A(t_1), \hat{B}_A(t_1)\}, rmax\{\hat{B}_C(t_1), \hat{B}_C(t_1)\}\}$

$= rmax\{\hat{B}_A(t_1), \hat{B}_C(t_1)\}$

$\hat{B}_{A\cup C}(0) = \hat{B}_{A\cup C}(t_1)$

and

$J_{A\cup C}(0) = J_{A\cup C}(t_1 * t_1)$

$= min\{J_A(t_1 * t_1), J_C(t_1 * t_1)\}$

$\leq min\{min\{J_A(t_1), J_A(t_1)\}, min\{J_C(t_1), J_C(t_1)\}\}$

$= min\{J_A(t_1), J_C(t_1)\}$

$J_{A\cup C}(0) = J_{A\cup C}(t_1)$

now

$M_{A\cup C}(t_1 * x) = max\{M_A(t_1 * x), M_C(t_1 * x)\}$

$\geq max\{max\{M_A((t_1 * t_2) * x), M_A(t_2)\}, max\{M_C((t_1 * t_2) * x), M_C(t_2)\}\}$

$= max\{max\{M_A((t_1 * t_2) * x), M_C((t_1 * t_2) * x)\}, max\{M_A(t_2), M_C(t_2)\}\}$

$= \{max\{M_{A\cup C}((t_1 * t_2) * x), M_{A\cup C}(t_2)\}$

and

$$\hat{B}_{A\cup C}(t_1 * x) = rmax\{\hat{B}_A(t_1 * x), \hat{B}_C(t_1 * x)\}$$

$$\geq rmax\{rmax\{\hat{B}_A((t_1 * t_2) * x), \hat{B}_A(t_2)\}, rmax\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}\}$$

$$= rmax\{rmax\{\hat{B}_A((t_1 * t_2) * x), \hat{B}_C((t_1 * t_2) * x)\}, rmax\{\hat{B}_A(t_2), \hat{B}_C(t_2)\}\}$$

$$= \{rmax\{\hat{B}_{A\cup B}((t_1 * t_2) * x), \hat{B}_{A\cup B}(t_2)\}$$

and

$$J_{A\cup C}(t_1 * x) = min\{J_A(t_1 * x), J_C(t_1 * x)\}$$

$$\leq min\{min\{J_A((t_1 * t_2) * x), J_A(t_2)\}, min\{J_C((t_1 * t_2) * x), J_C(t_2)\}\}$$

$$= min\{min\{J_A((t_1 * t_2) * x), J_C((t_1 * t_2) * x)\}, min\{J_A(t_2), J_C(t_2)\}\}$$

$$= \{min\{J_{A\cup C}((t_1 * t_2) * x), J_{A\cup C}(t_2)\}.$$

Hence the union $A \cup C$ is also MBJNTID of Y.

Theorem 3.6. Let A and C are the MBJ-neutrosophic sets in Y. The cartesian product $A \times C: Y \times Y \to [0,1]$ is defined by $M_A \times M_C(t_1, t_2) = max\{M_A(t_1), M_C(t_2)\}$, $\hat{B}_A \times \hat{B}_C(t_1, t_2) = rmax\{\hat{B}_A(t_1), \hat{B}_C(t_2)\}$ and $J_A \times J_C(t_1, t_2) = min\{J_A(t_1), J_C(t_2)\}$ for all $t_1, t_2 \in Y$. Let A and C are two MBJNTIDs of Y, then $A \times C$ is a MBJNTID of $Y \times Y$.

Proof. For any $t_1, t_2 \in Y \times Y$, we have $M_A \times M_C(0,0) = min\{M_A(0), M_C(0)\} \geq min\{M_A(t_1), M_C(t_2)\} = M_A \times M_C(t_1, t_2)$, $\hat{B}_A \times \hat{B}_C(0,0) = rmin\{\hat{B}_A(0), \hat{B}_C(0)\} \geq rmin\{\hat{B}_A(t_1), \hat{B}_C(t_2)\} = \hat{B}_A \times \hat{B}_C(t_1, t_2)$ and $J_A \times J_C(0,0) = max\{J_A(0), J_C(0)\} \leq max\{J_A(t_1), J_C(t_2)\} = J_A \times J_C(t_1, t_2)$. That is $M_A \times M_C(0,0) \geq M_A \times M_C(t_1, t_2)$, $\hat{B}_A \times \hat{B}_C(0,0) \geq \hat{B}_A \times \hat{B}_C(t_1, t_2)$ and $J_A \times J_C(0,0) \leq J_A \times J_C(t_1, t_2)$.

Now let $x, (t_1, t_2)$ and $(y_1, y_2) \in Y \times Y$. Then, $(M_A \times M_C)(t_1 * x, t_2 * x) = min\{M_A(t_1 * x), M_C(t_2 * x)\} \geq min\{min\{M_A((t_1 * y_1) * x), M_A(y_1)\}, min\{M_C((t_2 * y_2) * x), M_C(y_2)\}\} = min\{min\{M_A((t_1 * y_1) * x), M_C((t_2 * y_2) * x)\}, min\{M_A(y_1), M_C(y_2)\}\} = min\{(M_A \times M_C)(((t_1 * y_1) * x), ((t_2 * y_2) * x)), (M_A \times M_C)(y_1, y_2)\} = min\{(M_A \times M_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (M_A \times M_C)(y_1, y_2)$. That is $(M_A \times M_C)(t_1 * x, t_2 * x) \geq min\{(M_A \times M_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (M_A \times M_C)(y_1, y_2)$ and $(\hat{B}_A \times \hat{B}_C)(t_1 * x, t_2 * x) = rmin\{\hat{B}_A(t_1 * x), \hat{B}_C(t_2 * x)\} \geq rmin\{rmin\{\hat{B}_A((t_1 * y_1) * x), \hat{B}_A(y_1)\}, rmin\{\hat{B}_C((t_2 * y_2) * x), \hat{B}_C(y_2)\}\} = rmin\{rmin\{\hat{B}_A((t_1 * y_1) * x), \hat{B}_C((t_2 * y_2) * x)\}, rmin\{\hat{B}_A(y_1), \hat{B}_C(y_2)\}\} = rmin\{(\hat{B}_A \times \hat{B}_C)(((t_1 * y_1) * x), ((t_2 * y_2) * x)), (\hat{B}_A \times \hat{B}_C)(y_1, y_2)\} = rmin\{(\hat{B}_A \times \hat{B}_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (\hat{B}_A \times \hat{B}_C)(y_1, y_2)$. That is $(\hat{B}_A \times \hat{B}_C)(t_1 * x, t_2 * x) \geq rmin\{(\hat{B}_A \times \hat{B}_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (\hat{B}_A \times \hat{B}_C)(y_1, y_2)$ and $(J_A \times J_C)(t_1 * x, t_2 * x) = max\{J_A(t_1 * x), J_C(t_2 * x)\} \leq max\{max\{J_A((t_1 * y_1) * x), J_A(y_1)\}, max\{J_C((t_2 * y_2) * x), J_C(y_2)\}\} = max\{max\{J_A((t_1 * y_1) * x), J_C((t_2 * y_2) * x)\}, max\{J_A(y_1), J_C(y_2)\}\} = max\{(J_A \times J_C)(((t_1 * y_1) * x), ((t_2 * y_2) * x)), (J_A \times J_C)(y_1, y_2)\} = max\{(J_A \times J_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (J_A \times J_C)(y_1, y_2)$.
That is $(J_A \times J_C)(t_1 * x, t_2 * x) \leq max\{(J_A \times J_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (J_A \times J_C)(y_1, y_2)$.

Theorem3.7. Let A and C are two MBJNs in Y such that $A \times C$ is an MBJNTID of $Y \times Y$, then
1. $M_A(0) \geq M_A(t_1)$, $\hat{B}_A(0) \geq \hat{B}_A(t_1)$, $J_A(0) \leq J_A(t_1)$ and $M_C(0) \geq M_C(t_1)$, $\hat{B}_C(0) \geq \hat{B}_C(t_1)$, $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

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2. If $M_A(0) \geq M_A(t_1)$ then $M_C(0) \geq M_A(t_1)$ and $M_C(0) \geq M_C(t_1)$ for all $t_1 \in Y$ also if $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ then $\hat{B}_C(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$ also if $J_A(0) \leq J_A(t_1)$ then $J_C(0) \leq J_A(t_1)$ and $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

3. If $M_C(0) \geq M_C(t_1)$ then $M_A(0) \geq M_A(t_1)$ and $M_A(0) \geq M_C(t_1)$ for all $t_1 \in Y$ also if $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ then $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_A(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$ also if $J_C(0) \leq J_C(t_1)$ then $J_A(0) \leq J_A(t_1)$ and $J_A(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

Proof. 1. Suppose $M_A(t_1) > M_A(0)$ or $M_C(t_1) > M_C(0)$ for all $t_1 \in Y$ $(M_A \times M_C)(t_1, t_1) = \min\{M_A(t_1), M_C(t_1)\} > \min\{M_A(0), M_C(0)\} = (M_A \times M_C)(0,0)$. Thus $(M_A \times M_C)(t_1, t_1) > (M_A \times M_C)(0,0)$ for all $\in Y$, which is the contradiction to $(M_A \times M_C)$ is a MBJNTID of $Y \times Y$. Therefore $M_A(0) \geq M_A(t_1)$ and $M_C(0) \geq M_C(t_1)$ for all $t_1 \in Y$, also $\hat{B}_A(t_1) > \hat{B}_A(0)$ or $\hat{B}_C(t_1) > \hat{B}_C(0)$ for all $t_1 \in Y$, $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) = rmin\{\hat{B}_A(t_1), \hat{B}_C(t_1)\} > rmin\{\hat{B}_A(0), \hat{B}_C(0)\} = (\hat{B}_A \times \hat{B}_C)(0,0)$. Thus $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) > (\hat{B}_A \times \hat{B}_C)(0,0)$ for all $\in Y$, which is the contradiction to $(\hat{B}_A \times \hat{B}_C)$ is a MBJNTID of $Y \times Y$. Therefore $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$ and also $J_A(t_1) < J_A(0)$ or $J_C(t_1) < J_C(0)$ for all $t_1 \in Y$. $(J_A \times J_C)(t_1, t_1) = \max\{J_A(t_1), J_C(t_1)\} > max\{J_A(0), J_C(0)\} = (J_A \times J_C)(0,0)$. Thus $(J_A \times J_C)(t_1, t_1) < (J_A \times J_C)(0,0)$ for all $\in Y$, which is the contradiction to $(J_A \times J_C)$ is a MBJNTID of $Y \times Y$. Therefore $J_A(0) \leq J_A(t_1)$ and $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

2. Suppose $M_C(0) < M_A(t_1)$ or$M_A(0) < M_A(t_1)$ for all $t_1 \in Y$. Then $(M_A \times M_C)(0,0) = min\{M_A(0), M_C(0)\} = M_C(0)$ and $(M_A \times M_C)(t_1, t_1) = min\{M_A(t_1), M_C(t_1)\} > M_C(0) = (M_A \times M_C)(0,0)$. This implies $(M_A \times M_C)(t_1, t_1) = (M_A \times M_C)(0,0)$. Which is the contradiction to $(M_A \times M_C)$ is a MBJNTID of $Y \times Y$. Hence if $M_A(0) \geq M_A(t_1)$ then $M_C(0) \geq M_A(t_1)$ and $M_C(0) \geq M_C(t_1)$ for all $t_1 \in Y$. Now Suppose $\hat{B}_C(0) < \hat{B}_A(t_1)$ or $\hat{B}_A(0) < \hat{B}_A(t_1)$ for all $t_1 \in Y$. Then $(\hat{B}_A \times \hat{B}_C)(0,0) = rmin\{\hat{B}_A(0), \hat{B}_C(0)\} = \hat{B}_C(0)$ and $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) = rmin\{\hat{B}_A(t_1), \hat{B}_C(t_1)\} > \hat{B}_C(0) = (\hat{B}_A \times M_C)(0,0)$. This implies $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) = (\hat{B}_A \times \hat{B}_C)(0,0)$. Which is the contradiction to $(\hat{B}_A \times \hat{B}_C)$ is a MBJNTID of $Y \times Y$. Hence if $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ then $\hat{B}_C(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$. Now suppose $J_C(0) > J_A(t_1)$ or $J_A(0) > J_A(t_1)$ for all $t_1 \in Y$. Then $(J_A \times J_C)(0,0) = max\{J_A(0), J_C(0)\} = J_C(0)$ and $(J_A \times J_C)(t_1, t_1) = max\{J_A(t_1), J_C(t_1)\} > J_C(0) = (J_A \times J_C)(0,0)$. This implies $(J_A \times J_C)(t_1, t_1) = (J_A \times J_C)(0,0)$. Which is the contradiction to $(J_A \times J_C)$ is a MBJNTID of $Y \times Y$. Hence if $J_A(0) \leq J_A(t_1)$ then $J_C(0) \leq J_A(t_1)$ and $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

3. The proof is quit same to 2.

Theorm 3.8. Let $\Gamma: Y \to X$ is a B-homomorphism of B-algebra. If $C = (M_C, \hat{B}_C, J_C)$ is a MBJNTID of X then the pre-image $\Gamma^{-1}(C) = (\Gamma^{-1}(M_C), \Gamma^{-1}(\hat{B}_C),\ \Gamma^{-1}(J_C))$ of B under Γ is a MBJNTID in Y.

Proof. For any $t_1 \in Y$,

1. $\Gamma^{-1}(M_C)(t_1) = M_C(\Gamma(t_1)) \geq M_C(0) = M_C(\Gamma(0)) = \Gamma^{-1}(M_C)(0)$, $\Gamma^{-1}(\hat{B}_C)(t_1) = \hat{B}_C(\Gamma(t_1)) \geq \hat{B}_C(0) = \hat{B}_C(\Gamma(0)) = \Gamma^{-1}(\hat{B}_C)(0)$ and $\Gamma^{-1}(J_C)(t_1) = J_C(\Gamma(t_1)) \leq J_C(0) = J_C(\Gamma(0)) = \Gamma^{-1}(J_C)(0)$.

2. $\Gamma^{-1}(M_C)(t_1 * x) = M_C(\Gamma(t_1 * x)) \geq \min\{M_C((\Gamma(t_1) * \Gamma(t_2)) * x), M_C(\Gamma(t_2))\} = \min\{M_C(\Gamma((t_1 * t_2) * x), M_C(\Gamma(t_2))\} = \min\{\Gamma^{-1}(\tau)_B((t_1 * t_2) * x), \Gamma^{-1}(M_C)(t_2)\}$, also $\Gamma^{-1}(\hat{B}_C)(t_1 * x) = \hat{B}_C(\Gamma(t_1 * x)) \geq rmin\{\hat{B}_C((\Gamma(t_1) * \Gamma(t_2)) * x), \hat{B}_C(\Gamma(t_2))\} = rmin\{\hat{B}_C(\Gamma((t_1 * t_2) * x), \hat{B}_C(\Gamma(t_2))\} = rmin\{\Gamma^{-1}(\hat{B}_C)((t_1 * t_2) * x), \Gamma^{-1}(\hat{B}_C)(t_2)\}$ and $\Gamma^{-1}(J_C)(t_1 * x) = J_C(\Gamma(t_1 * x)) \leq \max\{J_C(\Gamma(((t_1) * \Gamma(t_2)) * x), J_C(\Gamma(t_2))\} = max \{J_C(\Gamma((t_1 * t_2) * x), J_C(\Gamma(t_2))\} = max\{\Gamma^{-1}(J_C)((t_1 * t_2) * x), \Gamma^{-1}(\sigma)(t_2)\}$.

Theorem 3.9. Let $\Gamma: Y \to X$ be an epimorphism of B-algebra. Then $C = (M_C, \hat{B}_C, J_C)$ is a MBJNTID of X, if $\Gamma^{-1}(C) = (\Gamma^{-1}(M_C), (\Gamma^{-1}(\hat{B}_C), \Gamma^{-1}(J_C))$ of B under Γ is a MBJNTID in Y.

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Proof. For any $t_1 \in X$, there exist $a \in Y$ such that $\Gamma(a) = t_1$. Then $M_C(t_1) = M_C(\Gamma(a)) = \Gamma^{-1}(M_C)(a) \geq \Gamma^{-1}(M_C)(0) = M_C(\Gamma(0)) = M_C(0)$, $\hat{B}_C(t_1) = \hat{B}_C(\Gamma(a)) = \Gamma^{-1}(\hat{B}_C)(a) \geq \Gamma^{-1}(\hat{B}_C)(0) = \hat{B}_C(\Gamma(0)) = \hat{B}_C(0)$ and $J_C(t_1) = J_C(\Gamma(a)) = \Gamma^{-1}(J_C)(a) \leq \Gamma^{-1}(J_C)(0) = J_C(\Gamma(0)) = J_C(0)$. Now let $t_1, t_2 \in X$ then $\Gamma(a) = t_1$ and $\Gamma(b) = t_2$ for some $a, b \in Y$, now $M_C(t_1 * x) = M_C(\Gamma(a * x)) = \Gamma^{-1}(M_C)(a * x) \geq min\{\Gamma^{-1}(M_C((a * b) * x)), \Gamma^{-1}(M_C(b))\} = min\{M_C(\Gamma((a * b) * x)), M_C(\Gamma(b))\} = min\{M_C(\Gamma(a) * \Gamma(b)) * x), M_C(\Gamma(b))\} = min\{M_C((t_1 * t_2) * x), M_C(t_2)\}$ and $\hat{B}_C(t_1 * x) = \hat{B}_C(\Gamma(a * x)) = \Gamma^{-1}(\hat{B}_C)(a * x) \geq rmin\{\Gamma^{-1}(\hat{B}_C((a * b) * x)), \Gamma^{-1}(\hat{B}_C(b))\} = rmin\{\hat{B}_C(\Gamma((a * b) * x)), \hat{B}_C(\Gamma(b))\} = rmin\{\hat{B}_C(\Gamma(a) * \Gamma(b)) * x), \hat{B}_C(\Gamma(b))\} = rmin\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}$ and $J_C(t_1 * x) = J_C(\Gamma(a * x)) = \Gamma^{-1}(J_C)(a * x) \leq max\{\Gamma^{-1}(J_C((a * b) * x)), \Gamma^{-1}(J_C(b))\} = max\{J_C(\Gamma((a * b) * x)), J_C(\Gamma(b))\} = max\{J_C((\Gamma(a) * \Gamma(b) * x)), J_C(\Gamma(b))\} = max\{J_C(((t_1 * t_2) * x)), J_C(t_2)\}$. Hence $C = (M_C, \hat{B}_C, J_C)$ is MBJ-neutrosphic T-ideal of Y.

7. Conclusion

In this paper, we studied the MBJ neutrosophic T-ideal, cartesian product of MBJ neutrosophic T-ideals and homomorphism of MBJ neutrosophic T-ideal through significent properties and results. This paper will give us new direction to use MBJ neutrosphic set in different atmosphere. In future, work can be done on MBJ-neutrosophic T-normal ideal,MBJ-neutrosophic cubic T-BMBJ ideal.

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