n-Cyclic Refined Neutrosophic Groups

Rozina Ali, Independent Researcher, Cairo, Egypt

e-mail: rozyyy123n@gmail.com

Abstract

This paper introduces for the first time the concept of n-cyclic refined neutrosophic group as a direct application of the concept of n-cyclic refined neutrosophic set. Also, it discusses some of its elementary properties such as AH-subgroups, kernels, and direct products.

Keywords: n-cyclic refined neutrosophic group , AH-subgroup, AH-homomorphism

1.Introduction

Neutrosophy is a philosophical concept founded by Smarandache to generalize the fuzzy logic [1,32] . Neutrosophic sets were very applicative in many areas of science such as topology [2,27,60,61,63], decision making [21,35], applied mathematics [3,4,20,25,28,57,64], and pure mathematics [9,10,19,39,43,44,47,50,51,52].

Neutrosophic sets were used in the study of algebraic structures such as modules [6], rings [12], groups [13], and matrices [6,53,62]. In the literature, we find three interesting generalizations of neutrosophic sets, where refined neutrosophic sets [13] n-refined neutrosophic sets [15,50], and n-cyclic refined neutrosophic sets [31]. These kinds lead us to many interesting generalizations of classical algebraic structures such as n-refined neutrosophic modules [5], n-refined spaces and matrices [8,46,54], refined neutrosophic ring [23,48,49], and n-cyclic refined neutrosophic modules [31].

In this work, we use the concept of n-cyclic refined neutrosophic set to defined n-cyclic refined neutrosophic groups. These groups will be studied carefully, and many elementary properties will be discussed through this paper, especially AH-subgroups, kernels, and homomorphisms.

2. Preliminaries

Definition 2.1: [31]

Let (R,+,×) be a ring and $I_k; 1 \leq k \leq n$ be n indeterminacies. We define R_n(I)={$a_0 + a_1 I + \cdots + a_n I_n$; $a_i \in R$} to be n-cyclic refined neutrosophic ring.

Operations on R_n(I) are defined as:

$$\sum_{i=0}^{n} x_i I_i + \sum_{i=0}^{n} y_i I_i = \sum_{i=0}^{n} (x_i + y_i) I_i \, , \sum_{i=0}^{n} x_i I_i \times \sum_{i=0}^{n} y_i I_i = \sum_{i,j=0}^{n} (x_i \times y_j) I_i I_j = \sum_{i,j=0}^{n} (x_i \times y_j) I_{(i+j \, mod n)}$$

Where × is the multiplication on the ring R.

DOI: 10.5281/zenodo.4924483

Received:January 10, 2021, Accepted: May 05, 2021

It is obvious that R_n(I) is a ring in the algebraic ordinary concept.

Definition 2.2: [31]

Let R_n(I) be an n-cyclic refined neutrosophic ring, it is called commutative if $xy = yx$ for all x , y ∈ R_n(I). If there is 1∈ R_n(I) such that $1.x = x.1 = x$, then it is called an n-cyclic refined neutrosophic ring with unity.

Definition 2.3: [31]

Let R_n(I) be a commutative n-cyclic refined neutrosophic ring and $P: R_n(\text{I}) \to R_n(\text{I})$ is a function defined as $P(x) = \sum_{i=0}^{m} a_i x^i$ such that $a_i \in R_n$(I), we call P an n-cyclic refined neutrosophic polynomial on R_n(I).

We denote by R_n(I)[x] to be the ring of n-cyclic refined neutrosophic polynomials over R_n(I).

Since R_n(I) is a classical ring, then R_n(I)[x] is a classical ring.

Definition 2.4 : [31]

Let (M,+,.) be a module over the ring R, we say that $M_n(I) = M + MI_1 + \cdots + MI_n = \{x_0 + x_1 I_1 + \cdots + x_n I_n;\ x_i \in M\}$ is a weak n-cyclic refined neutrosophic module over the ring R. Elements of $M_n(I)$ are called n-cyclic refined neutrosophic vectors, elements of R are called scalars.

If we take scalars from the n- cyclic refined neutrosophic ring $R_n(I)$, we say that $M_n(I)$ is a strong n-cyclic refined neutrosophic module over the n-cyclic refined neutrosophic ring $R_n(I)$. Elements of $M_n(I)$ are called n-refined neutrosophic scalars.

Remark 2.5: [31]

Addition on $M_n(I)$ is defined as:

$\sum_{i=0}^{n} a_i I_i + \sum_{i=0}^{n} b_i I_i = \sum_{i=0}^{n} (a_i + b_i) I_i$.

Multiplication by a scalar $m \in R$ is defined as:

$m. \sum_{i=0}^{n} a_i I_i = \sum_{i=0}^{n} (m. a_i) I_i$.

Multiplication by an n-cyclic refined neutrosophic scalar $m = \sum_{i=0}^{n} m_i I_i \in R_n(I)$ is defined as:

$\sum_{i=0}^{n} m_i I_i \ . \sum_{i=0}^{n} a_i I_i = \sum_{i=0}^{n} (m_i . a_i) I_i I_j$.

Where $\mathrm{a_i} \in \mathrm{M}, \mathrm{m_i} \in \mathrm{R}, \mathrm{I_i I_j} = \mathrm{I_{(i+j \bmod n)}}$.

3. Main Concepts and discussion

Definition 3.1:

Let I be the neutrosophic element, which refers to indeterminacy, we define n-cyclic refining system of I by the set L={$I^1 = I, I^2, \dots, I^{n-1}$}, where $I^i \neq I^j\ for\ all\ i \neq j\ and\ i, j < n$.

We define a binary operation on L as follows:

$I^j . I^i = I^{i+j(modn)}$. It is clear that L has a structure of the subset $(Z_n/\{0\}, +)$, where Z_n is the additive group of integers modulo n.

DOI: 10.5281/zenodo.4924483

Received:January 10, 2021, Accepted: May 05, 2021

Definition 3.2:

Let (G,*) be a group, and I is the neutrosophic element with property $I^n = I\, ; n \geq 2$, with

$I^i \neq I^j\ for\ all\ i \neq j\ and\ i,j < n$. We call M(G)=$G \cup GI \cup GI^2 \cup \ldots \cup GI^{n-1}$ an n-cyclic refined neutrosophic group.

It is easy to see that 2-cyclic refined neutrosophic group is the classical neutrosophic group.

Remark 3.3:

The sets GI^k are groups under the binary operation (xI^k)(yI^k)=(xyI^k) ; $k < n$ with identity I^k and each one of them must be isomorphic to G.

Definition 3.4:

Let M(G) be an n-cyclic refined neutrosophic group, H be a subset of M(G). We call H an AH-subgroup if H=$H_0 \cup H_1I \cup H_2I^2 \cup \ldots \cup H_{n-1}I^{n-1}$,where H_i is a subgroup of G for all i.

We call H an AHS-subgroup if $H_0 = H_1 = \cdots = H_{n-1}$.

We call H an AH-normal if H_i is normal subgroup of G for all i.

We call H an AHS-normal if it is AHS-subgroup and AH-normal.

Definition 3.5 :

Let M(G) , M(H) be two n , m-cyclic refined neutrosophic groups respectively, $f: M(G) \to M(H)$ be a map , we say that f is an AH- homomorphism if it is a homomrphism between G , H i.e. $f(xy) = f(x)f(y) \forall\ x,y \in G\ and\ f(I^k) = (I^{'})^k$ such $I^{'}$ is the neutrosophic element of H.

We define $AH - Ker(f)\ = Kerf_G \cup Kerf_G I \cup \ldots \cup Kerf_G I^{n-1}$, we regard that $AH - Ker(f)$ is an AHS-normal subgroup of M(G)

We say that f is an isomorphism if it is a correspondence one-to-one homomorphism.

If H = $H_0 \cup H_1I \cup \ldots \cup H_{n-1}I^{n-1}\ and\ K = K_0 \cup K_1I \cup \ldots \cup K_{n-1}I^{n-1}$ are two AH-subgroups of M(G). We say that they are isomorphic if $H_i \cong K_i$ for all i .

Definition 3.6 :

Let H , K be two AH-subgroups of M(G). We define

$HK = H_0K_0 \cup H_1K_1I^1 \cup \ldots \cup H_{n-1}K_{n-1}I^{n-1}$.

Definition 3.7 :

Let M(G) be an n-cyclic refined neutrosophic group, $H = H_0 \cup H_1I \cup H_2I^2 \cup \ldots \cup H_{n-1}I^{n-1}$ be an AH-normal subgroup of M(G). We define the corresponding AH-factor as M(G)/H = $(G/H_0) \cup (G/H_1)I \cup \ldots \cup (G/H_{n-1})I^{n-1}$.

Definition 3.8 :

Let M(G) be an n-cyclic refined neutrosophic group. We define the AH-center of M(G) by

DOI: 10.5281/zenodo.4924483

Received:January 10, 2021, Accepted: May 05, 2021

$Z(M(G)) = Z(G) \cup Z(G)I \cup \ldots \cup Z(G)I^{n-1}$.

It is easy to see that $Z(M(G))$ is an AHS-normal subgroup of M(G).

Definition 3.9 :

Let M(G) be an n-cyclic refined neutrosophic group. We say that M(G) is abelian if G is abelian, i.e. M(G)=Z(M(G)).

M(G) is said to be cyclic if G is cyclic.

Theorem 3.10 :

Let M(G) be an n-cyclic refined neutrosophic group, then

(a) If H is an AH-normal subgroup and M(G) is abelian, then M(G)/H is abelian.

(b) If M(G) is finite and H is an AHS-subgroup, then O(H) divides O(M(G))= n O(G).

(c) If H is an AH-normal subgroup and M(G) is cyclic then M(G)/H is cyclic.

Proof:

(a) Since M(G)/H = $(G/H_0) \cup (G/H_1)I \cup \ldots \cup (G/H_{n-1})I^{n-1}$ and G/H_i is abelian for all i, then M(G)/H is abelian.

(b) We have that O(H)= n O(H_0) and O(H_0) divides the order of G then O(H) divides O(M(G))= n O(G).

(c) Since G/H_i is cyclic for all i then M(G)/H is cyclic.

Theorem 3.11 :

Let M(G) be an n-cyclic refined neutrosophic group and H , K be two AH-subgroups, then

(a) $H \cap K$ is an AH-subgroup.

(b) If H , K are AHS-subgroups, then $H \cap K$ is an AHS-subgroup.

(c) If H , K are AH-normal subgroups, then $H \cap K$ and HK are AH-normal subgroups.

(d) If H, K are AHS-normal subgroups, then $H \cap K$ and HK are AHS-normal subgroups.

Proof : Suppose that H = $H_0 \cup H_1I \cup \ldots \cup H_{n-1}I^{n-1}$ *and* $K = K_0 \cup K_1I \cup \ldots \cup K_{n-1}I^{n-1}$ then $H \cap K = (H_0 \cap K_0) \cup (H_1 \cap K_1)I \cup \ldots \cup (H_{n-1} \cap K_{n-1})I^{n-1}$ by this argument we can easily find that the proof holds.

Theorem 3.12 :

Let M(G) , M(H) be two n , m-cyclic refined neutrosophic groups respectively, and $f: M(G) \to M(H)$ be a homomorphism, then

(a) $n \geq m$.

(b) If K is an AH-subgroup of M(G) then f(K) is an AH-subgroup of M(H).

(c) If K is an AHS-subgroup of M(G) then f(K) is an AHS-subgroup of M(H).

DOI: 10.5281/zenodo.4924483
Received:January 10, 2021, Accepted: May 05, 2021

(d) If K is an AH-normal subgroup of M(G) then f(K) is an AH-normal subgroup of f(M(G)).

(e) If K is an AHS-normal subgroup of M(G) then f(K) is an AHS-normal subgroup of f(M(G)).

(f) M(G)/Kerf ≅ f (M(G)).

Proof :

(a) Suppose that $n < m$ then $I' = f(I) = f(I^n) = (I')^n$ and this is a contradiction thus $n \geq m$.

(b) Suppose that $K = K_0 \cup K_1 I \cup \dots \cup K_{n-1} I^{n-1}$, then $f(K) = f(K_0) \cup f(K_1)I \cup \dots \cup f(K_{n-1})I^{n-1}$ with subgroups f(K_i) $for\ all\ i$ of M(H), so that $f(K)$ is an AH-subgroup of $M(H)$.

(c) It is obvious that if $K_i = K_j, then\ f(K_i) \cong f(K_j)$, thus $f(K)$ is an AHS-subgroup of M(H).

(d) , (e) hold directly from (b) and (c) and from the fact that if K_i is normal, then $f(K_i)$ is normal.

(f) From the definition, we find $M(G)/Kerf = (G/Kerf_G) \cup (G/Kerf_G)I \cup \dots \cup (G/Kerf_G)I^{n-1}$; but $G/Kerf_G \cong f(G)$, thus $M(G)/Kerf \cong f(G) \cup f(G)I \cup \dots \cup f(G)I^{n-1} = f(M(G))$.

Theorem 3.13 :

Let M(G) be an n-cyclic refined neutrosophic group and H , K be two AH-normal subgroups with K$\leq H$, then $(M(G)/K)/(H/K) \cong M(G)/H$.

Proof :

Suppose that = $H_0 \cup H_1 I \cup \dots \cup H_{n-1} I^{n-1}\ and\ K = K_0 \cup K_1 I \cup \dots \cup K_{n-1} I^{n-1}$ with K$\leq H$, then $(M(G)/K)/(H/K) = \ ((G/K_0) \cup \dots \cup (G/K_{n-1})I^{n-1})/((H_0/K_0) \cup \dots \cup (H_{n-1}/K_{n-1})I^{n-1}) \cong G/K_0/(H_0/K_0) \cup \dots \cup (G/K_{n-1})/(H_{n-1}/K_{n-1})I^{n-1} \cong G/H_0 \cup (G/H_1)I \cup \dots \cup (G/H_{n-1})I^{n-1} = M(G)/H$.

Theorem 3.14 :

Let M(G) be an n-cyclic refined neutrosophic group, and H is an AH-normal subgroup, then for each AH-subgroup T of M(G)/H there is an AH-subgroup of M(G) contains H.

Proof : It can be proved as the classical case.

Definition 3.15 :

Let M(G) , M(H) be two n-cyclic refined neutrosophic groups,

we define $M(G) \times M(H) = (G \times H) \cup (G \times H)II' \cup \dots \cup (G \times H)I^n(I')^n$ with $(II')^k = I^k(I')^k$ for all k , it is clear that $M(G) \times M(H)$ is an n-generalized neutrosophic group with neutrosophic element II'.

Theorem 3.16 :

Let M(G) , M(H) be two n-cyclic refined neutrosophic groups, then

(a) If M(G) , M(H) are abelian then $M(G) \times M(H)$ is abelian.

(b) If T,S are two AH-subgroups of M(G) , M(H) respectively, then $T \times S$ is an AH-subgroup of $M(G) \times M(H)$.

DOI: 10.5281/zenodo.4924483

Received:January 10, 2021, Accepted: May 05, 2021

(c) If T,S are two AH-normal subgroups of M(G) , M(H) respectively, then $T \times S$ is an AH-normal subgroup of $M(G) \times M(H)$.

(d) If T,S are two AHS-subgroups of M(G) , M(H) respectively, then $T \times S$ is an AHS-subgroup of $M(G) \times M(H)$.

(e) If T,S are two AHS-normal subgroups of M(G) , M(H) respectively then $T \times S$ is an AHS-normal subgroup of $M(G) \times M(H)$.

Proof :

(a) It is clear since $G \times H$ is abelian.

(b) Assume that $T = T_0 \cup \ldots \cup T_{n-1}I^{n-1}$ and $S = S_0 \cup \ldots \cup S_{n-1}(I^{'})^{n-1}$, then

$T \times S = (T_0 \times S_0) \cup \ldots \cup (T_{n-1} \times S_{n-1})(II^{'})^{n-1}$, we can regard that $T_i \times S_i$ is a subgroup of $G \times H$, so $T \times S$ is an AH-subgroup.

(c) It holds directly from (b).

(d) If $S_j \cong T_i$ *and* $S_j \cong S_i$ then $T_i \times S_i \cong T_j \times S_j$ and then $T \times S$ is an AHS-subgroup.

(e) It holds directly from (d) and (c).

Theorem 3.17 :

Let M(G) , M(H) be two n-cyclic refined neutrosophic groups and T,S be two AH-normal subgroups of M(G) , M(H) respectively, then

$M(G) \times M(H)/T \times S \ \cong M(G)/T \times M(H)/S.$

Proof :

Suppose that $T = T_0 \cup \ldots \cup T_{n-1}I^{n-1}$ and $S = S_0 \cup \ldots \cup S_{n-1}(I^{'})^{n-1}$, then

$M(G)/T \times M(H)/S = (G/T_0 \times H/S_0) \cup (G/T_1 \times H/S_1)II^{'} \cup \ldots \cup (G/T_{n-1} \times H/S_{n-1})(II^{'})^{n-1}$

$\cong \mathrm{G} \times \mathrm{H}/\mathrm{T}_0 \times \mathrm{S}_0 \times \ldots \times (G \times H/T_{n-1} \times S_{n-1})(II^{'})^{n-1} = M(G) \times M(H)/T \times S.$

Example 3.18:

Consider the additive group $(R^*,.)$ and the integer $n = 3$. The corresponding 3-cyclic refined neutrosophic group is

$M(R) = \{a, bI, cI^2; a, b, c \in R^*\}$.

1) We know that $(Q^*,.)$ is a subgroup of $(R^*,.)$, hence L=$Q^* \cup Q^*I \cup Q^*I^2$ =$\{x, yI, zI^2; x, y, z \in Q^*\}$ is a 3-cyclic refined AHS-subgroup.

2) The corresponding AH-factor is $M(R)/_L = R^*/_{Q^*} \cup R^*/_{Q^*} I \cup R^*/_{Q^*} I^2$.

Example 3.19:

DOI: 10.5281/zenodo.4924483
Received:January 10, 2021, Accepted: May 05, 2021

Consider the following two groups $G = (Z, +), H = (2Z, +)$, and the integer $n = 4$, the corresponding 4-cyclic refined neutrosophic groups are $M(G) = \{a, bI, cI^2, dI^3; a, b, c, d \in Z\}, M(H) = \{2a, 2bI, 2cI^2, 2dI^3; a, b, c, d \in Z\}$,

1) $g: G \to H; g(x) = 4x$ *is a group homomorphism*. $Ker(g) = \{0\}$.

2) The corresponding AH-homomorphism is $f: M(G) \to M(H); f(x) = 4x$, *and* $f(xI^l) = 4xI^l; 1 \leq l \leq 3$.

3) The corresponding AH-kernel is $AH - Ker(f) = Ker(g) \cup Ker(g)I \cup Ker(g)I^2 \cup Ker(g)I^3 = \{0, 0 + I, 0 + I^2, 0 + I^3\}$

Conclusion

In this paper, we have defined for the first time the concept of n-cyclic refined neutrosophic group as a new application of n-cyclic refined neutrosophic sets. We have discussed some of their elementary properties such as AH-subgroups, AH-kernels and direct products.

As a future research directions, we aim to study the n-cyclic refined neutrosophic semi groups and loops.

Funding: “This research received no external funding”

Conflicts of Interest: “The authors declare no conflict of interest.”

References

[1] Smarandache, F., " A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press. Rehoboth, 2003.

[2] Alhamido, R., and Gharibah, T., "Neutrosophic Crisp Tri-Topological Spaces", Journal of New Theory, Vol. 23 , pp.13-21. 2018.

[3] Edalatpanah. S.A., "Systems of Neutrosophic Linear Equations", Neutrosophic Sets and Systems, Vol. 33, pp. 92-104. 2020.

[4] Sankari, H., and Abobala, M., "Neutrosophic Linear Diophantine Equations With two Variables", Neutrosophic Sets and Systems, Vol. 38, pp. 22-30, 2020.

[5] Sankari, H., and Abobala, M." *n*-Refined Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 36, pp. 1-11. 2020.

[6] Alhamido, R., and Abobala, M., "AH-Substructures in Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 7, pp. 79-86 . 2020.

[7] Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 6 , pp. 80-86. 2020.

[8] Abobala, M.,. "A Study of AH-Substructures in *n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.

[9] Hatip, A., Alhamido, R., and Abobala, M., "A Contribution to Neutrosophic Groups", International Journal of Neutrosophic Science", Vol. 0, pp. 67-76 . 2019.

[10] Abobala, M., " *n*-Refined Neutrosophic Groups I", International Journal of Neutrosophic Science, Vol. 0, pp. 27-34. 2020.

DOI: 10.5281/zenodo.4924483

Received:January 10, 2021, Accepted: May 05, 2021

[11] Kandasamy, V.W.B., and Smarandache, F., "Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures", Hexis, Phonex, Arizona, 2006.

[12] Agboola, A.A.A., Akinola, A.D., and Oyebola, O.Y., " Neutrosophic Rings I" , International J.Mathcombin, Vol 4,pp 1-14. 2011

[13] Agboola, A.A.A., "On Refined Neutrosophic Algebraic Structures," Neutrosophic Sets and Systems,Vol.10, pp. 99-101. 2015.

[14] Abobala, M., "Classical Homomorphisms Between Refined Neutrosophic Rings and Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 72-75. 2020.

[15] Smarandache, F., and Abobala, M., n-Refined neutrosophic Rings, International Journal of Neutrosophic Science, Vol. 5 , pp. 83-90, 2020.

[16] Kandasamy, I., Kandasamy, V., and Smarandache, F., "Algebraic structure of Neutrosophic Duplets in Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 18, pp. 85-95. 2018.

[17] Yingcang, Ma., Xiaohong Zhang ., Smarandache, F., and Juanjuan, Z., "The Structure of Idempotents in Neutrosophic Rings and Neutrosophic Quadruple Rings", Symmetry Journal (MDPI), Vol. 11. 2019.

[18] Kandasamy, V. W. B,. Ilanthenral, K., and Smarandache, F., "Semi-Idempotents in Neutrosophic Rings", Mathematics Journal (MDPI), Vol. 7. 2019.

[19] Abobala, M., On Some Special Substructures of Neutrosophic Rings and Their Properties, International Journal of Neutrosophic Science", Vol. 4 , pp. 72-81, 2020.

[20] Smarandache, F., " An Introduction To neutrosophic Genetics", International Journal of neutrosophic Science, Vol.13, 2021.

[21] Martin, N, Smarandache, F, and Broumi, S., " Covid 19 Decision Making using Extended Plithogenic hypersoft Sets With Dual Dominent Attributes", International Journal of neutrosophic Science, Vol. 13, 2021.

[22]Agboola, A.A., "Introduction To Neutro groups", International Journal of neutrosophic Science, Vol. 6, 2020.

[23] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 59-66. 2020.

[24] Smarandache, F., and Ali, M., "Neutrosophic Triplet Group", Neural. Compute. Appl. 2019.

[25] Sankari, H., and Abobala, M.," AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 38, pp. 101-112, 2020.

[26] Smarandache, F., and Kandasamy, V.W.B., " Finite Neutrosophic Complex Numbers",·Source: arXiv. 2011.

[27]. Suresh, R., and S. Palaniammal,. "Neutrosophic Weakly Generalized open and Closed Sets", Neutrosophic Sets and Systems, Vol. 33, pp. 67-77,. 2020.

[28] Sahin, M., Olgun, N., Uluçay, V., Kargın, A., and Smarandache, F. , "A New Similarity Measure Based on Falsity Value between Single Valued Neutrosophic Sets Based on the Centroid Points of Transformed Single Valued Neutrosophic Numbers with Applications to Pattern Recognition", Neutrosophic Sets and Systems, vol. 15, pp. 31-48. 2017.

DOI: 10.5281/zenodo.4924483

Received:January 10, 2021, Accepted: May 05, 2021

[29] Ibrahim, M.A., Agboola, A.A.A, Badmus, B.S. and Akinleye, S.A., "On refined Neutrosophic Vector Spaces I", International Journal of Neutrosophic Science, Vol. 7, pp. 97-109. 2020.

[30] Ibrahim, M.A., Agboola, A.A.A, Badmus, B.S., and Akinleye, S.A., "On refined Neutrosophic Vector Spaces II", International Journal of Neutrosophic Science, Vol. 9, pp. 22-36. 2020.

[31] Abobala, M, "n-Cyclic Refined Neutrosophic Algebraic Systems Of Sub-Indeterminacies, An Application To Rings and Modules", International Journal of Neutrosophic Science, Vol. 12, pp. 81-95 . 2020.

[32] Smarandache, F., "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets", Inter. J. Pure Appl. Math., pp. 287-297. 2005.

[33] M. Ali, F. Smarandache, M. Shabir and L. Vladareanu., "Generalization of Neutrosophic Rings and Neutrosophic Fields", Neutrosophic Sets and Systems, vol. 5, pp. 9-14, 2014.

[34] Anuradha V. S., "Neutrosophic Fuzzy Hierarchical Clustering for Dengue Analysis in Sri Lanka", Neutrosophic Sets and Systems, vol. 31, pp. 179-199. 2020.

[35] Olgun, N., and Hatip, A., "The Effect Of The Neutrosophic Logic On The Decision Making, in Quadruple Neutrosophic Theory And Applications", Belgium, EU, Pons Editions Brussels,pp. 238-253. 2020.

[36] Zadeh, L. "Fuzzy Sets", Inform and Control, 8, pp.338-353. 1965.

[37] Turksen, I., "Interval valued fuzzy sets based on normal forms", Fuzzy Sets and Systems, 20, pp.191-210, 1986. 1986.

[38] Chalapathi, T., and Madhavi, L., "Neutrosophic Boolean Rings", Neutrosophic Sets and Systems, Vol. 33, pp. 57-66. 2020.

[39] Abobala, M., "Classical Homomorphisms Between n-refined Neutrosophic Rings", International Journal of Neutrosophic Science", Vol. 7, pp. 74-78. 2020.

[40] Agboola, A.A.A,. Akwu, A.D,. and Oyebo, Y.T., " Neutrosophic Groups and Subgroups", International .J .Math. Combin, Vol. 3, pp. 1-9. 2012.

[41] Smarandache, F., " n-Valued Refined Neutrosophic Logic and Its Applications in Physics", Progress in Physics, 143-146, Vol. 4, 2013.

[42] Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.

[43] Hatip, A., and Abobala, M., "AH-Substructures In Strong Refined Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 9, pp. 110-116 . 2020.

[44] Hatip, A., and Olgun, N., "On Refined Neutrosophic R-Module", International Journal of Neutrosophic Science, Vol. 7, pp.87-96. 2020.

[45] Chakraborty, A., Banik, B., Mondal, S.P., and Alam, S., "Arithmetic and Geometric Operators of Pentagonal Neutrosophic Number and its Application in Mobile Communication Service Based MCGDM Problem", Neutrosophic Sets and Systems, vol. 32, pp. 61-79. 2020.

[46] Smarandache F., and Abobala, M., "n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54. 2020.

DOI: 10.5281/zenodo.4924483

Received:January 10, 2021, Accepted: May 05, 2021

[47] Sankari, H., and Abobala, M., "Solving Three Conjectures About Neutrosophic Quadruple Vector Spaces", Neutrosophic Sets and Systems, Vol. 38, pp. 70-77. 2020.

[48] Adeleke, E.O., Agboola, A.A.A., and Smarandache, F., "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science, Vol. 2(2), pp. 89-94. 2020.

[49] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021

[50] Abobala, M., A Study of Maximal and Minimal Ideals of n-Refined Neutrosophic Rings, Journal of Fuzzy Extension and Applications, Vol. 2, pp. 16-22, 2021.

[51] Abobala, M., " Semi Homomorphisms and Algebraic Relations Between Strong Refined Neutrosophic Modules and Strong Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 39, 2021.

[52] Abobala, M., "On Some Neutrosophic Algebraic Equations", Journal of New Theory, Vol. 33, 2020.

[53] Abobala, M., On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations, Journal of Mathematics, Hindawi, 2021.

[54] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021

[55] Kandasamy V, Smarandache F., and Kandasamy I., Special Fuzzy Matrices for Social Scientists . Printed in the United States of America,2007, book, 99 pages.

[56] Khaled, H., and Younus, A., and Mohammad, A., " The Rectangle Neutrosophic Fuzzy Matrices", Faculty of Education Journal Vol. 15, 2019. (Arabic version).

[57] Abobala, M., Partial Foundation of Neutrosophic Number Theory, Neutrosophic Sets and Systems, Vol. 39 , 2021.

[58] F. Smarandache, *Neutrosophic Theory and Applications,* Le Quy Don Technical University, Faculty of Information technology, Hanoi, Vietnam, 17th May 2016.

[59] Sankari, H, and Abobala, M., " On A New Criterion For The Solvability of non Simple Finite Groups and m-Abelian Solvability, Journal of Mathematics, Hindawi, 2021.

[60] Giorgio, N, Mehmood, A., and Broumi, S.," Single Valued neutrosophic Filter", International Journal of Neutrosophic Science, Vol. 6, 2020.

[61] Es, Haydar, A., "A Note On neutrosophic Soft Menger Topological Spaces", International Journal of Neutrosophic Science, Vol.7, 2020.

[62] Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., The algebraic creativity In The Neutrosophic Square Matrices, Neutrosophic Sets and Systems, Vol. 40, pp. 1-11, 2021.

[63]Alhamido, K., R., "A New Approach of neutrosophic Topological Spaces", International Journal of neutrosophic Science, Vol.7, 2020.

[64] Chellamani, P., and Ajay, D., "Pythagorean neutrosophic Fuzzy Graphs", International Journal of Neutrosophic Science, Vol. 11, 2021.

DOI: 10.5281/zenodo.4924483

Received:January 10, 2021, Accepted: May 05, 2021

[65] Abobala, M., "On Some Special Elements In Neutrosophic Rings and Refined Neutrosophic Rings", Journal of New Theory, vol. 33, 2020.

[66]Milles, S, Barakat, M, and Latrech, A., " Completeness and Compactness In Standard Single Valued neutrosophic Metric Spaces", International Journal of Neutrosophic Science, Vol.12 , 2021.

[67] Agboola, A.A., "Introduction to Anti Groups", International Journal of Neutrosophic Science, vol.12, 2021.

[68] Sankari, H, and Abobala, M, " A Contribution to m-Power Closed Groups", UMM-Alqura University Journal for Applied Sciences, KSA, 2020.

DOI: 10.5281/zenodo.4924483

Received:January 10, 2021, Accepted: May 05, 2021