Introduction to Neutrosophic Soft Bitopological Spaces

Hasan Dadas [1]*, Sibel Demiralp[2]

[1]Department of Mathematics, Faculty of Arts and Sciences, Kastamonu University, Turkey;
Email: hsn.dad@gmail.com
[2]Department of Mathematics, Faculty of Arts and Sciences, Kastamonu University, Turkey;
Email sdemiralp@kastamonu.edu.tr

* Correspondence: hsn.dad@gmail.com

Abstract

In this study, the concept of neutrosophic soft bitopological space is defined and it is one of the few studies that have dealt with this concept. In addition, pairwise neutrosophic soft open (closed) set on neutrosophic soft bitopological spaces are studied. Supra neutrosophic soft topology is defined by pairwise neutrosophic soft open sets. Important theorems related to the subject supported with many examples for better understanding of the subject are given.

Keywords: Neutrosophic set; Neutrosophic soft set; Soft bitopological space; Neutrosophic soft bitopological space; Neutrosophic soft pairwise open (closed) set.

1.Introduction

In this world, there is a lot of complexity, neutrality, and lack of clarity. When we look at any topic, it is often described as true or false, while at other times it is defined as neutral or uncertain. This situation has no equivalent in standard classical logic. To describe this complexity and uncertainty, normal classical logic must evolve into a new logic. With this in mind, Zadeh [25] defined fuzzy logic in 1965. Then intuitionistic fuzzy set theory was defined by Attanasov [28] as a generalization of fuzzy theory. This theory does not address the vague and inconsistent information existing in the belief system. Smarandache [15,18,23] presented the notion of the neutrosophic set, which is a mathematical instrument to deal with problems including imprecise, uncertainty, and inconsistent data.

Molodtsov [16] characterized the concept of soft set, which is free from the parameterization. Then soft set theory was combined to fuzzy theory, intuitionistic fuzzy theory and neutrosophic theory.

In 1965, Kelly [13] expanded the notion of topological space to bitopological space. In 2019, bitopological space and its properties were extended to neutrosophic spaces by Öztürk et al [9].

In this study bitopological space is defined on neutrosophic soft set structure. The concepts of pairwise neutrosophic soft open set and pairwise neutrosophic soft closed set are given. Also, the neutrosophic soft topologies $\tau_1$and τ_2 are used to generate a family τ_{12}which is a supra neutrosophic soft topology on X.

2.Preliminary

In this section, basic definitions and theorems are given about neutrosophic set theory and neutrosophic soft set theory.

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2.1. Neutrosophic Sets

Let X be a space of points (objects), with a generic element in X denoted by x, a neutrosophic set A in X is characterized by a truth-membership function T, an indeterminacy-membership function I and a falsity-membership function F [23] . That are: $T, I, F: X \to]^{-}0, 1^{+}[$ where $T(x), I(x)$ and $F(x)$ are real standard or non-standard subsets of$]^{-}0,1^{+}[$. In general, there is no restriction on the sum of $T(x), I(x)$ and $F(x)$, so $^{-}0 \le T(x) + I(x) + F(x) \le 3^{+}$. T, I an d F are called neutrosophic components, the set of all neutrosophic sets in X is denoted by $N(X)$.

2.1.1 Definition [23]: Let $B, D \in N(X)$. Then

1. Subset: $D \subset B$ if $T_D(z) \le T_B(z), I_D(z) \le I_B(z), F_D(z) \ge F_B(z)$ for all $z \in X$.
2. Equality: $D = B$ if $D \subset B$ and $B \subset D$.
3. Intersection:

   $D \cap B = \{< z, min\{T_D(z), T_B(z)\}, \min\{I_D(z), I_B(z)\}, \max\{F_D(z), F_B(z)\} > : z \in X\}$.
4. Union:

   $D \cup B = \{< z, max\{T_D(z), T_B(z)\}, \max\{I_D(x), I_B(x)\}, \min\{F_D(z), F_B(z)\} > : z \in X\}$

More generally, the intersection and the union of a collection of neutrosophic sets $\{D_i\} \in I$ are defined by:

$$\bigcap_{i\in I} D_i = \{< z, \inf\{T_{D_i}(z)\}, \inf\{I_{D_i}(z)\}, \sup\{F_{D_i}(z)\} >: z \in X\}$$

$$\bigcup_{i\in I} D_i = \{< z, \sup\{T_{D_i}(z)\}, \sup\{I_{D_i}(z)\}, \inf\{F_{D_i}(z)\} >: z \in X\}$$

5. The neutrosophic set defined as $T_D(z) = 1, I_D(z) = 1$ and $F_D(z) = 0$ for all $z \in X$ is called the universal NS denoted by 1_X. Also, the neutrosophic set defined as $T_D(z) = 0, I_D(z) = 0$ and $F_D(z) = 1$ for all $z \in X$ is called the empty NS denoted by 0_X.
6. Difference: $D\backslash B = \{< z, |T_D(z) - T_B(z)|, |I_D(z) - I_B(z)|, |F_D(z) - F_B(z)| >: z \in X\}$
7. Complement: $D^c = 1_X \backslash D$

   Clearly, the complements of 1_X and 0_X are defined:

   $$(1_X)^c = 1_X \backslash 1_X = \{< z, 0,0,1 >: z \in X\} = 0_X$$
   $$(0_X)^c = 1_X \backslash 0_X = \{< z, 1,1,0 >: z \in X\} = 1_X$$

2.1.2. Proposition [27]: Let $D_1, D_2, D_3, D_4 \in N(X)$. Then the followings hold:

1. $D_1 \cap D_3 \subset D_2 \cap D_4$ and $D_1 \cup D_3 \subset D_2 \cup D_4$ if $D_1 \subset D_2$ and $D_3 \subset D_4$
2. $({D_1}^c)^c = D_1$ and $D_1 \subset D_2$ if ${D_2}^c \subset {D_1}^c$
3. $(D_1 \cap D_2)^c = {D_1}^c \cup {D_2}^c$ and $(D_1 \cup D_2)^c = {D_1}^c \cap {D_2}^c$

2.1.3 Definition [22]: Let $\Gamma \subset N(Y)$. Then Γ is named a neutrosophic topology on Y if the following conditions hold;

1. 0_X and 1_X are belong to Γ
2. Union of any number of neutrosophic sets in Γ is again belong to Γ
3. Intersection of finite number of neutrosophic sets in Γ is belong to Γ

Then the pair (Y, Γ) is named neutrosophic topology on Y.

2.2 Neutrosophic Soft Sets

2.2.1. Definition [27]: Let U be an initial universe set and E be a set of parameters. Then pair (H, E) is called as neutrosophic soft set (NSS) over U, where H is a mapping from E to $N(U)$.

The set of all NSS over U is denoted by $NSS(U)$. A neutrosophic set (H, E) can be written as:

$$(H, E) = \{(e, < x, T_{H(e)}(x), I_{H(e)}(x), F_{H(e)}(x) >: x \in U, e \in E\}$$

2.2.2 Definition [26]: Let X be an initial universe set and E be a set of parameters. Then the neutrosophic soft set ${x^e}_{(\alpha,\beta,\gamma)}$ defined as

$$ {x^e}_{(\alpha,\beta,\gamma)}(e')(y) = \begin{cases} (\alpha,\beta,\gamma) \text{ if } e = e' \text{and } x = y \\ (0,0,1) \text{ if } e \neq e' \text{and } x \neq y \end{cases} $$

for all $x \in X, 0 < \alpha,\beta,\gamma \leq 1, e \in E$, is called a neutrosophic soft point.

2.2.2. Definition [3]: Let $(H,E),(G,E) \in NSS(U)$. Then for all $x \in U$

1. Subset: $(H,E) \subset (G,E)$ if $T_{H(e)}(x) \leq T_{G(e)}(x), I_{H(e)}(x) \leq I_{G(e)}(x)$ and $F_{H(e)}(x) \geq F_{G(e)}(x)$ for all $e \in E$.
2. Equality: $(H,E) = (G,E)$ if $(H,E) \subset (G,E)$ and $(G,E) \subset (H,E)$.
3. Intersection:
$(H,E) \cap (G,E) = \{(e, \{< x, min\{T_{H(e)}(x), T_{G(e)}(x)\}, \max\{I_{H(e)}(x), I_{G(e)}(x)\}, \max\{F_{H(e)}(x), F_{G(e)}(x)\} >\} : e \in E\}$.
4. Union:
$(H,E) \cup (G,E) = \{(e, \{< x, max\{T_{H(e)}(x), T_{G(e)}(x)\}, \min\{I_{H(e)}(x), I_{G(e)}(x)\}, \min\{F_{H(e)}(x), F_{G(e)}(x)\} >\} : e \in E\}$

More generally, the intersection and the union of a collection of $\{(H_i, E)\} \subset NSS(U)$ are defined by:

$$ \bigcap_{i\in I}(H_i,E) = \left\{(e, \left\{< x, \inf\left\{T_{H_{i(e)}}(x)\right\}, \inf\{I_{H_i(e)}(x)\}, \sup\{F_{H_i(e)}(x)\} >\right\} : e \in E\right\} $$
$$ \bigcup_{i\in I}(H_i,E) = \left\{(e, \left\{< x, \sup\left\{T_{H_{i(e)}}(x)\right\}, \sup\left\{I_{H_{i(e)}}(x)\right\}, \inf\left\{F_{H_{i(e)}}(x)\right\} >\right\} : e \in E\right\} $$

5. The NSS defined as $T_{H(e)}(x) = 1, I_{H(e)}(x) = 0$ and $F_{H(e)}(x) = 0$, for all $e \in E$ and $x \in U$ is called the universal NSS denoted by $1_{(U,E)}$. Also, the neutrosophic set defined as $T_{H(e)}(x) = 0, I_{H(e)}(x) = 0$ and $F_{H(e)}(x) = 1$ for all $e \in E$ and $x \in U$ is called the empty NSS denoted by $0_{(U;E)}$.
6. Complement: $(H,E)^c = 1_{(X,E)} \backslash (H,E) = \{(e < e, F_{H(e)}(x), 1 - I_{H(e)}(x), T_{H(e)}(x) >: e \in E\}$
Clearly, the complements of $1_{(X,E)}$ and $0_{(X,E)}$ are defined:
$$ (1_{(X,E)})^c = 1_{(X,E)} \backslash 1_{(X,E)} = \{(e, \{< x, 0,0,1 >\} : e \in E\} = 0_{(X,E)} $$
$$ (0_{(X,E)})^c = 1_{(X,E)} \backslash 0_{(X,E)} = \{(e, \{< x, 1,0,0 >\} : e \in E\} = 1_{(X,E)} $$

2.2.3 Definition [4]: Let $\Gamma \subset NSS(Y)$. Then Γ is named a neutrosophic soft topology on Y if the following conditions hold

NST1) $0_{(U;E)}$ and $1_{(U,E)}$ are belong to Γ.

NST2) Union of any number of NSSs in Γ is again belong to Γ.

NST3) Intersection of finite number of NSSs in Γ is belong to Γ.

Then the pair (Y, Γ) is named neutrosophic soft topology on Y. Elements of Γ is called as neutrosophic soft open set. An NSS whose complement is neutrosophic soft open is called as neutrosophic soft closed set.

2.2.4 Definition [11]: Let $\Gamma \subset NSS(Y)$. Then Γ is named a neutrosophic soft supra topology on Y if $0_{(U;E)}, 1_{(U,E)} \in \Gamma$ and union of any number of NSSs in Γ is again belong to Γ.

2. Neutrosophic Soft Bitopological Spaces

In this part, the concept of neutrosophic soft bitopological space is defined. Furthermore, new types of open and closed sets have been introduced in neutrosophic soft bitopological spaces.

2.1 Definition: If (Y, τ_1, E) and (Y, τ_2, E) are two neutrosophic soft topological space, then (Y, τ_1, τ_2, E) is named as neutrosophic soft bitopological space. The sets belong to τ_i are called as neutrosophic soft i −open set for $i = 1,2$.

2.2 Example: Let $Y = \{y_1, y_2, y_3\}$, $E = \{e_1, e_2\}$ and $\tau_1 = \{0_{(Y,E)}, 1_{(Y,E)}, (M_1, E), (M_2, E)\}$,

$\tau_1 = \{0_{(Y,E)}, 1_{(Y,E)}, (N_1, E), (N_2, E)\}$ where $(M_1, E), (M_2, E), (N_1, E)$ and (N_2, E) being NSSs are as following:

$$f_{(M_1,E)}(e_1) = \{< y_1, 0.8,0.3,0.2 >, < y_2, 0.4,0.4,0.4 >, < y_3, 0.3,0.4,0.2 >\}$$

$$f_{(M_1,E)}(e_2) = \{< y_1, 0.6,0.2,0.3 >, < y_2, 0.5,0.5,0.1 >, < y_3, 0.5,0.3,0.4 >\}$$

$$f_{(M_2,E)}(e_1) = \{< y_1, 0.6,0.3,0.4 >, < y_2, 0.3,0.5,0.4 >, < y_3, 0.2,0.5,0.3 >\}$$

$$f_{(M_2,E)}(e_2) = \{< y_1, 0.5,0.4,0.3 >, < y_2, 0.4,0.6,0.2 >, < y_3, 0.3,0.6,0.4 >\}$$

$$f_{(N_1,E)}(e_1) = \{< y_1, 0.4,0.4,0.5 >, < y_2, 0.2,0.6,0.6 >, < y_3, 0.1,0.6,0.4 >\}$$

$$f_{(N_1,E)}(e_2) = \{< y_1, 0.3,0.6,0.4 >, < y_2, 0.3,0.7,0.3 >, < y_3, 0.1,0.7,0.5 >\}$$

$$f_{(N_2,E)}(e_1) = \{< y_1, 0.7,0.2,0.1 >, < y_2, 0.3,0.3,0.3 >, < y_3, 0.2,0.2,0.1 >\}$$

$$f_{(N_2,E)}(e_2) = \{< y_1, 0.7,0.2,0.1 >, < y_2, 0.3,0.3,0.3 >, < y_3, 0.2,0.2,0.1 >\}.$$

Then $(M_1, E) \cap (M_2, E) = (M_2, E)$, $(M_1, E) \cap (N_1, E) = (N_1, E)$, $(M_1, E) \cap (N_2, E) = (M_1, E)$, $(N_1, E) \cap (N_2, E) = (N_1, E)$, $(M_2, E) \cap (N_2, E) = (M_2, E)$ and $(M_1, E) \cup (M_2, E) = (M_2, E)$, $(M_1, E) \cup (N_1, E) = (N_1, E)$, $(M_1, E) \cup (N_2, E) = (M_2, E)$, $(N_1, E) \cup (N_2, E) = (N_1, E)$, $(M_2, E) \cup (N_2, E) = (N_2, E)$
Therefore τ_1 and τ_2 are neutrosophic soft topologies on Yand so (Y, τ_1, τ_2, E) is a neutrosophic soft bitopological space.

2.3 Theorem: Let (Y, τ_1, τ_2, E) be a neutrosophic soft bitopological space. Then $\tau_1 \cap \tau_2$ is a neutrosophic soft topology on Y.

Proof: NST1 and NST3 are clear. For NST2, let $\{(M_i, E); i \in I\} \in \tau_1 \cap \tau_2$. Then $(M_i, E) \in \tau_1$and$(M_i, E) \in \tau_2$. As τ_2 and τ_2 are neutrosophic soft topologies on Y, then $\cup_i (M_i, E) \in \tau_1and\cup_i (M_i, E) \in \tau_2$. Therefore $\cup_i (M_i, E) \in \tau_1 \cap \tau_2$.

2.4 Remark: Let (Y, τ_1, τ_2, E) be a neutrosophic soft bitopological space, then $\tau_1 \cup \tau_2$ need not be a neutrosophic soft topological space on Y.

2.5 Example: Let $Y = \{y_1, y_2, y_3\}$, $E = \{e_1, e_2\}$ and $\tau_1 = \{0_{(Y,E)}, 1_{(Y,E)}, (M_1, E), (M_2, E), (M_3, E)\}$,$\tau_1 = \{0_{(Y,E)}, 1_{(Y,E)}, (N_1, E), (N_2, E)\}$ where $(M_1, E), (M_2, E), (N_1, E)$ and (N_2, E) being NSSs are as following:

$$f_{(M_1,E)}(e_1) = \{< y_1, 0.8,0.3,0.2 >, < y_2, 0.4,0.4,0.4 >, < y_3, 0.3,0.4,0.2 >\}$$

$$f_{(M_1,E)}(e_2) = \{< y_1, 0.6,0.2,0.3 >, < y_2, 0.5,0.5,0.1 >, < y_3, 0.5,0.3,0.4 >\}$$

$$f_{(M_2,E)}(e_1) = \{< y_1, 0.6,0.3,0.4 >, < y_2, 0.3,0.5,0.4 >, < y_3, 0.2,0.5,0.3 >\}$$

$$f_{(M_2,E)}(e_2) = \{< y_1, 0.5,0.4,0.3 >, < y_2, 0.4,0.6,0.2 >, < y_3, 0.3,0.6,0.4 >\}$$

$$f_{(M_3,E)}(e_1) = \{< y_1, 0.4,0.4,0.5 >, < y_2, 0.2,0.6,0.6 >, < y_3, 0.1,0.6,0.4 >\}$$

$$f_{(M_3,E)}(e_2) = \{< y_1, 0.3,0.6,0.4 >, < y_2, 0.3,0.7,0.3 >, < y_3, 0.1,0.7,0.5 >\}$$

$$f_{(N_1,E)}(e_1) = \{< y_1, 0.4,0.4,0.5 >, < y_2, 0.2,0.6,0.6 >, < y_3, 0.1,0.6,0.4 >\}$$

$$f_{(N_1,E)}(e_2) = \{< y_1, 0.3,0.6,0.4 >, < y_2, 0.3,0.7,0.3 >, < y_3, 0.1,0.7,0.5 >\}$$

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$$f_{(N_2,E)}(e_1) = \{< y_1, 0.7,0.2,0.1 >, < y_2, 0.3,0.3,0.3 >, < y_3, 0.2,0.2,0.1 >\}$$

$$f_{(N_2,E)}(e_2) = \{< y_1, 0.7,0.2,0.1 >, < y_2, 0.3,0.3,0.3 >, < y_3, 0.2,0.2,0.1 >\}.$$

Here $\tau_1 \cup \tau_2 = \{0_{(Y,E)}, 1_{(Y,E)}, (M_1, E), (M_2, E), (M_3, E), (N_1, E), (N_2, E)\}$ is not a neutrosophic soft topology on Y. Because

$$(M_3, E) \cup (N_1, E) = \{(e_1, \{< x_1, 0.4,0.4,0.6 >, < x_2, 0.2,0.6,0.6 >, < x_3, 0.1,0.7,0.5 >\}),$$
$$(e_2, \{< x_1, 0.3,0.6,0.6 >, < x_2, 0.3,0.7,0.4 >, < x_3, 0.1,0.7,0.6 >\})\}$$

is not in $\tau_1 \cup \tau_2$.

2.6 Definition: Let (Y, τ_1, τ_2, E) be a neutrosophic soft bitopological space. Then an NSS

$$(H, E) = \{(e, \{< x, T_{H(e)}(x), I_{H(e)}(x), F_{H(e)}(x) >\}: x \in U, e \in E\}$$

is called as a pairwise neutrosophic soft open set if there exist a neutrosophic soft open (H_1, E) in τ_1 and a neutrosophic soft open (H_2, E) in τ_2 such that for all $x \in U$

$$(H, E) = (H_1, E) \cup (H_2, E)$$
$$= \{(e, \{< x, max\{T_{H(e)}(x), T_{G(e)}(x)\}, \min\{I_{H(e)}(x), I_{G(e)}(x)\}, \min\{F_{H(e)}(x), F_{G(e)}(x)\} >\}): e \in E\}$$

2.7 Definition: Let (Y, τ_1, τ_2, E) be a neutrosophic soft bitopological space. Then an NSS

$$(H, E) = \{(e, \{< x, T_{H(e)}(x), I_{H(e)}(x), F_{H(e)}(x) >\}): x \in U, e \in E\}$$

is called as a pairwise neutrosophic soft open set if there exist a neutrosophic soft open set (H_1, E) in τ_1 and a neutrosophic soft open set (H_2, E) in τ_2 such that for all $x \in U$

$$(H, E) = (H_1, E) \cup (H_2, E) = \{(e, \{< x, max\{T_{H(e)}(x), T_{G(e)}(x)\}, \min\{I_{H(e)}(x), I_{G(e)}(x)\}, \min\{F_{H(e)}(x), F_{G(e)}(x)\} >\}): e \in E\}$$

The set of all pairwise neutrosophic open sets in (Y, τ_1, τ_2, E) is denoted by $PNSO(Y, \tau_1, \tau_2, E)$.

2.8 Definition: Let (Y, τ_1, τ_2, E) be a neutrosophic soft bitopological space. Then an NSS

$$(H, E) = \{(e, \{< x, T_{H(e)}(x), I_{H(e)}(x), F_{H(e)}(x) >\}): x \in U, e \in E\}$$

is called as a pairwise neutrosophic soft closed set if $(H, E)^c$ is a pairwise neutrosophic soft open set. It is clear that (H, E) is a pairwise neutrosophic soft closed set if there exist a neutrosophic soft closed set (H_1, E) in τ_1 and a neutrosophic soft closed set(H_2, E) in τ_2 such that for all $x \in U$

$$(H, E) = (H_1, E) \cap (H_2, E) = \{(e, \{< x, min\{T_{H(e)}(x), T_{G(e)}(x)\}, \max\{I_{H(e)}(x), I_{G(e)}(x)\}, \max\{F_{H(e)}(x), F_{G(e)}(x)\} >\}): e \in E\}$$

The set of all pairwise neutrosophic closed sets in (Y, τ_1, τ_2, E) is denoted by $PNSC(Y, \tau_1, \tau_2, E)$.

2.9 Example: Let $X = \{x_1, x_2, x_3\}$,$E = \{e_1, e_2\}$, $\tau_1 = \{0_{(X,E)}, 1_{(X,E)}, (M_1, E)\}$, $\tau_2 = \{0_{(X,E)}, 1_{(X,E)}, (M_2, E)\}$ where $(M_1, E)$and (M_2, E) are defined as

$$f_{(M_1,E)}(e_1) = \{< y_1, 0.8,0.3,0.2 >, < y_2, 0.4,0.4,0.4 >, < y_3, 0.3,0.4,0.2 >\}$$

$$f_{(M_1,E)}(e_2) = \{< y_1, 0.6,0.2,0.3 >, < y_2, 0.5,0.5,0.1 >, < y_3, 0.5,0.3,0.4 >\}$$

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$$f_{(M_2,E)}(e_1) = \{< y_1, 0.6,0.2,0.4 >, < y_2, 0.5,0.5,0.1 >, < y_3, 0.5,0.3,0.4 >\}$$

$$f_{(M_2,E)}(e_2) = \{< y_1, 0.6,0.3,0.4 >, < y_2, 0.3,0.5,0.4 >, < y_3, 0.2,0.5,0.3 >\}.$$

Then

$$(M_1, E) \cup (M_2, E) = \{(e_1, \{\langle x_1, 0.8,0.2,0.2\rangle, \langle x_2, 0.5,0.4,0.1\rangle, \langle x_3, 0.5,0.3,0.2\rangle\}), (e_2, \{\langle x_1, 0.6,0.2,0.3\rangle, \langle x_2, 0.5,0.5,0.1\rangle, \langle x_3, 0.5,0.3,0.3\rangle\})\}$$

is a pairwise neutrosophic soft open set. Also

$$f_{(M_1,E)^c}(e_1) = \{< y_1, 0.2,0.7,0.8 >, < y_2, 0.6,0.6,0.6 >, < y_3, 0.2,0.6,0.3 >\}$$

$$f_{(M_1,E)^c}(e_2) = \{< y_1, 0.3,0.8,0.6 >, < y_2, 0.1,0.5,0.5 >, < y_3, 0.4,0.7,0.5 >\}$$

$$f_{(M_2,E)^c}(e_2) = \{< y_1, 0.3,0.8,0.6 >, < y_2, 0.1,0.5,0.5 >, < y_3, 0.4,0.7,0.3 >\}$$

$$f_{(M_2,E)^c}(e_1) = \{< y_1, 0.4,0.7,0.6 >, < y_2, 0.4,0.5,0.3 >, < y_3, 0.3,0.5,0.2 >\}.$$

Therefore

$$(M_1, E)^c \cap (M_2, E)^c = \{(e_1, \{\langle x_1, 0.2,0.7,0.8\rangle, \langle x_2, 0.6,0.6,0.6\rangle, \langle x_3, 0.7,0.6,0.8\rangle\}) (e_2, \{\langle x_1, 0.4,0.8,0.7\rangle, \langle x_2, 0.5,0.5,0.9\rangle, \langle x_3, 0.1,0.7,0.6\rangle\})\}$$

is a pairwise neutrosophic soft closed set.

2.10 Theorem: Let (X, τ_1, τ_2, E)be a neutrosophic soft bitopological space.

1. $0_{(X,E)}, 1_{(X,E)} \in PNSO(X, \tau_1, \tau_2, E)$.
2. If$\{(H_i, E)|i \in I\} \subseteq PNSO(X, \tau_1, \tau_2, E)$ then $\bigcup_{i\in I}(H_i, E) \in PNSO(X, \tau_1, \tau_2, E)$.
3. If$\{(G_i, E)|i \in I\} \subseteq PNSC(X, \tau_1, \tau_2, E)$ then $\bigcap_{i\in I}(G_i, E) \in PNSC(X, \tau_1, \tau_2, E)$.

Proof:

1. Since $0_{(X,E)} \cup 0_{(X,E)} = 0_{(X,E)}$and $1_{(X,E)} \cup 1_{(X,E)} = 1_{(X,E)}$ then $0_{(X,E)}$ and$1_{(X,E)}$ are pairwise neutrosophic soft closed sets.
2. Since $(H_i, E) \in PNSO(X, \tau_1, \tau_2, E)$, there exist $(H_i^1, E) \in \tau_1$ and $(H_i^1, E) \in \tau_2$ such that $(H_i, E) = (H_i^1, E) \cup (H_i^1, E)$ for all $i \in I$.
$$\bigcup_{i\in I}(H_i, E) = \bigcup_{i\in I}((H_i^1, E) \cup (H_i^1, E)) = (\bigcup_{i\in I}(H_i^1, E)) \cup (\bigcup_{i\in I}(H_i^2, E)).$$
As $\tau_1$andτ_2 are neutrosophic soft topologies on X, $\bigcup_{i\in I}(H_i^1, E) \in \tau_1$ and $\bigcup_{i\in I}(H_i^2, E) \in \tau_1$.
Therefore $\bigcup_{i\in I}(H_i, E) \in PNSO(X, \tau_1, \tau_2, E)$.
3. Since $(G_i, E) \in PNSC(X, \tau_1, \tau_2, E)$,there exist $(G_i^1, E)^c \in \tau_1$ and$(G_i^2, E)^c \in \tau_2$ such that $(G_i, E) = (G_i^1, E) \cap (G_i^2, E)$ for all $i \in I$. Then
$$\bigcap_{i\in I}(G_i, E) = \bigcap_{i\in I}((G_i^1, E) \cap (G_i^2, E)) = (\bigcap_{i\in I}(G_i^1, E)) \cap (\bigcap_{i\in I}(G_i^2, E)).$$
$$\bigcap_{i\in I}(G_i, E) \in PNSC(X, \tau_1, \tau_2, E) \text{as } (\bigcap_{i\in I}(G_i^1, E))^c \in \tau_1 \text{ and } (\bigcap_{i\in I}(G_i^2, E))^c \in \tau_1.$$

2.11 Corollary: Let (X, τ_1, τ_2, E)be a neutrosophic soft bitopological space. Then$PNSO(X, \tau_1, \tau_2, E)$ is asupra neutrosophic soft topology on X. This topology is denoted by τ_{12}.

2.12 Theorem: Let (X, τ_1, τ_2, E)be a neutrosophic soft bitopological space. Thenevery neutrosophic softi $-$open set is a pairwise neutrosophic soft open set.

Proof: Let$(H, E) \in \tau_I$ or $(H, E) \in \tau_2$. Since $(H, E) = (H, E) \cup 0_{(X,E)}$, then $(H, E) \in PNSO(X)$.

2.13 Corollary: Let (X, τ_1, τ_2, E)be a neutrosophic soft bitopological space. Then $\tau_1 \cup \tau_2 \subset \tau_{12}$.

The following example shows that the inverse of Theorem 2.12 does not hold.

2.13 Example: Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau_1 = \{\{0_{(X,E)}, 1_{(X,E)}, (M_1, E)\}$ $\tau_2 = \{0_{(X,E)}, 1_{(X,E)}, (M_2, E)\}$ where $(M_1, E)$and(M_2, E) be NSSs defined as:

$$f_{(M_1,E)}(e_1) = \{< x_1, 0.8,0.3,0.2 >, < x_2, 0.4,0.4,0.4 >, < x_3, 0.3,0.4,0.2 >\},$$

$$f_{(M_1,E)}(e_2) = \{< x_1, 0.6,0.2,0.3 >, < x_2, 0.5,0.5,0.1 >, < x_3, 0.5,0.3,0.4 >\}.$$

$$f_{(M_2,E)}(e_1) = \{< x_1, 0.6,0.2,0.3 >, < x_2, 0.5,0.5,0.1 >, < x_3, 0.5,0.3,0.4 >\}$$

$$f_{(M_2,E)}(e_2) = \{< x_1, 0.6,0.3,0.4 >, < x_2, 0.3,0.5,0.4 >< x_3, 0.2,0.5,0.3 >\}.$$

Then $\tau_{12} = \tau_1 \cup \tau_2 \cup \{(M_1, E) \cup (M_2, E)\}$ because the neutrosophic soft sets $(M_1, E) \cup (M_2, E)$ does not belong to either $\tau_1$nor τ_2.

2.13 Theorem: Let (X, τ_1, τ_2, E)be a neutrosophic soft bitopological space. Then every neutrosophic softi $-$closed set is a pairwise neutrosophic soft closed set.

Proof: Similar to proof of Theorem 2.12.

2.14 Theorem: Let (X, τ_1, τ_2, E)be a neutrosophic soft bitopological space. If $\tau_1 \subset \tau_2$, then $\tau_{12} = \tau_2$.

Proof: Let $\tau_1 \subset \tau_2$ and $(H, E) \in \tau_{12}$. Then there exist a neutrosophic soft open set (H_1, E) in τ_1 and a neutrosophic soft open set (H_2, E) in τ_2 such that $(H, E) = (H_1, E) \cup (H_2, E)$. Since $\tau_1 \subset \tau_2$, $(H_1, E) \in \tau_2$. Then $(H, E) \in \tau_2$, i.e. $\tau_{12} \subset \tau_2$. From theorem 2.12, $\tau_2 \subset \tau_{12}$.

2.15 Definition: Let (X, τ_1, τ_2, E) be a neutrosophic soft bitopological space and $(N, E) \in NSS(X)$. The pairwise neutrosophic soft closure of (N, E), denoted by $cl_P^{NSS}(N, E)$, is the intersection of all pairwise neutrosophic soft closed sets containing (N, E), i.e.,

$$cl_P^{NSS}(N, E) = \cap \{(M, E) \in PNSC(X) | (N, E) \subset (M, E)\}$$

It is clear that $cl_P^{NSS}(N, E)$ is the smallest pairwise neutrosophic soft closed set containing (N, E).

2.16 Example: Let (Y, τ_1, τ_2, E)be the same as in Example 2.5 and

$$(G, E) = \{(e_1, \{< y_1, 0.3,0.6,0.7 >, < y_2, 0.3,0.4,0.4 >, < y_3, 0.2,0.4,0.4 >\})$$
$$(e_2, \{< y_1, 0.2,0.5,0.6 >, < y_2, 0.1,0.3,0.7 >, < y_3, 0.3,0.3,0.4 \rangle\})\}.$$

be a neutrosophic soft set over Y. Now, we need to determine pairwise neutrosophic soft closed sets in(Y, τ_1, τ_2, E) to find $cl_P^{NSS}(G, E)$. Then,

$$f_{(M_2,E)}(e_1) = \{< y_1, 0.6,0.3,0.4 >, < y_2, 0.3,0.5,0.4 >, < y_3, 0.2,0.5,0.3 >\}$$

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$$f_{(M_2,E)}(e_2) = \{< y_1, 0.5, 0.4, 0.3 >, < y_2, 0.4, 0.6, 0.2 >, < y_3, 0.3, 0.6, 0.4 >\}$$

and

$$(M_2, E)^c = \{(e_1, \{< y_1, 0.4, 0.7, 0.6 >, < y_2, 0.4, 0.5, 0.3 >, < y_3, 0.3, 0.5, 0.2 >\}),$$

$$(e_2, \{< y_1, 0.3, 0.6, 0.5 >, < y_2, 0.2, 0.4, 0.6 >, < y_3, 0.4, 0.4, 0.3 >\})\}.$$

The pairwise neutrosophic soft closed sets which contains (G, E) are $(M_2, E)^c$and $1_{(X,E)}$. Therefore

$$cl_P^{NSS}(G, E) = (M_2, E)^c \cap 1_{(X,E)} = (M_2, E)^c.$$

2.17 Theorem: Let (X, τ_1, τ_2, E) be a neutrosophic soft bitopological spaceand $(N, E), (M, E) \in NSS(X)$. Then,

1. $cl_P^{NSS}(0_{(X,E)}) = 0_{(X,E)}$and $cl_P^{NSS}(1_{(X,E)}) = 1_{(X,E)}$
2. $(N, E) \subseteq cl_P^{NSS}(N, E)$
3. (N, E) is a pairwise neutrosophic soft closed set if $cl_P^{NSS}(N, E) = (N, E)$
4. $cl_P^{NSS}(N, E) \subseteq cl_P^{NSS}(M, E)$ if $(N, E) \subseteq (M, E)$
5. $cl_P^{NSS}(N, E) \cup cl_P^{NSS}(M, E) \subset cl_P^{NSS}((N, E) \cup (M, E))$
6. $cl_P^{NSS}\left(cl_P^{NSS}(N, E)\right) = cl_P^{NSS}(N, E)$, i.e., $cl_P^{NSS}(N, E)$ is a pairwise neutrosophic soft closed set.

Proof. Straightforward.

2.18 Theorem: Let (X, τ_1, τ_2, E) be a neutrosophic soft bitopological spaceand $(N, E) \in NSS(X)$. Then, $x^e{}_{(\alpha,\beta,\gamma)} \in cl_P^{NSS}(N, E)$if and only if for all $U_{x^e{}_{(\alpha,\beta,\gamma)}} \in \tau_{12}(x^e{}_{(\alpha,\beta,\gamma)})$ where $U_{x^e{}_{(\alpha,\beta,\gamma)}}$ is any pairwise neutrosophic soft open set contains $x^e{}_{(\alpha,\beta,\gamma)}$and $\tau_{12}(x^e{}_{(\alpha,\beta,\gamma)})$ is the family of all pairwise neutrosophic soft open sets contains $x^e{}_{(\alpha,\beta,\gamma)}$,

$$U_{x^e{}_{(\alpha,\beta,\gamma)}} \cap (N, E) \neq 0_{(X,E)}.$$

Proof: Let $x^e{}_{(\alpha,\beta,\gamma)} \in cl_P^{NSS}(N, E)$ and suppose that there exists $U_{x^e{}_{(\alpha,\beta,\gamma)}} \in \tau_{12}(x^e{}_{(\alpha,\beta,\gamma)})$ such that $U_{x^e{}_{(\alpha,\beta,\gamma)}} \cap (N, E) = 0_{(X,E)}$. Then $(N, E) \subset \left(U_{x^e{}_{(\alpha,\beta,\gamma)}}\right)^c$. Thus $cl_P^{NSS}(N, E) \subset cl_P^{NSS}\left(U_{x^e{}_{(\alpha,\beta,\gamma)}}\right)^c = \left(U_{x^e{}_{(\alpha,\beta,\gamma)}}\right)^c$ which implies $cl_P^{NSS}(N, E) \cap U_{x^e{}_{(\alpha,\beta,\gamma)}} = 0_{(X,E)}$, a contradiction.

Conversely, assume that $x^e{}_{(\alpha,\beta,\gamma)} \notin cl_P^{NSS}(N, E)$, then $x^e{}_{(\alpha,\beta,\gamma)} \in \left(cl_P^{NSS}(N, E)\right)^c \in \tau_{12}(x^e{}_{(\alpha,\beta,\gamma)})$. Therefore, by hypothesis,$\left(cl_P^{NSS}(N, E)\right)^c \cap (N, E) \neq 0_{(X,E)}$, a contradiction.

7. Conclusions and Future Work

In this study, the concept of bitopology is expanded to neutrosophic soft set theory. Pairwise neutrosophic soft open and pairwise neutrosophic soft closed sets are given. In addition, supra neutrosophic soft topology is defined by pairwise neutrosophic soft open sets. For the future work, neighbourhood structures will be studied and some separation axioms will be given on neutrosophic soft bitopological spaces.

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