Neutrosophic Soft Filter Structures Concerning Soft Points

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Abstract

In this paper, the concept of neutrosophic soft filter (NSF) and its basic properties are introduced. Later, we set up a neutrosophic soft topology with the help of a NSF. We also give the notions of the greatest lower bound and the least upper bound of the family of neutrosophic soft filters (NSFs), NSF subbase, NSF base and explore some basic properties of them.
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1 Introduction

We can not solve the problems by using mathematical tools generally in the social life since in mathematics, the concepts are precise and not subjective. To deal with this problem, researchers proposed several methods such as fuzzy set theory [28], rough set theory [20] and soft set theory [18]. Theories of fuzzy sets and rough sets can be considered as tools for dealing with vagueness but both of these theories have their own difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory as mentioned by Molodtsov [18] in 1999. Molodtsov initiated a novel concept of soft set theory which is a completely new approach for modeling uncertainities and succesfully applied it into several directions such as smoothness of functions, game theory, Riemann Integration, theory of measurement and so on. In 2018, Smarandache [24] generalized the soft set to the hypersoft set. The fundamental concepts of neutrosophic set were introduced by Smarandache [23]. This theory is a generalization of classical sets, fuzzy set theory [28], intuitionistic fuzzy set theory [2], etc. Later some researchers [21, 22] studied basic concepts and properties of neutrosophic sets. In order to practice neutrosophic set in real life applications conveniently, Wang et al. [25] introduced the concept of a single-valued neutrosophic sets, a subclass of the neutrosophic sets. Then single valued neutrosophic filters were introduced by Nordo et al. [19]. The notion of neutrosophic soft set (NSS) was first defined by Maji [13] and later, Deli and Broumi [5] modified it. Bera [3] introduced the concept of neutrosophic soft topological space (NSTS). Also, neutrosophic soft point concept and neutrosophic soft T_i-spaces were presented by Gündüz Aras et al. [10]. Recently, some researchers [6–9, 11, 12, 14–17] studied some of basic concepts and properties of NSSs and NSTSs. For more applications on neutrosophic logic the references are suggested [1, 4, 26].

The main purpose of this paper is to introduce NSFs. Later we study some basic properties of NSFs and set up a NSTS with the help of a NSF. Some new notions in NSFs such as the greatest lower bound and the least upper bound of the family of NSFs, NSF subbase and NSF base were introduced. Also, we give some basic properties of these concepts.

2 Preliminaries

In this section, we present the basic definitions and results of NSSs and NSTSs that we require in the next sections.

Definition 2.1. [5] Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X. Then a NSS $(\widetilde{F}, E)$ over X is a set defined by a set value function $\widetilde{F}$ representing

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a mapping $\widetilde{F}: E \to P(X)$, where $\widetilde{F}$ is called the approximate function of the NSS $(\widetilde{F}, E)$. In other words, the NSS is a parameterized family of some elements of the set $P(X)$ and it can be shown as a set of ordered pairs.

$$(\widetilde{F}, E) = \left\{ \left(e, \left\langle x, T_{\widetilde{F}(e)}(x), I_{\widetilde{F}(e)}(x), F_{\widetilde{F}(e)}(x) \right\rangle : x \in X \right) : e \in E \right\},$$

where $T_{\widetilde{F}(e)}(x),\ I_{\widetilde{F}(e)}(x),\ F_{\widetilde{F}(e)}(x) \in [0,1]$ are respectively called the truth-membership, indeterminacy-membership and falsity-membership function of $\widetilde{F}(e)$. Here $\widetilde{F}(e)$ may be considered as the set of e-approximate elements of $(\widetilde{F}, E)$. Since the supremum of each $T_{\widetilde{F}(e)}(x),\ I_{\widetilde{F}(e)}(x),\ F_{\widetilde{F}(e)}(x)$ is 1, the inequality $0 \leq T_{\widetilde{F}(e)}(x) + I_{\widetilde{F}(e)}(x) + F_{\widetilde{F}(e)}(x) \leq 3$ is clear.

Definition 2.2. [3] Let $(\widetilde{F}, E)$ be a NSS over the universe set X. The complement of $(\widetilde{F}, E)$ is denoted by $(\widetilde{F}, E)^c$ and is defined by:

$$(\widetilde{F}, E)^c = \left\{ \left(e, \left\langle x, F_{\widetilde{F}(e)}(x), 1 - I_{\widetilde{F}(e)}(x), T_{\widetilde{F}(e)}(x) \right\rangle : x \in X \right) : e \in E \right\}.$$

It is clear that $\left((\widetilde{F}, E)^c \right)^c = (\widetilde{F}, E)$.

Definition 2.3. [13] Let $(\widetilde{F}, E)$ and $(\widetilde{G}, E)$ be two NSSs over the universe set X. $(\widetilde{F}, E)$ is said to be a neutrosophic soft subset of $(\widetilde{G}, E)$ if $T_{\widetilde{F}(e)}(x) \leq T_{\widetilde{G}(e)}(x)$, $I_{\widetilde{F}(e)}(x) \leq I_{\widetilde{G}(e)}(x)$, $F_{\widetilde{F}(e)}(x) \geq F_{\widetilde{G}(e)}(x)$, $\forall e \in E, \forall x \in X$. It is denoted by $(\widetilde{F}, E) \subseteq (\widetilde{G}, E)$. $(\widetilde{F}, E)$ is said to be neutrosophic soft equal to $(\widetilde{G}, E)$ if $(\widetilde{F}, E)$ is a neutrosophic soft subset of $(\widetilde{G}, E)$ and $(\widetilde{G}, E)$ is a neutrosophic soft subset of $(\widetilde{F}, E)$. It is denoted by $(\widetilde{F}, E) = (\widetilde{G}, E)$.

Definition 2.4. [10] Let $(\widetilde{F_1}, E)$ and $(\widetilde{F_2}, E)$ be two NSSs over the universe set X. Then their union is denoted by $(\widetilde{F_1}, E) \cup (\widetilde{F_2}, E) = (\widetilde{F_3}, E)$ and is defined by:

$$(\widetilde{F_3}, E) = \left\{ \left(e, \left\langle x, T_{\widetilde{F_3}(e)}(x), I_{\widetilde{F_3}(e)}(x), F_{\widetilde{F_3}(e)}(x) \right\rangle : x \in X \right) : e \in E \right\},$$

where

$$\begin{aligned} T_{\widetilde{F_3}(e)}(x) &= \max\left\{ T_{\widetilde{F_1}(e)}(x), T_{\widetilde{F_2}(e)}(x) \right\}, \\ I_{\widetilde{F_3}(e)}(x) &= \max\left\{ I_{\widetilde{F_1}(e)}(x), I_{\widetilde{F_2}(e)}(x) \right\}, \\ F_{\widetilde{F_3}(e)}(x) &= \min\left\{ F_{\widetilde{F_1}(e)}(x), F_{\widetilde{F_2}(e)}(x) \right\}. \end{aligned}$$

Definition 2.5. [10] Let $(\widetilde{F_1}, E)$ and $(\widetilde{F_2}, E)$ be two NSSs over the universe set X. Then their intersection is denoted by $(\widetilde{F_1}, E) \cap (\widetilde{F_2}, E) = (\widetilde{F_3}, E)$ and is defined by:

$$(\widetilde{F_3}, E) = \left\{ \left(e, \left\langle x, T_{\widetilde{F_3}(e)}(x), I_{\widetilde{F_3}(e)}(x), F_{\widetilde{F_3}(e)}(x) \right\rangle : x \in X \right) : e \in E \right\},$$

where

$$\begin{aligned} T_{\widetilde{F_3}(e)}(x) &= \min\left\{ T_{\widetilde{F_1}(e)}(x), T_{\widetilde{F_2}(e)}(x) \right\}, \\ I_{\widetilde{F_3}(e)}(x) &= \min\left\{ I_{\widetilde{F_1}(e)}(x), I_{\widetilde{F_2}(e)}(x) \right\}, \\ F_{\widetilde{F_3}(e)}(x) &= \max\left\{ F_{\widetilde{F_1}(e)}(x), F_{\widetilde{F_2}(e)}(x) \right\}. \end{aligned}$$

Definition 2.6. [10] A NSS $(\widetilde{F}, E)$ over the universe set X is said to be a null NSS if $T_{\widetilde{F}(e)}(x) = 0$, $I_{\widetilde{F}(e)}(x) = 0$, $F_{\widetilde{F}(e)}(x) = 1$; $\forall e \in E, \forall x \in X$. It is denoted by $0_{(X,E)}$.

Definition 2.7. [10] A NSS $(\widetilde{F}, E)$ over the universe set X is said to be an absolute NSS if $T_{\widetilde{F}(e)}(x) = 1$, $I_{\widetilde{F}(e)}(x) = 1$, $F_{\widetilde{F}(e)}(x) = 0$; $\forall e \in E, \forall x \in X$. It is denoted by $1_{(X,E)}$.

Clearly, $0^c_{(X,E)} = 1_{(X,E)}$ and $1^c_{(X,E)} = 0_{(X,E)}$.

Definition 2.8. [10] Let $NSS(X, E)$ be the family of all NSSs over the universe set X and $\tau \subseteq NSS(X, E)$. Then τ is said to be a NST on X if:

1. $0_{(X,E)}$ and $1_{(X,E)}$ belong to τ,
2. the union of any number of NSSs in τ belongs to τ,
3. the intersection of a finite number of NSSs in τ belongs to τ.

Then (X, τ, E) is said to be a NSTS over X. Each member of τ is said to be a neutrosophic soft open set. A NSS $(\widetilde{F}, E)$ is called a neutrosophic soft closed set iff its complement $(\widetilde{F}, E)^c$ is a neutrosophic soft open set.

Definition 2.9. [10] Let $NSS(X, E)$ be the family of all NSSs over the universe set X. Then NSS $x^e_{(\alpha,\beta,\gamma)}$ is called a neutrosophic soft point, for every $x \in X$, $0 < \alpha, \beta, \gamma \leq 1$, $e \in E$ and is defined as follows:

$$x^e_{(\alpha,\beta,\gamma)}(e')(y) = \begin{cases} (\alpha, \beta, \gamma) & if\; e' = e \; and \; y = x, \\ (0, 0, 1) & if\; e' \neq e \; or \; y \neq x. \end{cases}$$

Definition 2.10. [10] Let $(\widetilde{F}, E)$ be a NSS over the universe set X. We say that $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{F}, E)$ read as belonging to the NSS $(\widetilde{F}, E)$ whenever $\alpha \leq T_{\widetilde{F}(e)}(x)$, $\beta \leq I_{\widetilde{F}(e)}(x)$ and $F_{\widetilde{F}(e)}(x) \leq \gamma$.

Definition 2.11. [10] Let (X, τ, E) be a NSTS over X. A NSS $(\widetilde{F}, E)$ in (X, τ, E) is called a neutrosophic soft neighborhood of the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{F}, E)$, if there exists a neutrosophic soft open set $(\widetilde{G}, E)$ such that $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{G}, E) \subseteq (\widetilde{F}, E)$.

Theorem 2.12. *[10] Let (X, τ, E) be a NSTS and $(\widetilde{F}, E)$ be a NSS over X. Then $(\widetilde{F}, E)$ is a neutrosophic soft open set if and only if $(\widetilde{F}, E)$ is a neutrosophic soft neighborhood of its neutrosophic soft points.*

The neighborhood system of a neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$, denoted by $U(x^e_{(\alpha,\beta,\gamma)}, E)$, is the family of all its neighborhoods.

Theorem 2.13. *[10] The neighborhood system $U(x^e_{(\alpha,\beta,\gamma)}, E)$ at $x^e_{(\alpha,\beta,\gamma)}$ in a NSTS (X, τ, E) has the following properties:*

1. If $(\widetilde{F}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$, then $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{F}, E)$,
2. If $(\widetilde{F}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$ and $(\widetilde{F}, E) \subseteq (\widetilde{H}, E)$ then $(\widetilde{H}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$,
3. If $(\widetilde{F}, E), (\widetilde{G}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$ then $(\widetilde{F}, E) \cap (\widetilde{G}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$,
4. If $(\widetilde{F}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$ then there exists a $(\widetilde{G}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$ such that $(\widetilde{G}, E) \in U(y^{e'}_{(\alpha',\beta',\gamma')}, E)$ for each $y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{G}, E)$.

Definition 2.14. Let (X, τ, E) be a NSTS and Ç$(x^e_{(\alpha,\beta,\gamma)}, E)$ be a family of some neutrosophic soft neighborhoods of neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$. If, for each neutrosophic soft neighborhood $(\widetilde{G}, E)$ of $x^e_{(\alpha,\beta,\gamma)}$, there exists a $(\widetilde{H}, E) \in$Ç$(x^e_{(\alpha,\beta,\gamma)}, E)$ such that $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{H}, E) \subseteq (\widetilde{G}, E)$, then we say that Ç$(x^e_{(\alpha,\beta,\gamma)}, E)$ is a neutrosophic soft neighborhood base at $x^e_{(\alpha,\beta,\gamma)}$.

Theorem 2.15. *If for each neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$ there corresponds a family $U(x^e_{(\alpha,\beta,\gamma)}, E)$ such that the properties 1. - 4. in Theorem 2.13 are satisfied, then there is a unique τ neutrosophic soft topological structure over X such that for each $x^e_{(\alpha,\beta,\gamma)}$, $U(x^e_{(\alpha,\beta,\gamma)}, E)$ is the family of τ-neutrosophic soft neighborhoods of $x^e_{(\alpha,\beta,\gamma)}$.*

Proof. Let $\tau = \left\{(\widetilde{G}, E) \in NSS(X, E) : x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{G}, E) \Longrightarrow (\widetilde{G}, E) \in \mathrm{U}(x^e_{(\alpha,\beta,\gamma)}, E)\right\}$. It is clear that, τ is a neutrosophic soft topology over X. The family τ certainly satisfies axioms 2. and 3. in Definition 2.8: for 3., this follows immediately from 2. in Theorem 2.13 and for 2., from 3. in Theorem 2.13. The axiom 1. in Definition 2.8 is a result of 2. and 3. in Theorem 2.13. It remains to show that, in the neutrosophic soft topology defined by τ, $U(x^e_{(\alpha,\beta,\gamma)}, E)$ is the set of τ-neutrosophic soft neighborhoods of $x^e_{(\alpha,\beta,\gamma)}$ for each $x^e_{(\alpha,\beta,\gamma)}$. It follows from 2. in Theorem 2.13 that every τ-neutrosophic soft neighborhood of $x^e_{(\alpha,\beta,\gamma)}$ belongs to $U(x^e_{(\alpha,\beta,\gamma)}, E)$. Conversely, let $(\widetilde{G}_1, E)$ be a NSS belonging to $U(x^e_{(\alpha,\beta,\gamma)}, E)$ and let $(\widetilde{G}_2, E)$ be the NSS of neutrosophic soft points $y^{e'}_{(\alpha',\beta',\gamma')}$ such that $(\widetilde{G}_1, E) \in U(y^{e'}_{(\alpha',\beta',\gamma')}, E)$. If we can show that $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{G}_2, E)$, $(\widetilde{G}_2, E) \subseteq (\widetilde{G}_1, E)$ and $(\widetilde{G}_2, E) \in \tau$, then the proof will be complete. Since for every neutrosophic soft point $y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{G}_2, E)$ belongs to $(\widetilde{G}_1, E)$ by reason of 1. in Theorem 2.13 and the hypothesis $(\widetilde{G}_1, E) \in U(y^{e'}_{(\alpha',\beta',\gamma')}, E)$, we obtain $(\widetilde{G}_2, E) \subseteq (\widetilde{G}_1, E)$. Since $(\widetilde{G}_1, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$ and $(\widetilde{G}_2, E) \subseteq (\widetilde{G}_1, E)$, we have $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{G}_2, E)$. It remains to show that $(\widetilde{G}_2, E) \in \tau$, i.e. that $(\widetilde{G}_2, E) \in U(y^{e'}_{(\alpha',\beta',\gamma')}, E)$ for each $y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{G}_2, E)$. If $y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{G}_2, E)$ then by 4. in Theorem 2.13 there is a NSS $(\widetilde{G}_3, E)$ such that for each $z^{e''}_{(\alpha'',\beta'',\gamma'')} \in (\widetilde{G}_3, E)$ we have $(\widetilde{G}_1, E) \in U(z^{e''}_{(\alpha'',\beta'',\gamma'')}, E)$. Since $(\widetilde{G}_1, E) \in U(z^{e''}_{(\alpha'',\beta'',\gamma'')}, E)$ means that $z^{e''}_{(\alpha'',\beta'',\gamma'')} \in (\widetilde{G}_2, E)$, it follows that $(\widetilde{G}_3, E) \subseteq (\widetilde{G}_2, E)$ and therefore, by 2. in Theorem 2.13, that $(\widetilde{G}_2, E) \in U(y^{e'}_{(\alpha',\beta',\gamma')}, E)$. □

3 Neutrosophic soft filters

Definition 3.1. Let $\aleph \subseteq NSS(X, E)$, then $\aleph$ is called a NSF on X if $\aleph$ satisfies the following properties:

($\aleph_1$) $0_{(X,E)} \notin \aleph$,
($\aleph_2$) $\forall(\widetilde{F}, E)$, $(\widetilde{G}, E) \in \aleph \Longrightarrow (\widetilde{F}, E) \cap (\widetilde{G}, E) \in \aleph$,
($\aleph_3$) $\forall(\widetilde{F}, E) \in \aleph$ and $(\widetilde{F}, E) \subseteq (\widetilde{G}, E) \Longrightarrow (\widetilde{G}, E) \in \aleph$.

Proposition 3.2. *The condition* ($\aleph_2$) *is equivalent to the following two conditions:*

($\aleph_{2a}$) The intersection of two members of $\aleph$ belongs to $\aleph$.
($\aleph_{2b}$) $1_{(X,E)}$ belongs to $\aleph$.

Example 3.3. The family $\aleph = \{1_{(X,E)}\}$ is a NSF over X.

Theorem 3.4. *Let* $0_{(X,E)} \neq (\widetilde{F}, E) \in NSS(X, E)$. *Then the family*
$\aleph_{(\widetilde{F},E)} = \left\{(\widetilde{G}, E) : (\widetilde{F}, E) \subseteq (\widetilde{G}, E) \in NSS(X, E)\right\}$ *is a NSF over* X.

Proof. Since $1_{(X,E)} \in \aleph$ and $0_{(X,E)} \notin \aleph$, $\emptyset \neq \aleph \neq NSS(X, E)$. Suppose $(\widetilde{H}_1, E)$, $(\widetilde{H}_2, E) \in \aleph$, then $(\widetilde{F}, E) \subseteq (\widetilde{H}_1, E)$, $(\widetilde{F}, E) \subseteq (\widetilde{H}_2, E)$. Thus $T_{\widetilde{F}(e)}(x) \leq \min\left\{T_{\widetilde{H}_1(e)}(x), T_{\widetilde{H}_2(e)}(x)\right\}$, $I_{\widetilde{F}(e)}(x) \leq \min\left\{I_{\widetilde{H}_1(e)}(x), I_{\widetilde{H}_2(e)}(x)\right\}$ and $F_{\widetilde{F}(e)}(x) \leq \max\left\{F_{\widetilde{H}_1(e)}(x), F_{\widetilde{H}_2(e)}(x)\right\}$ for all $x \in X$. So $(\widetilde{F}, E) \subseteq (\widetilde{H}_1, E) \cap (\widetilde{H}_2, E)$ and hence $(\widetilde{H}_1, E) \cap (\widetilde{H}_2, E) \in \aleph$. □

Theorem 3.5. *Let* (X, τ, E) *be a NSTS over* X. *The neighborhood system* $U(x^e_{(\alpha,\beta,\gamma)}, E)$ *is a NSF for every neutrosophic soft point* $x^e_{(\alpha,\beta,\gamma)}$. *Also, it is called neutrosophic soft neighborhoods filter of the neutrosophic soft point* $x^e_{(\alpha,\beta,\gamma)}$.

Proof. ($\aleph_1$) By 1. in Theorem 2.13, since $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{G}, E)$, we obtain
$0_{(X,E)} \notin U(x^e_{(\alpha,\beta,\gamma)}, E)$.
($\aleph_2$) This is clearly seen by 3. in Theorem 2.13.
($\aleph_3$) This is clearly seen by 2. in Theorem 2.13. □

Now, we set up a neutrosophic soft topology with the help of a NSF.

Theorem 3.6. *If, for every $x^e_{(\alpha,\beta,\gamma)}$, there exists a NSF $\aleph(x^e_{(\alpha,\beta,\gamma)}) = U(x^e_{(\alpha,\beta,\gamma)}, E)$ which satisfies the following two properties, then there exists a unique neutrosophic soft topology τ such that $\aleph(x^e_{(\alpha,\beta,\gamma)})$ consists of the τ-neutrosophic soft neighborhoods of the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$.*

(1) Every NSS in the NSF $\aleph(x^e_{(\alpha,\beta,\gamma)})$ contains the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$,

(2) For every $(\widetilde{G}, E) \in \aleph(x^e_{(\alpha,\beta,\gamma)})$ there exists a $(\widetilde{H}, E) \in \aleph(x^e_{(\alpha,\beta,\gamma)})$ such that for every $y^{e'}_{(\alpha',\beta',\gamma')} \in (\widetilde{H}, E)$, $(\widetilde{G}, E) \in \aleph(y^{e'}_{(\alpha',\beta',\gamma')})$.

Proof. Since the axioms $(\aleph_1)$, $(\aleph_2)$, $(\aleph_3)$, (1) and (2) are equivalent to the neighborhood axioms $1. - 4.$, by Theorem 2.15, there exists a neutrosophic soft topology τ such that $\aleph(x^e_{(\alpha,\beta,\gamma)})$ consists of the τ-neutrosophic soft neighborhoods of the neutrosophic soft point $x^e_{(\alpha,\beta,\gamma)}$. □

Example 3.7. Let (X, τ, E) be a NSTS and $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point over X. Since $(\widetilde{G}, E)$ cannot be an element of $Ç(x^e_{(\alpha,\beta,\gamma)}, E)$ for every $(\widetilde{H}, E) \in Ç(x^e_{(\alpha,\beta,\gamma)}, E)$ and $(\widetilde{H}, E) \subseteq (\widetilde{G}, E)$, then the neutrosophic soft neighborhood base $Ç(x^e_{(\alpha,\beta,\gamma)}, E)$ is not a NSF over X.

4 Comparison of neutrosophic soft filters

Definition 4.1. Let $\aleph_1$ and $\aleph_2$ be NSFs over X. If $\aleph_1 \subseteq \aleph_2$, then $\aleph_2$ is said to be finer that $\aleph_1$ or $\aleph_1$ coarser than $\aleph_2$.

If also $\aleph_1 \neq \aleph_2$, then $\aleph_2$ is strictly finer than $\aleph_1$ or $\aleph_1$ is strictly coarser than $\aleph_2$. If either $\aleph_1 \subseteq \aleph_2$ or $\aleph_2 \subseteq \aleph_1$, then $\aleph_1$ is comparable with $\aleph_2$.

Theorem 4.2. *Let $(\aleph_i)_{i\in I}$ be a family of NSFs over X. Then $\aleph = \bigcap_{i\in I} \aleph_i$ is a NSF over X.*

In fact $\aleph$ is the greatest lower bound of the family $(\aleph_i)_{i\in I}$.

Proof. $(\aleph_1)$ Since $0_{(X,E)} \notin \aleph_i$ for each $i \in I$, then $0_{(X,E)}$ does not belong to $\aleph = \bigcap_{i\in I} \aleph_i$.

$(\aleph_2)$ Let $(\widetilde{F}, E)$, $(\widetilde{G}, E) \in \aleph = \bigcap_{i\in I} \aleph_i$. Then $(\widetilde{F}, E)$, $(\widetilde{G}, E) \in \aleph_i$ for each $i \in I$. Since $(\widetilde{F}, E) \cap (\widetilde{G}, E) \in \aleph_i$ for each $i \in I$, so we obtain $(\widetilde{F}, E) \cap (\widetilde{G}, E) \in \aleph = \bigcap_{i\in I} \aleph_i$.

$(\aleph_3)$ Let $(\widetilde{F}, E) \in \aleph = \bigcap_{i\in I} \aleph_i$ and $(\widetilde{F}, E) \subseteq (\widetilde{G}, E)$. Since $(\widetilde{F}, E) \in \aleph_i$ for each $i \in I$ and $(\widetilde{F}, E) \subseteq (\widetilde{G}, E)$, we get $(\widetilde{G}, E) \in \aleph_i$ for each $i \in I$. Hence $(\widetilde{G}, E) \in \aleph = \bigcap_{i\in I} \aleph_i$. □

Now, we investigate the least upper bound of the family of NSFs over X.

Theorem 4.3. *Let $S \subseteq NSS(X, E)$. Then there exists a NSF $\aleph$ which contains the family S, if S has the following property: "The all finite intersections of NSSs of S are not $0_{(X,E)}$".*

Proof. Let $S = \left\{(\widetilde{F_i}, E) : \forall i \in J \ (J \ is \ finite), \ \bigcap_{i\in J} (\widetilde{F_i}, E) \neq 0_{(X,E)}\right\}$. Then we give the family which consists of finite intersections of elements of S;

$\beta = \left\{(\widetilde{G}, E) : \forall i \in J \ (J \ is \ finite), \ (\widetilde{F_i}, E) \in S \ and \ (\widetilde{G}, E) = \bigcap_{i\in J} (\widetilde{F_i}, E)\right\}$.

Then the family $\aleph(S) = \left\{(\widetilde{H}, E) : (\widetilde{G}, E) \in \beta \ and \ (\widetilde{G}, E) \subseteq (\widetilde{H}, E)\right\}$ is a NSF over X.

$(\aleph_1)$ $0_{(X,E)} \in \beta$, for every $(\widetilde{H}, E) \in \aleph(S)$, $(\widetilde{H}, E) \neq 0_{(X,E)}$ and so $0_{(X,E)} \notin \aleph(S)$.

$(\aleph_2)$ Let $(\widetilde{H_1}, E)$, $(\widetilde{H_2}, E) \in \aleph(S)$. There exist NSSs $(\widetilde{G_1}, E)$, $(\widetilde{G_2}, E) \in \beta$ such that $(\widetilde{G_1}, E) \subseteq (\widetilde{H_1}, E)$ and $(\widetilde{G_2}, E) \subseteq (\widetilde{H_2}, E)$. From the definition of β, $0_{(X,E)} \neq (\widetilde{G_1}, E) \cap (\widetilde{G_2}, E) \in \beta$. Since $(\widetilde{G_1}, E) \cap (\widetilde{G_2}, E) \subseteq (\widetilde{H_1}, E) \cap (\widetilde{H_2}, E)$, we obtain $(\widetilde{H_1}, E) \cap (\widetilde{H_2}, E) \in \aleph(S)$.

$(\aleph_3)$ Let $(\widetilde{H}_1, E) \in \aleph(S)$ and $(\widetilde{H}_1, E) \subseteq (\widetilde{H}_2, E)$. Then there exists a NSS $(\widetilde{G}, E) \in \beta$ such that $(\widetilde{G}, E) \subseteq (\widetilde{H}_1, E)$. Since $(\widetilde{H}_1, E) \subseteq (\widetilde{H}_2, E)$, we obtain $(\widetilde{H}_2, E) \in \aleph(S)$. $\square$

Remark 4.4. The NSF $\aleph(S)$ in Theorem 4.3 is said to be generated by S and S is said to be NSF subbase of $\aleph(S)$. It is clear that $S \subseteq \aleph(S)$.

Theorem 4.5. *The NSF $\aleph(S)$ which is generated by S is the coarsest NSF which contains S.*

Proof. Suppose that $S \subseteq \aleph_1$. By Theorem 4.3, $S \subseteq \beta \subseteq \aleph_1$. By Remark 4.4, for every $(\widetilde{H}, E) \in \aleph(S)$ there exists a $(\widetilde{G}, E) \in \beta$ such that $(\widetilde{G}, E) \subseteq (\widetilde{H}, E)$. Since $\beta \subseteq \aleph_1$, then $(\widetilde{G}, E) \in \aleph_1$. Since $\aleph_1$ is a NSF, $(\widetilde{H}, E) \in \aleph_1$ by $(\aleph_3)$ in Definition 3.1. Hence we obtain $\aleph(S) \subseteq \aleph_1$. $\square$

Theorem 4.6. *The family $(\aleph_i)_{i\in I}$ of NSFs over X has a least upper bound if and only if for all finite subfamilies $(\aleph_i)_{1\leq i\leq n}$ of $(\aleph_i)_{i\in I}$ and all $(\widetilde{G}_i, E) \in \aleph_i$ $(1 \leq i \leq n)$, $(\widetilde{G}_1, E) \cap \ldots \cap (\widetilde{G}_n, E) \neq 0_{(X,E)}$.*

Proof. $\Longrightarrow$: If there exists a least upper bound of the family $(\aleph_i)_{i\in I}$, by $(\aleph_1)$ and $(\aleph_2)$ in Definition 3.1, for all finite subfamilies $(\aleph_i)_{1\leq i\leq n}$ of $(\aleph_i)_{i\in I}$ and all $(\widetilde{G}_i, E) \in \aleph_i$ $(1 \leq i \leq n)$, the intersection $(\widetilde{G}_1, E) \cap \ldots \cap (\widetilde{G}_n, E) \neq 0_{(X,E)}$.

$\Longleftarrow$: Let $(\widetilde{G}_1, E) \cap \ldots \cap (\widetilde{G}_n, E) \neq 0_{(X,E)}$ for all finite subfamilies $(\aleph_i)_{1\leq i\leq n}$ of $(\aleph_i)_{i\in I}$ and all $(\widetilde{G}_i, E) \in \aleph_i$ $(1 \leq i \leq n)$. Then the NSF $\aleph(S)$ generated by

$$S = \bigcup_{i\in I} \aleph_i = \left\{ (\widetilde{F}, E) : (\exists i \in I)\ (\widetilde{F}, E) \in \aleph_i \right\}$$

is the least upper bound of the family $(\aleph_i)_{i\in I}$ by Theorem 4.5. $\square$

Definition 4.7. Let $\beta \subseteq NSS(X, E)$, then β is said to be a NSF base on X if

(β_1) $\beta \neq \emptyset$ and $0_{(X,E)} \notin \beta$.
(β_2) The intersection of two members of β contain a member of β.

Remark 4.8. β which is in Theorem 4.3 is a NSF base.

Remark 4.9. It is clear that, every NSF is a NSF base.

Example 4.10. Let (X, τ, E) be a NSTS and $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point over X. The neutrosophic soft neighborhood base $Ç(x^e_{(\alpha,\beta,\gamma)}, E)$ is a NSF base over X.

(β_1) Clearly, $Ç(x^e_{(\alpha,\beta,\gamma)}, E) \neq \emptyset$. For every $(\widetilde{H}, E) \in Ç(x^e_{(\alpha,\beta,\gamma)}, E)$, $x^e_{(\alpha,\beta,\gamma)} \in (\widetilde{H}, E)$. Then $(\widetilde{H}, E) \neq 0_{(X,E)}$. Hence we obtain $0_{(X,E)} \notin Ç(x^e_{(\alpha,\beta,\gamma)}, E)$.

(β_2) Let $(\widetilde{G}, E)$, $(\widetilde{H}, E) \in Ç(x^e_{(\alpha,\beta,\gamma)}, E)$. Since $(\widetilde{G}, E)$, $(\widetilde{H}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$, we get $(\widetilde{G}, E) \cap (\widetilde{H}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$. By Definition 2.14, there exists a $(\widetilde{K}, E) \in Ç(x^e_{(\alpha,\beta,\gamma)}, E)$ such that $(\widetilde{K}, E) \subseteq (\widetilde{G}, E) \cap (\widetilde{H}, E)$. Hence we get $Ç(x^e_{(\alpha,\beta,\gamma)}, E)$ is a NSF base of neutrosophic soft neighborhoods filter $U(x^e_{(\alpha,\beta,\gamma)}, E)$ by Definition 4.7.

Theorem 4.11. *Let $\aleph$ be a NSF over X and $\beta \subseteq \aleph$. Then β is a base of $\aleph$ if and only if every member of $\aleph$ contains a member of β.*

Proof. It is obvious from Theorem 4.3. $\square$

Definition 4.12. Two NSF bases β_1 and β_2 over X are equivalent if and only if every member of β_1 contains a member of β_2 and every member of β_2 contains a member of β_1.

Remark 4.13. Two equivalent NSF bases generate the same NSF.

Theorem 4.14. *Let (X, τ, E) be a NSTS and $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point over X. If $Ç_1(x^e_{(\alpha,\beta,\gamma)}, E)$ and $Ç_2(x^e_{(\alpha,\beta,\gamma)}, E)$ are different neutrosophic soft neighborhood bases of $x^e_{(\alpha,\beta,\gamma)}$, then $Ç_1(x^e_{(\alpha,\beta,\gamma)}, E)$ and $Ç_2(x^e_{(\alpha,\beta,\gamma)}, E)$ are two equivalent NSF bases.*

Proof. For each $(\widetilde{F_1}, E) \in Ç_1(x^e_{(\alpha,\beta,\gamma)}, E)$, by Example 4.10, $(\widetilde{F_1}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$. Also, since $Ç_2(x^e_{(\alpha,\beta,\gamma)}, E) \subseteq U(x^e_{(\alpha,\beta,\gamma)}, E)$ there exists a $(\widetilde{F_2}, E) \in Ç_2(x^e_{(\alpha,\beta,\gamma)}, E)$ such that $(\widetilde{F_2}, E) \subseteq (\widetilde{F_1}, E)$. Similarly, for each $(\widetilde{F_2}, E) \in Ç_2(x^e_{(\alpha,\beta,\gamma)}, E)$, by Example 4.10, $(\widetilde{F_2}, E) \in U(x^e_{(\alpha,\beta,\gamma)}, E)$. Since $Ç_1(x^e_{(\alpha,\beta,\gamma)}, E) \subseteq U(x^e_{(\alpha,\beta,\gamma)}, E)$, there exists a $(\widetilde{F_1}, E) \in Ç_1(x^e_{(\alpha,\beta,\gamma)}, E)$ such that $(\widetilde{F_1}, E) \subseteq (\widetilde{F_2}, E)$. Hence we obtain $Ç_1(x^e_{(\alpha,\beta,\gamma)}, E)$ and $Ç_2(x^e_{(\alpha,\beta,\gamma)}, E)$ are equivalent by Definition 4.12. $\square$

Theorem 4.15. *Let β_1, β_2 be NSF bases and $\aleph_1$, $\aleph_2$ be NSFs over X such that $\beta_1 \subseteq \aleph_1$ and $\beta_2 \subseteq \aleph_2$. Then $\aleph_2 \subseteq \aleph_1$ if and only if every member of β_2 contains a member of β_1.*

Proof. $\Longrightarrow$: Let $\aleph_2 \subseteq \aleph_1$ and $(\widetilde{G_2}, E) \in \beta_2$. Since $\beta_2 \subseteq \aleph_2 \subseteq \aleph_1$, then $(\widetilde{G_2}, E) \in \aleph_1$. Since $\beta_1 \subseteq \aleph_1$, there exists a $(\widetilde{G_1}, E) \in \beta_1$ such that $(\widetilde{G_1}, E) \subseteq (\widetilde{G_2}, E)$ by Theorem 4.11.

$\Longleftarrow$: Let $(\widetilde{F_2}, E) \in \aleph_2$. From Theorem 4.11, there exists a $(\widetilde{G_2}, E)$ such that $(\widetilde{G_2}, E) \subseteq (\widetilde{F_2}, E)$. By hypothesis, there exists a $(\widetilde{G_1}, E) \in \beta_1$ such that $(\widetilde{G_1}, E) \subseteq (\widetilde{G_2}, E)$. Then we obtain $(\widetilde{G_1}, E) \subseteq (\widetilde{F_2}, E)$. Since $\beta_1 \subseteq \aleph_1$, $(\widetilde{F_2}, E) \in \aleph_1$ by Definition 4.7. Hence we obtain $\aleph_2 \subseteq \aleph_1$. $\square$

5 Conclusion

In the present study, we have introduced NSFs which are defined over an initial universe with a fixed set of parameters. We set up a neutrosophic soft topology with the help of a NSF. We further investigate some essential features and basic concepts of NSFs. We expect that results in this paper will be helpfull for future studies in NSSs.

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