Decision Support Modeling For Agriculture Land Selection Based On Sine Trigonometric Single Valued Neutrosophic Information

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Abstract

A single valued neutrosophic set (SVNS) is a useful tool to portray uncertainty in multi attribute decision-making. In this article, we develop hybrid averaging and hybrid geometric aggregation operator using sine trigonometric function to handle uncertainty in single valued neutrosophic information, which are, sine trigonometric-single valued neutrosophic hybrid weighted averaging (ST-SVNHWA) operator and , sine trigonometric-single valued neutrosophic hybrid weighted geometric (ST-SVNHWG) operator. We investigate properties, namely, idempotancy, monotonicity and boundedness for the proposed operators. Moreover, we give an algorithm to solve multi-criteria decision-making issues which involve SVN information with ST-SVNHWA and ST-SVNHWG operators. Finally, an illustrative example of agricultural land selection is provided to verify the effectiveness. Sensitivity and comparative analyses are also implemented to assess the stability and validity of our method.

Keywords:

Single valued neutrosophic set, Sine trigonometric single valued neutrosophic information,Agriculture land selection,Decision Support.

1 Introduction

Multi-criteria decision-making (MCDM) is performing a vital role in different areas, including social, physical, medical and environmental sciences. MCDM methods use not only to evaluate an appropriate object but also to rank the objects in a given problem. To solve different uncertain problems for decision-making, Atanassov[13] presented the concept of intuitionistic fuzzy set (IFS) to include both membership and non membership parts, an extension of fuzzy set[47] in which simply membership part is characterized. After that, various hybrid models of fuzzy sets (FSs) have been presented and investigated such as, Pythagorean fuzzy sets (PyFSs),[46] Picture fuzzy sets (PFSs),[17] Spherical fuzzy sets (SFSs)[1,2] and single-valued neutrosophic set (SVNS).[38,40]

Aggregation operators (AOs) perform an important role in order to combine data into a single form and solve MCDM problems. Aggregation implies the invention of a numeral of things to a cluster or a bunch of objects that have come or been taken together. In the past few years, aggregation operators based on FSs and its various hybrid compositions have made very much attention and become attractive because they can quickly execute functional areas of diverse regions. For example, Yager[45] introduced weighted aggregation operators (AOs). Khan et al.,[23] presented the probabilistic hesitant based DM technique. Xu[41] proposed some new AOs under IFSs. Khan et al.,[25] established the novel decision making (DM) methodology under generalized intuitionistic fuzzy soft information. Khan et al.,[24] established the Dombi AOs under PyFSs. Ashraf et al.,[5] proposed the fuzzy decision support modelling for internet finance soft power evaluation based on sine trigonometric Pythagorean fuzzy information. Batool et al., established the entropy based DM method under probabilistic Pythagorean hesitant fuzzy information. Sajjad et al.,[39] established the TOPSIS approach under PyFSs. Ashraf et al.,[6] presented the AOs based on algebraic norm under PFSs. Khan et al.,[26] presented the AOs based on Einstein norm under PFSs. Ashraf et al.,[7] introduced the DM method under picture cubic fuzzy sets (PCFSs). Qiyas et al.,[34,35] established the AOs based on the Dombi and algebraic norm using

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linguistic picture fuzzy information respectively. Rafiq et al.,[37] proposed the cosine trigonometric function based similarity measure under SFSs. Zang et al.,[48] proposed the DM approach using TOPSIS under spherical fuzzy rough set. Ashraf et al.,[8] introduced the norm representation under SFSs. Ashraf et al.,[9] presented the AOs using Dombi norm under SFSs. Jin et al.,[20] introduced the AOs for hybrid structure of linguistic and SFSs. Ashraf et al.,[10] presented the AOs for SFSs using the symmetric sum technique and discussed their application in DM. Jin et al.,[21] established the logarithmic function based AOs under SFSs. Ashraf et al.,[11] presented the DM technique using distance measures under SFSs. Barukab et al.,[14] introduced the entropy measure based extended TOPSIS under SFSs. Ashraf et al.,[12] established the AOs using logarithmic function under SVNSs. Ye[42] established the correlation coefficient based DM approach under SVNSs. Liu et al.,[28] proposed the Dombi norm based DM methodology under linguistic SVNSs. Liu et al.,[29] established the muirhead mean based power AOs under SVNSs. Liu et al.,[30,31] presented the Heronian mean based power AOs using linguistic SVNSs and cubic neutrosophic information respectively. Ji et al.,[22] presented the Bonferroni mean AOs using Frank norm under SVNSs. Ye[43] introduced the list of novel AOs using exponential function under SVNSs and discussed their application to tackle the uncertainty in decision making problems.

Since single valued neutrosophic model is more general than the fuzzy sets, IFS, PyFS, PFS and SFSs due to the wider range of applicability over different complex problems. The SVN sets can explain uncertainties concurrently more precisely than the other existing methods, like fuzzy set and existing structures.[16,27,36]

The motivation of developed AOs is summarized as below.

(1) A very difficult MCDM problem is the estimation of the supreme option in a single valued neutrosophic environment due to the involvement of several imprecise factors. Assessment of information in different MCDM techniques is simply depicted through fuzzy and their existing structures which may not consider all the data in a real-world problem.

(2) As a general theory, single valued neutrosophic numbers describes efficient execution in the assessment process about uncertain, imprecise and vague information. Thus, single valued neutrosophic theory provide an excellent approach for the assessment of objects under multinary data.

(3) In view of the fact that sine trigonometric hybrid AOs are simple but provide a pioneering tool for solving MCDM problems when combine with other powerful mathematical tools, this article aims to develop sine trigonometric hybrid AOs in a single valued neutrosophic environment to handle complex problems.

(4) A single valued neutrosophic model is different from the mathematical tools like fuzzy sets and their extensions. Because the fuzzy set and their extensions can only handle one dimensional data, two dimensional data and three dimensional data, respectively, which may prompt a loss in data. Nevertheless, in many daily life problems, we handle the situations having higher dimension to sort out all the attributes.

(5) The sine trigonometric hybrid AOs employed in the construction of SVN sine trigonometric hybrid AOs are more suitable than all other aggregation approaches to tackle the MCDM situations as developed AOs have ability to consider all the information within the aggregation procedure.

(6) sine trigonometric AOs make the optimal outcomes more accurate and definite when utilized in practical MCDM problems under single valued neutrosophic environment. However, the proposed single valued neutrosophic operators handle the drawbacks of AOs present in the literature.

Therefore, some single valued neutrosophic sine trigonometric hybrid AOs are developed to choose the best option in different decision-making situations. The developed operators has some advantages over other approaches which are given as below:

(1) Our proposed methods explain the problems more accurately which involve multiple attributes because they consider single valued neutrosophic numbers.

(2) The developed AOs are more precise and efficient with single attribute.

(3) To solve practical problems by using sine trigonometric hybrid AOs with single valued neutrosophic numbers is very significant.

The rest of this article is structured as follows: Section 2 recalls some fundamental definitions and operations of the SVNSs. Section 3 presents novel sine hybrid aggregation operators. Section 4 develops a methodology of these AOs to MCDM problems under single valued nuetrosophic environment. Section 5 discusses a application of the selection of best agricultural land. Section 6 provides comparative analysis of developed approaches with different aggregating methodologies. Section 7 discusses the conclusions and future directions.

2 Preliminaries

In this section some essential notions of PFS, SFS, and SVNS are examined.

Definition 2.1. [17]A PFS $\mathcal{U}$ in fixed universe set $\gimel$ is defined as

$$\mathcal{U}=\left\{\left\langle\flat,\widetilde{\rho}_{\wp}\left(\flat\right),\daleth_{\wp}\left(\flat\right),\tilde{n}_{\wp}\left(\flat\right)\right\rangle|\flat\in\gimel\right\},$$

where the positive, neutral and negative membership grades, $\widetilde{\rho}_{\wp}:\gimel\rightarrow\Theta$, $\daleth_{\wp}:\gimel\rightarrow\Theta$ and $\tilde{n}_{\wp}:\gimel\rightarrow\Theta$, respectively and also, $\Theta=[0,1]$ is the unit interval. Furthermore, $0\leq\widetilde{\rho}_{\wp}\left(\flat\right)+\daleth_{\wp}\left(\flat\right)+\tilde{n}_{\wp}\left(\flat\right)\leq1$, for each $\flat\in\gimel$.

Definition 2.2. [1,2]A SFS $\mathcal{U}$ in fixed universe set $\gimel$ is defined as

$$\mathcal{U}=\left\{\left\langle\flat,\widetilde{\rho}_{\wp}\left(\flat\right),\daleth_{\wp}\left(\flat\right),\tilde{n}_{\wp}\left(\flat\right)\right\rangle|\flat\in\gimel\right\},$$

where the positive, neutral and negative membership grades, $\widetilde{\rho}_{\wp}:\gimel\rightarrow\Theta$, $\daleth_{\wp}:\gimel\rightarrow\Theta$ and $\tilde{n}_{\wp}:\gimel\rightarrow\Theta$, respectively and also, $\Theta=[0,1]$ is the unit interval. Furthermore, $0\leq\widetilde{\rho}^{2}_{\wp}\left(\flat\right)+\daleth^{2}_{\wp}\left(\flat\right)+\tilde{n}^{2}_{\wp}\left(\flat\right)\leq1$, for each $\flat\in\gimel$.

Definition 2.3. [38] A neutrosophic set $\mathcal{U}$ in a fixed universe set $\gimel$ is defined as

$$\mathcal{U}=\left\{\left\langle\flat,\widetilde{\rho}_{\wp}\left(\flat\right),\daleth_{\wp}\left(\flat\right),\tilde{n}_{\wp}\left(\flat\right)\right\rangle|\flat\in\gimel\right\},$$

where the positive, neutral and negative membership grades, $\widetilde{\rho}_{\wp}:\gimel\rightarrow\left]0^{-},1^{+}\right[$, $\daleth_{\wp}:\gimel\rightarrow\left]0^{-},1^{+}\right[$ and $\tilde{n}_{\wp}:\gimel\rightarrow\left]0^{-},1^{+}\right[$, respectively. Furthermore, $0^{-}\leq\widetilde{\rho}_{\wp}\left(\flat\right)+\daleth_{\wp}\left(\flat\right)+\tilde{n}_{\wp}\left(\flat\right)\leq3^{+}$, for each $\flat\in\gimel$.

Definition 2.4. [40] A SVNS $\mathcal{U}$ in a fixed universe set $\gimel$ is defined as

$$\mathcal{U}=\left\{\left\langle\flat,\widetilde{\rho}_{\wp}\left(\flat\right),\daleth_{\wp}\left(\flat\right),\tilde{n}_{\wp}\left(\flat\right)\right\rangle|\flat\in\gimel\right\},$$

where the positive, neutral and negative membership grades, $\widetilde{\rho}_{\wp}:\gimel\rightarrow\Theta$, $\daleth_{\wp}:\gimel\rightarrow\Theta$ and $\tilde{n}_{\wp}:\gimel\rightarrow\Theta$, respectively and also, $\Theta=[0,1]$ is the unit interval. Furthermore, $0\leq\widetilde{\rho}_{\wp}\left(\flat\right)+\daleth_{\wp}\left(\flat\right)+\tilde{n}_{\wp}\left(\flat\right)\leq3$, for each $\flat\in\gimel$.

In simplification, throughout the whole work, the triplet $\mathcal{U}=\{\widetilde{\rho}_{\wp},\daleth_{\wp},\tilde{n}_{\wp}\}$ called single valued neutrosophic number (SVNN) and their collection denoted by $SVNN\left(\gimel\right)$.

Here we highlight the some basic operations for SVNNs,[4044] &[50] as follows:

Definition 2.5. [50] Let $\mathcal{U}_1=\{\widetilde{\rho}_{\wp_1},\daleth_{\wp_1},\tilde{n}_{\wp_1}\}$ and $\mathcal{U}_2=\{\widetilde{\rho}_{\wp_2},\daleth_{\wp_2},\tilde{n}_{\wp_2}\}\in SVNN\left(\gimel\right)$. than,
(1) $\mathcal{U}_1\subseteq\mathcal{U}_2$ if and only if $\widetilde{\rho}_{\wp_1}\leq\widetilde{\rho}_{\wp_2}$, $\daleth_{\wp_1}\geq\daleth_{\wp_2}$ and $\tilde{n}_{\wp_1}\geq\tilde{n}_{\wp_2}$ for each $\flat\in\gimel$.
(2) $\mathcal{U}_1=\mathcal{U}_2$ if and only if $\mathcal{U}_1\subseteq\mathcal{U}_2$ and $\mathcal{U}_2\subseteq\mathcal{U}_1$.
(2) $\mathcal{U}_1\cap\mathcal{U}_2=\left\{\min\left(\widetilde{\rho}_{\wp_1},\widetilde{\rho}_{\wp_2}\right),\max\left(\daleth_{\wp_1},\daleth_{\wp_2}\right),\max\left(\tilde{n}_{\wp_1},\tilde{n}_{\wp_2}\right)\right\}$,
(3) $\mathcal{U}_1\cup\mathcal{U}_2=\left\{\max\left(\widetilde{\rho}_{\wp_1},\widetilde{\rho}_{\wp_2}\right),\min\left(\daleth_{\wp_1},\daleth_{\wp_2}\right),\min\left(\tilde{n}_{\wp_1},\tilde{n}_{\wp_2}\right)\right\}$,
(4) $\mathcal{U}_1^{c}=\left\{\tilde{n}_{\wp_1},\daleth_{\wp_1},\widetilde{\rho}_{\wp_1}\right\}$.

Definition 2.6. [40,44] Let $\mathcal{U}_1=\{\widetilde{\rho}_{\wp_1},\daleth_{\wp_1},\tilde{n}_{\wp_1}\}$ and $\mathcal{U}_2=\{\widetilde{\rho}_{\wp_2},\daleth_{\wp_2},\tilde{n}_{\wp_2}\}\in SVNN\left(\gimel\right)$ with $\eth>0$. than,
(1) $\mathcal{U}_1\otimes\mathcal{U}_2=\left\{\widetilde{\rho}_{\wp_1}\widetilde{\rho}_{\wp_2},\daleth_{\wp_1}+\daleth_{\wp_2}-\daleth_{\wp_1}\cdot\daleth_{\wp_2},\tilde{n}_{\wp_1}+\tilde{n}_{\wp_2}-\tilde{n}_{\wp_1}\cdot\tilde{n}_{\wp_2}\right\}$;
(2) $\mathcal{U}_1\oplus\mathcal{U}_2=\left\{\widetilde{\rho}_{\wp_1}+\widetilde{\rho}_{\wp_2}-\widetilde{\rho}_{\wp_1}\widetilde{\rho}_{\wp_2},\daleth_{\wp_1}\daleth_{\wp_2},\tilde{n}_{\wp_1}\tilde{n}_{\wp_2}\right\}$;
(3) $\left(\mathcal{U}_1\right)^{\eth}=\left\{\left(\widetilde{\rho}_{\wp_1}\right)^{\eth},1-\left(1-\daleth_{\wp_1}\right)^{\eth},1-\left(1-\tilde{n}_{\wp_1}\right)^{\eth}\right\}$;
(4) $\eth\cdot\mathcal{U}_1=\left\{1-\left(1-\widetilde{\rho}_{\wp_1}\right)^{\eth},\left(\daleth_{\wp_1}\right)^{\eth},\left(\tilde{n}_{\wp_1}\right)^{\eth}\right\}$.

Definition 2.7. [4] Let $\mathcal{U}_\hbar=\{\widetilde{\rho}_{\wp_\hbar},\daleth_{\wp_\hbar},\tilde{n}_{\wp_\hbar}\}\in SVNN\left(\gimel\right)$ $(\hbar=1,2,3,...,n)$. Then, the Algebraic averaging aggregation operator for $SVNN\left(\gimel\right)$ is denoted by $SVNWA$ and defined as follows:

$$\begin{aligned}SVNWA\left(\mathcal{U}_1,\mathcal{U}_2,\mathcal{U}_3,...,\mathcal{U}_n\right)&=\sum_{\hbar=1}^{n}\eth_\hbar\mathcal{U}_\hbar,\\&=\left\{1-\Pi_{\hbar=1}^{n}(1-\widetilde{\rho}_{\wp_\hbar})^{\eth_\hbar},\Pi_{\hbar=1}^{n}(\daleth_{\wp_\hbar})^{\eth_\hbar},\Pi_{\hbar=1}^{n}(\tilde{n}_{\wp_\hbar})^{\eth_\hbar}\right\}\end{aligned}$$

where the weights of $\mathcal{U}_\hbar$ represented by $\eth_\hbar$ $(\hbar=1,2,...,n)$ having $\eth_\hbar\geq0$ and $\sum_{\hbar=1}^{n}\eth_\hbar=1$.

Definition 2.8. [4] Let $\mathcal{U}_\hbar=\{\widetilde{\rho}_{\wp_\hbar},\daleth_{\wp_\hbar},\tilde{n}_{\wp_\hbar}\}\in SVNN\left(\gimel\right)$ $(\hbar=1,2,3,...,n)$. Then, the Algebraic geometric aggregation operator for $SVNN\left(\gimel\right)$ is denoted by $SVNWG$ and defined as follows:

$$\begin{aligned}SFWG\left(\mathcal{U}_1,\mathcal{U}_2,\mathcal{U}_3,...,\mathcal{U}_n\right)&=\prod_{\hbar=1}^{n}\left(\mathcal{U}_\hbar\right)^{\eth_\hbar},\\&=\left\{\Pi_{\hbar=1}^{n}(\widetilde{\rho}_{\wp_\hbar})^{\eth_\hbar},1-\Pi_{\hbar=1}^{n}(1-\daleth_{\wp_\hbar})^{\eth_\hbar},1-\Pi_{\hbar=1}^{n}(1-\tilde{n}_{\wp_\hbar})^{\eth_\hbar}\right\}\end{aligned}$$

where $\eth_\hbar$ $(\hbar=1,2,...,n)$ represents the weights of $\mathcal{U}_\hbar$ $(\hbar=1,2,3,...,n)$ with $\eth_\hbar\geq0$ and $\sum_{\hbar=1}^{n}\eth_\hbar=1$.

Ashraf et al.,[4] introduced the sine trigonometric operational laws for single valued neutrosophic environments.

Definition 2.9. [4] Let $\mathcal{U} = \{\widetilde{\rho}_{\wp}, \daleth_{\wp}, \tilde{n}_{\wp}\} \in SVNN(\beth)$. If

$$\sin(\mathcal{U}) = \left\{ \left(\begin{array}{c} \beth, \sin\left(\frac{\pi}{2}\widetilde{\rho}_{\wp}(\beth)\right), 1 - \sin\left(\frac{\pi}{2}1 - \daleth_{\wp}(\beth)\right), \\ 1 - \sin\left(\frac{\pi}{2}1 - \tilde{n}_{\wp}(\beth)\right) \end{array} \right) | \flat \in \beth \right\}$$

then, $\sin(\mathcal{U})$ is said to be sine trigonometric operator and their value known to be sine trigonometric SVNN (ST-SVNN).

Definition 2.10. [4] Let $\sin(\mathcal{U}_1) = \left\{ \left(\begin{array}{c} \sin\left(\frac{\pi}{2}\widetilde{\rho}_{\wp_1}\right), \\ 1 - \sin\left(\frac{\pi}{2}1 - \daleth_{\wp_1}\right), \\ 1 - \sin\left(\frac{\pi}{2}1 - \tilde{n}_{\wp_1}\right) \end{array} \right) \right\}$ and $\sin(\mathcal{U}_2) = \left\{ \left(\begin{array}{c} \sin\left(\frac{\pi}{2}\widetilde{\rho}_{\wp_2}\right), \\ 1 - \sin\left(\frac{\pi}{2}1 - \daleth_{\wp_2}\right), \\ 1 - \sin\left(\frac{\pi}{2}1 - \tilde{n}_{\wp_2}\right) \end{array} \right) \right\}$ be two ST-SVNNs. The basic operations for SVNNs are as follows

(1) $\sin(\mathcal{U}_1) \oplus \sin(\mathcal{U}_2) = \left(\begin{array}{c} 1 - \left(1 - \sin\left(\frac{\pi}{2}\widetilde{\rho}_{\wp_1}\right)\right)\left(1 - \sin\left(\frac{\pi}{2}\widetilde{\rho}_{\wp_2}\right)\right), \\ \left(1 - \sin\left(\frac{\pi}{2}1 - \daleth_{\wp_1}\right)\right)\left(1 - \sin\left(\frac{\pi}{2}1 - \daleth_{\wp_2}\right)\right), \\ \left(1 - \sin\left(\frac{\pi}{2}1 - \tilde{n}_{\wp_1}\right)\right)\left(1 - \sin\left(\frac{\pi}{2}1 - \tilde{n}_{\wp_2}\right)\right) \end{array} \right)$,

(2) $\psi \cdot \sin(\mathcal{U}_1) = \left(\begin{array}{c} 1 - \left(1 - \sin\left(\frac{\pi}{2}\widetilde{\rho}_{\wp_1}\right)\right)^{\psi}, \left(1 - \sin\left(\frac{\pi}{2}1 - \daleth_{\wp_1}\right)\right)^{\psi}, \\ \left(1 - \sin\left(\frac{\pi}{2}1 - \tilde{n}_{\wp_1}\right)\right)^{\psi} \end{array} \right)$,

(3) $\sin(\mathcal{U}_1) \otimes \sin(\mathcal{U}_2) = \left(\begin{array}{c} \sin\left(\frac{\pi}{2}\widetilde{\rho}_{\wp_1}\right)\sin\left(\frac{\pi}{2}\widetilde{\rho}_{\wp_2}\right), \\ 1 - \left(\sin\left(\frac{\pi}{2}1 - \daleth_{\wp_1}\right)\right)\left(\sin\left(\frac{\pi}{2}1 - \daleth_{\wp 2}\right)\right), \\ 1 - \left(\sin\left(\frac{\pi}{2}1 - \tilde{n}_{\wp_1}\right)\right)\left(\sin\left(\frac{\pi}{2}1 - \tilde{n}_{\wp 2}\right)\right) \end{array} \right)$,

(4) $(\sin(\mathcal{U}_1))^{\psi} = \left(\begin{array}{c} \left(\sin\left(\frac{\pi}{2}\widetilde{\rho}_{\wp_1}\right)\right)^{\psi}, 1 - \left(\sin\left(\frac{\pi}{2}1 - \daleth_{\wp_1}\right)\right)^{\psi}, \\ 1 - \left(\sin\left(\frac{\pi}{2}1 - \tilde{n}_{\wp_1}\right)\right)^{\psi} \end{array} \right)$.

Definition 2.11. Let $\mathcal{U} = \{\widetilde{\rho}_{\wp}, \daleth_{\wp}, \tilde{n}_{\wp}\} \in SVNN(\beth)$. Then, the score and accuracy of $\mathcal{U}$ is denoted and defined as
(1) $\check{s}\check{c}(\mathcal{U}) = \widetilde{\rho}_{\wp} - \daleth_{\wp} - \tilde{n}_{\wp}$, and
(2) $\tilde{a}\check{c}(\mathcal{U}) = \widetilde{\rho}_{\wp} + \daleth_{\wp} + \tilde{n}_{\wp}$.

Definition 2.12. Let $\mathcal{U}_1 = \{\widetilde{\rho}_{\wp_1}, \daleth_{\wp_1}, \tilde{n}_{\wp_1}\}$ and $\mathcal{U}_2 = \{\widetilde{\rho}_{\wp_2}, \daleth_{\wp_2}, \tilde{n}_{\wp_2}\} \in SVNN(\beth)$. Then,
(1) If $\check{s}\check{c}(\mathcal{U}_1) < \check{s}\check{c}(\mathcal{U}_2)$ then $\mathcal{U}_1 < \mathcal{U}_2$,
(2) If $\check{s}\check{c}(\mathcal{U}_1) > \check{s}\check{c}(\mathcal{U}_2)$ then $\mathcal{U}_1 > \mathcal{U}_2$,
(3) If $\check{s}\check{c}(\mathcal{U}_1) = \check{s}\check{c}(\mathcal{U}_2)$ then
(a) $\tilde{a}\check{c}(\mathcal{U}_1) < \tilde{a}\check{c}(\mathcal{U}_2)$ then $\mathcal{U}_1 < \mathcal{U}_2$,
(b) $\tilde{a}\check{c}(\mathcal{U}_1) > \tilde{a}\check{c}(\mathcal{U}_2)$ then $\mathcal{U}_1 > \mathcal{U}_2$,
(c) $\tilde{a}\check{c}(\mathcal{U}_1) = \tilde{a}\check{c}(\mathcal{U}_2)$ then $\mathcal{U}_1 = \mathcal{U}_2$.

3 Novel Sine Trigonometric Hybrid AOs for SVNNs

This section propose the novel sine trigonometric hybrid AOs under SVN information.

Definition 3.1. Let $\mathcal{U}_{\hbar} = \{\widetilde{\rho}_{\wp_{\hbar}}(\flat), \daleth_{\wp_{\hbar}}(\flat), \tilde{n}_{\wp_{\hbar}}(\flat)\} \in SVNN(\beth)$ $(\hbar \in \mathbb{N})$. The sine trigonometric hybrid weighted averaging AO for $SVNN(\beth)$ is represented by $ST - SVNHWA$ and discribed as follows:

$$\begin{aligned} ST - SVNHWA(\mathcal{U}_1, \mathcal{U}_2, ..., \mathcal{U}_n) &= \sigma_1 \sin\left(\mathcal{U}'_{\upsilon(1)}\right) \oplus \sigma_2 \sin\left(\mathcal{U}'_{\upsilon(2)}\right) \oplus ... \oplus \sigma_n \sin\left(\mathcal{U}\prime_{\upsilon(n)}\right) \\ &= \sum_{\hbar=1}^{n} \sigma_{\hbar} \sin\left(\mathcal{U}'_{\upsilon(\hbar)}\right), \end{aligned}$$

where the weights of $\mathcal{U}_{\hbar}$ is represented by $\eth_{\hbar}$ $(\hbar = 1, 2, ..., n)$ having $\eth_{\hbar} \geq 0$ and $\sum_{\hbar=1}^{n} \eth_{\hbar} = 1$ and gth biggest weighted value is $\mathcal{U}'_{\upsilon(\hbar)}$ $\left(\mathcal{U}'_{\upsilon(\hbar)} = n\eth_{\hbar}\mathcal{U}_{\upsilon(\hbar)} | \hbar = 1, 2, ..., n\right)$ consequently by total order $(\upsilon(1), \upsilon(2), \upsilon(3), ..., \upsilon(n))$. Also the associated weight vector $\sigma = \sigma_{\hbar}$ $(\hbar = 1, 2, ..., n)$ with $\sigma_{\hbar} \geq 0$ and $\sum_{\hbar=1}^{n} \sigma_{\hbar} = 1$.

Theorem 3.2. *Let* $\mathcal{U}_{\hbar} = \{\widetilde{\rho}_{\wp_{\hbar}}(\flat), \daleth_{\wp_{\hbar}}(\flat), \tilde{n}_{\wp_{\hbar}}(\flat)\} \in SVNN(\beth)$ $(\hbar \in \mathbb{N})$ *and the weights of* $\mathcal{U}_{\hbar}$ *represented by* $(\eth_1, \eth_2, ..., \eth_n)^T$ *subject to* $\sum_{\hbar=1}^{n} \eth_{\hbar} = 1$. *The* $ST - SVNHWA$ *AO is defined by a mapping* $G^n \longrightarrow G$

with associated weight vector $\sigma_\hbar$ $(\hbar = 1, 2, ..., n)$ *having* $\sigma_\hbar \geq 0$ *and* $\sum_{\hbar=1}^{n} \sigma_\hbar = 1$*:*

$$ST - SVNHWA\left(\mathcal{U}_1, \mathcal{U}_2, ..., \mathcal{U}_n\right) = \sum_{\hbar=1}^{n} \sigma_\hbar \sin\left(\mathcal{U}'_{\upsilon(\hbar)}\right) = \begin{pmatrix} 1 - \prod_{\hbar=1}^{n}\left(1 - \sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ \prod_{\hbar=1}^{n}\left(1 - \sin\left(\frac{\pi}{2}1 - \daleth'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ \prod_{\hbar=1}^{n}\left(1 - \sin\left(\frac{\pi}{2}1 - \tilde{n}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar} \end{pmatrix} \quad (1)$$

Proof. By using mathematical induction on n we prove Theorem 3.2. Then the mathematical induction steps below were implemented.

Step-1: For $n = 2$, we obtained

$$ST - SVNHWA\left(\mathcal{U}_1, \mathcal{U}_2\right) = \sigma_1 \sin\left(\mathcal{U}'_{\upsilon(1)}\right) \oplus \sigma_2 \sin\left(\mathcal{U}'_{\upsilon(2)}\right).$$

Since by the Definition 2.9, $\sin(\mathcal{U}_1)$ and $\sin(\mathcal{U}_2)$ are SVNNs and hence $\sigma_1 \sin\left(\mathcal{U}'_{\upsilon(1)}\right) \oplus \sigma_2 \sin\left(\mathcal{U}'_{\upsilon(2)}\right)$ is also SVNN. Further, for $\mathcal{U}_1$ and $\mathcal{U}_2$, we have

$$\begin{aligned} & ST - SVNHWA\left(\mathcal{U}_1, \mathcal{U}_2\right) \\ = \; & \sigma_1 \sin\left(\mathcal{U}'_{\upsilon(1)}\right) \oplus \sigma_2 \sin\left(\mathcal{U}'_{\upsilon(2)}\right) \\ = \; & \begin{pmatrix} 1 - \left(1 - \sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(1)}}\right)\right)^{\sigma_1}, \\ \left(1 - \sin\left(\frac{\pi}{2}1 - \daleth'_{\wp_{\upsilon(1)}}\right)\right)^{\sigma_1}, \\ \left(1 - \sin\left(\frac{\pi}{2}1 - \tilde{n}'_{\wp_{\upsilon(1)}}\right)\right)^{\sigma_1} \end{pmatrix} \oplus \begin{pmatrix} 1 - \left(1 - \sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(2)}}\right)\right)^{\sigma_2}, \\ \left(1 - \sin\left(\frac{\pi}{2}1 - \daleth'_{\wp_{\upsilon(2)}}\right)\right)^{\sigma_2}, \\ \left(1 - \sin\left(\frac{\pi}{2}1 - \tilde{n}'_{\wp_{\upsilon(2)}}\right)\right)^{\sigma_2} \end{pmatrix} \\ = \; & \begin{pmatrix} 1 - \prod_{\hbar=1}^{2}\left(1 - \sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ \prod_{\hbar=1}^{2}\left(1 - \sin\left(\frac{\pi}{2}1 - \daleth'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ \prod_{\hbar=1}^{2}\left(1 - \sin\left(\frac{\pi}{2}1 - \tilde{n}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar} \end{pmatrix} \end{aligned}$$

Step-2: Suppose that Equation 1 is holds for $n = \kappa$, we have

$$ST - SVNWA\left(\mathcal{U}_1, \mathcal{U}_2, ...\mathcal{U}_\kappa\right) = \begin{pmatrix} 1 - \prod_{\hbar=1}^{\kappa}\left(1 - \sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ \prod_{\hbar=1}^{\kappa}\left(1 - \sin\left(\frac{\pi}{2}1 - \daleth'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ \prod_{\hbar=1}^{\kappa}\left(1 - \sin\left(\frac{\pi}{2}1 - \tilde{n}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar} \end{pmatrix}$$

Step-3: Now we have to prove that Equation 1 is holds for $n = \kappa + 1$.

$$\begin{aligned}
& ST - SVNHWA\left(\mathcal{U}_1,\mathcal{U}_2,...\mathcal{U}_{\kappa+1}\right) \\
= & \sum_{\hbar=1}^{\kappa}\sigma_\hbar \sin\left(\mathcal{U}'_{\upsilon(\hbar)}\right) \oplus \sigma_{\kappa+1}\sin\left(\mathcal{U}'_{\upsilon(\kappa+1)}\right) \\
= & \begin{pmatrix} 1-\prod_{\hbar=1}^{\kappa}\left(1-\sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ \prod_{\hbar=1}^{\kappa}\left(1-\sin\left(\frac{\pi}{2}1-\daleth'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ \prod_{\hbar=1}^{\kappa}\left(1-\sin\left(\frac{\pi}{2}1-\tilde{n}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar} \end{pmatrix} \oplus \begin{pmatrix} 1-\left(1-\sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\kappa+1)}}\right)\right)^{\sigma_{\kappa+1}}, \\ \left(1-\sin\left(\frac{\pi}{2}1-\daleth'_{\wp_{\upsilon(\kappa+1)}}\right)\right)^{\sigma_{\kappa+1}}, \\ \left(1-\sin\left(\frac{\pi}{2}1-\tilde{n}'_{\wp_{\upsilon(\kappa+1)}}\right)\right)^{\sigma_{\kappa+1}} \end{pmatrix} \\
= & \begin{pmatrix} 1-\prod_{\hbar=1}^{\kappa+1}\left(1-\sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ \prod_{\hbar=1}^{\kappa+1}\left(1-\sin\left(\frac{\pi}{2}1-\daleth'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ \prod_{\hbar=1}^{\kappa+1}\left(1-\sin\left(\frac{\pi}{2}1-\tilde{n}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar} \end{pmatrix}
\end{aligned}$$

that is, when $n = z + 1$, Equation 1 also holds.

Hence, Equation 1 holds for any n. The proof is completed. □

Next, we give some properties that are apparently carried by the proposed ST-SVNHWA aggregation operator.

(1) Let $\mathcal{U}_\hbar = \{\widetilde{\rho}_{\wp_\hbar}, \daleth_{\wp_\hbar}, \tilde{n}_{\wp_\hbar}\} \in SVNN(\beth)$ $(\hbar = 1, 2, 3, ..., n)$ such that $\mathcal{U}_\hbar = \mathcal{U}$. Then

$$ST - SVNHWA\left(\mathcal{U}_1,\mathcal{U}_2,...,\mathcal{U}_n\right) = \sin\left(\mathcal{U}\right).$$

(2) Let $\mathcal{U}_\hbar = \{\widetilde{\rho}_{\wp_\hbar}, \daleth_{\wp_\hbar}, \tilde{n}_{\wp_\hbar}\}, \mathcal{U}_\hbar^- = \{\min\left(\widetilde{\rho}_{\wp_\hbar}\right), \max\left(\daleth_{\wp_\hbar}\right), \max\left(\tilde{n}_{\wp_\hbar}\right)\}$ and $\mathcal{U}_\hbar^+ = \{\max\left(\widetilde{\rho}_{\wp_\hbar}\right), \min\left(\daleth_{\wp_\hbar}\right), \min\left(\tilde{n}_{\wp_\hbar}\right)\}$ $\in SVNN(\beth)$ $(\hbar = 1, 2, 3, ..., n)$. Then,

$$\sin\left(\mathcal{U}_\hbar^-\right) \le ST - SVNHWA\left(\mathcal{U}_1,\mathcal{U}_2,...,\mathcal{U}_n\right) \le \sin\left(\mathcal{U}_\hbar^+\right).$$

(3) Let $\mathcal{U}_\hbar = \{\widetilde{\rho}_{\wp_\hbar}, \daleth_{\wp_\hbar}, \tilde{n}_{\wp_\hbar}\}, \mathcal{U}_\hbar^* = \left\{\widetilde{\rho}^*_{\wp_\hbar}, \daleth^*_{\wp_\hbar}, \tilde{n}^*_{\wp_\hbar}\right\} \in SVNN(\beth)$ $(\hbar = 1, 2, 3, ..., n)$. If $\widetilde{\rho}_{\wp_\hbar} \le \widetilde{\rho}^*_{\wp_\hbar}, \daleth_{\wp_\hbar} \le \daleth^*_{\wp_\hbar}$ and $\tilde{n}_{\wp_\hbar} \le \tilde{n}^*_{\wp_\hbar}$, then

$$ST - SVNHWA\left(\mathcal{U}_1,\mathcal{U}_2,...,\mathcal{U}_n\right) \le ST - SVNHWA\left(\mathcal{U}_1^*,\mathcal{U}_2^*,...,\mathcal{U}_n^*\right).$$

Definition 3.3. Let $\mathcal{U}_\hbar = \{\widetilde{\rho}_{\wp_\hbar}(\flat), \daleth_{\wp_\hbar}(\flat), \tilde{n}_{\wp_\hbar}(\flat)\} \in SVNN(\beth)$ $(\hbar \in \mathbb{N})$. The sine trigonometric hybrid weighted geometric AO for $SVNN(\beth)$ is represented by $ST - SVNHWG$ and discribed as follows:

$$\begin{aligned}
ST - SVNHWG\left(\mathcal{U}_1,\mathcal{U}_2,...,\mathcal{U}_n\right) &= \left(\sin\left(\mathcal{U}'_{\upsilon(1)}\right)\right)^{\sigma_1} \otimes \left(\sin\left(\mathcal{U}'_{\upsilon(2)}\right)\right)^{\sigma_2} \otimes ... \otimes \left(\sin\left(\mathcal{U}'_{\upsilon(n)}\right)\right)^{\sigma_n} \\
&= \prod_{\hbar=1}^{n}\left(\sin\left(\mathcal{U}'_{\upsilon(\hbar)}\right)\right)^{\sigma_\hbar}
\end{aligned}$$

where the weights of $\mathcal{U}_\hbar$ is represented by $\eth_\hbar$ $(\hbar = 1, 2, ..., n)$ having $\eth_\hbar \ge 0$ and $\sum_{\hbar=1}^{n}\eth_\hbar = 1$ and gth biggest weighted value is $\mathcal{U}'_{\upsilon(\hbar)}$ $\left(\mathcal{U}'_{\upsilon(\hbar)} = n\eth_\hbar\mathcal{U}_{\upsilon(\hbar)} | \hbar = 1, 2, ..., n\right)$ consequently by total order $(\upsilon(1), \upsilon(2), \upsilon(3), ..., \upsilon(n))$. Also the associated weight vector $\sigma = \sigma_\hbar$ $(\hbar = 1, 2, ..., n)$ with $\sigma_\hbar \ge 0$ and $\sum_{\hbar=1}^{n}\sigma_\hbar = 1$.

Theorem 3.4. *Let $\mathcal{U}_\hbar = \{\widetilde{\rho}_{\wp_\hbar}(\flat), \daleth_{\wp_\hbar}(\flat), \tilde{n}_{\wp_\hbar}(\flat)\} \in SVNN(\beth)$ $(\hbar \in \mathbb{N})$ and the weights of $\mathcal{U}_\hbar$ represented by $(\eth_1, \eth_2, ..., \eth_n)^T$ subject to $\sum_{\hbar=1}^{n}\eth_\hbar = 1$. The $ST - SVNHWG$ AO is defined by a mapping $G^n \longrightarrow G$ with associated weight vector $\sigma_\hbar$ $(\hbar = 1, 2, ..., n)$ having $\sigma_\hbar \ge 0$ and $\sum_{\hbar=1}^{n}\sigma_\hbar = 1$:*

$$\begin{aligned}
ST - SVNHWG\left(\mathcal{U}_1,\mathcal{U}_2,...,\mathcal{U}_n\right) &= \prod_{\hbar=1}^{n}\left(\sin\left(\mathcal{U}'_{\upsilon(\hbar)}\right)\right)^{\sigma_\hbar} \\
&= \begin{pmatrix} \prod_{\hbar=1}^{n}\left(\sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ 1-\prod_{\hbar=1}^{n}\left(\sin\left(\frac{\pi}{2}1-\daleth'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ 1-\prod_{\hbar=1}^{n}\left(\sin\left(\frac{\pi}{2}1-\tilde{n}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar} \end{pmatrix} \qquad (2)
\end{aligned}$$

Proof. By using mathematical induction on n we prove Theorem 3.4. Then the mathematical induction steps below were implemented.

Step-1: For $n = 2$, we get

$$ST - SVNHWG\,(\mathcal{U}_1,\mathcal{U}_2) = \left(\sin\left(\mathcal{U}'_{\upsilon(1)}\right)\right)^{\sigma_1} \otimes \left(\sin\left(\mathcal{U}'_{\upsilon(2)}\right)\right)^{\sigma_2}.$$

Since by the Definition 2.9, $\sin(\mathcal{U}_1)$ and $\sin(\mathcal{U}_2)$ are SFNs and hence $\left(\sin\left(\mathcal{U}'_{\upsilon(1)}\right)\right)^{\sigma_1} \otimes \left(\sin\left(\mathcal{U}'_{\upsilon(2)}\right)\right)^{\sigma_2}$ is also SVNN. Further, for $\mathcal{U}_1$ and $\mathcal{U}_2$, we have

$$\begin{aligned}
& ST - SVNHWG\,(\mathcal{U}_1,\mathcal{U}_2)\\
=\; & \left(\sin\left(\mathcal{U}'_{\upsilon(1)}\right)\right)^{\sigma_1} \otimes \left(\sin\left(\mathcal{U}'_{\upsilon(2)}\right)\right)^{\sigma_2}\\
=\; & \begin{pmatrix} \left(\sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(1)}}\right)\right)^{\sigma_1}, \\ 1-\left(\sin\left(\frac{\pi}{2}1-\daleth'_{\wp_{\upsilon(1)}}\right)\right)^{\sigma_1}, \\ 1-\left(\sin\left(\frac{\pi}{2}1-\tilde{n}'_{\wp_{\upsilon(1)}}\right)\right)^{\sigma_1} \end{pmatrix} \otimes \begin{pmatrix} \left(\sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(1)}}\right)\right)^{\sigma_1}, \\ 1-\left(\sin\left(\frac{\pi}{2}1-\daleth'_{\wp_{\upsilon(1)}}\right)\right)^{\sigma_1}, \\ 1-\left(\sin\left(\frac{\pi}{2}1-\tilde{n}'_{\wp_{\upsilon(1)}}\right)\right)^{\sigma_1} \end{pmatrix}\\
=\; & \begin{pmatrix} \prod_{\hbar=1}^{2}\left(\sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ 1-\prod_{\hbar=1}^{2}\left(\sin\left(\frac{\pi}{2}1-\daleth'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ 1-\prod_{\hbar=1}^{2}\left(\sin\left(\frac{\pi}{2}1-\tilde{n}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar} \end{pmatrix}
\end{aligned}$$

Step-2: Suppose that Equation 2 is holds for $n = \kappa$, we have

$$ST - SVNHWG\,(\mathcal{U}_1,\mathcal{U}_2,...\mathcal{U}_\kappa) = \begin{pmatrix} \prod_{\hbar=1}^{\kappa}\left(\sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ 1-\prod_{\hbar=1}^{\kappa}\left(\sin\left(\frac{\pi}{2}1-\daleth'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ 1-\prod_{\hbar=1}^{\kappa}\left(\sin\left(\frac{\pi}{2}1-\tilde{n}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar} \end{pmatrix}$$

Step-3: Now we have to prove that Equation 2 is holds for $n = \kappa + 1$.

$$\begin{aligned}
& ST - SVNHWG\,(\mathcal{U}_1,\mathcal{U}_2,...\mathcal{U}_{\kappa+1})\\
=\; & \prod_{\hbar=1}^{\kappa}\left(\sin\left(\mathcal{U}'_{\upsilon(\hbar)}\right)\right)^{\sigma_\hbar} \otimes \left(\sin\left(\mathcal{U}'_{\upsilon(\kappa+1)}\right)\right)^{\sigma_{\kappa+1}}\\
=\; & \begin{pmatrix} \prod_{\hbar=1}^{\kappa}\left(\sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ 1-\prod_{\hbar=1}^{\kappa}\left(\sin\left(\frac{\pi}{2}1-\daleth'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ 1-\prod_{\hbar=1}^{\kappa}\left(\sin\left(\frac{\pi}{2}1-\tilde{n}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar} \end{pmatrix} \otimes \begin{pmatrix} \left(\sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\kappa+1)}}\right)\right)^{\sigma_{\kappa+1}}, \\ 1-\left(\sin\left(\frac{\pi}{2}1-\daleth'_{\wp_{\upsilon(\kappa+1)}}\right)\right)^{\sigma_{\kappa+1}}, \\ 1-\left(\sin\left(\frac{\pi}{2}1-\tilde{n}'_{\wp_{\upsilon(\kappa+1)}}\right)\right)^{\sigma_{\kappa+1}} \end{pmatrix}\\
=\; & \begin{pmatrix} \prod_{\hbar=1}^{\kappa+1}\left(\sin\left(\frac{\pi}{2}\widetilde{\rho}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ 1-\prod_{\hbar=1}^{\kappa+1}\left(\sin\left(\frac{\pi}{2}1-\daleth'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar}, \\ 1-\prod_{\hbar=1}^{\kappa+1}\left(\sin\left(\frac{\pi}{2}1-\tilde{n}'_{\wp_{\upsilon(\hbar)}}\right)\right)^{\sigma_\hbar} \end{pmatrix}
\end{aligned}$$

that is, when $n = z + 1$, Equation 2 also holds.

Hence, Equation 2 holds for any n. The proof is completed. □

Next, we give some properties that are apparently carried by the proposed ST-SVNHWG aggregation operator.

(1) Let $\mathcal{U}_\hbar = \{\widetilde{\rho}_{\wp_\hbar}, \daleth_{\wp_\hbar}, \tilde{n}_{\wp_\hbar}\} \in SVNN\,(\beth)$ $(\hbar = 1, 2, 3, ..., n)$ such that $\mathcal{U}_\hbar = \mathcal{U}$. Then

$$ST - SVNHWG\,(\mathcal{U}_1,\mathcal{U}_2,...,\mathcal{U}_n) = \sin(\mathcal{U}).$$

(2) Let $\mathcal{U}_\hbar = \{\widetilde{\rho}_{\wp_\hbar}, \daleth_{\wp_\hbar}, \tilde{n}_{\wp_\hbar}\}$, $\mathcal{U}_\hbar^- = \{\min(\widetilde{\rho}_{\wp_\hbar}), \max(\daleth_{\wp_\hbar}), \max(\tilde{n}_{\wp_\hbar})\}$ and $\mathcal{U}_\hbar^+ = \{\max(\widetilde{\rho}_{\wp_\hbar}), \min(\daleth_{\wp_\hbar}), \min(\tilde{n}_{\wp_\hbar})\}$ $\in SVNN(\beth)$ $(\hbar = 1, 2, 3, ..., n)$. Then,

$$\sin(\mathcal{U}_\hbar^-) \leq ST - SVNHWG(\mathcal{U}_1, \mathcal{U}_2, ..., \mathcal{U}_n) \leq \sin(\mathcal{U}_\hbar^+).$$

(3) Let $\mathcal{U}_\hbar = \{\widetilde{\rho}_{\wp_\hbar}, \daleth_{\wp_\hbar}, \tilde{n}_{\wp_\hbar}\}$, $\mathcal{U}_\hbar^* = \{\widetilde{\rho}_{\wp_\hbar}^*, \daleth_{\wp_\hbar}^*, \tilde{n}_{\wp_\hbar}^*\} \in SVNN(\beth)$ $(\hbar = 1, 2, 3, ..., n)$. If $\widetilde{\rho}_{\wp_\hbar} \leq \widetilde{\rho}_{\wp_\hbar}^*$, $\daleth_{\wp_\hbar} \leq \daleth_{\wp_\hbar}^*$ and $\tilde{n}_{\wp_\hbar} \leq \tilde{n}_{\wp_\hbar}^*$, then

$$ST - SVNHWG(\mathcal{U}_1, \mathcal{U}_2, ..., \mathcal{U}_n) \leq ST - SVNHWG(\mathcal{U}_1^*, \mathcal{U}_2^*, ..., \mathcal{U}_n^*).$$

4 Decision Making Algorithm

This section describes a decision-making algorithm to address the uncertainty of decision-making problems (DMPs) in the SVN environment, supported by an illustrative example. The following notions are utilized to tackle the MCDM situations having SVN information. Suppose that $\{\beth_1, \beth_2, ..., \beth_k\}$ is a universal set and $\{r_1, r_2, r_3, ..., r_n\}$ is the universe of attributes. Assume $\eth = \{\eth_1, \eth_2, ..., \eth_n\}$ is a weigh-vector with $\eth_\hbar \in [0, 1]$ such that $\sum_{\hbar=1}^{n} \eth_\hbar = 1$. Consider $D^{(k)} = \left(\mathcal{U}_{ij}^{(k)}\right)_{k\times n}$ is a single valued nuetrosophic decision-matrix, which represents the membership values evaluated by the experts.

We construct an algorithmic method to handle MCDM problems by proposed aggregation operators. The algorithm includes steps below:

Step-1 Summarize the values of each alternative in term of decision matrix $D^{(k)} = \left(\mathcal{U}_{ij}^{(k)}\right)_{n\times m}$ with SVNNs.

Step-2 Construct the normalized decision matrix $P = (p_{ij})$ from $D = (\mathcal{U}_{ij})$, where p_{ij} is calculated as

$$p_{ij} = \begin{cases} (\widetilde{\rho}_{ij}, \daleth_{ij}, \tilde{n}_{ij}) & \text{If criteria are benefit type} \\ (\tilde{n}_{ij}, \daleth_{ij}, \widetilde{\rho}_{ij}) & \text{If criteria are cost type} \end{cases} \quad (3)$$

Step-3 If the attribute weights are known as a prior then utilize them. Otherwise, we compute them by utilizing the concept of the entropy measure. For it, the information entropy of criteria t_j is computed as

$$En_j = \frac{1 + \frac{1}{n}\sum_{i=1}^{n}\left(\widetilde{\rho}_{\wp_{ij}}\log\left(\widetilde{\rho}_{\wp_{ij}}\right) + \daleth_{\wp_{ij}}\log\left(\daleth_{\wp_{ij}}\right) + \tilde{n}_{\wp_{ij}}\log\left(\tilde{n}_{\wp_{ij}}\right)\right)}{\sum_{j=1}^{m}\left(1 + \frac{1}{n}\sum_{i=1}^{n}\widetilde{\rho}_{\wp_{ij}}\log\left(\widetilde{\rho}_{\wp_{ij}}\right) + \daleth_{\wp_{ij}}\log\left(\daleth_{\wp_{ij}}\right) + \tilde{n}_{\wp_{ij}}\log\left(\tilde{n}_{\wp_{ij}}\right)\right)}$$

Step-4 Using proposed aggregation operators defined in Theorem 3.2 & 3.4 and attributes weight vector, the aggregated single valued neutrosophic numbers of the each alternative $\{\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, ..., \mathcal{U}_n\}$ are obtained.

Step-6 Evaluate the scores values $\check{s}\check{c}(\mathcal{U}_\hbar)$ using the Definition 2.11 of collective spherical fuzzy numbers $\mathcal{U}_\hbar$ $(\hbar = 1, 2, ..., n)$ and rank using the Definition 2.12 according the maximum score values.

Step-6 Select the optimal alternative according the maximum score value or accuracy degree.

5 Application Decision Making Algorithm

In this segment, the numerical implementation of agricultural land selection is used to demonstrate the MCDM methodology developed.

5.1 Practical case study

Agriculture is an important component of the Economic System of Pakistan. This area directly supports the population of the country and accounts for 26% of gross domestic product (GDP). The major agricultural crops are sugarcane, wheat , rice, cotton, vegetables and fruit. A businessman wants to invest in the agriculture sector and to look for appropriate land. The options in his brain are $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3, \mathcal{Y}_4$ and $\mathcal{Y}_5$. He consults to an expert to get his suggestion about the alternatives based on the following desired parameters:

(1) Location (r_1) ,
(2) Climate (r_2) ,
(3) Fertility (r_3) ,
(4) Price (r_4) and
(5) Water Availability (r_5) .

The expert was asked to use SVN data in this assessment and weights of the attributes are $(0.15, 0.28, 0.20, 0.22, 0.15)^T$. The expert's findings are summarized in Table-1:

Table-2.: Expert Information

$$D^1 = \begin{array}{c} \\ \mathcal{Y}_1 \\ \mathcal{Y}_2 \\ \mathcal{Y}_3 \\ \mathcal{Y}_4 \\ \mathcal{Y}_5 \end{array} \begin{array}{c} \begin{array}{ccccc} r_1 & r_2 & r_3 & r_4 & r_5 \end{array} \\ \begin{pmatrix} (0.5,0.3,0.4) & (0.3,0.2,0.5) & (0.2,0.2,0.6) & (0.4,0.2,0.3) & (0.3,0.3,0.4) \\ (0.7,0.1,0.3) & (0.3,0.2,0.7) & (0.6,0.3,0.2) & (0.2,0.4,0.6) & (0.7,0.1,0.2) \\ (0.5,0.3,0.4) & (0.4,0.2,0.6) & (0.6,0.1,0.2) & (0.3,0.1,0.5) & (0.6,0.4,0.3) \\ (0.7,0.3,0.2) & (0.2,0.2,0.7) & (0.4,0.5,0.2) & (0.2,0.2,0.5) & (0.4,0.5,0.4) \\ (0.4,0.1,0.3) & (0.2,0.1,0.5) & (0.4,0.1,0.5) & (0.6,0.3,0.4) & (0.3,0.2,0.4) \end{pmatrix} \end{array}$$

Step-2 The r_1, r_3 & r_5 are benefits type and r_2 & r_4 are cost type criteria. According to defined formula 3, the normalized decision matrix is summarized in Table-2:

Table-2.: Normalized Information

$$P = \begin{array}{c} \\ \mathcal{Y}_1 \\ \mathcal{Y}_2 \\ \mathcal{Y}_3 \\ \mathcal{Y}_4 \\ \mathcal{Y}_5 \end{array} \begin{array}{c} \begin{array}{ccccc} r_1 & r_2 & r_3 & r_4 & r_5 \end{array} \\ \begin{pmatrix} (0.5,0.3,0.4) & (0.5,0.2,0.3) & (0.2,0.2,0.6) & (0.3,0.2,0.4) & (0.3,0.3,0.4) \\ (0.7,0.1,0.3) & (0.7,0.2,0.3) & (0.6,0.3,0.2) & (0.6,0.4,0.2) & (0.7,0.1,0.2) \\ (0.5,0.3,0.4) & (0.6,0.2,0.4) & (0.6,0.1,0.2) & (0.5,0.1,0.3) & (0.6,0.4,0.3) \\ (0.7,0.3,0.2) & (0.7,0.2,0.2) & (0.4,0.5,0.2) & (0.5,0.2,0.2) & (0.4,0.5,0.4) \\ (0.4,0.1,0.3) & (0.5,0.1,0.2) & (0.4,0.1,0.5) & (0.4,0.3,0.6) & (0.3,0.2,0.4) \end{pmatrix} \end{array}$$

Step-3 The Expert provides the following parameters weights

$$\kappa = \{\kappa_1 = 0.15, \kappa_2 = 0.28, \kappa_3 = 0.20, \kappa_4 = 0.22, \kappa_5 = 0.15\}$$

Step-4 In this step, we used proposed AOp namely, ST-SVHWA and ST-SVHWG to aggregate the single valued neutrosophic information as follows:

Firstly, we find out the weighted matrix shown in Table-3:

Table-3.: $\left(n\eth_{\hbar}\mathcal{U}_{\upsilon(\hbar)}|\hbar = 1, 2, ..., n\right)$

$$\begin{pmatrix} (0.60,0.09,0.14) & (0.8,0.004,0.01) & (0.30,0.02,0.17) & (0.48,0.01,0.06) & (0.36,0.09,0.14) \\ (0.81,0.01,0.09) & (0.9,0.004,0.01) & (0.80,0.04,0.02) & (0.83,0.06,0.01) & (0.81,0.01,0.05) \\ (0.60,0.09,0.14) & (0.9,0.004,0.02) & (0.8,0.004,0.02) & (0.7,0.002,0.03) & (0.71,0.14,0.09) \\ (0.81,0.09,0.05) & (0.9,0.004,0.004) & (0.58,0.12,0.02) & (0.74,0.01,0.01) & (0.48,0.20,0.14) \\ (0.48,0.01,0.09) & (0.8,0.005,0.04) & (0.5,0.004,0.12) & (0.62,0.03,0.14) & (0.36,0.05,0.14) \end{pmatrix}$$

Table-4.: Corresponding Score Values

$$\begin{pmatrix} (0.3557) & (0.8037) & (0.1144) & (0.4113) & (0.1187) \\ (0.6943) & (0.9380) & (0.7444) & (0.7634) & (0.7384) \\ (0.3557) & (0.8687) & (0.7840) & (0.7052) & (0.4650) \\ (0.6601) & (0.9467) & (0.4454) & (0.7140) & (0.1296) \\ (0.3696) & (0.8160) & (0.4603) & (0.4433) & (0.1628) \end{pmatrix}$$

Now, according to the score values, the $\left(\mathcal{U}'_{\upsilon(\hbar)}:\hbar=1,2,...,n\right)$ values are shows as follows:

Table-5.: $\left(\mathcal{U}'_{\upsilon(\hbar)}:\hbar=1,2,...,n\right)$

$\mathcal{U}'_{\upsilon(11)}=\mathcal{U}_{12}=(0.5,0.2,0.3)$	$\mathcal{U}'_{\upsilon(12)}=\mathcal{U}_{14}=(0.3,0.2,0.4)$	$\mathcal{U}'_{\upsilon(13)}=\mathcal{U}_{11}=(0.5,0.3,0.4)$
$\mathcal{U}'_{\upsilon(14)}=\mathcal{U}_{15}=(0.3,0.3,0.4)$	$\mathcal{U}'_{\upsilon(15)}=\mathcal{U}_{13}=(0.2,0.2,0.6)$	
$\mathcal{U}'_{\upsilon(21)}=\mathcal{U}_{22}=(0.7,0.2,0.3)$	$\mathcal{U}'_{\upsilon(22)}=\mathcal{U}_{24}=(0.6,0.4,0.2)$	$\mathcal{U}'_{\upsilon(23)}=\mathcal{U}_{23}=(0.6,0.3,0.2)$
$\mathcal{U}'_{\upsilon(24)}=\mathcal{U}_{25}=(0.7,0.1,0.2)$	$\mathcal{U}'_{\upsilon(25)}=\mathcal{U}_{21}=(0.7,0.1,0.3)$	
$\mathcal{U}'_{\upsilon(31)}=\mathcal{U}_{32}=(0.6,0.2,0.4)$	$\mathcal{U}'_{\upsilon(32)}=\mathcal{U}_{33}=(0.6,0.1,0.2)$	$\mathcal{U}'_{\upsilon(33)}=\mathcal{U}_{34}=(0.5,0.1,0.3)$
$\mathcal{U}'_{\upsilon(34)}=\mathcal{U}_{35}=(0.6,0.4,0.3)$	$\mathcal{U}'_{\upsilon(35)}=\mathcal{U}_{31}=(0.5,0.3,0.4)$	
$\mathcal{U}'_{\upsilon(41)}=\mathcal{U}_{42}=(0.7,0.2,0.2)$	$\mathcal{U}'_{\upsilon(42)}=\mathcal{U}_{44}=(0.5,0.2,0.2)$	$\mathcal{U}'_{\upsilon(43)}=\mathcal{U}_{41}=(0.7,0.3,0.2)$
$\mathcal{U}'_{\upsilon(44)}=\mathcal{U}_{43}=(0.4,0.5,0.2)$	$\mathcal{U}'_{\upsilon(45)}=\mathcal{U}_{45}=(0.4,0.5,0.2)$	
$\mathcal{U}'_{\upsilon(51)}=\mathcal{U}_{52}=(0.5,0.1,0.2)$	$\mathcal{U}'_{\upsilon(52)}=\mathcal{U}_{53}=(0.4,0.1,0.5)$	$\mathcal{U}'_{\upsilon(53)}=\mathcal{U}_{54}=(0.4,0.3,0.6)$
$\mathcal{U}'_{\upsilon(54)}=\mathcal{U}_{51}=(0.4,0.1,0.3)$	$\mathcal{U}'_{\upsilon(55)}=\mathcal{U}_{55}=(0.3,0.2,0.4)$	

Aggregated information of the alternatives are calculated in Table-6 as follows:

Table-6.: Aggregated Information

	$ST-SVNHWA$	$ST-SVNHWG$
$\mathcal{Y}_1$	$(0.54515,0.02797,0.08164)$	$(0.50042,0.03039,0.08899)$
$\mathcal{Y}_2$	$(0.85733,0.02063,0.02539)$	$(0.85066,0.03635,0.02741)$
$\mathcal{Y}_3$	$(0.77818,0.01567,0.04227)$	$(0.77177,0.02991,0.04829)$
$\mathcal{Y}_4$	$(0.76483,0.04584,0.02450)$	$(0.71602,0.06397,0.02901)$
$\mathcal{Y}_5$	$(0.59151,0.00952,0.07508)$	$(0.58134,0.01529,0.09549)$

Step-6 Score value of the aggregated alternatives are calculated in Table-7:

Table-7.: Score Values

	$\check{s}\check{c}(\mathcal{Y}_1)$	$\check{s}\check{c}(\mathcal{Y}_2)$	$\check{s}\check{c}(\mathcal{Y}_3)$	$\check{s}\check{c}(\mathcal{Y}_4)$	$\check{s}\check{c}(\mathcal{Y}_5)$
$ST-SVNHWA$	0.43553	0.81130	0.72023	0.69448	0.50690
$ST-SVNHWG$	0.38103	0.78688	0.69357	0.62302	0.47055

Step-7 Ranking of the alternatives are shown in Table-8:

Table-8.: Ranking

	Score Ranking	Best Alternative
$ST-SVNHWA$	$\check{s}\check{c}(\mathcal{Y}_2)>\check{s}\check{c}(\mathcal{Y}_3)>\check{s}\check{c}(\mathcal{Y}_4)>\check{s}\check{c}(\mathcal{Y}_5)>\check{s}\check{c}(\mathcal{Y}_1)$	$\mathcal{Y}_2$
$ST-SVNHWG$	$\check{s}\check{c}(\mathcal{Y}_2)>\check{s}\check{c}(\mathcal{Y}_3)>\check{s}\check{c}(\mathcal{Y}_4)>\check{s}\check{c}(\mathcal{Y}_5)>\check{s}\check{c}(\mathcal{Y}_1)$	$\mathcal{Y}_2$

From this above computational process, we can conclude that the alternative $\mathcal{Y}_2$ is the best among the others and hence it is highly recommendable to select for the required task/plan.

5.2 Comparison Study

In the section, we include some appropriate examples to demonstrate the feasibility and efficacy of the established decision-making approach and make a comparison with existing studies. Here, we presented the different existing aggregated information of the SVNNs namely, SVNWA,[33] SVNOWA,[33] NWA,[44] SVNFWA,[18] SVNHWA,[32] L-SVNWA,[19] L-SVNOWA,[19] ST-SVNWA[4] and ST-SVNWG[4] in Table-9, 10 and 11:

Table-9: Aggregated Information

	SVNWA[33]	SVNOWA[33]	NWA[44]	SVNFWA[18]
$\mathcal{Y}_1$	$(0.37,0.22,0.40)$	$(0.38,0.24,0.40)$	$(0.37,0.23,0.42)$	$(0.37,0.22,0.40)$
$\mathcal{Y}_2$	$(0.66,0.20,0.23)$	$(0.66,0.18,0.24)$	$(0.66,0.24,0.24)$	$(0.66,0.20,0.23)$
$\mathcal{Y}_3$	$(0.56,0.17,0.31)$	$(0.55,0.18,0.31)$	$(0.56,0.21,0.32)$	$(0.56,0.17,0.31)$
$\mathcal{Y}_4$	$(0.57,0.29,0.22)$	$(0.57,0.31,0.22)$	$(0.57,0.33,0.23)$	$(0.56,0.29,0.22)$
$\mathcal{Y}_5$	$(0.41,0.14,0.36)$	$(0.39,0.13,0.36)$	$(0.41,0.16,0.41)$	$(0.41,0.14,0.36)$

Table-10: Aggregated Information

	SVNHWA[32] $\gamma = 2$	SVNHWA[32] $\gamma = 3$	L-SVNWA[19]	L-SVNOWA[19]
$\mathcal{Y}_1$	$(0.37, 0.22, 0.40)$	$(0.36, 0.22, 0.40)$	$(0.31, 0.17, 0.35)$	$(0.32, 0.19, 0.36)$
$\mathcal{Y}_2$	$(0.66, 0.20, 0.23)$	$(0.66, 0.20, 0.23)$	$(0.64, 0.19, 0.23)$	$(0.65, 0.17, 0.23)$
$\mathcal{Y}_3$	$(0.56, 0.17, 0.31)$	$(0.56, 0.18, 0.31)$	$(0.49, 0.17, 0.33)$	$(0.48, 0.18, 0.33)$
$\mathcal{Y}_4$	$(0.56, 0.29, 0.22)$	$(0.56, 0.30, 0.22)$	$(0.55, 0.27, 0.19)$	$(0.55, 0.29, 0.19)$
$\mathcal{Y}_5$	$(0.41, 0.14, 0.36)$	$(0.41, 0.14, 0.37)$	$(0.28, 0.12, 0.37)$	$(0.24, 0.12, 0.38)$

Table-11: Aggregated Information

	ST-SVNWA[4]	ST-SVNWA[4]
$\mathcal{Y}_1$	$(0.56, 0.02, 0.07)$	$(0.50, 0.02, 0.08)$
$\mathcal{Y}_2$	$(0.86, 0.02, 0.02)$	$(0.85, 0.03, 0.03)$
$\mathcal{Y}_3$	$(0.77, 0.01, 0.04)$	$(0.76, 0.02, 0.05)$
$\mathcal{Y}_4$	$(0.78, 0.04, 0.02)$	$(0.73, 0.06, 0.02)$
$\mathcal{Y}_5$	$(0.60, 0.009, 0.06)$	$(0.59, 0.01, 0.08)$

Also, their corresponding ranking are calculated in Table-12:

Table-12: Ranking

Existing Operators	Ranking	Best Alternative
NWA[43]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
SVNWA[33]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
SVNOWA[33]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
SVNWG[33]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
SVNOWG[33]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
SVNFWA[18]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
SVNHWA[32] $\gamma = 2$	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
SVNHWA[32] $\gamma = 3$	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
NWG[43]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
SVNFWG[18]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
SVNHWG[32] $\gamma = 2$	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
SVNHWG[32] $\gamma = 3$	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
SNWEA[44]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_5 > \mathcal{Y}_4 > \mathcal{Y}_1$	$\mathcal{Y}_2$
L-SVNWA[19]	$\mathcal{Y}_2 > \mathcal{Y}_4 > \mathcal{Y}_3 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
L-SVNOWA[19]	$\mathcal{Y}_2 > \mathcal{Y}_4 > \mathcal{Y}_3 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
L-SVNWG[19]	$\mathcal{Y}_2 > \mathcal{Y}_4 > \mathcal{Y}_3 > \mathcal{Y}_1 > \mathcal{Y}_5$	$\mathcal{Y}_2$
L-SVNOWG[19]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
L-SVNWA[4]	$\mathcal{Y}_2 > \mathcal{Y}_4 > \mathcal{Y}_3 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
L-SVNWG[4]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
L-SVNOWA[4]	$\mathcal{Y}_2 > \mathcal{Y}_4 > \mathcal{Y}_3 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
L-SVNOWG[4]	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$

Table-9.: Overall ranking of the alternatives

Proposed Operators	Ranking	Best Alternative
L-SVNHWA	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$
L-SVNHWG	$\mathcal{Y}_2 > \mathcal{Y}_3 > \mathcal{Y}_4 > \mathcal{Y}_5 > \mathcal{Y}_1$	$\mathcal{Y}_2$

It is evident that the above conversation confirms the effectiveness and applicability of the proposed decision-making methodology based on new sine trigonometric aggregation operators under SVN environments.

6 Conclusion

Due to the existence of multiple attributes/criteria in many real-world problems, classical MCDM methods are not useful to tackle complicated decision-making situations. To overcome the difficulties of existing models, we have combined SVNSs with sine trigonometric AOs.

In this article, we have discussed MCDM issues based on single valued neutrosophic information. Motivated by sine trigonometric function based operational laws, we have proposed different AOs, namely, ST-SVNHWA and ST-SVNHWG aggregation operators. We have investigated different features of these operators. We have employed these AOs to enlarged the applicability scope of MCDM. We have given real-life application for the selection of best agricultural land for best investment. At the end, we have provided a comparison of developed AOs with existing aggregation techniques in the literatures and authenticate the proposed strategy by effectiveness comparison study. In the future, we are extending our work in interval valued single valued neutrosophic environment.

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