Neutrosophic Crisp β- Functions

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Abstract

The purpose of the present paper is to introduce and study the concept of β-continuous function and β-open function in neutrosophic crisp topological spaces. Finally, some characterizations concerning neutrosophic crisp functions are presented and one obtains several properties.

Keywords: Neutrosophic crisp β-continuous function, Neutrosophic crisp β-open function and Neutrosophic crisp β-closed function

1. Introduction

Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^-0, 1^+[$ is non-standard unit interval. After the introduction of the neutrosophic crisp set concepts in [1-8] and after haven given the fundamental definitions of neutrosophic crisp set operations. Some applications of neutrosophic theory can be found in [12-16]. We generalize the crisp functions to the notion of neutrosophic crisp set. Finally, we introduce the definitions of neutrosophic crisp β-continuous function and neutrosophic crisp β-open function, and we obtain several properties and some characterizations concerning the neutrosophic crisp topologicl space.

2. Preliminaries

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [9-11]. Salama et al. [1-8] considered some possible definitions for basic concepts of the neutrosophic crisp set and its operations. We now improve some results by the following.

Definition 2.1 [4]

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For any non-empty fixed set X, a neutrosophic crisp set (NC-set, for short) A is an object having the form $A=\langle A_{1,2}, A_3\rangle$, where A_1, A_2 and A_3 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$, $A_1 \cap A_3 = \emptyset$ and $A_3 \cap A_2 = \emptyset$. Several relations and operations between NC-sets were defined in [3, 6, 8].

Definition 2.2 [3]

A neutrosophic crisp topology (NCT, for short) on a non-empty set X is a family Γ of neutrosophic crisp subsets of X satisfying the following axioms

i) $\emptyset$, $X_N \in \Gamma$.

ii) $A_1 \cap A_2 \in \Gamma$ for any A_1 and $A_2 \in \Gamma$.

iii) $\cup A_j \in \Gamma$ for any $\{A_j : j \in J\} \subseteq \Gamma$.

In this case the pair $(X,)$ is called a neutrosophic crisp topological space ($NCTS$, for short) in X. The elements in Γ are called neutrosophic crisp open sets (NC-open sets for short) in X. A NC-set F is said to be neutrosophic crisp closed set (NC-closed set, for short) if and only if its complement F^c is a NC-open set.

Definition 2.3 [3]

Let $(X,)$ be a $NCTS$ and $A=\langle A_{1,2}, A_3\rangle$ be a NC-set in X. Then the neutrosophic crisp closure of A ($NC(A)$ for short) and neutrosophic crisp interior ($NCint(A)$ for short) of A are defined by:

(i) $NCcl(A)=\cap\{K:K$ is a NC-closed set in X and $A\subseteq K\}$

(ii) $NCint(A)=\cup\{G:G$ is a NC-open set in X and $G\subseteq A)$,

It can be also shown that $NC(A)$ is a NC-closed set, and $NCint(A)$ is a NC-open set in X .

Definition 2.4 [1]

Let $(X,)$ be a $NCTS$ and $A=\langle A_{1,2}, A_3\rangle$ be a NCS in X, then A is called:

i) Neutrosophic crisp α-open set iff $A \subseteq NCi(NCcl(NCint(A))$.

ii) Neutrosophic crisp semi-open set iff $A \subseteq NC(NCint(A))$.

iii) Neutrosophic crisp pre-open set iff $A \subseteq NCi(NCcl(A))$.

iv) Neutrosophic crisp β- open set iff $A \subseteq (NCcl\ (NCi(NCcl\ (A))$.

Definition 2.5 [1]

Let $(X,)$ be a $NCTS$ and $A=\langle A_{1,2}, A_3\rangle$ be a NCS in X, and f: $X \longrightarrow X$ then:

1) If $f NC\alpha$-continuous $\Longrightarrow$ inverse image of $NC\alpha$ open set is $NC\alpha$- open set
2) If f NCpre-continuous $\Longrightarrow$ inverse image of NCpre-open set is NCpre- open set
3) If f NCsemi-continuous $\Longrightarrow$ inverse image of NCsemi-open set is NCsemi- open set
4) If f $NC\beta$-continuous $\Longrightarrow$ inverse image of $NC\beta$-open set is $NC\beta$- open set

Definition 2.6 [3]

(a) If $A=\langle A_1,A_2,A_3\rangle$ is a NC-set in X, then the NC-image of A under f denoted by $f(A)$ is the a NC-set in Y defined by $f(A)=\langle f(A_1),f(A_2),f(A_3)\rangle$

(b) If f is a bijective map then $f:(X, \Gamma_1)\longrightarrow(Y, \Gamma_2)$ is a map defined such that: for any NC-set $B=\langle B_1,B_2,B_3\rangle$ in Y, the NC-preimage of B, denoted by $f^{-1}(B)$ is a NC-set in X defined by $f^{-1}(B)=\langle f^{-1}(B_1),f^{-1}(B_2),f^{-1}(B_3)\rangle$.

Here we introduce the properties of NC-images and NC-preimages, some of which we shall frequently use in the following sections.

Corollary 2.1 [3]

Let $A=\{A_i$:$i\in J\}$ be NC-open sets in X, and $B=\{B_j$: $j\in K\}$ be NC-open sets in Y, and $f:(X, \Gamma_1)\longrightarrow(Y, \Gamma_2)$ a function. Then

(i) $A_1\subseteq A_2 \Leftrightarrow f\ (A_1)\subseteq f\ (A_2)$, $B_1\subseteq B_2 \Leftrightarrow f^{-1}(B_1)\subseteq f^{-1}(B_2)$,

(ii) $A\subseteq f^{-1}((A))$ and if f is injective, then $A_1= f^{-1}((A))$.

(iii) $f^{-1}((B))\subseteq B$ and if f is injective, then $f^{-1}(f(B))= B$.

(iv) $f^{-1}(\cup B_i)= \cup f^{-1}(B_i)$, $f^{-1}(\cap B_i)\subseteq \cap f^{-1}(B_i)$,

(v) $f(\cup A_i)= \cup f(A_i)$, $f(\cap A_i)\subseteq \cap f(A_i)$.

(vi) $f^{-1}(Y_N)= X_N$, $f^{-1}(\Phi_N)= \Phi_N$,

(vii) $(\Phi_N)= \Phi_N$, $(X_N)= Y_N$, if f is subjective.

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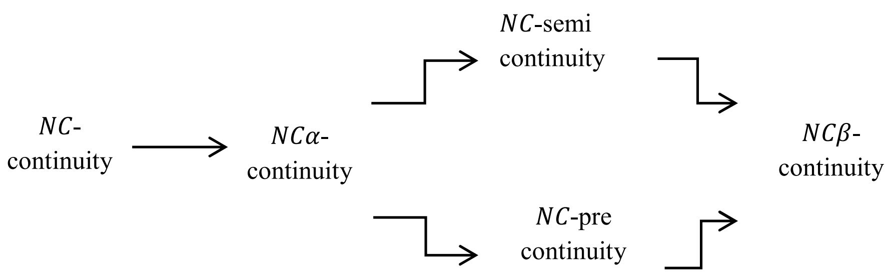
3. Neutrosophic crisp β-continuous function

Definition 3.1

A function $:(,\Gamma_1)\longrightarrow(Y,\Gamma_2)$ is said to be $NC\beta$-continuous (briefly $NC\beta$-cont) if the inverse image of each NC-open set in Y is $NC\beta$-open in X.

It is clear that the class of $NC\beta$-continuity contains each of classes NC-semiopen and NC-preopen the implication between then and other type of continuities are given by the following diagram.



The converses of these implication not hold, in general, as shown in the following example.

Example 3.1

Let X=Y= {a,b,c,d} and let the NCT on X be indiscrete and on Y be discrete. The identity function $:(,\Gamma_1)\longrightarrow(Y,\Gamma_2)$ is $NC\beta$-continuous but not NC-semi continuous.

Example 3.2

Let X=Y={a,b,c,d} with NC-topologies $\Gamma_x=\{X_N,\Phi_N,A\}$, $\Gamma_y=\{Y_N,\Phi_N,D\}$ where

A=⟨{a,b}, {b,d},{c}⟩,

D= ⟨{a,d},{a,d},{c}⟩.

A function $:(,\Gamma_1)\longrightarrow(Y,\Gamma_2)$ which defined as

(a)=a, (b)=c and f(c)=b f(d)=a, is $NC\beta$-cont. but not NC-pre cont.

The following theorem gives easy characterization of $NC\beta$-continuity.

Theorem 3.3

Each $NC\beta$-open set which is also NCsemi-closed set is NCsemi-open.

Proof. Let A= ⟨A_1,A_2,A_3⟩ be a $NC\beta$-open set which is also $NC\beta$-closed set then, $A\subseteq NCcl\ NCint\ NCclA$ and $A\subseteq NCint\ NCclA$. Thus $NCint\ NCclA\subseteq A\subseteq NC(NCintNCclA)$. Therefore, A= ⟨$A_{1,2}$,A_3⟩ is NC-semiopen.

Theorem 3.4

Let $:(,\Gamma_1)\longrightarrow(Y,\Gamma_2)$ be a function. The following statement are equivalent.

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(i) f is $NC\beta$-cont.

For each NC-set $x \epsilon X$ and each NC-open set $= \langle V_1,V_2,V_3\rangle \subseteq Y$ containing $f(x)$, there exists a $NC\beta$-open set $W= \langle W_1,W_2,W_3\rangle \subseteq X$ containing (x) such that $f(w)\subseteq V$.

(ii) the inverse image of each NC-closed set in Y is $NC\beta$-closed set in X.

(iii) $NCint\ NCcl\ NCi(f^{-1}(A)))\subseteq f^{-1}(NCcl(A))$.

(iv) $f(NCint\ NCcl\ NCint\ D) \subseteq NCcl(f(D))$. For each $D= \langle D_{1,2},D_3\rangle \subseteq X$.

Proof. (i)⟹(ii). Since $V= \langle V_{1,2},V_3\rangle \subseteq Y$ containing f(x) is NC-open, then $f^{-1}(V) \epsilon NC\beta$(x). NC-set $W= f^{-1}(V)$. which containing X, therefore $f(w) \subseteq V$.

(i)⟸(ii). Let $V= \langle V_{1,2},V_3\rangle \subseteq Y$ be NC-open, and let x$\epsilon f^{-1}(V)$, then $f(x)\epsilon V$ and thus there exists $W_x\epsilon NC\beta$(x).such that xϵW_x and $f(W_x) \subseteq V$. then x$\epsilon W_x\subseteq f^{-1}(V)$, and so, $f^{-1}(V)= \cup W_x$, x$\epsilon f^{-1}(V)$. but $\cup W_x \epsilon NC\beta$(x) By Theorem 2.5. hence $f^{-1}(V)\subseteq NC\beta$(x). and therefore f is $NC\beta$-cont.

(i)⟹(iii). Let $F= \langle F_{1,2},F_3\rangle\subseteq$ be NC-closed, Y-F is NC-open, $f^{-1}(Y\text{-}F)\epsilon$ $NC\beta$(x). i.e., X-$f^{-1}(F)$ $NC\beta$(x). then $f^{-1}(F)=\langle f^{-1}(F_1),f^{-1}(F_2),f^{-1}(F_3)\rangle$ is $NC\beta$-closed set in X.

(iii) ⟹ (iv). Let $A= \langle A_{1,2},A_3\rangle \subseteq Y$, then $f^{-1}(NCcl(A))$ is $NC\beta$-closed set in X, i.e., $f^{-1}(NCcl(A))\supseteq NCint\ NCcl\ NCint(f^{-1}(NCcl(A))) \supseteq NCint\ NCcl\ NCint(f^{-1}(A)))$.

(iv)⟹(v). Let $D=\langle D_1,D_2,D_3\rangle\subseteq X$, NC-set $A=f(D)$ in (iv), that $NCintNCcl\ NCint(f^{-1}(f(D)) \subseteq f^{-1}(NCcl(f(D)))$, $NCint\ NCcl\ int(D)\subseteq f^{-1}(NCcl(f(D)))$, then $f(NCint\ NCcl\ NCint(D)) \subseteq (NCcl(f(D)))$.

(v)⟹(i). Let $V= \langle V_1,V_2,V_3\rangle\subseteq Y$ be a NC-open, W=Y-V and $D= f^{-1}(W)$, by $f(NCint\ NCcl\ NCint(f^{-1}(W)))\subseteq NCcl(f(f^{-1}(W)))\subseteq NCcl(W)= W$.

so, $NCint\ NCcl\ NCint(f^{-1}(W))\subseteq f^{-1}(W)$, i.e., $f^{-1}(W)= \langle f^{-1}(W_1),f^{-1}(W_2),f^{-1}(W_3)\rangle$ is $NC\beta$-closed set in X, thus f is $NC\beta$-cont.

Theorem 3. 5

If :(, Γ_1)⟶(Y, Γ_2) be a $NC\beta$-cont. and $NC\alpha$-open function. Then the inverse image of any $NC\beta$-open set in Y is $NC\beta$-open set of X.

Proof. Let $W= \langle W_{1,2},W_3\rangle \epsilon NC\beta$(y), then $W \subseteq NCcl\ NCint\ NCcl(W)$ and so, $f^{-1}(W)\subseteq f^{-1}(NCcl\ NCint\ NCcl(W))\subseteq NCcl(f^{-1}(NCint\ NCcl\ (W)))$. because f is $NC\alpha$-open and $NCint\ NCcl(W)$ is NC-preopen. Since f is $NC\beta$-cont. $f^{-1}(W)\subseteq NCcl\ NCint\ NCcl\ (f^{-1}(NCint\ NCcl\ (W))\subseteq NCcl\ NCint\ NCcl(f^{-1}(NCcl\ NCint\ NCcl(W))\subseteq NCcl\ NCint\ NCcl(f^{-1}((W))$ - $NCint\ NCcl(f^{-1}((W))$. Because f is $NC\alpha$-open.

Theorem 3. 6

If :(, Γ_1)⟶(Y, Γ_2) be a $NC\beta$-cont. and NC-open function, Then the

following statements hold.

(i) The inverse image of each NC-preopen in Y is $NC\beta$-open in X.

(ii) The inverse image of each NC-semiopen in Y is $NC\beta$-open in X.

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Proof. (i) Let $A=\langle A_1,A_2,A_3\rangle \in NC$-preopen (Y), $A\subseteq NCint\ NCcl\ A$, then $f^{-1}(A)\subseteq NCcl\ NCint\ NCcl(f^{-1}(NCint\ NCclf^{-1}(A)))\subseteq NCcl\ NCint\ NCcl(f^{-1}(NCclf(A)))\subseteq NCcl\ NCint\ NCcl\ f^{-1}(A)= NCcl\ NCint\ NCcl\ f^{-1}(A)$.

(ii) Let $D\epsilon\ NC$-semiopen (Y), $D\subseteq NCcl\ NCint\ D$, and so, $f^{-1}(D)\subseteq f^{-1}(NCcl\ NCintD\subseteq NCcl(f^{-1}(NCint(D)))\subseteq NCclNCintNCcl(f^{-1}(NCint(D))) = NCclNCintNCclf^{-1}(NCint(D))\subseteq NCcl\ NCint\ NCcl\ f^{-1}(D)$.

Theorem 3. 7

Let :(, $\Gamma_1)\longrightarrow(Y, \Gamma_2)$ be $NC\beta$-cont. surjective such that $NCcl\ NCint\ NCclf^{-1}(V)\subseteq f^{-1}(NCclV)$, for each NC-open set $V=\langle V_1,V_2,V_3\rangle \subseteq$Y. if X is connected, then Y is connected.

Proof. Let Y is not connected, i.e., there exists two NC-open sets $_1$ and V_2 such that $V_1\cup V_2=Y$ and $V_1\cap V_2=\phi$. Since f is $NC\beta$-cont. then $f^{-1}(V_i)\subseteq (NCcl\ NCintNCcl(f^{-1}(V_i)))\subseteq f^{-1}(NCcl\ (V_i))= f^{-1}(V_i)$, $i\in\{1,2\}$. so, $\cap f^{-1}(V_i)\subseteq\cap NCclNCintNCcl(f^{-1}(V_i))\subseteq\cap f^{-1}(NCcl(V_i))\subseteq f^{-1}(\cap V_i) = f^{-1}(\phi)=\phi$, and $\cup f^{-1}(V_i)\subseteq \cup(NCclNCint\ NCcl(f^{-1}(V_i)))\subseteq\cup f^{-1}(NCcl\ V_i))=\cup f^{-1}(V_i)= f^{-1}(\cup V_i)= f^{-1}(Y)= X$.

Therefore X is not connected which leads to a contradiction. Then Y is connected. The relation between $NC\beta$-cont. and θ-cont. will be clear by the following theorem.

Remark 3.1

The composition of two $NC\beta$-cont. functions need not be $NC\beta$-cont. in general, as shown by the following example.

Example 3.3

Let $X=Z=\{a,b,c,d\}$ and $Y=\{a,b,c,d,e\}$with NC-topologies $\Gamma_x=\{X_N,\Phi_N,A\}$, $\Gamma_y=\{Y_N,\Phi_N,C\}$, $\Gamma_z=\{Z_N,\Phi_N,D\}$ where

$A=\langle\{a,b\},\{c\},\{b\}\rangle$,

$C=\langle\{d\},\{c\},\{a,b\}\rangle$, and

$D=\langle\{a,d\},\{c,d\},\{a,b\}\rangle$.

Let the identity function $f:(X, \Gamma_1)\longrightarrow(Y, \Gamma_2)$ and $i:(X, \Gamma_2)\longrightarrow(Y, \Gamma_3)$ defined as $f(a)=a$, $f(b)=b=f(d)$ and $f(c)=e$. it is clear that each of f and i is $NC\beta$-cont. but $f\circ i$ is not $NC\beta$-cont.

The next theorem shown that under what condition that composition is $NC\beta$-cont.

Theorem 3.8

If $f:(X, \Gamma_1)\longrightarrow(Y, \Gamma_2)$ and $g:(Y, \Gamma_2)\longrightarrow(Z, \Gamma_3)$ be two functions, if f is$NC\beta$-cont. and $NC\alpha$-open and g is a $NC\beta$-cont., then gof is $NC\beta$-cont.

Proof. Let $V\subseteq Z$ be a NC-open set, then $(gof)^{-1}(V) = f^{-1}(g^{-1}(V)$, but $g^{-1}(V)\in NC\beta(x)$ for g is $NC\beta$-cont., and by Theorem 2.6, $f^{-1}(g^{-1}(V)\in NC\beta(x)$.

Therefore gof is $NC\beta$-cont. The following lemma is very useful in the sequel.

Lemma 3.1

If $=\langle U_1,U_2,U_3\rangle \in NC\alpha$-(x) and $V=\langle V_1,V_2,V_3\rangle \in NC\beta(x)$, then $U\cap V\in NC\beta(u)$.

Proof. Since $U\cap V\subseteq NCint\ NCcl\ NCintU\cap NCcl\ NCint\ NCclV\subseteq NCcl\ (NCint\ NCcl\ NCintU\cap NCintNC\ clV)\subseteq NCcl\ (NCcl\ NCintU\cap NCint\ NCclV)\subseteq NCcl(NCintU\cap NCint\ NCclV)$, $U\cap V\subseteq NCcl(NCintU\cap\ NCint\ NCclV\cap U = NCcl\ (NCintU\cap NCint\ NCclV)$. but $NCintU\cap\ NCint\ NCclV\subseteq U$ is NC-open in X, then $NCint\ (NCintU\cap NCint$

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$NCclV$)= $NCint$ $U\cap NCint$ $NCclV$, thus $U\cap V\subseteq$ $NCcl$ ($NCint$($NCintU$ $\cap NCclV$))$\subseteq$ $NCcl$($NCint$ ($NCcl$ ($NCintU\cap V$)$\cap U$))$\subseteq$ $NCcl$($NCint$($NCcl$ ($U\cap V$)$\cap U$))=$NCcl$($NCint$($NCcl$($U\cap V$)$\cap U$)).

Therefore $U\cap V\in$ (u).

Theorem 3.9

If :(, Γ_1)⟶(Y, Γ_2) be a $NC\beta$-cont. and $NC\alpha$(x). Then $f\backslash U$ is $NC\beta$-cont.

Proof. Let V = ⟨$V_{1,2}$,V_3⟩$\subseteq Y$ be a NC-open set, then $f^{-1}(V)\in NC\beta$(x), since U=⟨U_1,U_2,U_3⟩$\in NC\alpha$(x), by Lemma 2.12 $U\cap f^{-1}(V) = (f\backslash U)^{-1}(V)\in NC\beta$(x) therefore $f\backslash U$ is $NC\beta$-cont.

Lemma 3.2

Let A= ⟨$A_{1,2}$,A_3⟩$\subseteq Y\subseteq X$, $Y\in NC\beta$(x) and $A\in NC\beta$(y), then $A\in NC\beta$(x).

Proof. Since A= ⟨A_1,A_2,A_3⟩$\subseteq$ $NC\beta$(y)$\subseteq$ $NCcl$($NCint$($NCcl$(A)))$\subseteq NCcl$($NCint$($NCcl$($A\cup Y$)) $\subseteq$ $NCcl$($NCint$($NCcl$(A))). since $NCint$ $NCclA$ is NC-open in Y, then exists a NC-open set $U\subseteq X$ such that $NCint$ $NCclA$= $U\cap Y$, thus $A\subseteq$ $NCcl$($U\cap$ $NCcl$ $NCint$ $NCclY$)$\subseteq$ $NCcl$($NCcl$ $NCint$ $NCcl$($U\cap Y$)))= $NCcl$($NCcl$ $NCint$ $NCcl$($NCint$ $NCclA$)))$\subseteq$ $NCcl$($NCcl$ $NCint$ $NCclA$) $\subseteq$ $NCcl$ $NCint$ $NCclA$= $NCcl$ $NCint$ $NCclA$. Therefore, $A\subseteq$ $NC\beta$(x).

Theorem 3.10

If :(, Γ_1)⟶(Y, Γ_2) be a function, and {U_i ,$i\in$I} be a cover of X by $NC\beta$-open set of X, then f is $NC\beta$-cont. if ($f\backslash U$) is $NC\beta$-cont. for each $i\in$I.

Proof

Let ⟨$V_{1,2}$,V_3⟩$\subseteq$ Y be a NC-open set, then $(f\backslash U)^{-1}(V)\in NC\beta(U_i)$ since $U_i\in NC\beta$(x). by Lemma 2.12, $(f\backslash U)^{-1}(V)\in$ $NC\beta$(x) for each $i\in$I. but $f^{-1}(V)$=$\cup$ $(f\backslash U_i)^{-1}$ (V), by Remark 2.9 $f^{-1}(V)\in NC\beta$(x). this implies that f is $NC\beta$-cont.

4. Neutrosophic crisp β-open (closed) function.

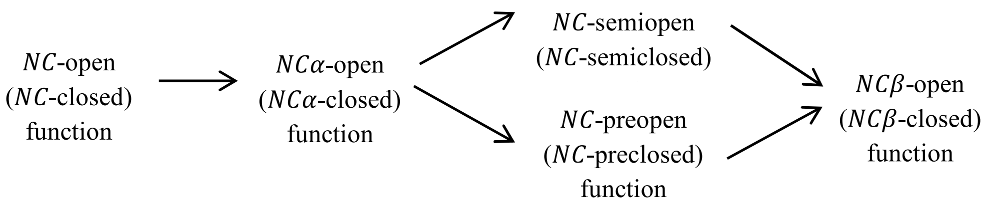
Definition 4.1

A function :(, Γ_1)⟶(Y, Γ_2) is said to be $NC\beta$-open If the image of any NC-open set in X is $NC\beta$-open in Y.

Definition 4.2

A function :(, Γ_1)⟶(Y, Γ_2) is said to be $NC\beta$-closed If the image of any NC-closed set in X is $NC\beta$-closed set in Y.

The implications between $NC\beta$-open ($NC\beta$-closed) function and other types of NC-open (NC-closed) function are given by the following diagram.



The converses of these statements may be not necessarily true, as shown by the following examples.

Example 4.1

Let X=Y={a,b,c,d} with NC-topologies Γ_x={X_N,Φ_N,A} and Γ_y be an indiscrete NCT.

Where A=⟨{a,b},{c},{b}⟩.

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The identity function f:X⟶Y is $NC\beta$-open ($NC\beta$-closed) but not may be NC-semiopen (NC-semiclosed).

Example 4.2

Let X= {a,b,c,d}, Y= {a,b,c,d,e} with NC-topologies Γ_x={X_N,Φ_N,A}, Γ_y= {Y_N,Φ_N,D}.where A=⟨{a,b},{c},{b,d}⟩, D= ⟨{b,c},{a,c}, {a}⟩.

A function f:X⟶Y defined as (a)= b, f(b)= d and f(c)= e =f(d), it is clear that f is $NC\beta$-open ($NC\beta$-closed) but not NC-preopen (NC-preclosed).

Remark 4.1

A one to one function is $NC\beta$-open iff it is $NC\beta$-closed.

The following theorem gives easy characterization of a $NC\beta$-open function.

Theorem 4.1

Let :(, Γ_1)⟶(Y, Γ_2) be a function. The following statements may be equivalent.

(i) f is $NC\beta$-open.

(ii) For each x∈X and U each neighborhood U of X, there exists a $NC\beta$-open set W= ⟨W_1, $W_{2,3}$⟩⊆ Y containing f(x) such that W⊆ f(u).

Proof. (i)⟹(ii). Let x∈X and U be a neighborhood $U$of X, then there exists a NC-open set = ⟨V_1,V_2,V_3⟩⊆ X such that x∈V⊆U. NC-set W= (V), since f is $NC\beta$-open, (V)=W∈ $NC\beta$(y) and so f(x)∈ W⊆f(u).

(ii)⟹(i). Following directly from the Definition 3.1

Theorem 4.2

Let :(, Γ_1)⟶(Y, Γ_2) be a function. The following statement may be equivalent.

(i) f is $NC\beta$-open.

(ii) $NCint\ NCcl\ NCint\ A$⊆ NC(f^{-1}(A)); for each A⊆ Y.

(iii) if f is bijective, $NCint\ NCcl\ NCint$(f(D))⊆ (f($NCcl$(D)); for each D⊆ X.

Proof. (i)⟹(ii). Since f is $NC\beta$-open and A= ⟨A_1,A_2,A_3⟩⊆ Y, then $NCcl$ (f^{-1}(A))⊆ X containing f^{-1}(A)=⟨f^{-1}(A_1),f^{-1}(A_2),f^{-1}(A_2)⟩ by Theorem 3.9 there is a $NC\beta$-closed set W= ⟨W_1,W_2,W_3⟩ ⊇A such that $NCcl$(f^{-1}(A)) ⊇f^{-1}(W)) ⊇$NCint\ NCcl\ NCint$ f^{-1}(W) ⊇f^{-1}($NCint\ NCclNCint$ (A)).

(ii)⟹(iii). Let D⊆ X, (D)⊆ Y. NC-set A= f(D) in (ii), then f^{-1}($NCint\ NCcl\ NCint$ (f(D)))⊆ $NCcl$(f^{-1}(f(D))⊆ $NCclD$ and so, $NCint\ NCcl\ NCint$(f(D))⊆ f($NCcl$(D).

(iii)⟹(i). Suppose U=⟨$U_{1,2}$,U_3⟩ is a NC-open set in X, then $NCcl$(f(X-U))=f(X-U) ⊇$NCint\ NCcl\ NCint$(f(X-U)). Since f is bijective, (U) ⊆ $NCint\ NCcl\ NCin$(f(U)) i.e., f(U)∈ $NC\beta$(y), hence f is $NC\beta$-open. Now we try to construct some new connection between $NC\beta$-open ($NC\beta$-closed) functions and other types of NC-open (NC-closed) functions.

Theorem 4.3

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Let f:(X, Γ_1)⟶(Y, Γ_2) be a $NC\beta$-open ($NC\beta$-closed) function if W= ⟨W_1, W_2,W_3⟩⊆ Y and F= ⟨F_1,F_2,F_3⟩⊆ X is a NC-open(NC-closed) set containing $f^{-1}(W)$, then there exists $NC\beta$-closed ($NC\beta$-open) H= ⟨H_1,H_2,H_3⟩⊆ Y containing W such that f^{-1}(H)⊆ F.

Proof. H= ⟨H_1,H$_2$,H_3⟩ =Y-f(X-F), since $f^{-1}(W)$⊆ F, W⊆ H, hence H is $NC\beta$-closed and $f^{-1}(H)$ = X-f^{-1}(f(X-F))⊆ F. the second side of the theorem can be prove by the same manner.

Theorem 4.4

If :(, Γ_1)⟶(Y, Γ_2) be a $NC\alpha$-cont and $NC\beta$-open function, Then the inverse image of any $NC\alpha$-open set in Y is $NC\alpha$-open set of X.

Proof. Let V be a $NC\alpha$-open set of Y. so, A∈ $NC\beta$(x), A⊆ $NCint$ $NCcl$ $NCint(A)$ and so, $f^{-1}(V)$⊆ f^{-1}($NCint$ $NCcl$ $NCint(V)$)⊆ $NCint$ $NCcl$ $NCint$(f^{-1}($NCint$ $NCcl$ $NCint(V)$)). Since f is $NC\beta$-open by Theorem 3.7.(ii) we have $f^{-1}(V)$⊆ $NCint$ $NCcl$ $NCint$(f^{-1}($NCint$ $NCcl$ $NCint$ (V)))⊆ $NCint$ $NCcl$ $NCint$($NCcl$(f^{-1}($NCint(V)$))⊆ $NCint$ $NCclf^{-1}$($NCint(V)$). Since f is NC-αcont., $f^{-1}(V)$⊆ $NCint$ $NCcl$(f^{-1}($NCint$ (V)))⊆ $NCint$ $NCcl$($NCint$ $NCcl$ $NCintf^{-1}$($NCint$ (V)))⊆ $NCint$ $NCcl$ $NCint$($f^{-1}(V)$). Hence $f^{-1}(V)$ is a $NC\alpha$-open set of X.

Corollary 4.1

If :(, Γ_1)⟶(Y, Γ_2) be a $NC\alpha$-cont. and $NC\beta$-open function, Then the inverse image of any $NC\alpha$-closed set in Y is $NC\alpha$-closed set of X.

Proof. Obvious.

Theorem 4.5

If :(, Γ_1)⟶(Y, Γ_2) be a $NC\alpha$-cont. and $NC\beta$-open function, Then the image of any $NC\beta$-open set in X may be $NC\beta$-open set of Y.

Proof. Let A∈ $NC\beta$(x), A⊆ $NCint$ $NCcl$ $NCint(A)$ and so, $f(A)$⊆ f($NCcl$ $NCint$ $NCcl(A)$)⊆ $NCcl$(f($NCint$ $NCcl(A)$))⊆ $NCcl$ $NCint$ $NCcl$ (f($NCint$ $NCcl(A)$))⊆ $NCcl$ $NCint$ $NCcl$ (f($NCcl(A)$))⊆ $NCcl$ $NCint$ $NCclf(A)$= $NCcl$ $NCint$ $NCclf(A)$.

Corollary 4.2

If :(, Γ_1)⟶(Y, Γ_2) be a $NC\alpha$-cont. and $NC\beta$-open and injective, Then the image of each $NC\beta$-closed set in X may be it is $NC\beta$-closed set of Y.

Proof. Let D⊆ X be $NC\beta$-closed, then (X-D) ∊ $NC\beta$(x) by Theorem 3.8 (X-D)⊆ $NCcl$ $NCint$ $NCcl$ (f(X-D)), Y-$f(D)$⊆ Y- $NCcl$ $NCint$ $NCcl$($f(D)$). So, (D) ⊇ $NCint$ $NCclNCin$($f(D)$).

5. Conclusion

In this paper, we introduce both the neutrosophic crisp nearly continuous functions, the neutrosophic crisp nearly open functions, and we present properties related to them. This paper, will promote the future study on

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neutrosophic crisp topological functions and many other general frameworks.

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