Generalized Weighted Exponential Similarity Measures of Single Valued Neutrosophic Sets

Abhijit Saha [1*] and Arnab Paul [2]

[1,2] Dept. of Mathematics, Techno College of Engg. Agartala, Maheshkhola, Tripura, INDIA; abhijit84.math@gmail.com[1], mrarnabpaul87@gmail.com[2]

* Correspondence: abhijit84.math@gmail.com

Abstract

A single valued neutrsophic set is one of the most successful extensions of the classical set, fuzzy set, intuitionistic fuzzy set, Pythagorean fuzzy set and q-rung orthopair fuzzy set due to the fact that it can handle uncertain data in more wider way. In this paper, we introduce some new generalized weighted similarity measures based on the exponential functions defined on truth-membership function, indeterminacy membership function and falsity membership function of a single valued neutrosophic set to study the independent influences of the truth-membership function, indeterminacy membership function and falsity membership function. The salient features of these proposed similarity measures are studied in detail. Based on the proposed similarity measures, we propose a multi attribute decision making method. To show the feasibility and effectiveness of the proposed method, an investment decision making problem is demonstrated.

Keywords: Single valued neutrosophic set, weighted exponential similarity measures, decision making.

1. Introduction

In our daily life, we come across various types of multi-attribute decision making problems with non-crisp/ uncertain data. Fuzzy set theory is one such extremely useful tool that helps us to deal with non-crisp data. In 1965, Lotfi A. Zadeh **[1]** first published the famous research paper on fuzzy sets that originated due to mainly the inclusion of vague human assessments in computing problems and it can deal with uncertainty, vagueness, partially trueness, impreciseness, Sharpless boundaries etc. Basically, the theory of fuzzy set is founded on the concept of partial belongings of an element in a set in order to process inexact information. Later on, fuzzy sets have been generalized to intuitionistic fuzzy sets **[2]** by adding a non-membership function by Atanassov in 1986 in order to deal with problems that possess incomplete information. In case of fuzzy sets or intuitionistic fuzzy sets, it is known that the membership (or non-membership) value of an element in a set takes a unique value in the closed interval [0,1]. However, the application range of intuitionistic fuzzy set is narrow in the sense that it has the constraint that sum of membership degree and non-membership degree of an element cannot exceed '1'. But, in complex decision-making problems, decision makers/experts may choose the preferences in such a way that the above condition gets violated. For instance, if an expert gives his preference with membership degree 0.8 and non-membership degree 0.7, then clearly their sum is 1.5, which is greater than 1. Therefore, intuitionistic fuzzy sets are not able to deal with this situation. To solve this problem, Yager **[3, 4]** introduced the non-standard fuzzy set named as Pythagorean fuzzy sets with membership degree ζ and non-membership degree ϑ with the condition $\zeta^2 + \vartheta^2 \leq 1$. Obviously, the Pythagorean fuzzy sets accommodate more uncertainties than the intuitionistic fuzzy sets. Yager **[5]** defined q-rung

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orthopair fuzzy sets (*q-ROFSs*) by enlarging the scope of Pythagorean fuzzy sets. The q-rung orthopair fuzzy sets allows the result of the qth power of the membership grade plus the qth power of the non-membership grade to be limited in interval [0,1]. If q=1, the q-rung orthopair fuzzy set transforms into the intuitionistic fuzzy set; if q=2, the q-rung orthopair fuzzy set transforms into the Pythagorean fuzzy set, which means that the q-rung orthopair fuzzy sets are extensions of intuitionistic fuzzy sets and Pythagorean fuzzy sets.

In 1999, Smarandache **[6]** introduced the notion neutrsophic set as a generalization of the classical set, fuzzy set, intuitionistic fuzzy set, Pythagorean fuzzy set and q-rung orthopair fuzzy set. The characterization of this neutrosophic set is explicitly done by truth-membership function, indeterminacy membership function and falsity membership function. The concept of single valued neutrosophic set was developed by Wang et al. **[7]** as an extension of fuzzy sets, Pythagorean fuzzy sets, q-rung orthopair fuzzy sets, intuitionistic fuzzy sets, single valued spherical neutrosophic sets **[8]**, n-hyperspherical neutrosophic sets **[8]**. The possible applications of neutrosophic sets and single valued neutrosophic sets on image segmentation have been studied in Gou and Cheng [9], Gou and Sensur [10]. Also, we find their probable infliction on clustering analysis in Karaaslan [11] and on medical diagnosis problems in Ansari et al. [12] respectively. Furthermore, the subject of the neutrosophic set theory has been practiced in Wang et al. [13], Gou et al. [14], Ye [15], Sun et al. [16], Ye [17, 18, 19] and Abdel Basset et al. [20, 21]. Some recent studies on this area can be found in [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].

Similarity measure plays a significant role for measuring the uncertain information. The fuzzy similarity measure is a measure that depicts the closeness among fuzzy sets. Many researchers have conducted extensive studies on similarity measures between fuzzy sets. Zwick et al. [35] reviewed and compared several similarity measures between fuzzy sets based on both geometric and set-theoretic ways. Pappis and Karacapilidis [36] introduced three similarity measures between fuzzy sets. Some more works on similarity measures in fuzzy environment can be found in [37], [38], [39], [40], [41]. Apart from these, some similarity measures in intuitionistic fuzzy environment are summarized in **[42, 43, 44, 45, 46, 47, 48]**. Similarity measures of single valued neutrosophic sets were introduced by Majumdar and Samanta **[49]**. Some authors **[50, 51, 52]** studied the concept of similarity measure between the two single valued neutrosophic sets which are useful to identify whether two sets are identical or atleast to what degree they are identical.

In case of the existing similarity measures **[49, 50, 51, 52]** of single valued neutrosophic sets, the independent influences of the truth-membership function, indeterminacy membership function and falsity membership function are completely ignored. To extend the existing similarity measures, in this paper, we introduce some new generalized weighted similarity measures based on the exponential functions defined on truth-membership function, indeterminacy membership function and falsity membership function. We call them "Generalized weighted exponential similarity measures" of single valued neutrosophic sets.

The rest of the paper is arranged as follows:
Some relevant definitions and results are given in Section 2. In Section 3, different types of generalized weighted exponential similarity measures between two single valued neutrosophic sets are introduced. The salient features of these proposed similarity measures are studied in detail. In Section 4, we propose a multi attribute decision making method. To show the feasibility and effectiveness of the proposed method, an investment decision making problem is considered. Section 5 is devoted to comparative study. Section 6 concludes the paper.

2. Preliminaries

In this section, first we recall some basic notions that are relevant to our study.

2.1 Definition: **[7]** A single-valued neutrosophic set (SVNS) ς on the universe set U is given by

$$\varsigma = \{< x, \xi(x), \vartheta(x), \eta(x) >: x \in U\}$$

where the functions $\xi, \vartheta, \eta : U \to [0,1]$ satisfy the condition $0 \le \xi(x) + \vartheta(x) + \eta(x) \le 3$ for every $x \in U$. The functions $\xi(x), \vartheta(x), \eta(x)$ define the degree of truth-membership, indeterminacy-membership and falsity-membership, respectively of $x \in U$.

2.2 Definition: **[7]** Suppose ς and ς' be two single-valued neutrosophic sets on U and are given by

$\varsigma = \{< x, \xi(x), \vartheta(x), \eta(x) >: x \in U\}$ and $\varsigma' = \{< x, \xi'(x), \vartheta'(x), \eta'(x) >: x \in U\}$. Then

(i) $\varsigma \subseteq \varsigma'$ if and only if $\xi(x) \le \xi'(x), \vartheta(x) \ge \vartheta'(x), \eta(x) \ge \eta'(x)$ $\forall x \in U$.

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(ii) $\varsigma^c = \{< x, \eta(x), 1-\vartheta(x), \xi(x) >: x \in U\}$

(iii) $\varsigma \cup \varsigma' = \{< x, \max(\xi(x), \xi'(x)), \min(\vartheta(x), \vartheta'(x)), \min(\eta(x), \eta'(x)) >: x \in U\}$.

(iv) $\varsigma \cap \varsigma' = \{< x, \min(\xi(x), \xi'(x)), \max(\vartheta(x), \vartheta'(x)), \max(\eta(x), \eta'(x)) >: x \in U\}$.

2.3 Definition: [49] Let $SVNS^U$ be the collection of all single-valued neutrosophic sets on U. Suppose ς, $\varsigma' \in SVNS^U$ and are given by: $\varsigma = \{< x, \xi(x), \vartheta(x), \eta(x) >: x \in U\}$ and $\varsigma' = \{< x, \xi'(x), \vartheta'(x), \eta'(x) >: x \in U\}$. Then, a similarity measure between ς and ς' is a function defined as $S: SVNS^U \to [0,1]$ which satisfies the following properties:

(I) $0 \le S(\varsigma, \varsigma') \le 1$

(II) $S(\varsigma, \varsigma') = S(\varsigma', \varsigma)$

(III) $S(\varsigma, \varsigma') = 1$ if and only if $\varsigma = \varsigma'$

(IV) $\varsigma \subseteq \varsigma' \subseteq \varsigma'' \Rightarrow S(\varsigma, \varsigma'') \le \min\{S(\varsigma, \varsigma'), S(\varsigma', \varsigma'')\}$

2.3 Definition: [49] Let $SVNS^U$ be the collection of all single-valued neutrosophic sets on U. Suppose ς, $\varsigma' \in SVNS^U$ and are given by: $\varsigma = \{< x, \xi(x), \vartheta(x), \eta(x) >: x \in U\}$ and $\varsigma' = \{< x, \xi'(x), \vartheta'(x), \eta'(x) >: x \in U\}$. Then, a weighted similarity measure between ς and ς' is defined as:

$$S(\varsigma, \varsigma') = \frac{\sum_x \omega_x \left(\xi(x)\xi'(x) + \vartheta(x)\vartheta'(x) + \eta(x)\eta'(x)\right)^2}{\sum_x \omega_x \left\{\left((\xi(x))^2 + (\vartheta(x))^2 + (\eta(x))^2\right) \times \left((\xi'(x))^2 + (\vartheta'(x))^2 + (\eta'(x))^2\right)\right\}} \quad (x \in U)$$

3. **Exponential similarity measures of *SVNSs*:**

This sections presents various types of generalized weighted exponential similarity measures of *SVNSs*. The basic properties of these newly defined similarity measures are discussed.

3.1 Definition: Let $\Delta_1 = \{< x, \mu_{\Delta_1}(x), \gamma_{\Delta_1}(x), \delta_{\Delta_1}(x) >: x \in U\}$ and $\Delta_2 = \{< x, \mu_{\Delta_2}(x), \gamma_{\Delta_2}(x), \delta_{\Delta_2}(x) > : x \in U\}$ be two *SVNSs* over U. For $k \ge 1$ and $x \in U$, let us define three exponential functions:

$$S_x^{\mu}(\Delta_1, \Delta_2) = e^{-\left|\mu^k{}_{\Delta_1}(x) - \mu^k{}_{\Delta_2}(x)\right|}, \; S_x^{\gamma}(\Delta_1, \Delta_2) = e^{-\left|\gamma^k{}_{\Delta_1}(x) - \gamma^k{}_{\Delta_2}(x)\right|}, \; S_x^{\delta}(\Delta_1, \Delta_2) = e^{-\left|\delta^k{}_{\Delta_1}(x) - \delta^k{}_{\Delta_2}(x)\right|}.$$

3.2 Theorem: Let $\Delta_1 = \{< x, \mu_{\Delta_1}(x), \gamma_{\Delta_1}(x), \delta_{\Delta_1}(x) >: x \in U\}$ and $\Delta_2 = \{< x, \mu_{\Delta_2}(x), \gamma_{\Delta_2}(x), \delta_{\Delta_2}(x) > : x \in U\}$ be two *SVNSs* over U. Then

(a) $0 \le S_x^{\mu}(\Delta_1, \Delta_2), S_x^{\gamma}(\Delta_1, \Delta_2), S_x^{\delta}(\Delta_1, \Delta_2) \le 1$

(b) $S_x^{\mu}(\Delta_1, \Delta_2) = S_x^{\mu}(\Delta_2, \Delta_1), S_x^{\gamma}(\Delta_1, \Delta_2) = S_x^{\gamma}(\Delta_2, \Delta_1)$ and $S_x^{\delta}(\Delta_1, \Delta_2) = S_x^{\delta}(\Delta_2, \Delta_1)$,

(c) $S_x^{\mu}(\Delta_1, \Delta_2) = S_x^{\gamma}(\Delta_1, \Delta_2) = S_x^{\delta}(\Delta_1, \Delta_2) = 1$ if and only if $\Delta_1 = \Delta_2$

(d) If $\Delta_1 \subseteq \Delta_2 \subseteq \Delta_3$, then $S_x^{\mu}(\Delta_1, \Delta_3) \le \min\{S_x^{\mu}(\Delta_1, \Delta_2), S_x^{\mu}(\Delta_2, \Delta_3)\}$, $S_x^{\gamma}(\Delta_1, \Delta_3) \le \min\{S_x^{\gamma}(\Delta_1, \Delta_2), S_x^{\gamma}(\Delta_2, \Delta_3)\}$, $S_x^{\delta}(\Delta_1, \Delta_3) \le \min\{S_x^{\delta}(\Delta_1, \Delta_2), S_x^{\delta}(\Delta_2, \Delta_3)\}$.

Proof: (a)- (c) straight forward.

(d) As $\Delta_1 \subseteq \Delta_2 \subseteq \Delta_3$, we have, $0 \le \mu_{\Delta_1}(x) \le \mu_{\Delta_2}(x) \le \mu_{\Delta_3}(x) \le 1, 1 \ge \gamma_{\Delta_1}(x) \ge \gamma_{\Delta_2}(x) \ge \gamma_{\Delta_3}(x) \ge 0, 1 \ge \delta_{\Delta_1}(x)$

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$\geq \delta_{\Delta_2}(x) \geq \delta_{\Delta_3}(x) \geq 0.$ This gives,

$$0 \leq \mu^k{}_{\Delta_1}(x) \leq \mu^k{}_{\Delta_2}(x) \leq \mu^k{}_{\Delta_3}(x) \leq 1, 1 \geq \gamma^k{}_{\Delta_1}(x) \geq \gamma^k{}_{\Delta_2}(x) \geq \gamma^k{}_{\Delta_3}(x) \geq 0, 1 \geq \delta^k{}_{\Delta_1}(x) \geq \delta^k{}_{\Delta_2}(x), \geq \delta^k{}_{\Delta_3}(x) \geq 0.$$

Now $\left|\mu^k{}_{\Delta_1}(x) - \mu^k{}_{\Delta_3}(x)\right| = \left|\mu^k{}_{\Delta_1}(x) - \mu^k{}_{\Delta_2}(x) + \mu^k{}_{\Delta_2}(x) - \mu^k{}_{\Delta_3}(x)\right|$

$$\leq \left|\mu^k{}_{\Delta_1}(x) - \mu^k{}_{\Delta_2}(x)\right| + \left|\mu^k{}_{\Delta_2}(x) - \mu^k{}_{\Delta_3}(x)\right|$$

$$\Rightarrow -\left|\mu^k{}_{\Delta_1}(x) - \mu^k{}_{\Delta_3}(x)\right| \geq -\left|\mu^k{}_{\Delta_1}(x) - \mu^k{}_{\Delta_2}(x)\right| - \left|\mu^k{}_{\Delta_2}(x) - \mu^k{}_{\Delta_3}(x)\right|$$

$$\Rightarrow e^{-\left|\mu^k{}_{\Delta_1}(x) - \mu^k{}_{\Delta_3}(x)\right|} \leq e^{-\left|\mu^k{}_{\Delta_1}(x) - \mu^k{}_{\Delta_2}(x)\right|} \times e^{-\left|\mu^k{}_{\Delta_2}(x) - \mu^k{}_{\Delta_3}(x)\right|}$$

$$\Rightarrow S_x^{\mu}(\Delta_1, \Delta_3) \leq S_x^{\mu}(\Delta_1, \Delta_2) \times S_x^{\mu}(\Delta_2, \Delta_3) \leq \min\{S_x^{\mu}(\Delta_1, \Delta_2), S_x^{\mu}(\Delta_2, \Delta_3)\}$$

Similarly, $S_x^{\gamma}(\Delta_1, \Delta_3) \leq \min\{S_x^{\gamma}(\Delta_1, \Delta_2), S_x^{\gamma}(\Delta_2, \Delta_3)\}$, $S_x^{\delta}(\Delta_1, \Delta_3) \leq \min\{S_x^{\delta}(\Delta_1, \Delta_2), S_x^{\delta}(\Delta_2, \Delta_3)\}$.

Next, we define the generalized weighted exponential similarities measures for *SVNSs* using the exponential functions defined in definition 3.1.

3.3 Definition: Let $\Delta_1 = \{< x, \mu_{\Delta_1}(x), \gamma_{\Delta_1}(x), \delta_{\Delta_1}(x) >: x \in U\}$ and $\Delta_2 = \{< x, \mu_{\Delta_2}(x), \gamma_{\Delta_2}(x), \delta_{\Delta_2}(x) > : x \in U\}$ be two *SVNSs* over U. Also let $\omega_x > 0$ denotes the weight of the element $x \in U$ such that $\sum_x \omega_x = 1$ .Then we define the generalized weighted exponential similarity measure between the *SVNSs* Δ_1 and Δ_2 as:

$$S_{\omega}^k(\Delta_1, \Delta_2) = \sum_x \omega_x \times S_x^{\mu}(\Delta_1, \Delta_2) \times S_x^{\gamma}(\Delta_1, \Delta_2) \times S_x^{\delta}(\Delta_1, \Delta_2)$$

3.4 Theorem: Let $\Delta_1 = \{< x, \mu_{\Delta_1}(x), \gamma_{\Delta_1}(x), \delta_{\Delta_1}(x) >: x \in U\}$ and $\Delta_2 = \{< x, \mu_{\Delta_2}(x), \gamma_{\Delta_2}(x), \delta_{\Delta_2}(x) > : x \in U\}$ be two *SVNSs* over U. Then

(a) $0 \leq S_{\omega}^k(\Delta_1, \Delta_2) \leq 1$

(b) $S_{\omega}^k(\Delta_1, \Delta_2) = S_{\omega}^k(\Delta_2, \Delta_1)$

(c) $S_{\omega}^k(\Delta_1, \Delta_2) = 1$ if and only if $\Delta_1 = \Delta_2$

(d) If $\Delta_1 \subseteq \Delta_2 \subseteq \Delta_3$, then $S_{\omega}^k(\Delta_1, \Delta_3) \leq \min\{S_{\omega}^k(\Delta_1, \Delta_2), S_{\omega}^k(\Delta_2, \Delta_3)\}$.

Proof: (a)-(c) straight forward.

(d) For the *SVNSs* $\Delta_1, \Delta_2, \Delta_3$ satisfying $\Delta_1 \subseteq \Delta_2 \subseteq \Delta_3$, we observe from theorem 3.2 that,

$S_x^{\mu}(\Delta_1, \Delta_3) \leq \min\{S_x^{\mu}(\Delta_1, \Delta_2), S_x^{\mu}(\Delta_2, \Delta_3)\}, S_x^{\gamma}(\Delta_1, \Delta_3) \leq \min\{S_x^{\gamma}(\Delta_1, \Delta_2), S_x^{\gamma}(\Delta_2, \Delta_3)\}$, and $S_x^{\delta}(\Delta_1, \Delta_3) \leq \min\{S_x^{\delta}(\Delta_1, \Delta_2), S_x^{\delta}(\Delta_2, \Delta_3)\}\ \forall x \in U$.

Using these, we get from definition 3.3,

$$S_{\omega}^k(\Delta_1, \Delta_3)$$

$$\leq \sum_x \omega_x \times \min\{S_x^{\mu}(\Delta_1, \Delta_2), S_x^{\mu}(\Delta_2, \Delta_3)\} \times \min\{S_x^{\gamma}(\Delta_1, \Delta_2), S_x^{\gamma}(\Delta_2, \Delta_3)\} \times \min\{S_x^{\delta}(\Delta_1, \Delta_2), S_x^{\delta}(\Delta_2, \Delta_3)\}$$

$$\leq \min\left\{\sum_x \omega_x \times S_x^{\mu}(\Delta_1, \Delta_2) \times S_x^{\gamma}(\Delta_1, \Delta_2) \times S_x^{\delta}(\Delta_1, \Delta_2), \sum_x \omega_x \times S_x^{\mu}(\Delta_2, \Delta_3) \times S_x^{\gamma}(\Delta_2, \Delta_3) \times S_x^{\delta}(\Delta_2, \Delta_3)\right\}$$

$$= \min\left\{S_{\omega}^k(\Delta_1, \Delta_2), S_{\omega}^k(\Delta_2, \Delta_3)\right\}$$

Next we define the generalized weighted average exponential similarity measure of *SVNSs*.

3.5 Definition: Let $\Delta_1 = \{< x, \mu_{\Delta_1}(x), \gamma_{\Delta_1}(x), \delta_{\Delta_1}(x) >: x \in U\}$ and $\Delta_2 = \{< x, \mu_{\Delta_2}(x), \gamma_{\Delta_2}(x), \delta_{\Delta_2}(x) >$

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$: x \in U\}$ be two *SVNSs* over U. Then the generalized weighted average exponential similarity measure between Δ_1 and Δ_2 is defined as:

$$ {}_A S_\omega^k(\Delta_1, \Delta_2) = \sum_x \omega_x \times \left\{ \frac{S_x^\mu(\Delta_1,\Delta_2) + S_x^\gamma(\Delta_1,\Delta_2) + S_x^\delta(\Delta_1,\Delta_2)}{3} \right\} $$

where $\omega_x > 0$ denotes weight of $x \in U$ such that $\sum_x \omega_x = 1$.

3.6 Theorem: Let $\Delta_1 = \{< x, \mu_{\Delta_1}(x), \gamma_{\Delta_1}(x), \delta_{\Delta_1}(x) >: x \in U\}$ and $\Delta_2 = \{< x, \mu_{\Delta_2}(x), \gamma_{\Delta_2}(x), \delta_{\Delta_2}(x) > : x \in U\}$ be two *SVNSs* over U. Then

(a) $0 \le {}_A S_\omega^k(\Delta_1, \Delta_2) \le 1$

(b) ${}_A S_\omega^k(\Delta_1, \Delta_2) = {}_A S_\omega^k(\Delta_2, \Delta_1)$

(c) ${}_A S_\omega^k(\Delta_1, \Delta_2) = 1$ if and only if $\Delta_1 = \Delta_2$

(d) If $\Delta_1 \subseteq \Delta_2 \subseteq \Delta_3$, then ${}_A S_\omega^k(\Delta_1, \Delta_3) \le \min\{{}_A S_\omega^k(\Delta_1, \Delta_2), {}_A S_\omega^k(\Delta_2, \Delta_3)\}$.

Proof: (a)-(c) straight forward.

(d) For the *SVNSs* $\Delta_1, \Delta_2, \Delta_3$ satisfying $\Delta_1 \subseteq \Delta_2 \subseteq \Delta_3$, we observe from theorem 3.2 that,

$S_x^\mu(\Delta_1,\Delta_3) \le \min\{S_x^\mu(\Delta_1,\Delta_2), S_x^\mu(\Delta_2,\Delta_3)\}$, $S_x^\gamma(\Delta_1,\Delta_3) \le \min\{S_x^\gamma(\Delta_1,\Delta_2), S_x^\gamma(\Delta_2,\Delta_3)\}$, and $S_x^\delta(\Delta_1,\Delta_3) \le \min\{S_x^\delta(\Delta_1,\Delta_2), S_x^\delta(\Delta_2,\Delta_3)\}$ $\forall x \in U$.

Using these, we get from definition 3.5,

$$ \begin{aligned} &{}_A S_\omega^k(\Delta_1, \Delta_3) \\ &= \sum_x \omega_x \times \left\{ \frac{S_x^\mu(\Delta_1,\Delta_3) + S_x^\gamma(\Delta_1,\Delta_3) + S_x^\delta(\Delta_1,\Delta_3)}{3} \right\} \\ &\le \frac{1}{3}\left\{ \sum_x \omega_x \times \min\{S_x^\mu(\Delta_1,\Delta_2), S_x^\mu(\Delta_2,\Delta_3)\} + \sum_x \omega_x \times \min\{S_x^\gamma(\Delta_1,\Delta_2), S_x^\gamma(\Delta_2,\Delta_3)\} \times \right. \\ &\quad \left. \sum_x \omega_x \times \min\{S_x^\delta(\Delta_1,\Delta_2), S_x^\delta(\Delta_2,\Delta_3)\} \right\} \\ &\le \min\left\{ \sum_x \omega_x \times \frac{S_x^\mu(\Delta_1,\Delta_2) + S_x^\gamma(\Delta_1,\Delta_2) + S_x^\delta(\Delta_1,\Delta_2)}{3}, \sum_x \omega_x \times \frac{S_x^\mu(\Delta_2,\Delta_3) + S_x^\gamma(\Delta_2,\Delta_3) + S_x^\delta(\Delta_2,\Delta_3)}{3} \right\} \\ &= \min\left\{ {}_A S_\omega^k(\Delta_1, \Delta_2), {}_A S_\omega^k(\Delta_2, \Delta_3) \right\} \end{aligned} $$

4. Multi attribute decision making:

Let $A = \{A_1, A_2, A_3, \ldots\ldots, A_m\}$ be a set of m alternatives and $C = \{C_1, C_2, C_3, \ldots\ldots, C_n\}$ be a sets of n attributes. Suppose ω_j is the weight of the attribute C_j with $\omega_j > 0$ and $\sum_{j=1}^n \omega_j = 1$. These alternatives are evaluated by an expert and evaluation values are presented in terms of SVNSs $\xi_{ij} = < \mu_{ij}, \gamma_{ij}, \delta_{ij} >$ such that $\mu_{ij}, \gamma_{ij}, \delta_{ij} \ge 0$ and $\mu_{ij} + \gamma_{ij} + \delta_{ij} \le 3$ are satisfied for each *i, j*.

To determine the best alternatives, the following steps are followed based on the proposed similarity measures:

Step-1: Determine the weight of each criterion.

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The weight vector $\omega_{j,r}\left(r=0,1,2,...\right)$ of criteria C_j is determined by using the formula:

$$\omega_{j,r}=\frac{\left(\beta_j\right)^r}{\sum_{j=1}^{n}\left(\beta_j\right)^r},\ r=0,1,2,3....$$

Where $\beta_j=\beta_{1j}+\beta_{2j}+\beta_{3j}$ in which , $\beta_{1j}=\max_i \mu_{ij},\beta_{2j}=\min_i \gamma_{ij},\beta_{3j}=\min_i \delta_{ij}$ for all $i=1,2,3...,n$ such that $\sum_{j=1}^{n}\omega_{j,r}=1,\text{for } r=0,1,2,3.....$

Step-2: Determine the ideal values.

Let $C=C'\cup C''$ where C' denotes the set of all cost criteria and C'' denotes the set of all benefit criteria.

The triplets (0,1,1) and (1,0,0) are considered as ideal values corresponding to cost criteria and benefit criteria respectively.

If $A_I(j)$ represent the ideal value for the criteria C_j, then

$$A_I(j)=\begin{cases}(1,0,0) & \text{if } C_j\in C'' \\ (0,1,1) & \text{if } C_j\in C'\end{cases}\qquad (j=1,2,3,.....,n)$$

Suppose A_I denotes the ideal values for all criteria i.e; $A_I=\{A_I(1),A_I(1),A_I(3),A_I(4),A_I(5),A_I(6)\}$.

Step-3: Calculate the similarity measures using S_ω^k or ${}_A S_\omega^k$ between each alternative and it's ideal values .

Step-4: Based on the values of similarity measures, rank the alternatives using the following rule:

$A_p\prec A_q$ if and only if $S_\omega^k(A_p,A_I)<S_\omega^k(A_q,A_I)$ or ${}_A S_\omega^k(A_p,A_I)<{}_A S_\omega^k(A_q,A_I)$ for $p,q\in\{1,2,....,m\}$ $\left(p\neq q\right)$

An illustrative example:

We consider a investment decision making problem given below adapted from **[53]**.

"There are five possible companies A_i $(i=1,2,3,4,5)$ which are considered as alternatives. To evaluate these alternatives, a person hires an investment expert who evaluates these companies under the set of six criteria, namely –technical ability$\left(C_1\right)$, expected benefit $\left(C_2\right)$,competitive power on the market$\left(C_3\right)$, ability to bear risk $\left(C_4\right)$, management capacity $\left(C_5\right)$ and organizational culture$\left(C_6\right)$".

The expert(s) evaluation result for each alternative based on each criteria is depicted in **Table-1**:

Table-1: Initial evaluation result

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	<0.3,0.4,0.2>	<0.4,0.7,0.6>	<0.1,0.4,0.3>	<0.5,0.5,0.2>	<0.4,0.3,0.5>	<0.6,0.1,0.4>
A_2	<0.6,0.1,0.4>	<0.2,0.4,0.5>	<0.5,0.3,0.4>	<0.7,0.4,0.6>	<0.6,0.3,0.6>	<0.5,0.4,0.2>
A_3	<0.7,0.6,0.3>	<0.5,0.4,0.5>	<0.8,0.6,0.3>	<0.7,0.4,0.6>	<0.8,0.6,0.3>	<0.9,0.5,0.3>
A_4	<0.5,0.5,0.2>	<0.3,0.4,0.2>	<0.4,0.3,0.5>	<0.7,0.3,0.5>	<0.1,0.4,0.3>	<0.5,0.5,0.2>
A_5	<0.1,0.4,0.3>	<0.6,0.6,0.4>	<0.5,0.5,0.2>	0.4,0.3,0.5>	<0.3,0.4,0.2>	<0.1,0.4,0.3>

Then to find the best alternative(s), the following steps are executed:

Step-1: Take *r*=1.

Then we have,

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$$\beta_1 = \beta_{11} + \beta_{21} + \beta_{31} = 0.7 + 0.1 + 0.2 = 1, \quad \beta_2 = \beta_{12} + \beta_{22} + \beta_{32} = 0.8 + 0.4 + 0.2 = 1.4,$$
$$\beta_3 = \beta_{13} + \beta_{23} + \beta_{33} = 0.8 + 0.3 + 0.2 = 1.3, \quad \beta_4 = \beta_{14} + \beta_{24} + \beta_{34} = 0.7 + 0.3 + 0.2 = 1.2,$$
$$\beta_5 = \beta_{15} + \beta_{25} + \beta_{35} = 0.8 + 0.3 + 0.2 = 1.3, \quad \beta_6 = \beta_{16} + \beta_{26} + \beta_{36} = 0.9 + 0.1 + 0.2 = 1.2.$$

$$\therefore \; \omega_{1,1} = \frac{1}{1+1.4+1.3+1.2+1.3+1.2} = 0.135, \quad \omega_{2,1} = \frac{1.4}{1+1.4+1.3+1.2+1.3+1.2} = 0.189,$$
$$\omega_{3,1} = \frac{1.3}{1+1.4+1.3+1.2+1.3+1.2} = 0.176, \quad \omega_{4,1} = \frac{1.2}{1+1.4+1.3+1.2+1.3+1.2} = 0.162,$$
$$\omega_{5,1} = \frac{1.3}{1+1.4+1.3+1.2+1.3+1.2} = 0.176, \quad \omega_{6,1} = \frac{1.2}{1+1.4+1.3+1.2+1.3+1.2} = 0.162.$$

Step-2: As $C_4 \in C'$ and $C_1, C_2, C_3, C_5, C_6 \in C''$, so the ideal values are given by:

$$A_I(1) = (1,0,0), A_I(2) = (1,0,0) >, A_I(3) = (1,0,0), A_I(4) = (0,1,1), A_I(5) = (1,0,0), A_I(6) = (1,0,0).$$

Step-3: Using the similarity measure S_ω^k, (for k=2) we get,

$$S_\omega^2(A_1, A_I) = 0.2792, S_\omega^2(A_2, A_I) = 0.3172, S_\omega^2(A_3, A_I) = 0.3854, S_\omega^2(A_4, A_I) = 0.2911, S_\omega^2(A_5, A_I) = 0.2916.$$

Step-4: Since $S_\omega^2(A_1, A_I) < S_\omega^2(A_4, A_I) < S_\omega^2(A_5, A_I) < S_\omega^2(A_2, A_I) < S_\omega^2(A_3, A_I)$, the best alternative is A_3 i.e; the best company is A_3. However the overall ranking is: $A_1 \prec A_4 \prec A_5 \prec A_2 \prec A_3$.

In another aspect, if we apply the other proposed similarity measure namely, ${}_AS_\omega^k$, then the problem can be solved similarly as above. If we utilize the similarity measure S_ω^k or ${}_AS_\omega^k$ for different values of k, then the final the ranking order of the given alternatives are summarized in **Table-2**. We can conclude from table-2 that although the ranking orders of the alternatives are slightly different; the most desirable alternative is still A_3 in all cases.

Table-2: Ranking of alternatives

		Overall measure values					
Value of *k*	**Similarity measures used**	A_1	A_2	A_3	A_4	A_5	**Ranking order**
1	S_ω^k	0.2332	0.2731	**0.3139**	0.2487	0.2474	$A_1 \prec A_5 \prec A_4 \prec A_2 \prec A_3$
	${}_AS_\omega^k$	0.6186	0.6489	**0.6716**	0.6305	0.6304	$A_1 \prec A_5 \prec A_4 \prec A_2 \prec A_3$
2	S_ω^k	0.2792	0.3172	**0.3854**	0.2911	0.2916	$A_1 \prec A_4 \prec A_5 \prec A_2 \prec A_3$
	${}_AS_\omega^k$	0.6756	0.6985	**0.7239**	0.6889	0.6954	$A_1 \prec A_4 \prec A_5 \prec A_2 \prec A_3$
3	S_ω^k	0.2997	0.3292	**0.4162**	0.3114	0.3151	$A_1 \prec A_4 \prec A_5 \prec A_2 \prec A_3$
	${}_AS_\omega^k$	0.7091	0.7244	**0.7506**	0.7191	0.7277	$A_1 \prec A_4 \prec A_2 \prec A_5 \prec A_3$
4	S_ω^k	0.3105	0.3316	**0.4199**	0.3203	0.3245	$A_1 \prec A_4 \prec A_5 \prec A_2 \prec A_3$
	${}_AS_\omega^k$	0.7275	0.7373	**0.7629**	0.7343	0.7422	$A_1 \prec A_4 \prec A_2 \prec A_5 \prec A_3$
5	S_ω^k	0.3170	0.3314	**0.4131**	0.3245	0.3280	$A_1 \prec A_4 \prec A_5 \prec A_2 \prec A_3$
	${}_AS_\omega^k$	0.7380	0.7441	**0.7681**	0.7424	0.7488	$A_1 \prec A_4 \prec A_2 \prec A_5 \prec A_3$

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5. Comparative study

In pursuance of performance comparison of the weighted exponential similarity measures developed by us with the existing weighted similarity measure **[49]**, a comparative study alongside their corresponding final ranking are summarized in tabular form, numbered by 3. It is very much translucent from table 3 that in spite of appearance of slight difference occur to the respective ranking order of the alternatives, the best i.e. most desirable alternative is absolutely same.

Table-3: Comparative study

		Overall measure values					
Value of *k*	**Similarity measures used**	A_1	A_2	A_3	A_4	A_5	**Ranking order**
K=2	S_ω^k	0.2792	0.3172	**0.3854**	0.2911	0.2916	$A_1 \prec A_4 \prec A_5 \prec A_2 \prec A_3$
	${}_A S_\omega^k$	0.6756	0.6985	**0.7239**	0.6889	0.6954	$A_1 \prec A_4 \prec A_5 \prec A_2 \prec A_3$
	S_ω [49]	0.2843	0.3661	**0.4823**	0.2342	0.2629	$A_4 \prec A_5 \prec A_1 \prec A_2 \prec A_3$

6. Conclusion

In this paper, some new weighted exponential similarity measures between single valued neutrosophic sets have been introduced. The desirable properties of these proposed similarity measures are demonstrated. To show the efficiency of the proposed similarity measures, a multi-attribute decision making method is constructed. The proposed approach is examined on a investment decision making problem. Finally we did a comparative analysis of the proposed approach and get ensured about its best performance.

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