A Study On Neutrosophic UP-algebra

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Abstract

In this paper we apply the neutrosophic set on the concept of the UP-algebra to obtain some types of neurosophic sets satisfies certain conditions which are called neutrosophic Up-subalgebras. Several types of these neutrosophic Up-subalgebras are introduced and their properties are investigated. Also, illustrative examples are given when they are needed.

Keywords: UP-algebra; neutrosophic set; neutrosophic UP-subalgebra; $(\mathscr{e}, \mathscr{s})$-sets

1 Introduction

Mathematicians frequently apply algebraic structures across diverse fields, including theoretical physics, computer science, control engineering, information science, coding theory, and topology. This extensive use provides strong motivation for researchers to revisit key concepts and findings in abstract algebra within the broader framework of fuzzy settings. UP-algebras, a class of logical algebras introduced by Iampan,[4] exhibit a close relationship with posets. Recently, many researchers have focused on UP-algebras, utilizing them in various mathematical domains such as group theory, fuzzy set theory, probability theory, topology, and functional analysis. Numerous system identification problems involve inherently non-probabilistic characteristics.[15] Neutrosophic set theory has also been applied to multiple algebraic structures, with concepts like neutrosophic points and various UP-subalgebras (ideals) explored in studies[5,6,8,11] and.[7] The falling shadow representation theory guides selection based on joint degree distributions, offering a reasonable and practical approach to advancing the theory and application of fuzzy sets and fuzzy logic.

2 Definitions and Useful Results

In this section, we give the basic definitions and results on UP-algebras which are needed in the next chapters. Also, we present some definitions and propositions of neutrosophic sets.

Definition 2.1. [12] Let Ω be a non-empty set. The neutrosophic set $\mathscr{A}$ (NS set) is written as $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ and the functions $\mathscr{T}_{\mathscr{A}} : \Omega \to [0,1]$, $\mathscr{I}_{\mathscr{A}} : \Omega \to [0,1]$, $\mathscr{F}_{\mathscr{A}} : \Omega \to [0,1]$ denotes the degree of truth membership, the degree of indeterminacy membership, the degree of falsity membership of elements of Ω respectively.

Definition 2.2. Let Ω be a non-empty set. The neutrosophic set $\mathscr{A}$ is called the null set if $\mathscr{A} = \{< x, 0, 0, 1 >: x \in \Omega\}$ (denoted by $\mathbf{0}$) and it is absolute if $\mathscr{A} = \{< x, 1, 1, 0 >: x \in \Omega\}$ (denoted by $\mathbf{1}$). By $\mathscr{A}^c$ we mean the neutrosophic set $\mathscr{B} = \{< x, 1 - \mathscr{T}_{\mathscr{A}}(x), 1 - \mathscr{I}_{\mathscr{A}}(x), 1 - \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$. $\mathscr{T}_{\mathscr{A}^c}(x) = 1 - \mathscr{T}_{\mathscr{A}}(x)$, $\mathscr{I}_{\mathscr{A}^c}(x) = 1 - \mathscr{I}_{\mathscr{A}}(x)$ and, $\mathscr{F}_{\mathscr{A}^c}(x) = 1 - \mathscr{T}_{\mathscr{A}}(x)$ for all $x \in \Omega$. Obviously, $\mathbf{0}^c = \mathbf{1}$.

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Definition 2.3. [3] A UP-algebra is defined as $(X, \curlyvee, 0)$ where $X \neq \varphi$, $\curlyvee$ is a binary operation and 0 is a constant element which satisfies the following axioms: for all $x, y, z \in \Omega$,

1. $(x \curlyvee y) \curlyvee ((z \curlyvee x) \curlyvee (z \curlyvee y)) = 0$,
2. $0 \curlyvee x = x$,
3. $x \curlyvee 0 = 0$, and
4. $x \curlyvee y = 0$ and $y \curlyvee x = 0$ imply $x = y$.

In a UP-algebra $(X, \curlyvee, 0)$ a binary relation $\leq$ on Ω is defined as follows: for all $x, y \in \Omega$,

$$x \leq y \quad \Longleftrightarrow \quad x \curlyvee y = 0.$$

Proposition 2.4. [3] In a UP-algebra Ω, the following properties hold: for all $x, y, z \in \Omega$,

1. $x \curlyvee x = 0$,
2. $x \curlyvee y = 0$, and $y \curlyvee z = 0$ imply $x \curlyvee z = 0$,
3. $x \curlyvee y = 0$ implies $(y \curlyvee z) \curlyvee (x \curlyvee z) = 0$, and
4. $x \curlyvee (y \curlyvee x) = 0$.

Proposition 2.5. [1] In a UP-algebra Ω, the following properties hold: for all $x, y, z \in \Omega$,

1. $x \leq x$,
2. $x \leq y$ and $y \leq x$ imply $x = y$,
3. $x \leq y$ and $y \leq z$ imply $x \leq z$,
4. $x \leq y$ implies $z \curlyvee x \leq z \curlyvee y$,
5. $x \leq y$ implies $y \curlyvee z \leq x \curlyvee z$, and
6. $x \leq y \curlyvee x$.

Definition 2.6. [3] A subset S of a UP-algebra Ω is called a UP-subalgebra of Ω if it satisfies:

1. $0 \in S$, and
2. $x \curlyvee y \in S$ for all $x, y \in S$.

Lemma 2.7. [10] If U and V are two subsets of a UP-algebra Ω such that $U \cap V \neq \varphi$, then $0 \in U \curlyvee V$.

Proposition 2.8. [13] Let Ω be a non-empty set and $b \in \Omega$. Define an operation $\curlyvee$ on Ω as:

$$x \curlyvee y = \begin{cases} y & : \quad x \neq y, \\ b & : \quad otherwise. \end{cases}$$

For all $x, y \in \Omega$, then $(X, \curlyvee, b)$ is a UP-algebra.

Proposition 2.9. [13] Let Ω be a non-empty totally ordered set and $b \in \Omega$. Define an operation $\curlyvee$ on Ω as:

$$x \curlyvee y = \begin{cases} y & : \quad x < y \quad or \quad x = b, \\ b & : \quad otherwise. \end{cases}$$

For all $x, y \in \Omega$, then $(X, \curlyvee, b)$ is a UP-algebra.

Proposition 2.10. [13] Let Ω be a non-empty totally ordered set and $b \in \Omega$. Define an operation $\curlyvee$ on Ω as:

$$x \curlyvee y = \begin{cases} y & : \quad x > y \quad or \quad x = b, \\ b & : \qquad otherwise. \end{cases}$$

For all $x, y \in \Omega$, then $(X, \curlyvee, b)$ is a UP-algebra.

Definition 2.11. [14] A fuzzy set λ in Ω is called a fuzzy UP-subalgebra of Ω, if for any $x, y \in \Omega$, $\lambda(x \curlyvee y) \geq \min\{\lambda(x), \lambda(y)\}$.

Definition 2.12. [9] If $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is a neutrosophic set and for $\alpha, \beta \in (0, 1]$, $\gamma \in [0, 1)$, the following neutrosphic subsets of Ω are defined:

$$\mathscr{T}_{\epsilon}(\mathscr{A}, \alpha) = \{x \in \Omega : \mathscr{T}_{\mathscr{A}}(x) \geq \alpha\},$$
$$\mathscr{I}_{\epsilon}(\mathscr{A}, \beta) = \{x \in \Omega : \mathscr{I}_{\mathscr{A}}(x) \geq \beta\},$$
$$\mathscr{F}_{\epsilon}(\mathscr{A}, \gamma) = \{x \in \Omega : \mathscr{F}_{\mathscr{A}}(x) \leq \gamma\}.$$

$$\mathscr{T}_{q}(\mathscr{A}, \alpha) = \{x \in \Omega : \mathscr{T}_{\mathscr{A}}(x) + \alpha > 1\},$$
$$\mathscr{I}_{q}(\mathscr{A}, \beta) = \{x \in \Omega : \mathscr{I}_{\mathscr{A}}(x) + \beta > 1\},$$
$$\mathscr{F}_{q}(\mathscr{A}, \gamma) = \{x \in \Omega : \mathscr{F}_{\mathscr{A}}(x) + \gamma < 1\}.$$

Definition 2.13. [9] A neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ of Ω is called (ψ, ξ)-neutrosophic UP-subalgebra of Ω , if for all $x, y \in \Omega$:

$$x \in \mathscr{T}_{\psi}(\mathscr{A}, \alpha_x), y \in \mathscr{T}_{\psi}(\mathscr{A}, \alpha_y) \text{ implies } x \curlyvee y \in \mathscr{T}_{\xi}(\mathscr{A}, \alpha_x \wedge \alpha_y),$$
$$x \in \mathscr{I}_{\psi}(\mathscr{A}, \beta_x), y \in \mathscr{I}_{\psi}(\mathscr{A}, \beta_y) \text{ implies } x \curlyvee y \in \mathscr{I}_{\xi}(\mathscr{A}, \beta_x \wedge \beta_y),$$
$$x \in \mathscr{F}_{\psi}(\mathscr{A}, \gamma_x), y \in \mathscr{F}_{\psi}(\mathscr{A}, \gamma_y) \text{ implies } x \curlyvee y \in \mathscr{F}_{\xi}(\mathscr{A}, \gamma_x \vee \gamma_y).$$

for all $\gamma_x, \gamma_y \in [0, 1)$, $\beta_x, \beta_y, \alpha_x, \alpha_y \in (0, 1]$ where $\psi, \xi \in \{\epsilon, q\}$.

3 Some Types of Neurosphic UP-subalgebras

Definition 3.1. Given a neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ and for $\gamma \in (0, 1]$, $\beta, \alpha \in [0, 1)$, we define the following neutrosphic subsets of Ω:

$$\mathscr{T}_{p}(\mathscr{A}, \alpha) = \{x \in \Omega : \mathscr{T}_{\mathscr{A}}(x) \leq \alpha\},$$
$$\mathscr{I}_{p}(\mathscr{A}, \beta) = \{x \in \Omega : \mathscr{I}_{\mathscr{A}}(x) \leq \beta\},$$
$$\mathscr{F}_{p}(\mathscr{A}, \gamma) = \{x \in \Omega : \mathscr{F}_{\mathscr{A}}(x) \geq \gamma\}$$

Definition 3.2. Given a neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ and for $\gamma \in (0, 1]$, $\beta, \alpha \in [0, 1)$, we define the following neutrosphic subsets of Ω:

$$\mathscr{T}_{k}(\mathscr{A}, \alpha) = \{x \in \Omega : \mathscr{T}_{\mathscr{A}}(x) + \alpha < 1\},$$
$$\mathscr{I}_{k}(\mathscr{A}, \beta) = \{x \in \Omega : \mathscr{I}_{\mathscr{A}}(x) + \beta < 1\},$$
$$\mathscr{F}_{k}(\mathscr{A}, \gamma) = \{x \in \Omega : \mathscr{F}_{\mathscr{A}}(x) + \gamma > 1\}.$$

Definition 3.3. Given a neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ and for $\gamma \in (0, 1]$, $\beta, \alpha \in [0, 1)$, we define the following neutrosphic subsets of Ω:

$$\mathscr{T}_{p \vee k}(\mathscr{A}, \alpha) = \{x \in \Omega : \mathscr{T}_{\mathscr{A}}(x) + \alpha < 1 \text{ or } \mathscr{T}_{\mathscr{A}}(x) \leq \alpha\},$$
$$\mathscr{I}_{p \vee k}(\mathscr{A}, \beta) = \{x \in \Omega : \mathscr{I}_{\mathscr{A}}(x) + \beta < 1 \text{ or } \mathscr{I}_{\mathscr{A}}(x) \leq \beta\},$$
$$\mathscr{F}_{p \vee k}(\mathscr{A}, \gamma) = \{x \in \Omega : \mathscr{F}_{\mathscr{A}}(x) + \gamma > 1 \text{ or } \mathscr{F}_{\mathscr{A}}(x) \geq \gamma\}.$$

Definition 3.4. A neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ of Ω is called $(\mathscr{e}, \mathscr{s})$-neutrosophic UP-subalgebra of Ω, if for all $x, y \in \Omega$:

$$x \in \mathscr{T}_{e}(\mathscr{A}, \alpha_x), y \in \mathscr{T}_{e}(\mathscr{A}, \alpha_y) \text{ implies } x \curlyvee y \in \mathscr{T}_{s}(\mathscr{A}, \alpha_x \vee \alpha_y)$$
$$x \in \mathscr{I}_{e}(\mathscr{A}, \beta_x), y \in \mathscr{I}_{e}(\mathscr{A}, \beta_y) \text{ implies } x \curlyvee y \in \mathscr{I}_{s}(\mathscr{A}, \beta_x \vee \beta_y),$$
$$x \in \mathscr{F}_{e}(\mathscr{A}, \gamma_x), y \in \mathscr{F}_{e}(\mathscr{A}, \gamma_y) \text{ implies } x \curlyvee y \in \mathscr{F}_{s}(\mathscr{A}, \gamma_x \wedge \gamma_y).$$

for all $\gamma_x, \gamma_y \in (0, 1]$, $\beta_x, \beta_y, \alpha_x, \alpha_y \in [0, 1)$ where $\mathscr{e}, \mathscr{s} \in \{\mathscr{p}, \mathscr{k}\}$.

Obviously, the null neutrosophic set is $(\mathscr{p}, \mathscr{p})$-neutrosophic UP-subalgebra of Ω.

Proposition 3.5. Given a neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ and for $\gamma \in (0, 1]$, $\beta, \alpha \in [0, 1)$, then

$$\mathscr{T}_{\mathscr{p}}(\mathscr{A}, \alpha) = \mathscr{T}_{\epsilon}(\mathscr{A}^c, 1 - \alpha),$$
$$\mathscr{I}_{\mathscr{p}}(\mathscr{A}, \beta) = \mathscr{I}_{\epsilon}(\mathscr{A}^c, 1 - \beta),$$
$$\mathscr{F}_{\mathscr{p}}(\mathscr{A}, \gamma) = \mathscr{F}_{\epsilon}(\mathscr{A}^c, 1 - \gamma).$$

Proof. Suppose that $x \in \mathscr{T}_{\mathscr{p}}(\mathscr{A}, \alpha) \iff \mathscr{T}_{\mathscr{A}}(x) \leq \alpha \iff 1 - \mathscr{T}_{\mathscr{A}}(x) \geq 1 - \alpha \iff \mathscr{T}_{\mathscr{A}^c}(x) \geq 1 - \alpha \iff x \in \mathscr{T}_{\epsilon}(\mathscr{A}^c, 1 - \alpha)$. Hence, $\mathscr{T}_{\mathscr{p}}(\mathscr{A}, \alpha) = \mathscr{T}_{\epsilon}(\mathscr{A}^c, 1 - \alpha)$. By similar statements we can prove that $\mathscr{I}_{\mathscr{p}}(\mathscr{A}, \beta) = \mathscr{I}_{\epsilon}(\mathscr{A}^c, 1 - \beta)$.
Suppose that $x \in \mathscr{F}_{\mathscr{p}}(\mathscr{A}, \gamma) \iff \mathscr{F}_{\mathscr{A}}(x) \geq \gamma \iff 1 - \mathscr{F}_{\mathscr{A}}(x) \leq 1 - \gamma \iff \mathscr{F}_{\mathscr{A}^c}(x) \leq 1 - \gamma \iff x \in \mathscr{F}_{\epsilon}(\mathscr{A}^c, 1 - \gamma)$. Hence, $\mathscr{F}_{\mathscr{p}}(\mathscr{A}, \gamma) = \mathscr{F}_{\epsilon}(\mathscr{A}^c, 1 - \gamma)$. □

Proposition 3.6. Given a neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ and for $\gamma \in (0, 1]$, $\beta, \alpha \in [0, 1)$, then

$$\mathscr{T}_{q}(\mathscr{A}, \alpha) = \mathscr{T}_{\mathscr{k}}(\mathscr{A}^c, 1 - \alpha),$$
$$\mathscr{I}_{q}(\mathscr{A}, \beta) = \mathscr{I}_{\mathscr{k}}(\mathscr{A}^c, 1 - \beta),$$
$$\mathscr{F}_{q}(\mathscr{A}, \gamma) = \mathscr{F}_{\mathscr{k}}(\mathscr{A}^c, 1 - \gamma).$$

Proof. Suppose that $x \in \mathscr{T}_{q}(\mathscr{A}, \alpha) \iff \mathscr{T}_{\mathscr{A}}(x) + \alpha > 1 \iff 1 - \mathscr{T}_{\mathscr{A}}(x) - \alpha < 0 \iff \mathscr{T}_{\mathscr{A}^c}(x) + (1 - \alpha) < 1 \iff x \in \mathscr{T}_{\mathscr{k}}(\mathscr{A}^c, 1 - \alpha)$. Hence, $\mathscr{T}_{q}(\mathscr{A}, \alpha) = \mathscr{T}_{\mathscr{k}}(\mathscr{A}^c, 1 - \alpha)$. By similar statements we can prove that $\mathscr{I}_{q}(\mathscr{A}, \beta) = \mathscr{I}_{\mathscr{k}}(\mathscr{A}^c, 1 - \beta)$.
Suppose that $x \in \mathscr{F}_{q}(\mathscr{A}, \gamma) \iff \mathscr{F}_{\mathscr{A}}(x) + \gamma < 1 \iff 1 - \mathscr{F}_{\mathscr{A}}(x) - \gamma > 0 \iff \mathscr{F}_{\mathscr{A}^c}(x) + (1 - \gamma) > 1 \iff x \in \mathscr{F}_{\mathscr{k}}(\mathscr{A}^c, 1 - \gamma)$. Hence, $\mathscr{F}_{q}(\mathscr{A}, \gamma) = \mathscr{F}_{\mathscr{k}}(\mathscr{A}^c, 1 - \gamma)$. □

Proposition 3.7. If $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is a netrosophic set of Ω and for each $x, y \in \Omega$ with $\alpha_x, \alpha_y, \beta_x.\beta_y \in (0, 1]$, $\gamma_x, \gamma_y \in [0, 1)$, then

$$\mathscr{T}_{\mathscr{p}}\big(\mathscr{A}, (1 - \alpha_x) \vee (1 - \alpha_y)\big) = \mathscr{T}_{\epsilon}(\mathscr{A}^c, \alpha_x \wedge \alpha_y).$$
$$\mathscr{I}_{\mathscr{p}}\big(\mathscr{A}, (1 - \beta_x) \vee (1 - \beta_y)\big) = \mathscr{I}_{\epsilon}(\mathscr{A}^c, \beta_x \wedge \beta_y).$$
$$\mathscr{F}_{\mathscr{p}}\big(\mathscr{A}, (1 - \gamma_x \wedge (1 - \gamma_y)\big) = \mathscr{F}_{\epsilon}(\mathscr{A}^c, \gamma_x \vee \gamma_y).$$

Proof. Let $x \in \mathscr{T}_{\mathscr{p}}\big(\mathscr{A}, (1 - \alpha_x) \vee (1 - \alpha_y)\big) \iff x \in \mathscr{T}_{\mathscr{p}}\big(\mathscr{A}, 1 - (\alpha_x \wedge \alpha_y)\big) \iff \mathscr{T}_{\mathscr{A}}(x) \leq 1 - (\alpha_x \wedge \alpha_y) \iff 1 - \mathscr{T}_{\mathscr{A}}(x) \geq \alpha_x \wedge \alpha_y \iff \mathscr{T}_{\mathscr{A}^c} \geq \alpha_x \wedge \alpha_y \iff x \in \mathscr{T}_{\epsilon}(\mathscr{A}^c, \alpha_x \wedge \alpha_y)$. Hence, $\mathscr{T}_{\mathscr{p}}\big(\mathscr{A}, (1 - \alpha_x) \vee (1 - \alpha_y)\big) = \mathscr{T}_{\epsilon}(\mathscr{A}^c, \alpha_x \wedge \alpha_y)$.
The proof of the second condition is similar.
Now $x \in \mathscr{F}_{\mathscr{p}}\big(\mathscr{A}, (1 - \gamma_x \wedge (1 - \gamma_y)\big) \iff x \in \mathscr{F}_{\mathscr{p}}\big(\mathscr{A}, 1 - (\gamma_x \vee \gamma_y)\big) \iff \mathscr{F}_{\mathscr{A}}(x) \geq 1 - (\gamma_x \vee \gamma_y) \iff 1 - \mathscr{F}_{\mathscr{A}}(x) \leq \gamma_x \vee \gamma_y \iff \mathscr{F}_{\mathscr{A}^c}(x) \leq \gamma_x \vee \gamma_y$ $x \in \mathscr{F}_{\epsilon}(\mathscr{A}^c, \gamma_x \vee \gamma_y)$. Therefore, $\mathscr{F}_{\mathscr{p}}\big(\mathscr{A}, (1 - \gamma_x \wedge (1 - \gamma_y)\big) = \mathscr{F}_{\epsilon}(\mathscr{A}^c, \gamma_x \vee \gamma_y)$. □

Proposition 3.8. If $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is a netrosophic set of Ω and for each $x, y \in \Omega$ with $\alpha_1, \alpha_2, \beta_1.\beta_2 \in [0, 1)$, $\gamma_1, \gamma_2 \in (0, 1]$, then

$$\mathscr{T}_{k}(\mathscr{A}, \alpha_1 \vee \alpha_2) = \mathscr{T}_q(\mathscr{A}^c, (1-\alpha_1) \wedge (1-\alpha_2)).$$
$$\mathscr{I}_{k}(\mathscr{A}, \beta_1 \vee \beta_2) = \mathscr{I}_q(\mathscr{A}^c, (1-\beta_1) \wedge (1-\beta_2)).$$
$$\mathscr{F}_{k}(\mathscr{A}, \gamma_1 \wedge \gamma_2) = \mathscr{F}_q(\mathscr{A}^c, (1-\gamma_1) \vee (1-\gamma_2)).$$

Proof. Let $x \in \mathscr{T}_{k}(\mathscr{A}, \alpha_1 \vee \alpha_2)$, then $\mathscr{T}_{\mathscr{A}}(x) + \alpha_1 \vee \alpha_2 < 1$ if and only if $1 - \mathscr{T}_{\mathscr{A}}(x) - (\alpha_1 \vee \alpha_2) > 0$ if and only if $\mathscr{T}_{\mathscr{A}^c}(x) + (1 - (\alpha_1 \vee \alpha_2)) > 1$ if and only if $\mathscr{T}_{\mathscr{A}^c}(x) + (1-\alpha_1) \wedge (1-\alpha_2)) > 1$ if and only if $x \in \mathscr{T}_q(\mathscr{A}^c, (1-\alpha_1) \wedge (1-\alpha_2))$. □

Proposition 3.9. *A neutrosophic set $\mathscr{A} = \{x :< \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >, x \in \Omega\}$ of Ω is $(\not{p}, \not{p})$-neutrosophic UP-subalgebra of Ω if and only if $\mathscr{A}^c = \{x :< 1 - \mathscr{T}_{\mathscr{A}}(x), 1 - \mathscr{I}_{\mathscr{A}}(x), 1 - \mathscr{F}_{\mathscr{A}}(x) >, x \in \Omega\}$ is (ϵ, ϵ)-neutrosophic UP-subalgebra of Ω.*

Proof. Suppose that $\mathscr{A} = \{x :< \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >, x \in \Omega\}$ is $(\not{p}, \not{p})$-neutrosophic subalgebra of Ω and let $x \in \mathscr{T}_{\epsilon}(\mathscr{A}^c, \alpha_x)$, $y \in \mathscr{T}_{\epsilon}(\mathscr{A}^c, \alpha_y)$. Hence, $\mathscr{T}_{\mathscr{A}^c}(x) \geq \alpha_x$ and $\mathscr{T}_{\mathscr{A}^c}(y) \geq \alpha_y$ implies that $1 - \mathscr{T}_{\mathscr{A}}(x) \geq \alpha_x$ and $1 - \mathscr{T}_{\mathscr{A}}(y) \geq \alpha_y$. Hence, $\mathscr{T}_{\mathscr{A}}(x) \leq 1 - \alpha_x$ and $\mathscr{T}_{\mathscr{A}}(y) \leq 1 - \alpha_y$. Since $\mathscr{A} = \{x :< \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >, x \in \Omega\}$ is $(\not{p}, \not{p})$-neutrosophic subalgebra of Ω, so $x \curlyvee y \in \mathscr{T}_{\not{p}}(\mathscr{A}, (1-\alpha_x) \vee (1-\alpha_y)) = \mathscr{T}_{\not{p}}(\mathscr{A}, 1 - (\alpha_x \wedge \alpha_y)) = \mathscr{T}_{\epsilon}(\mathscr{A}^c, \alpha_x \wedge \alpha_y)$. By similar statements we prove that if $x \in \mathscr{I}_{\epsilon}(\mathscr{A}^c, \beta_x)$, $y \in \mathscr{I}_{\epsilon}(\mathscr{A}^c, \beta_y)$, then $x \curlyvee y \in \mathscr{I}_{\epsilon}(\mathscr{A}^c, \beta_x \wedge \beta_y)$. Now, let $x \in \mathscr{F}_{\epsilon}(\mathscr{A}^c, \gamma_x)$, $y \in \mathscr{F}_{\epsilon}(\mathscr{A}^c, \gamma_y)$, then $\mathscr{F}_{\mathscr{A}^c}(x) \leq \gamma_x$ and $\mathscr{F}_{\mathscr{A}^c}(y) \leq \gamma_y$. Hence, $1 - \mathscr{F}_A(x) \leq \gamma_x$ and $1 - \mathscr{F}_A(y) \leq \gamma_y$ implies $1 - \gamma_x \leq \mathscr{F}_A(x)$ and $1 - \gamma_y \leq \mathscr{F}_A(y)$. Therefore, $x \in \mathscr{F}_{\not{p}}(\mathscr{A}, 1-\gamma_x)$ and $y \in \mathscr{F}_{\not{p}}(\mathscr{A}, 1-\gamma_y)$, so by hypothesis $x \curlyvee y \in \mathscr{F}_{\not{p}}(\mathscr{A}, (1-\gamma_x) \wedge (1-\gamma_y)) = \mathscr{F}_{\not{p}}(\mathscr{A}, (1 - (\gamma_x \vee \gamma_y)) = \mathscr{F}_{\epsilon}(\mathscr{A}^c, \alpha_x \vee \alpha_y)$. This completes the proof. □

Theorem 3.10. *If $\mathscr{A} = \{x : \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x), x \in \Omega\}$ is $(\not{p}, \not{p})$-neutrosophic subalgebra of Ω, then each of $\mathscr{T}_{\not{p}}(\mathscr{A}, \alpha)$, $\mathscr{I}_{\not{p}}(\mathscr{A}, \beta)$ and $\mathscr{F}_{\not{p}}(\mathscr{A}, \Gamma)$ contain 0 whenever they are nonempty.*

Proof. If $\mathscr{T}_{\not{p}}(\mathscr{A}, \alpha) \neq \varphi$, then there exists a point $x \in \Omega$ such that $x \in \mathscr{T}_{\not{p}}(\mathscr{A}, \alpha)$. From the definition, we have $0 = x \curlyvee x \in \mathscr{T}_{\not{p}}(\mathscr{A}, \alpha)$. Similarly, $\mathscr{I}_{\not{p}}(\mathscr{A}, \beta)$ and $\mathscr{F}_{\not{p}}(\mathscr{A}, \Gamma)$ contain 0. □

Theorem 3.11. A neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ of Ω is $(\not{p}, \not{p})$-neutrosophic subalgebra of Ω if and only if for all $x, y \in \Omega$:

$$\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \leq \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y),$$
$$\mathscr{I}_{\mathscr{A}}(x \curlyvee y) \leq \mathscr{I}_{\mathscr{A}}(x) \vee \mathscr{I}_{\mathscr{A}}(y),$$
$$\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \geq \mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y).$$

Proof. Suppose that $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ of Ω is $(\not{p}, \not{p})$-neutrosophic subalgebra of Ω. If there exist $x, y \in \Omega$ such that $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) > \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y)$. Then there exists an $\alpha \in [0, 1)$ such that $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) > \alpha \geq \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y)$. This implies that $x \in \mathscr{T}_{\not{p}}(\mathscr{A}, \alpha)$, $y \in \mathscr{T}_{\not{p}}(\mathscr{A}, \alpha)$ but $x \curlyvee y \notin \mathscr{T}_{\not{p}}(\mathscr{A}, \gamma \vee \alpha)$ which is contradiction. If $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) < \mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y)$ for some $x, y \in \Omega$, so there exists $\gamma \in (0, 1]$ such that $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) < \gamma \leq \mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y)$ implies $x, y \in \mathscr{F}_{\not{p}}(\mathscr{A}, \gamma)$ but $x \curlyvee y \notin \mathscr{F}_{\not{p}}(\mathscr{A}, \gamma)$ which is contradiction. The third condition is similar. □

Theorem 3.12. *If $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is an $(\not{p}, \not{p})$-neutrosophic UP-subalgebra of Ω, then $\mathscr{T}_{k}(\mathscr{A}, \alpha)$, $\mathscr{I}_{k}(\mathscr{A}, \beta)$ and $\mathscr{F}_{k}(\mathscr{A}, \gamma)$ are UP-subalgebras of Ω for all $\gamma \in (0, 1]$ and $\alpha, \beta \in [0, 1)$ whenever they are nonempty.*

Proof. Suppose that $x, y \in \mathscr{T}_{k}(\mathscr{A}, \alpha)$, then by definition, $\mathscr{T}_{\mathscr{A}}(x) + \alpha < 1$ and $\mathscr{T}_{\mathscr{A}}(y) + \alpha < 1$. Since $A = \{x : \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x), x \in \Omega\}$ of Ω is $(\not{p}, \not{p})$-neutrosophic subalgebra of Ω, so by Theorem 3.11, $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \leq \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y)$. Hence, $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) + \alpha \leq \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y) + \alpha \leq \{\mathscr{T}_{\mathscr{A}}(x) + \alpha\} \vee \{\mathscr{T}_{\mathscr{A}}(y) + \alpha\} < 1$. Therefore, $x \curlyvee y \in \mathscr{T}_{k}(\mathscr{A}, \alpha)$ implies that $\mathscr{T}_{k}(\mathscr{A}, \alpha)$ is a UP-subalgebras of Ω. □

Example 3.13. Let $X = \{0, a, b, c\}$ and $\curlyvee$ be defined as following Cayley table:

Let $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ be a neutrosophic set defined as:

$\curlyvee$	0	a	b	c
0	0	a	b	c
a	0	0	c	b
b	0	a	0	c
c	0	a	a	0

Table 1: UP-algebra

Ω	$\mathscr{T}_{\mathscr{A}}(x)$	$\mathscr{I}_{\mathscr{A}}(x)$	$\mathscr{F}_{\mathscr{A}}(x)$
0	0.6	0.6	0.5
a	0.7	0.6	0.5
b	0.7	0.7	0.5
c	0.8	0.7	0.5

Table 2: UP-algebra

For all $\gamma \in (0,1]$ and $\alpha, \beta \in [0,1)$, we have

$$\mathscr{T}_{k}(\mathscr{A},\alpha) = \begin{cases} X & When \quad \alpha \in [0,0.2), \\ \{0,a,b\} & When \quad \alpha \in [0.2,0.3), \\ \varphi \quad or \quad \{0\} & When \quad \alpha \in [0.3,1). \end{cases}$$

$$\mathscr{I}_{k}(\mathscr{A},\beta) = \begin{cases} X & When \quad \beta \in [0,0.3), \\ \{0,a\} & When \quad \beta \in [0.3,0.4), \\ \varphi & When \quad \beta \in [0.4,1). \end{cases}$$

$$\mathscr{F}_{k}(\mathscr{A},\gamma) = \begin{cases} X & When \quad \gamma \in (0.5,1], \\ \varphi & When \quad \gamma \in (0,0.5]. \end{cases}$$

Obviously, $\mathscr{T}_{k}(\mathscr{A},\alpha)$, $\mathscr{I}_{k}(\mathscr{A},\beta)$ and $\mathscr{F}_{k}(\mathscr{A},\gamma)$ are UP-subalgebras of Ω. But, we have $\mathscr{T}_{\mathscr{A}}(a) = \mathscr{T}_{\mathscr{A}}(b) = 0.7$ and $\mathscr{T}_{\mathscr{A}}(a \curlyvee b) = \mathscr{T}_{\mathscr{A}}(c) = 0.8$, so $\mathscr{T}_{\mathscr{A}}(b \curlyvee a) \nleq \mathscr{T}_{\mathscr{A}}(b) \vee \mathscr{T}_{\mathscr{A}}(a)$. Hence, $A = \{x : \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x), x \in \Omega\}$ is not a neutrosophic (p, p) UP-subalgebra.

Theorem 3.14. *If $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is an $(k, p \vee k)$-neutrosophic UP-subalgebra of Ω, then $\mathscr{T}_{k}(\mathscr{A},\alpha)$, $\mathscr{I}_{k}(\mathscr{A},\beta)$ and $\mathscr{F}_{k}(\mathscr{A},\gamma)$ are UP-subalgebras of Ω for all $\gamma \in (0.5,1]$ and $\alpha, \beta \in [0,0.5)$ whenever they are nonempty.*

Proof. Let $x, y \in \mathscr{T}_{k}(\mathscr{A},\alpha)$, then by hypothesis either $x \curlyvee y \in \mathscr{T}_{k}(\mathscr{A},\alpha)$ or $x \curlyvee y \in \mathscr{T}_{p}(\mathscr{A},\alpha)$, so $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \leq \alpha$. Since $\alpha < 0.5$, so $\alpha < 1 - \alpha$ implies that $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) + \alpha < 1$. Therefore, $x \curlyvee y \in \mathscr{T}_{k}(\mathscr{A},\alpha)$.
By similar statements we can prove that $\mathscr{I}_{k}(\mathscr{A},\beta)$ is a UP-subalgebras of Ω.
Let $x, y \in \mathscr{F}_{k}(\mathscr{A},\gamma)$, so by hypothesis, either $x \curlyvee y \in \mathscr{F}_{k}(\mathscr{A},\gamma)$, or $x \curlyvee y \in \mathscr{F}_{p}(\mathscr{A},\gamma)$. Hence, $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \geq \gamma$. Since $\gamma > 0.5$, so $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) + \gamma \geq 2\gamma > 1$. Therefore, $x \curlyvee y \in \mathscr{F}_{k}(\mathscr{A},\gamma)$ implies that $\mathscr{F}_{k}(\mathscr{A},\gamma)$ is a UP-subalgebras of Ω. □

If $A = \{x : \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x), x \in \Omega\}$ is an (k, p)-neutrosophic UP-subalgebra of Ω, then $\mathscr{T}_{k}(\mathscr{A},\alpha)$, $\mathscr{I}_{k}(\mathscr{A},\beta)$ and $\mathscr{F}_{k}(\mathscr{A},\gamma)$ may not be subalgebras as it is shown in the following example.

Example 3.15. Let $X = \{0, a, b, c\}$ and $\curlyvee$ be defined as following Cayley table:

Let $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ be a neutrosophic set defined as:

$\curlyvee$	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	0	0	c
c	0	a	b	0

Table 3: UP-algebra

Ω	$\mathscr{T}_{\mathscr{A}}(x)$	$\mathscr{I}_{\mathscr{A}}(x)$	$\mathscr{F}_{\mathscr{A}}(x)$
0	0.4	0.4	0.7
a	0.3	0.7	0.5
b	0.4	0.6	0.5
c	0.8	0.6	0.5

Table 4: $(\hslash, \wp \vee \hslash)$-neutrosophic UP-subalgebra of Ω

By simple calculation, we can see that $\mathscr{A}$ is an $(\hslash, \wp \vee \hslash)$-neutrosophic UP-subalgebra of Ω. Now we have

$$\mathscr{T}_{\hslash}(\mathscr{A}, \alpha) = \begin{cases} X & When \quad \alpha \in [0, 0.2), \\ \{0, a, b\} & When \quad \alpha \in [0.2, 0.6), \\ \{a\} & When \quad \alpha \in [0.6, 0.7). \end{cases}$$

We have when $\alpha = 0.6$, $a \in \mathscr{T}_{\hslash}(\mathscr{A}, 0.6)$ but $a \curlyvee a = 0 \notin \mathscr{T}_{\hslash}(\mathscr{A}, 0.6)$.

Theorem 3.16. A neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is an $(\wp, \wp \vee \hslash)$-neutrosophic UP-subalgebra of Ω if and only if for all $x, y \in \Omega$:

$$\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \leq \bigvee\{\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y), 0.5\},$$
$$\mathscr{I}_{\mathscr{A}}(x \curlyvee y) \leq \bigvee\{\mathscr{I}_{\mathscr{A}}(x) \vee \mathscr{I}_{\mathscr{A}}(y), 0.5\},$$
$$\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \geq \bigwedge\{\mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y), 0.5\}.$$

Proof. Suppose that $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ of Ω is $(\wp, \wp \vee \hslash)$-neutrosophic subalgebra of Ω. If there exist $x, y \in \Omega$ such that $\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y) \geq 0.5$. Then, we get $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \leq \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y)$. Suppose that $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) > \bigvee\{\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y), 0.5\}$ for some $x, y \in \Omega$. Hence, we have the following cases:
(Case 1) If $\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y) \geq 0.5$, then $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) > \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y)$. If we choose $0.5 < \alpha \in [0, 1)$, then we get $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) > \alpha \geq \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y)$. Also, we have $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) + \alpha > 1$ implies that $x \curlyvee y \notin \mathscr{T}_{\hslash}(\mathscr{A}, \alpha)$. Therefore, $x \in \mathscr{T}_{\wp}(\mathscr{A}, \alpha)$, $y \in \mathscr{T}_{\wp}(\mathscr{A}, \alpha)$ but $x \curlyvee y \notin \mathscr{T}_{\wp \vee \hslash}(\mathscr{A}, \gamma \vee \alpha)$ which is contradiction.
(Case 2) If $\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y) \leq 0.5$, then $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) > 0.5$. Hence, $x \in \mathscr{T}_{\wp}(\mathscr{A}, 0.5)$, $y \in \mathscr{T}_{\wp}(\mathscr{A}, 0.5)$ and $x \curlyvee y \notin \mathscr{T}_{\wp}(\mathscr{A}, 0.5)$. Also, we have $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) + 0.5 > 1$ implies that $x \curlyvee y \notin \mathscr{T}_{\hslash}(\mathscr{A}, 0.5)$. Hence, $x \curlyvee y \notin \mathscr{T}_{\wp \vee \hslash}(\mathscr{A}, 0.5)$ which is contradiction. By similar statements, we can prove $\mathscr{I}_{\mathscr{A}}(x \curlyvee y) \leq \bigvee\{\mathscr{I}_{\mathscr{A}}(x) \vee \mathscr{I}_{\mathscr{A}}(y), 0.5\}$.
Now, suppose that there exist $x, y \in \Omega$ such that $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) < \bigwedge\{\mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y), 0.5\}$. Then we have the following cases:
Case 1, if $\mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y) \leq 0.5$, then $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) < \mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y)$, so there exists $\gamma < 0.5$ such that $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) < \gamma \leq \mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y)$. Therefore, $x \in \mathscr{F}_{\wp}(\mathscr{A}, \gamma)$, $y \in \mathscr{F}_{\wp}(\mathscr{A}, \gamma)$ but $x \curlyvee y \notin \mathscr{F}_{\wp}(\mathscr{A}, \gamma \wedge \gamma)$. Also, we have $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) + \gamma < 1$, so $x \curlyvee y \notin \mathscr{F}_{\hslash}(\mathscr{A}, \gamma \wedge \gamma)$. Hence, $x \in \mathscr{F}_{\wp}(\mathscr{A}, \gamma)$, $y \in \mathscr{F}_{\wp}(\mathscr{A}, \gamma)$ but $x \curlyvee y \notin \mathscr{F}_{\wp \vee \hslash}(\mathscr{A}, \gamma \wedge \gamma)$ which is contradiction.
Case 2, if $\mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y) \geq 0.5$, then $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) < 0.5$. This implies that $x \in \mathscr{F}_{\wp}(\mathscr{A}, 0.5)$, $y \in \mathscr{F}_{\wp}(\mathscr{A}, 0.5)$ and $x \curlyvee y \notin \mathscr{F}_{\wp}(\mathscr{A}, 0.5)$. Also, we have $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) + 0.5 < 1$ implies that $x \curlyvee y \notin \mathscr{F}_{\hslash}(\mathscr{A}, 0.5)$. Hence, $x \in \mathscr{F}_{\wp}(\mathscr{A}, 0.5)$, $y \in \mathscr{F}_{\wp}(\mathscr{A}, 0.5)$ and $x \curlyvee y \notin \mathscr{F}_{\wp \vee \hslash}(\mathscr{A}, 0.5)$ again we get a contradiction. Therefore, $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \geq \bigwedge\{\mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y), 0.5\}$ for all $x, y \in \Omega$.
Conversely, suppose that the neutrosophic set $A = \{x : \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x), x \in \Omega\}$ satisfies the condition of the theorem. Let $x, y \in \Omega$ and $\alpha_x, \alpha_y \in [0, 1)$. Let $x \in \mathscr{T}_{\wp}(\mathscr{A}, \alpha_x)$ and $y \in \mathscr{T}_{\wp}(\mathscr{A}, \alpha_y)$, then $\mathscr{T}_{\mathscr{A}}(x) \leq \alpha_x$ and $\mathscr{T}_{\mathscr{A}}(y) \leq \alpha_y$ and by hypothesis, we have $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \leq \bigvee\{\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y), 0.5\}$. If $\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y) \geq 0.5$,

then we get $\mathscr{T}_{\mathscr{A}}(x\curlyvee y) \leq \mathscr{T}_{\mathscr{A}}(x)\vee\mathscr{T}_{\mathscr{A}}(y) \leq \alpha_x\vee\alpha_y$. Hence, $x\curlyvee y \in \mathscr{T}_{p}(\mathscr{A}, \alpha_x\vee\alpha_y)$. If $\mathscr{T}_{\mathscr{A}}(x)\vee\mathscr{T}_{\mathscr{A}}(y) \leq 0.5$, then $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \leq 0.5$. Now for each α_x, α_y with $\alpha_x \vee \alpha_y < 0.5$, we have $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) + \alpha_x \vee \alpha_y < 1$. Hence, $x \curlyvee y \in \mathscr{T}_{k}(\mathscr{A}, \alpha_x \vee \alpha_y)$. In both cases, we obtain that $x \curlyvee y \in \mathscr{T}_{p\vee k}(\mathscr{A}, \alpha_x \vee \alpha_y)$. Similar statements can be done for $\mathscr{I}_{\mathscr{A}}$. Now, if $x, y \in \Omega$ and $\gamma_x, \gamma_y \in (0, 1]$. Let $x \in \mathscr{F}_{p}(\mathscr{A}, \gamma_x)$ and $y \in \mathscr{F}_{p}(\mathscr{A}, \gamma_y)$, then $\mathscr{F}_{\mathscr{A}}(x) \geq \gamma_x$ and $\mathscr{F}_{\mathscr{A}}(y) \geq \gamma_y$ and by hypothesis, we have $\mathscr{F}_{\mathscr{A}}(x\curlyvee y) \geq \bigwedge\{\mathscr{F}_{\mathscr{A}}(x)\wedge\mathscr{F}_{\mathscr{A}}(y), 0.5\}$. If $\mathscr{F}_{\mathscr{A}}(x)\wedge\mathscr{F}_{\mathscr{A}}(y) \leq 0.5$, then $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \geq \mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y)$. Hence, $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \geq \gamma_x \wedge \gamma_y$, so $x \curlyvee y \in \mathscr{F}_{p}(\mathscr{A}, \gamma_x \wedge \gamma_y)$. If $\mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y) \geq 0.5$, then $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \geq 0.5$. Hence for each γ_x, γ_y with $\gamma_x \wedge \gamma_y > 0.5$, we have $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) + \gamma_x \wedge \gamma_y > 1$. Therefore, $x \curlyvee y \in \mathscr{T}_{k}(\mathscr{A}, \gamma_x \wedge \gamma_y)$. Hence, in both cases we obtain that $x \curlyvee y \in \mathscr{T}_{p\vee k}(\mathscr{A}, \gamma_x \wedge \gamma_y)$. □

Theorem 3.17. A neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is an $(p, p \vee k)$-neutrosophic UP-subalgebra of Ω if and only if the neutrosophic subsets $\mathscr{T}_{k}(\mathscr{A}, \alpha)$, $\mathscr{I}_{k}(\mathscr{A}, \beta)$ and $\mathscr{F}_{k}(\mathscr{A}, \gamma)$ are UP-subalgebras of Ω for for all $\alpha, \beta \in [0, 0.5)$ and $\gamma \in (0.5, 1]$ whenever they are nonempty.

Proof. Assume that $\mathscr{T}_{k}(\mathscr{A}, \alpha)$, $\mathscr{I}_{k}(\mathscr{A}, \beta)$ and $\mathscr{F}_{k}(\mathscr{A}, \gamma)$ are nonempty for all $\alpha, \beta \in [0, 0.5)$ and $\gamma \in (0.5, 1]$. Let $x, y \in \mathscr{T}_{k}(\mathscr{A}, \alpha)$. Then $\mathscr{T}_{\mathscr{A}}(x)+\alpha < 1$ and $\mathscr{T}_{\mathscr{A}}(y)+\alpha < 1$. Then by Theorem 3.16, we have $\mathscr{T}_{\mathscr{A}}(x\curlyvee y) \leq \bigvee\{\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y), 0.5\}$ and hence, $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) + \alpha \leq \bigvee\{\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y), 0.5\} + \alpha$. Thus, $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) + \alpha \leq \bigvee\{\mathscr{T}_{\mathscr{A}}(x) + \alpha \vee \mathscr{T}_{\mathscr{A}}(y) + \alpha, 0.5 + \alpha\}$ and since $\alpha < 1$, so we obtain that $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) + \alpha < 1$. Therefore, $x \curlyvee y \in \mathscr{T}_{k}(\mathscr{A}, \alpha)$ implies that $\mathscr{T}_{k}(\mathscr{A}, \alpha)$ is a UP-subalgebras of Ω.
By similar statements we can prove that $\mathscr{I}_{k}(\mathscr{A}, \alpha)$ is a UP-subalgebras of Ω.
Now, let $x, y \in \mathscr{F}_{k}(\mathscr{A}, \gamma)$. Then $\mathscr{F}_{\mathscr{A}}(x) + \gamma > 1$ and $\mathscr{F}_{\mathscr{A}}(y) + \gamma > 1$. By Theorem 3.16, $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \geq \bigwedge\{\mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y), 0.5\}$ and hence $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) + \gamma \geq \bigwedge\{\mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y), 0.5\} + \gamma$ implies that $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) + \gamma \geq \bigwedge\{\mathscr{F}_{\mathscr{A}}(x) + \gamma \wedge \mathscr{F}_{\mathscr{A}}(y) + \gamma, 0.5 + \gamma\}$. Since $\gamma > 0.5$, so each factor in the right side of the inequality is greater than 1. Thus, $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) + \gamma > 1$ which implies that $x \curlyvee y \in \mathscr{F}_{k}(\mathscr{A}, \gamma)$. Therefore, $\mathscr{F}_{k}(\mathscr{A}, \gamma)$ is a UP-subalgebras of Ω.
Conversely, let $\alpha_x, \alpha_y \in [0, 0.5)$ and let $x \in \mathscr{T}_{p}(\mathscr{A}, \alpha_x)$, $y \in \mathscr{T}_{p}(\mathscr{A}, \alpha_y)$. Thus, $\mathscr{T}_{\mathscr{A}}(x) \leq \alpha_x$ and $\mathscr{T}_{\mathscr{A}}(y) \leq \alpha_y$ and because $\alpha_x, \alpha_y \in [0, 0.5)$, so $\mathscr{T}_{\mathscr{A}}(x) + \alpha_x < 1$ and $\mathscr{T}_{\mathscr{A}}(y) + \alpha_y < 1$ implies that $x \in \mathscr{T}_{k}(\mathscr{A}, \alpha_x)$, $y \in \mathscr{T}_{k}(\mathscr{A}, \alpha_y)$. Since $\mathscr{T}_{k}(\mathscr{A}, \alpha)$ is a UP-subalgebra of Ω for for all $\alpha \in [0, 0.5)$, so $x \curlyvee y \in \mathscr{T}_{k}(\mathscr{A}, \alpha_x \vee \alpha_y)$.
By similar statements we can prove that if $x \in \mathscr{I}_{p}(\mathscr{A}, \beta_x)$, $y \in \mathscr{I}_{p}(\mathscr{A}, \beta_y)$, then $x \curlyvee y \in \mathscr{I}_{k}(\mathscr{A}, \beta_x \vee \beta_y)$.
Now, suppose that $x \in \mathscr{F}_{p}(\mathscr{A}, \gamma_x)$, $y \in \mathscr{F}_{p}(\mathscr{A}, \gamma_y)$ where $\gamma_x, \gamma_y \in (0.5, 1]$. Thus, $\mathscr{F}_{\mathscr{A}}(x) \geq \gamma_x$ and $\mathscr{F}_{\mathscr{A}}(y) \geq \gamma_y$, so $\mathscr{F}_{\mathscr{A}}(x) + \gamma_x > 1$ and $\mathscr{F}_{\mathscr{A}}(y) + \gamma_y > 1$. Hence, $x \in \mathscr{F}_{k}(\mathscr{A}, \gamma_x)$, $y \in \mathscr{F}_{k}(\mathscr{A}, \gamma_y)$ implies that $x \curlyvee y \in \mathscr{F}_{k}(\mathscr{A}, \gamma_x \wedge \gamma_y)$. Therefore, $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is an $(p, p \vee k)$-neutrosophic UP-subalgebra of Ω. □

The following example shows that if $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is not $(p, p \vee k)$-neutrosophic UP-subalgebra of Ω, then at least one of the sets $\mathscr{T}_{k}(\mathscr{A}, \alpha)$, $\mathscr{I}_{k}(\mathscr{A}, \beta)$ or $\mathscr{F}_{k}(\mathscr{A}, \gamma)$ is not a UP-subalgebra of Ω.

Example 3.18. Let $X = \{0, a, b, c\}$ and $\curlyvee$ be defined as following Cayley table:

$\curlyvee$	0	a	b	c
0	0	a	b	c
a	0	0	a	a
b	0	0	0	a
c	0	a	0	0

Table 5: UP-algebra

Let $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ be a neutrosophic set defined as:

For all $\gamma \in (0.5, 1]$ and $\alpha, \beta \in [0, 0.5)$, we have $\mathscr{T}_{k}(\mathscr{A}, \alpha) = \{0, c\}$, $\mathscr{I}_{k}(\mathscr{A}, \beta) = \{0, b\}$ and $\mathscr{F}_{k}(\mathscr{A}, \gamma) = \{0, b, c\}$ or Ω. Obviously, $\mathscr{F}_{k}(\mathscr{A}, \gamma)$ is not a UP-subalgebra of Ω. Also, we have $b, c \in \mathscr{F}_{p}(\mathscr{A}, 0.6)$ but $b \curlyvee c \notin \mathscr{F}_{p}(\mathscr{A}, 0.6)$ and $b \curlyvee c \notin \mathscr{F}_{k}(\mathscr{A}, 0.6)$. Hence, $b \curlyvee c \notin \mathscr{F}_{p\vee k}(\mathscr{A}, 0.6)$. Therefore, $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is not $(p, p \vee k)$-neutrosophic UP-subalgebra of Ω.

Ω	$\mathscr{T}_{\mathscr{A}}(x)$	$\mathscr{I}_{\mathscr{A}}(x)$	$\mathscr{F}_{\mathscr{A}}(x)$
0	0.4	0.3	0.8
a	0.8	0.7	0.3
b	0.7	0.4	0.6
c	0.5	0.6	0.7

Table 6: UP-algebra

Theorem 3.19. A neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is an $(\not p, \not p \vee \not k)$-neutrosophic UP-subalgebra of Ω if and only if the neutrosophic subsets $\mathscr{T}_{\not p}(\mathscr{A}, \alpha)$, $\mathscr{I}_{\not p}(\mathscr{A}, \beta)$ and $\mathscr{F}_{\not p}(\mathscr{A}, \gamma)$ are UP-subalgebras of Ω for for all $\alpha, \beta \in [0.5, 1)$ and $\gamma \in (0, 0.5]$ whenever they are nonempty.

Proof. Assume that $\mathscr{T}_{\not p}(\mathscr{A}, \alpha)$, $\mathscr{I}_{\not p}(\mathscr{A}, \beta)$ and $\mathscr{F}_{\not p}(\mathscr{A}, \gamma)$ are nonempty for all $\alpha, \beta \in [0.5, 1)$ and $\gamma \in (0, 0.5]$. Let $x, y \in \mathscr{T}_{\not p}(\mathscr{A}, \alpha)$. Then $\mathscr{T}_{\mathscr{A}}(x) \leq \alpha$ and $\mathscr{T}_{\mathscr{A}}(y) \leq \alpha$. By Theorem 3.16, we have $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \leq \bigvee\{\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y), 0.5\} = \alpha$ because $\alpha > 0.5$. Therefore, $x \curlyvee y \in \mathscr{T}_{\not p}(\mathscr{A}, \alpha)$.
$\mathscr{I}_{\not p}(\mathscr{A}, \beta)$ is a UP-subalgebras of Ω can be proved similarly.
Let $x, y \in \mathscr{F}_{\not p}(\mathscr{A}, \alpha)$. Then $\mathscr{F}_{\mathscr{A}}(x) \geq \gamma$ and $\mathscr{T}_{\mathscr{A}}(y) \geq \gamma$. By Theorem 3.16, we have $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \geq \bigwedge\{\mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y), 0.5\} = \gamma$ because $\gamma \leq 0.5$. Therefore, $x \curlyvee y \in \mathscr{F}_{\not p}(\mathscr{A}, \gamma)$.
The converse is obvious. □

Proposition 3.20. For each neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$. If $\mathscr{T}_{\not k}(\mathscr{A}, \alpha)$, $\mathscr{I}_{\not k}(\mathscr{A}, \beta)$ and $\mathscr{F}_{\not k}(\mathscr{A}, \gamma)$ are UP-subalgebras of Ω, then $\mathscr{T}_{\not p \vee \not k}(\mathscr{A}, \alpha)$, $\mathscr{I}_{\not p \vee \not k}(\mathscr{A}, \beta)$ and $\mathscr{F}_{\not p \vee \not k}(\mathscr{A}, \gamma)$ are UP-subalgebras of Ω for for all $\alpha, \beta \in [0, 0.5)$ and $\gamma \in (0.5, 1]$.

Proof. Let $x, y \in \mathscr{T}_{\not p}(\mathscr{A}, \alpha)$, then $\mathscr{T}_{\mathscr{A}}(x) \leq \alpha$ and $\mathscr{T}_{\mathscr{A}}(y) \leq \alpha$. Hence, $\mathscr{T}_{\mathscr{A}}(x) + \alpha \leq 2\alpha < 1$ and $\mathscr{T}_{\mathscr{A}}(y) + \alpha \leq 2\alpha < 1$. By hypothesis, $x \curlyvee y \in \mathscr{T}_{\not k}(\mathscr{A}, \alpha)$, so $x \curlyvee y \in \mathscr{T}_{\not p \vee \not k}(\mathscr{A}, \alpha)$. Therefore, $\mathscr{T}_{\not p \vee \not k}(\mathscr{A}, \alpha)$ is a UP-subalgebra of Ω.
The other proofs are similar. □

Theorem 3.21. Let $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ be a neutrosophic set. The nonempty sets $\mathscr{T}_{\not p}(\mathscr{A}, \alpha)$, $\mathscr{I}_{\not p}(\mathscr{A}, \beta)$ and $\mathscr{F}_{\not p}(\mathscr{A}, \gamma)$ are UP-subalgebras of Ω for for all $\alpha, \beta \in [0.5, 1)$ and $\gamma \in (0, 0.5]$ if and only if for all $x, y \in \Omega$:

$$\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \wedge 0.5 \leq \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y),$$
$$\mathscr{I}_{\mathscr{A}}(x \curlyvee y) \wedge 0.5 \leq \mathscr{I}_{\mathscr{A}}(x) \vee \mathscr{I}_{\mathscr{A}}(y),$$
$$\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \vee 0.5 \geq \mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y).$$

Proof. If there is $x, y \in \Omega$ such that $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \vee 0.5 > \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y) = \alpha$, so $\mathscr{T}_{\mathscr{A}}(x) \leq \alpha$ and $\mathscr{T}_{\mathscr{A}}(y) \leq \alpha$. Hence $x, y \in \mathscr{T}_{\not p}(\mathscr{A}, \alpha)$ and by hypothesis, $\mathscr{T}_{\not p}(\mathscr{A}, \alpha)$ is a UP-subalgebra of Ω implies that $x \curlyvee y \in \mathscr{T}_{\not p}(\mathscr{A}, \alpha)$ but we have $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \wedge 0.5 > \alpha$ and since $\alpha > 0.5$ implies that $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) > \alpha$ which is contradiction. Therefore, $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \wedge 0.5 \leq \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y)$. By similar statements, we prove that $\mathscr{I}_{\mathscr{A}}(x \curlyvee y) \vee 0.5 \leq \mathscr{I}_{\mathscr{A}}(x) \vee \mathscr{I}_{\mathscr{A}}(y)$.
Suppose there is $x, y \in \Omega$ such that $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \vee 0.5 < \mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y) = \gamma$, so $\mathscr{F}_{\mathscr{A}}(x) \geq \gamma$ and $\mathscr{F}_{\mathscr{A}}(y) \geq \gamma$. Hence $x, y \in \mathscr{F}_{\not p}(\mathscr{A}, \gamma)$ and by hypothesis, $\mathscr{F}_{\not p}(\mathscr{A}, \gamma)$ is a UP-subalgebra of Ω implies that $x, y \in \mathscr{F}_{\not p}(\mathscr{A}, \gamma)$ but we have $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \vee 0.5 < \gamma$ and $\gamma < 0.5$ implies that $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) < \gamma$ which is contradiction. Thus $\mathscr{F}_{\mathscr{A}}(x \curlyvee y) \vee 0.5 \geq \mathscr{F}_{\mathscr{A}}(x) \wedge \mathscr{F}_{\mathscr{A}}(y)$.
Conversely, Let $x, y \in \mathscr{T}_{\not p}(\mathscr{A}, \alpha)$, then $\mathscr{T}_{\mathscr{A}}(x) \leq \alpha$ and $\mathscr{T}_{\mathscr{A}}(y) \leq \alpha$. Therefore, $\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y) \leq \alpha$ and by hypothesis, $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \wedge 0.5 \leq \mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y) \leq \alpha$. Therefore, $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \wedge 0.5 \leq \alpha$ but we have $\alpha > 0.5$, so $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \leq \alpha$ implies that $x \curlyvee y \in \mathscr{T}_{\not p}(\mathscr{A}, \alpha)$. Thus, $\mathscr{T}_{\not p}(\mathscr{A}, \alpha)$ is a UP-subalgebra of Ω. Similarly, $\mathscr{I}_{\not p}(\mathscr{A}, \beta)$ and $\mathscr{F}_{\not p}(\mathscr{A}, \gamma)$ are UP-subalgebras of Ω. □

Remark 3.22. In order the conditions of Theorem 3.21 to be true, we must have $\mathscr{T}_{\mathscr{A}}(x) \geq \mathscr{T}_{\mathscr{A}}(0)$, $\mathscr{I}_{\mathscr{A}}(x) \geq \mathscr{I}_{\mathscr{A}}(0)$ and $\mathscr{F}_{\mathscr{A}}(x) \leq \mathscr{F}_{\mathscr{A}}(0)$ for each $x \in \Omega$.

In the following example, we have $\mathscr{T}_{\mathscr{A}}(a) < \mathscr{T}_{\mathscr{A}}(0)$ and $\mathscr{I}_{\mathscr{A}}(b) < \mathscr{I}_{\mathscr{A}}(0)$.

Example 3.23. Let $X = \{0, a, b, c\}$ and $\curlyvee$ be defined as following Cayley table:

$\curlyvee$	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	a	0	c
c	0	a	b	0

Table 7: A UP-algebra

Let $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ be a neutrosophic set defined as:

Ω	$\mathscr{T}_{\mathscr{A}}(x)$	$\mathscr{I}_{\mathscr{A}}(x)$	$\mathscr{F}_{\mathscr{A}}(x)$
0	0.6	0.3	0.8
a	0.2	0.7	0.3
b	0.7	0.1	0.4
c	0.5	0.6	0.5

Table 8: $\mathscr{T}_{\mathscr{A}}(a) < \mathscr{T}_{\mathscr{A}}(0)$ and $\mathscr{I}_{\mathscr{A}}(b) < \mathscr{I}_{\mathscr{A}}(0)$

In this example, we have $\mathscr{T}_{\not p}(\mathscr{A}, \alpha) = \{a\}$ when $\alpha \in [0.5, 0.8)$ which is not a a UP-subalgebras of Ω. Also, $\mathscr{T}_{\mathscr{A}}(a \curlyvee a) \wedge 0.5 \nleq \mathscr{T}_{\mathscr{A}}(a) \vee \mathscr{T}_{\mathscr{A}}(a)$. Also, $\mathscr{I}_{\mathscr{A}}(b \curlyvee b) \wedge 0.5 \nleq \mathscr{I}_{\mathscr{A}}(b) \vee \mathscr{I}_{\mathscr{A}}(b)$. Here, $\mathscr{I}_{\not p}(\mathscr{A}, \beta) = \{0, b\}$ when $\beta \in [0.5, 0.7)$ which is a UP-subalgebra but $\mathscr{I}_{\not p}(\mathscr{A}, \beta)$ is not a UP-subalgebra for all $\beta \in [0.5, 1)$ because, $\mathscr{I}_{\not p}(\mathscr{A}, \beta) = \{b\}$ when $\beta \in [0.7, 0.9)$ which is not a UP-subalgebra.

Proposition 3.24. For a neutrosophic set $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$. If the subsets $\mathscr{T}_{\not p \vee \not k}(\mathscr{A}, \alpha)$, $\mathscr{I}_{\not p \vee \not k}(\mathscr{A}, \beta)$ and $\mathscr{F}_{\not p \vee \not k}(\mathscr{A}, \gamma)$ are UP-subalgebras of Ω for all $\alpha, \beta \in [0, 1)$ and $\gamma \in (0, 1]$, then $\mathscr{A} = \{< x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x) >: x \in \Omega\}$ is a $(\not p, \not p \vee \not k)$-neutrosophic UP-subalgebras of Ω.

Proof. Suppose that $\mathscr{T}_{\not p \vee \not k}(\mathscr{A}, \alpha)$ is a UP-algebra and if there is $x, y \in \Omega$ such that $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) > \bigvee\{\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y), 0.5\}$. Hence, there exists $\alpha \in [0.5, 1)$ such that $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) > \alpha \geq \bigvee\{\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y), 0.5\}$. Therefore, $x, y \in \mathscr{T}_{\not p}(\mathscr{A}, \alpha)$. Also, we have $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) > \alpha$ and since $\alpha \in [0.5, 1)$, so $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) + \alpha > 1$. THus, neither $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \leq \alpha$ nor $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) + \alpha < 1$ implies $x \curlyvee y \notin \mathscr{T}_{\not p \vee \not k}(\mathscr{A}, \alpha)$ which is contradiction. Therefore, $\mathscr{T}_{\mathscr{A}}(x \curlyvee y) \leq \bigvee\{\mathscr{T}_{\mathscr{A}}(x) \vee \mathscr{T}_{\mathscr{A}}(y), 0.5\}$. □

References

[1] M. A. Ansari, A. Haidar, and N. A. Koam, *On a graph associated to UP-algebras*, Mathe- matical and Computational Applications **23**.4 (2018), pp. 1–12.

[2] V. Traneva, S. Tranev, and V. Todorov, ”An Elliptic Intuitionistic Fuzzy Analysis of Regional Samsung Net Revenue Variations,” in Proceedings of the International Conference on Intelligent and Fuzzy Systems (INFUS), 2024, pp. 522–531.

[3] A. Iampan. “*A new branch of the logical algebra: UP-algebras*, Journal of Algebra and Related Topics **5**.1 (2017), pp. 35–54.

[4] A. Iampan, *Fuzzy translations of a fuzzy set in UP-algebras*, Journal of the Indonesian Mathematical Society **23**.2 (2017), pp. 1–19.

[5] Y. B. Jun, *Neutrosophic subalgebras of several types in BCK/BCI-algebras*, Annals of Fuzzy Mathematics and Informatics **14**.1 (2017), pp. 75–86.

[6] G. Muhiuddin and Y. B. Jun *Further results of Neutrosophic subalgebras in BCK/BCI-algebras based on neutrosophic points*, TWMS J. App. Eng. Math. **10**.1 (2020), pp. 232–240.

[7] G. Muhiuddin, H. Bordbar, F. Smarandache and Y. B. Jun, *Further results on* (ϵ, ϵ)*-neutrosophic subalgebras and ideals in BCK/BCI-algebras*, Neutrosophic Sets and Systems **20** (2018), pp. 36–43.

[8] M. A. Ozturk and Y. B. Jun, *Neutrosophic ideals in BCK/BCI-algebras based on neutrosophic points*, Journal of International Mathematical Virtual Institute **8**.1 (2018), pp. 1–17.

[9] V. Rajam, *A study of neutrosophic algebra*". PhD Dissertation, Bharathidasan University, India, 2024.

[10] U. Rivieccio, T. Flaminio, and T. Nascimento da Silva, "On the Representation of (Weak) Nilpotent Minimum Algebras," in Proceedings of the 2020 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), 2020, pp. 1–8.

[11] A. B. Saeid and Y. B. Jun, *Neutrosophic subalgebras of BCK/BCI-algebras based on neutro- sophic points*, Annals of Fuzzy Mathematics and Informatics **14**.2 (2017), pp. 87–97.

[12] S. Soundararajan, "On Fuzzy B-Volterra Spaces," Journal of Analysis and Mathematical Information, vol. 42, pp. 189–197, Jan. 2024.

[13] S. Majbour, M. Barbier, and J.-M. Le Bars, "Fuzzy Vault Security Enhancement to Avoid Statistical Biases," in Proceedings of the International Conference on Security and Cryptography (SECRYPT), 2024, pp. 1–2.

[14] S. R. Sree and Ramesh, "A Fuzzy Logic-Based Model for Software Effort Estimation Using NASA93 Dataset," Applied Sciences, vol. 13, no. 8, p. 5014, 2021.

[15] L. F. Zadeh, *Fuzzy sets*, Information and Control **8**.3 (1965), pp. 338–353.