Matrices and Correlation Coefficient for possibility interval-valued neutrosophic hypersoft sets and their applications in real-life

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Abstract

In this careful study , through the concept possibility interval valued neutrosophic hyper soft set (abbreviated as piv-NHSS) which is combined from the hypersoft set (HSS) and Interval-valued neutrosophic set under the posobolity degree and each iv-NHSS is assigned a possibility degree in the interval $[0, 1]$. Based on this concept, we present a more flexible, expanded method for a previous concept named possibility interval valued neutrosophic hyper soft matrix (piv-NHSM) as a new generalization of piv-NHSS. In this work, we also present several algebraic operations and also all the mathematical properties associated with this model. In addition to the above, we have presented a clear algorithm based on the matrix properties of this model, which has been used to solve one of the multi-property decision-making problems. Finally, the correlation coefficient for this concept was defined and explained in detail according to an approved mechanism, with a numerical example provided to illustrate the mechanism of use. Moreover, we develop a new algorithm for solving the decision-making issue based on the proposed correlation coefficient for piv-NHSS .

Keywords: Interval-valued neutrosophic set; Soft set; Hypersoft set; Interval-valued neutrosophic hypersoft set; Possibility interval-valued neutrosophic hypersoft matrix; Correlation coefficient for possibility interval-valued neutrosophic hypersoft sets; Decision-making

1 Introduction

Correlation is crucial in statistics and engineering, helping to assess the relationship between two variables. While probabilistic methods are used in engineering, they face challenges, such as relying on large, random datasets and dealing with uncertainties in complex systems, which makes accurate probabilities difficult to obtain. As a result, limited data can make probability-based results less useful for experts. In real-world scenarios, insufficient data may prevent accurate processing of standard statistical information. Consequently, probabilistic methods are often inadequate for handling uncertainties, leading researchers to propose alternative solutions for uncertain problems. In response to this requirement, Smarandache[1] presented an innovative mathematical model called neutrosophic to deal with data that includes issues of ambiguity, uncertainty, and inconsistency. Neutrosophic sets represent a groundbreaking advancement in the realm of mathematical modeling and decision-making, particularly in situations characterized by uncertainty and imprecision. Unlike traditional sets that rely solely on crisp boundaries and binary classifications, neutrosophic sets embrace the complexity of real-world scenarios by allowing for degrees of truth, indeterminacy, and falsehood. This triadic approach opens a myriad of possibilities across various fields, from artificial intelligence to risk assessment in finance.

The concept of neutrosophic sets is rooted in the idea that knowledge is not always complete or precise. For example, in decision-making processes, information may often be ambiguous or contradictory. Neutrosophic sets provide a framework to represent this ambiguity mathematically, enabling more nuanced analyses. By incorporating the possibility of indeterminate states, neutrosophic sets facilitate a more comprehensive understanding of problems, allowing for solutions that better reflect the complexities of reality.In real-world applications where the universal set is used, it is essential to define the three functions of NS. To address this, Wang et al.[2] introduced the concept of single value-NS (sv-NS) and demonstrated its operations along with some algebraic properties.Subsequently, to leverage the flexibility offered by interval structures, Wang et al.[3] expanded sv-NS into interval-valued NS (iv-NS), where each NS structure is represented as an interval value.

In practical applications, neutrosophic sets can be utilized in areas such as medical diagnosis and,[4,5] where symptoms may not clearly indicate a specific condition. Here, neutrosophic sets can help model the uncertainty inherent in diagnosis, allowing healthcare professionals to make more informed decisions. Similarly, in engineering and environmental studies, neutrosophic sets can aid in evaluating risks and uncertainties, leading to more robust and resilient designs.[6–9]

The potential of neutrosophic sets lies in their ability to model and analyze uncertainty in a way that traditional sets cannot. As researchers continue to explore and expand the applications of neutrosophic theory, we can expect to see innovative solutions that address the complexities of various domains, ultimately enhancing our understanding and management of uncertainty in an increasingly intricate world.

This work was a great incentive for researchers to integrate this concept with many other mathematical concepts such as algebra[14] , geometry,[10]Statistically[11] , complex analysis,[12] and algebraic topology,[13] in addition to discovering many life applications such as medicine, economics, computer science, and many other applications.[15]

The possibility of neutrosophic sets lies in their ability to capture the complexity of human reasoning and the ambiguity present in decision-making processes. For instance, consider a scenario in which a manager must choose between multiple projects. Each project may have varying levels of feasibility, potential profit, and associated risks. Traditional methods might struggle to adequately represent the uncertainties involved, leading to suboptimal choices. In contrast, neutrosophic sets enable the manager to express varying degrees of truth regarding the projects' success, while simultaneously accounting for the uncertainties and contradictions inherent in the evaluation process.

This flexibility extends beyond mere representation; it empowers decision-makers to apply advanced mathematical tools, such as neutrosophic logic and neutrosophic probability, to analyze and synthesize information. As a result, neutrosophic sets pave the way for more informed and nuanced decisions, enhancing outcomes in fields ranging from finance and engineering to healthcare and social sciences. Thus, the exploration of neutrosophic sets not only enriches theoretical frameworks but also holds practical implications for improving decision-making processes in an increasingly complex world. This idea inspired many researchers and made them present many ideas, including: Karaaslan introduced the operation on the Possibility of neutrosophic soft sets[16] that were as a combination of neutrosophic sets and soft sets. Possibility degree of single-valued neutrosophic hypersoft set (psv-NHSS) integrated by Al-Hijjawi and Alkhazaleh[17] and used resolve the solid waste site selection problem. Recently, Romdhini et al.[18] discussed three decision-making algorithms based on possibility interval-valued fuzzy hypersoft set (PIVFHS-set). To overcome the above-mentioned limitations Eman et al.[19] extended the concept of Possibility single-valued neutrosophic hypersoft set (psv-NHSS) to possibility interval valued neutrosophic hyper soft set (piv-NHSS) by merging and combining both Hypersoft set and Interval-valued neutrosophic set under posobolity degree when every iv-NHSS given posobolity degree in [0; 1]. Based on what was mentioned above and what was mentioned in these previous studies, the main contributions that were given to this work as following:

1.Set up piv-NHSM to merge both iv-NSMs and HSS with the probability degree.

2. Explainall the basic set operations on piv-NHSM and show all numerical examples.
3. Try to Introduce some mathematical properties of all set operations .

4.Build an algorithm to solve a decision-making problem based on piv-NHSMs.
5. Define the Correlation Coefficient for two piv-NHSSs and build a one example to show its work.
6.Build an algorithm to solve a decision-making problem based on Correlation Coefficient for two piv-NHSSs.

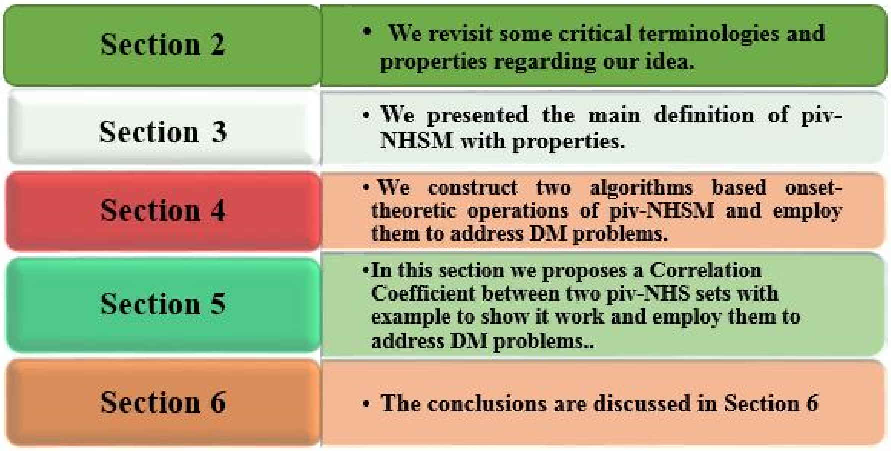

Figure 1: Represents of paper organization.

Creating correlation coefficients based on piv-NHSS is necessary for dealing with the environment mentioned before. In contrast to works using fuzzy and intuitionistic hypersoft sets, the constructed piv-NHSS Correlations handle difficulties with uncertainty.

2 Preliminaries

In this part, we revisit some critical definitions and properties for our idea

Definition 2.1. [3] An iv-NS $\widehat{\Gamma}$ on a fixed set $\ddot{Z} = \left\{\ddot{z}^1, \ddot{z}^2, \ddot{z}^3, ..., \ddot{z}^n\right\}$ is stated as $\widehat{\Gamma}\left(\ddot{z}^n\right) = \left\langle \dagger_{\mathbb{T}}\left(\ddot{z}^n\right), \dagger_{\mathbb{I}}\left(\ddot{z}^n\right), \dagger_{\mathbb{F}}\left(\ddot{z}^n\right)\right\rangle$ where $\dagger_{\mathbb{T}}\left(\ddot{z}^n\right) = \left\langle\left[\dagger_{\mathbb{T}}^l\left(\ddot{z}^n\right), \dagger_{\mathbb{T}}^u\left(\ddot{z}^n\right)\right]\right\rangle$,$\dagger_{\mathbb{I}}\left(\ddot{z}^n\right) = \left\langle\left[\dagger_{\mathbb{I}}^l\left(\ddot{z}^n\right), \dagger_{\mathbb{I}}^u\left(\ddot{z}^n\right)\right]\right\rangle$ and $\dagger_{\mathbb{F}}\left(\ddot{z}^n\right) = \left\langle\left[\dagger_{\mathbb{F}}^l\left(\ddot{z}^n\right), \dagger_{\mathbb{F}}^u\left(\ddot{z}^n\right)\right]\right\rangle$ respectively, with the condition $0 \leq \dagger_{\mathbb{T}}^l\left(\ddot{z}^n\right) + \dagger_{\mathbb{I}}^l\left(\ddot{z}^n\right) + \dagger_{\mathbb{F}}^l\left(\ddot{z}^n\right) \leq 3$ and $0 \leq \dagger_{\mathbb{T}}^u\left(\ddot{z}^n\right) + \dagger_{\mathbb{I}}^u\left(\ddot{z}^n\right) + \dagger_{\mathbb{F}}^u\left(\ddot{z}^n\right) \leq 3$. Here the terms $\dagger_{\mathbb{T}}^l\left(\ddot{z}^n\right), \dagger_{\mathbb{I}}^l\left(\ddot{z}^n\right), \dagger_{\mathbb{F}}^l\left(\ddot{z}^n\right), \dagger_{\mathbb{T}}^u\left(\ddot{z}^n\right), \dagger_{\mathbb{I}}^u\left(\ddot{z}^n\right), \dagger_{\mathbb{F}}^u\left(\ddot{z}^n\right)$ are declared as lower and upper three NS-memberships.

Definition 2.2. [19] Let $\ddot{Z} = \left\{\ddot{z}^1, \ddot{z}^2, \ddot{z}^3, ..., \ddot{z}^n\right\}$be a non-empty softhyper fixed set and $\Lambda_1, \Lambda_2, \Lambda_3, ..., \Lambda_n$ be sets of defferent traits that have $n \geq 1$ sub-value and $\Omega = \prod_{i=1}^{n} \Lambda_i$ such that every $\Lambda_i = \{\eta_1, \eta_2, \eta_3, ..., \eta_r\}$ for $i = 1, 2, 3, ..., r$ Then a piv-NHSS $\widehat{\Upsilon}_{\hat{\mu}}$ define over a non-empty hypersoft fixed set $\left(\ddot{Z}, \Omega\right)$ as the following form $(\Omega, \dagger_{\Omega})$ where

(i) $\dagger_{\Omega} : \Omega \rightarrow iv - NS \times \mathcal{I}_{\hat{Z}}$ such that it define as
$\dagger_{\Omega}(\eta) = \left\{\left(\frac{\ddot{z}^n}{\widehat{\Gamma}(\eta)(\ddot{z}^n)}, \hat{\mu}(\eta)\left(\ddot{z}^n\right)\right) : \ddot{z}^n \in \ddot{Z} \ \ \& \ \ \eta \in \Omega\right\}$ and
$\widehat{\Gamma}(\eta)\left(\ddot{z}^n\right) = \left\langle \dagger_{\mathbb{T}}(\eta)\left(\ddot{z}^n\right), \dagger_{\mathbb{I}}(\eta)\left(\ddot{z}^n\right), \dagger_{\mathbb{F}}(\eta)\left(\ddot{z}^n\right)\right\rangle$
where $\dagger_{\mathbb{T}}(\eta)\left(\ddot{z}^n\right) = \left\langle\left[\dagger_{\mathbb{T}}^l(\eta)\left(\ddot{z}^n\right), \dagger_{\mathbb{T}}^u(\eta)\left(\ddot{z}^n\right)\right]\right\rangle$,$\dagger_{\mathbb{I}}(\eta)\left(\ddot{z}^n\right) = \left\langle\left[\dagger_{\mathbb{I}}^l(\eta)\left(\ddot{z}^n\right), \dagger_{\mathbb{I}}^u(\eta)\left(\ddot{z}^n\right)\right]\right\rangle$ and $\dagger_{\mathbb{F}}(\eta)\left(\ddot{z}^n\right) = \left\langle\left[\dagger_{\mathbb{F}}^l(\eta)\left(\ddot{z}^n\right), \dagger_{\mathbb{F}}^u(\eta)\left(\ddot{z}^n\right)\right]\right\rangle$ respectively, with the condition $0 \leq \dagger_{\mathbb{T}}^l(\eta)\left(\ddot{z}^n\right) + \dagger_{\mathbb{I}}^l(\eta)\left(\ddot{z}^n\right) + \dagger_{\mathbb{F}}^l(\eta)\left(\ddot{z}^n\right) \leq 3$ and $0 \leq \dagger_{\mathbb{T}}^u(\eta)\left(\ddot{z}^n\right) + \dagger_{\mathbb{I}}^u(\eta)\left(\ddot{z}^n\right) + \dagger_{\mathbb{F}}^u(\eta)\left(\ddot{z}^n\right) \leq 3$.

(ii)$I_{\hat{Z}} : \hat{Z} \rightarrow [0, 1]$ and $\hat{\mu}(\eta)\left(\ddot{z}^n\right) \in I_{\hat{Z}}$ denotes to the degree of possibility of $\ddot{z}^n \in \hat{Z}$ in $\widehat{\Upsilon}\left(\ddot{z}^n\right)$.
Based on the above a piv-NHSS $\widehat{\Upsilon}_{\hat{\mu}}$ define over a non-empty hypersoft fixed set $\left(\ddot{Z}, \Omega\right)$ as the following structure:

$$\widehat{\Upsilon}_{\hat{\mu}} = \left\{\left(\eta, \left\{\left(\frac{\ddot{z}^n}{\widehat{\Gamma}(\eta)(\ddot{z}^n)}, \hat{\mu}(\eta)\left(\ddot{z}^n\right)\right) : \ddot{z}^n \in \ddot{Z}\right\}\right) \eta \in \Omega\right\}$$

3 Possibility interval-valued neutrosophic hypersoft matrix (piv-NHSM)

This part explores the characterization of a possibility interval-valued neutrosophic hypersoft matrix (piv-NHSM) with its basic set operations and some examples for all its operations.

Definition 3.1. Assume that $\ddot{\mathcal{Z}} = \{\ddot{z}^1, \ddot{z}^2, \ddot{z}^3, ..., \ddot{z}^n\}$ be a nonempty unvirsal set and $\Omega = \prod_{i=1}^{n} \Lambda_i$ such that every $\Lambda_i = \{\eta_1, \eta_2, \eta_3, ..., \eta_r\}$ for $i = 1, 2, 3, ..., r$ Then a piv-NHSM $\widehat{\Upsilon}_{\hat{\mu}}$ define as a matrix over a non-empty universal set $\left(\ddot{\mathcal{Z}}, \Omega\right)$ as the following form

$$\left(\widehat{\Upsilon}^1_{\hat{\mu}}\right)_{m\times n} =$$
$$\begin{bmatrix} \left((\dagger^1_T)_{1\times 1}, (\dagger^1_I)_{1\times 1}, (\dagger^1_F)_{1\times 1}, \hat{\mu}_{1\times 1}\right)\cdots & \left((\dagger^1_T)_{1\times m}, (\dagger^1_I)_{1\times m}, (\dagger^1_F)_{1\times m}, \hat{\mu}_{1\times m}\right) \\ \left((\dagger^1_T)_{2\times 1}, (\dagger^1_I)_{2\times 1}, (\dagger^1_F)_{2\times 1}, \hat{\mu}_{2\times 1}\right) & \left((\dagger^1_T)_{2\times m}, (\dagger^1_I)_{2\times m}, (\dagger^1_F)_{2\times m}, \hat{\mu}_{2\times 2}\right) \\ \vdots & \vdots \\ \left((\dagger^1_T)_{n\times 1}, (\dagger^1_I)_{n\times 1}, (\dagger^1_F)_{n\times 1}, \hat{\mu}_{n\times 1}\right) & \left((\dagger^1_T)_{n\times m}, (\dagger^1_I)_{n\times m}, (\dagger^1_F)_{n\times m}, \hat{\mu}_{n\times m}\right) \end{bmatrix}_{m\times n}$$

where

$$(\dagger^1_T)_{n\times m} = \left[\left(\dagger^{1,l}_T\right)_{n\times m}, \left(\dagger^{1,u}_T\right)_{n\times m}\right]$$
$$(\dagger^1_I)_{n\times m} = \left[\left(\dagger^{1,l}_I\right)_{n\times m}, \left(\dagger^{1,u}_I\right)_{n\times m}\right]$$
$$(\dagger^1_F)_{n\times m} = \left[\left(\dagger^{1,l}_F\right)_{n\times m}, \left(\dagger^{1,u}_F\right)_{n\times m}\right]$$

Example 3.2. Consder the example given in[19] and consider the piv-NHS given as following

$\widehat{\Upsilon}_{\hat{\mu}}(\eta_1) = \Big\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.7],[0.6,0.8],[0.4,0.4]\rangle}, 0.4\right), \left(\frac{\ddot{z}^2}{\langle[0.5,0.6],[0.3,0.5],[0.3,0.8]\rangle}, 0.7\right),$
$\left(\frac{\ddot{z}^3}{\langle[0.2,0.4],[0.1,0.3],[0.5,0.6]\rangle}, 0.6\right), \left(\frac{\ddot{z}^4}{\langle[0.3,0.6],[0.4,0.5],[0.2,0.5]\rangle}, 0.8\right)\Big\}$
$\widehat{\Upsilon}_{\hat{\mu}}(\eta_2) = \Big\{\left(\frac{\ddot{z}^1}{\langle[0.3,0.6],[0.2,0.5],[0.2,0.3]\rangle}, 0.2\right), \left(\frac{\ddot{z}^2}{\langle[0.4,0.7],[0.4,0.8],[0.1,0.5]\rangle}, 0.5\right),$
$\left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.3,0.7],[0.5,0.6]\rangle}, 0.3\right), \left(\frac{\ddot{z}^4}{\langle[0.5,0.7],[0.8,0.6],[0.7,0.7]\rangle}, 0.6\right)\Big\}$
$\widehat{\Upsilon}_{\hat{\mu}}(\eta_3) = \Big\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.7,0.9],[0.6,0.8]\rangle}, 0.3\right), \left(\frac{\ddot{z}^2}{\langle[0.2,0.4],[0.6,0.9],[0.5,0.7]\rangle}, 0.5\right),$
$\left(\frac{\ddot{z}^3}{\langle[0.4,0.7],[0.8,0.8],[0.6,0.6]\rangle}, 0.9\right), \left(\frac{\ddot{z}^4}{\langle[0.5,0.5],[0.7,0.7],[0.3,0.3]\rangle}, 0.5\right)\Big\}$
$\widehat{\Upsilon}_{\hat{\mu}}(\eta_4) = \Big\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.5,0.7],[0.2,0.2]\rangle}, 0.4\right), \left(\frac{\ddot{z}^2}{\langle[0.1,0.1],[0.6,0.6],[0.7,0.8]\rangle}, 0.1\right),$
$\left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.7,0.7],[0.9,0.9]\rangle}, 0.5\right), \left(\frac{\ddot{z}^4}{\langle[0.2,0.3],[0.4,0.8],[0.4,0.7]\rangle}, 0.6\right)\Big\}$

The piv-NHSS is formed as

$\widehat{\Upsilon}_{\hat{\mu}} = \Big\{ \widehat{\Upsilon}_{\hat{\mu}}(\eta_1) = \Big\{\left(\frac{\ddot{z}^1}{\langle[0.2,0.5],[0.1,0.3],[0.4,0.4]\rangle}, 0.4\right), \left(\frac{\ddot{z}^2}{\langle[0.5,0.6],[0.3,0.5],[0.3,0.8]\rangle}, 0.7\right),$
$\left(\frac{\ddot{z}^3}{\langle[0.2,0.4],[0.1,0.3],[0.5,0.6]\rangle}, 0.6\right), \left(\frac{\ddot{z}^4}{\langle[0.3,0.6],[0.4,0.5],[0.2,0.5]\rangle}, 0.8\right)\Big\}$
$\widehat{\Upsilon}_{\hat{\mu}}(\eta_2) = \Big\{\left(\frac{\ddot{z}^1}{\langle[0.3,0.6],[0.2,0.5],[0.2,0.3]\rangle}, 0.2\right), \left(\frac{\ddot{z}^2}{\langle[0.4,0.7],[0.4,0.8],[0.1,0.5]\rangle}, 0.5\right),$

$$\left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.3,0.7],[0.5,0.6]\rangle},0.3\right),\left(\frac{\ddot{z}^4}{\langle[0.5,0.7],[0.8,0.6],[0.7,0.7]\rangle},0.6\right)\Big\}$$

$$\widehat{\Upsilon}_{\hat{\mu}}(\eta_3)=\Big\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.7,0.9],[0.6,0.8]\rangle},0.3\right),\left(\frac{\ddot{z}^2}{\langle[0.2,0.4],[0.6,0.9],[0.5,0.7]\rangle},0.5\right),$$

$$\left(\frac{\ddot{z}^3}{\langle[0.4,0.7],[0.8,0.8],[0.6,0.6]\rangle},0.9\right),\left(\frac{\ddot{z}^4}{\langle[0.5,0.5],[0.7,0.7],[0.3,0.3]\rangle},0.5\right)\Big\}$$

$$\widehat{\Upsilon}_{\hat{\mu}}(\eta_4)=\Big\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.5,0.7],[0.2,0.2]\rangle},0.4\right),\left(\frac{\ddot{z}^2}{\langle[0.1,0.1],[0.6,0.6],[0.7,0.8]\rangle},0.1\right),$$

$$\left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.7,0.7],[0.9,0.9]\rangle},0.5\right),\left(\frac{\ddot{z}^4}{\langle[0.2,0.3],[0.4,0.8],[0.4,0.7]\rangle},0.6\right)\Big\}\Bigg\}$$

Furthermore, the values mentioned above can be given as a matrix as follows:

$$\widehat{\Upsilon}_{\hat{\mu}}=\begin{pmatrix}\Omega/\ddot{Z} & \ddot{z}^1 & \ddot{z}^2 & \ddot{z}^3 & \ddot{z}^4\\ \eta_1 & (\langle[.2,.5],[.1,.3],[.4,.4]\rangle,.4) & (\langle[.5,.6],[.3,.5],[.3,.8]\rangle,.7) & (\langle[.2,.4],[.1,.3],[.5,.6]\rangle,.6) & (\langle[.3,.6],[.4,.5],[.2,.5]\rangle,.8)\\ \eta_2 & (\langle[.3,.6],[.2,.5],[.2,.3]\rangle,.2) & (\langle[.4,.7],[.4,.8],[.1,.5]\rangle,.5) & (\langle[.6,.6],[.3,.7],[.5,.6]\rangle,.3) & (\langle[.5,.7],[.6,.8],[.7,.7]\rangle,.6)\\ \eta_3 & (\langle[.1,.8],[.7,.9],[.6,.8]\rangle,.3) & (\langle[.2,.4],[.6,.9],[.5,.7]\rangle,.5) & (\langle[.4,.7],[.8,.8],[.6,.6]\rangle,.9) & (\langle[.5,.5],[.7,.7],[.3,.3]\rangle,.5)\\ \eta_4 & (\langle[.1,.8],[.5,.7],[.2,.2]\rangle,.4) & (\langle[.1,.1],[.6,.6],[.7,.8]\rangle,.1) & (\langle[.6,.6],[.7,.7],[.9,.9]\rangle,.5) & (\langle[.2,.3],[.4,.8],[.4,.7]\rangle,.6)\end{pmatrix}_{4\times 4}$$

Definition 3.3. (**piv-NHS-null-matrex**) A piv-NHSM $\left(\widehat{\Upsilon}^1_{\hat{\mu}^1}\right)_{m\times n}$ on $\left(\ddot{\mathcal{Z}},\Omega\right)$ is said to be piv-NHS-null-matrex and denotes $\left(\hat{\Phi}^0_{\hat{\mu}^0}\right)_{m\times n}$ and its given as following

$$\left(\widehat{\Upsilon}^1_{\hat{\mu}^1}(\eta)(\hat{z})\right)_{m\times n}=([0,0],[0,0],[0,0],0)_{m\times n}$$

.

Example 3.4. If $\left(\hat{\Phi}^0_{\hat{\mu}^0}\right)_{2\times 2}$ given as following matrix:

$$\left(\hat{\Phi}^0_{\hat{\mu}^0}\right)_{2\times 2}=\begin{pmatrix}(\langle[0.0],[0.0],[0.0]\rangle,0) & (\langle[0.0],[0.0],[0.0]\rangle,0)\\ (\langle[0.0],[0.0],[0.0]\rangle,0) & (\langle[0.0],[0.0],[0.0]\rangle,0)\end{pmatrix}_{2\times 2}$$

Then here $\left(\hat{\Phi}^0_{\hat{\mu}^0}\right)_{2\times 2}$ named piv-NHS-null-matrex.

Definition 3.5. (**piv-NHS-absolute-matrex**) A piv-NHSM $\left(\widehat{\Upsilon}^1_{\hat{\mu}^1}\right)_{m\times n}$ on $\left(\ddot{\mathcal{Z}},\Omega\right)$ is said to be piv-NHS-absolute-matrex and denotes $\left(\hat{\Phi}^1_{\hat{\mu}^1}\right)_{m\times n}$ and its given as following

$$\left(\widehat{\Upsilon}^1_{\hat{\mu}^1}(\eta)(\hat{z})\right)_{m\times n}=([1,1],[1,1],[1,1],1)_{m\times n}$$

.

Definition 3.6. (**piv-NHS-absolute set**) A piv-NHSS $\widehat{\Upsilon}^1_{\hat{\mu}^1}$ on $\left(\ddot{\mathcal{Z}},\Omega\right)$ is said to be piv-NHS-nullset and denotes $\widehat{\Phi}^1_{\hat{\mu}^1}$ if
$\widehat{\Upsilon}^1_{\hat{\mu}}(\eta)(\ddot{z}^n)=([1,1],[1,1],[1,1])$ and for possibility membership part $\hat{\mu}^1(\eta)(\ddot{z}^n)=1$.

Example 3.7. If $\left(\hat{\Phi}^1_{\hat{\mu}^1}\right)_{2\times 2}$ given as following matrix:

$$\left(\hat{\Phi}^1_{\hat{\mu}^1}\right)_{2\times 2}=\begin{pmatrix}(\langle[1,1],[1,1],[1,1]\rangle,1) & (\langle[1,1],[1,1],[1,1]\rangle,1)\\ (\langle[1,1],[1,1],[1,1]\rangle,1) & (\langle[1,1],[1,1],[1,1]\rangle,1)\end{pmatrix}_{2\times 2}$$

Then here $\left(\hat{\Phi}^1_{\hat{\mu}^1}\right)_{2\times 2}$ named piv-NHS-absolute-matrex.

Definition 3.8. (**Complement in piv-NHSMs**) A piv-NHSM $\widehat{\Upsilon}_{\hat{\mu}}$ on $\left(\ddot{\mathcal{Z}},\Omega\right)$ have a complement in the following form $\widehat{\Upsilon}^c_{\hat{\mu}^c}$and define as

$$\widehat{\Upsilon}^c_{\hat{\mu}^c}=\left\{\left(\eta,\left\{\left(\frac{\ddot{z}^n}{\dagger^c(\eta)(\ddot{z}^n)},\hat{\mu^c}(\eta)(\ddot{z}^n)\right):\ddot{z}^n\in\ddot{Z}\right\}\right)\eta\in\Omega\right\}$$

where
$\dagger_{\mathbb{T}}^{c}(\eta)(\ddot{z}^{n})=\left\langle\left[\dagger_{\mathbb{F}}^{l}(\eta)(\ddot{z}^{n}),\dagger_{\mathbb{F}}^{u}(\eta)(\ddot{z}^{n})\right]\right\rangle$, $\dagger_{\mathbb{I}}^{c}(\eta)(\ddot{z}^{n})=\left\langle\left[1-\dagger_{\mathbb{I}}^{u}(\eta)(\ddot{z}^{n}),1-\dagger_{\mathbb{I}}^{l}(\eta)(\ddot{z}^{n})\right]\right\rangle$ and
$\dagger_{\mathbb{F}}^{c}(\eta)(\ddot{z}^{n})=\left\langle\left[\dagger_{\mathbb{T}}^{l}(\eta)(\ddot{z}^{n}),\dagger_{\mathbb{T}}^{u}(\eta)(\ddot{z}^{n})\right]\right\rangle$
and for possibility fuzzy membership part $\hat{\mu}^{c}(\eta)(\ddot{z}^{n})=1-\hat{\mu}(\eta)(\ddot{z}^{n})$.

Example 3.9. Consider the piv-NHSM that given in example 3.2 $\left(\widehat{\Upsilon}_{\hat{\mu}^1}^{1}\right)^{c}$ given as following:

$$\left(\widehat{\Upsilon}_{\hat{\mu}^1}^{1}\right)=\left\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.7],[0.6,0.8],[0.4,0.4]\rangle},0.4\right),\left(\frac{\ddot{z}^2}{\langle[0.5,0.6],[0.3,0.5],[0.3,0.8]\rangle},0.7\right)\right\}$$

then the $\left(\widehat{\Upsilon}_{\hat{\mu}^1}^{1}\right)^{c}$ is given based on above definition as following:

$$\left(\widehat{\Upsilon}_{\hat{\mu}^1}^{1}\right)^{c}=\left\{\left(\frac{\ddot{z}^1}{\langle[0.4,0.4],[0.2,0.4],[0.1,0.7]\rangle},0.6\right),\left(\frac{\ddot{z}^2}{\langle[0.3,0.8],[0.4,0.5],[0.5,0.6]\rangle},0.3\right)\right\}$$

4 Application of piv-NHSM in the most qualified employee selection

In this section, we deal with one of the daily life DM scenarios, specifically the process of selecting a competent employee to fill a position, by presenting a multi-step algorithm, which is an extension or expansion of the algorithm presented in.?

4.1 Cuse studty

Jobs that are open for application are publicised by the HR department. Given the high volume of applicants, it is necessary to establish an expert committee to evaluate each candidate's credentials and select the best fit for the open positions. Consequently, in order to use our suggested model to examine all candidates' data, piv-NHSS, we assume that the fixed set contains four candidates, i.e $\ddot{\mathcal{Z}}=\left\{\ddot{z}^1,\ddot{z}^2,\ddot{z}^3,\ddot{z}^4\right\}$ with members of the attributes (Criteria) i.e η_1 = Academic Certificate,η_2 = English language level and η_3 = Computer skills, these attributes (Criteria) having their sub-attribute values given as following: $\Lambda_1=\{\eta_{1,1}=Diploma,\eta_{1,2}=BA\}$, $\Lambda_2=\{\eta_{2,1}=intermediate,\eta_{2,2}=High\}$, $\Lambda_2=\{\eta_{3,1}=High\}$ then $\Omega=\Lambda_1\times\Lambda_2\times\Lambda_3=\{\eta_1,\eta_2,\eta_3,\eta_4\}$ where every component η_i has a 3-tuple entity. Then accordingly, a committee was formed to interview each candidate individually, and the committee adopted the following steps to accomplish its work:

Step 1. Analyze the information of each candidate based on piv-NHSS $\widehat{\Upsilon}_{\hat{\mu}}$ are given as following:

$$\widehat{\Upsilon}_{\hat{\mu}}=\left\{\begin{array}{l}\widehat{\Upsilon}_{\hat{\mu}}(\eta_1)=\left\{\left(\frac{\ddot{z}^1}{\langle[0.2,0.5],[0.1,0.3],[0.4,0.4]\rangle},0.4\right),\left(\frac{\ddot{z}^2}{\langle[0.5,0.6],[0.3,0.5],[0.3,0.8]\rangle},0.7\right),\right.\\ \left.\left(\frac{\ddot{z}^3}{\langle[0.2,0.4],[0.1,0.3],[0.5,0.6]\rangle},0.6\right),\left(\frac{\ddot{z}^4}{\langle[0.3,0.6],[0.4,0.5],[0.2,0.5]\rangle},0.8\right)\right\}\\ \widehat{\Upsilon}_{\hat{\mu}}(\eta_2)=\left\{\left(\frac{\ddot{z}^1}{\langle[0.3,0.6],[0.2,0.5],[0.2,0.3]\rangle},0.2\right),\left(\frac{\ddot{z}^2}{\langle[0.4,0.7],[0.4,0.8],[0.1,0.5]\rangle},0.5\right),\right.\\ \left.\left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.3,0.7],[0.5,0.6]\rangle},0.3\right),\left(\frac{\ddot{z}^4}{\langle[0.5,0.7],[0.8,0.6],[0.7,0.7]\rangle},0.6\right)\right\}\\ \widehat{\Upsilon}_{\hat{\mu}}(\eta_3)=\left\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.7,0.9],[0.6,0.8]\rangle},0.3\right),\left(\frac{\ddot{z}^2}{\langle[0.2,0.4],[0.6,0.9],[0.5,0.7]\rangle},0.5\right),\right.\\ \left.\left(\frac{\ddot{z}^3}{\langle[0.4,0.7],[0.8,0.8],[0.6,0.6]\rangle},0.9\right),\left(\frac{\ddot{z}^4}{\langle[0.5,0.5],[0.7,0.7],[0.3,0.3]\rangle},0.5\right)\right\}\\ \widehat{\Upsilon}_{\hat{\mu}}(\eta_4)=\left\{\left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.5,0.7],[0.2,0.2]\rangle},0.4\right),\left(\frac{\ddot{z}^2}{\langle[0.1,0.1],[0.6,0.6],[0.7,0.8]\rangle},0.1\right),\right.\\ \left.\left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.7,0.7],[0.9,0.9]\rangle},0.5\right),\left(\frac{\ddot{z}^4}{\langle[0.2,0.3],[0.4,0.8],[0.4,0.7]\rangle},0.6\right)\right\}\end{array}\right\}$$

Step 2. Represent piv-NHS in matrix form as a following:

$$\left(\widehat{\Upsilon}_{\hat{\mu}}\right)_{4\times 4} = \begin{pmatrix} \Omega/\ddot{Z} & \ddot{z}^1 & \ddot{z}^2 & \ddot{z}^3 & \ddot{z}^4 \\ \eta_1 & (\langle[.2,.5],[.1,.3],[.4,.4]\rangle,.4) & (\langle[.5,.6],[.3,.5],[.3,.8]\rangle,.7) & (\langle[.2,.4],[.1,.3],[.5,.6]\rangle,.6) & (\langle[.3,.6],[.4,.5],[.2,.5]\rangle,.8) \\ \eta_2 & (\langle[.3,.6],[.2,.5],[.2,.3]\rangle,.2) & (\langle[.4,.7],[.4,.8],[.1,.5]\rangle,.5) & (\langle[.6,.6],[.3,.7],[.5,.6]\rangle,.3) & (\langle[.5,.7],[.6,.8],[.7,.7]\rangle,.6) \\ \eta_3 & (\langle[.1,.8],[.7,.9],[.6,.8]\rangle,.3) & (\langle[.2,.4],[.6,.9],[.5,.7]\rangle,.5) & (\langle[.4,.7],[.8,.8],[.6,.6]\rangle,.9) & (\langle[.5,.5],[.7,.7],[.3,.3]\rangle,.5) \\ \eta_4 & (\langle[.1,.8],[.5,.7],[.2,.2]\rangle,.4) & (\langle[.1,.1],[.6,.6],[.7,.8]\rangle,.1) & (\langle[.6,.6],[.7,.7],[.9,.9]\rangle,.5) & (\langle[.2,.3],[.4,.8],[.4,.7]\rangle,.6) \end{pmatrix}_{4\times 4}$$

Step 3. Convert piv-NHS-matrix $\left(\widehat{\Upsilon}_{\hat{\mu}}\right)_{i\times j}$ to the core matrix $\vec{\Pi}_{i\times j}$ as follows:

$$\vec{\Pi}_{i\times j} = \begin{pmatrix} \widehat{\theta}_{\hat{\varpi}_1}(\eta_1) & \widehat{\theta}_{\hat{\varpi}_1}(\eta_2) & \cdots & \widehat{\theta}_{\hat{\varpi}_1}(\eta_j) \\ \widehat{\theta}_{\hat{\varpi}_2}(\eta_1) & \widehat{\theta}_{\hat{\varpi}_2}(\eta_1) & \cdots & \widehat{\theta}_{\hat{\varpi}_2}(\eta_j) \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{\theta}_{\hat{\varpi}_i}(\eta_1) & \widehat{\theta}_{\hat{\varpi}_i}(\eta_2) & \cdots & \widehat{\theta}_{\hat{\varpi}_i}(\eta_j) \end{pmatrix}$$

Where

$$\widehat{\theta}_{\varpi_i}(\eta_j) = \left(\left[\frac{\dagger^l_{\Re}(\eta_j) + \dagger^u_{\Re}(\eta_j)}{2}\right] + \left[\frac{\dagger^l_{\Re}(\eta_j) + \dagger^u_{\Re}(\eta_j)}{2}\right] + \left[\frac{\dagger^l_{\Re}(\eta_j) + \dagger^u_{\Re}(\eta_j)}{2}\right] \times \hat{\mu}(\eta_j)\right)$$

Step 4. Find out all $\zeta^{Max}(\eta_j)$ (Max. limit), $\zeta^{Min}(\eta_j)$ (Min. limit), and c (scoring value) of all $\widehat{\xi}(\eta_j)$ as following format:

$$\zeta^{Max}(\eta_j) = \sum_{i=1}^{s}\left(1 - \widehat{\theta}_{\varpi_i}(\eta_j)\right)^2, \qquad \zeta^{Min}(\eta_j) = \sum_{i=1}^{s}\left(\widehat{\theta}_{\varpi_i}(\eta_j)\right)^2$$

$$\widehat{\xi}(\eta_j) = \zeta^{Max}(\eta_j) + \zeta^{Min}(\eta_j)$$

Step 5. The optimum selection is a maximum score of $\partial(\eta_j)$ where

$$\partial_k = Max\left\{\widehat{\xi}(\eta_1), \widehat{\xi}(\eta_1), ..\widehat{\xi}(\eta_j)\right\}$$

.

Now for the purpose of implementing the algorithm, starting from step 3.

$$\vec{\Pi}_{4\times 4} = \begin{pmatrix} .12 & .52 & .81 & .74 \\ .23 & .28 & .64 & .46 \\ .41 & .71 & .65 & .91 \\ .32 & .64 & .69 & .84 \end{pmatrix}_{4\times 4}$$

Now, Table 1 illustrates step 4.

Table 1: The values of $\zeta^{Max}(\eta_j)$,$\zeta^{Min}(\eta_j)$ and ∂_k

	z^1	z^2	z^3	z^4
$\zeta^{Max}(\eta_j)$	2.177	2.764	3.843	4.008
$\zeta^{Min}(\eta_j)$	0.337	0.527	0.674	0.876
∂_k	2.514	3.291	4.517	4.884
Sum($\ddot{Z}^i$) = 4.884				

The optimal choice is z^4.

5 Correlation Coefficient for piv-NHSSs

Now in this section, we will explain and know the techniques of correlation coefficient for piv-NHSSs.

Definition 5.1. (**Correlation Coefficient for piv-NHSSs**) Let $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ be two piv-NHSSs. Then the correlation coefficient of $\mathbb{C}_{ivp-NHSs}$ between $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ given as following formalh:

$$\mathbb{C}_{ivp-NHSs}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1},\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}\right)=\frac{1}{6}\left(\mathbb{C}_{\mathbb{T}}\left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{T}},\overset{\frown}{\Upsilon}^{2}_{\mathbb{T}}\right)+\mathbb{C}_{\mathbb{I}}\left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{I}},\overset{\frown}{\Upsilon}^{2}_{\mathbb{I}}\right)+\mathbb{C}_{\mathbb{F}}\left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{F}},\overset{\frown}{\Upsilon}^{2}_{\mathbb{F}}\right)\right)\mathbb{C}_{P}\left(\hat{\mu}^1,\hat{\mu}^2\right)$$

Where

$$\mathbb{C}_{\mathbb{T}}\left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{T}},\overset{\frown}{\Upsilon}^{2}_{\mathbb{T}}\right)=$$

$$\frac{1}{|\Omega|}\sum_{\eta_i\in\Omega}\left(\frac{\sum_{j=1}^{n}\left(\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)}\right)\left(\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)}\right)+\left(\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)}\right)\left(\dagger^{2,u}_{\Re_j}(\eta_i)-\overline{\dagger^{2,u}_{\mathbb{T}_j}(\eta_i)}\right)}{\sqrt{\sum_{j=1}^{n}\left(\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)}\right)^2\sum_{j=1}^{n}\left(\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)}\right)^2+\sum_{j=1}^{n}\left(\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)}\right)^2\sum_{j=1}^{n}\left(\dagger^{2,u}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{2,u}_{\mathbb{T}_j}(\eta_i)}\right)^2}}\right)$$

$$\mathbb{C}_{\mathbb{I}}\left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{I}},\overset{\frown}{\Upsilon}^{2}_{\mathbb{I}}\right)=$$

$$\frac{1}{|\Omega|}\sum_{\eta_i\in\Omega}\left(\frac{\sum_{j=1}^{n}\left(\dagger^{1,l}_{\mathbb{I}_j}(\eta_i)-\overline{\dagger^{1,l}_{\mathbb{I}_j}(\eta_i)}\right)\left(\dagger^{2,l}_{\mathbb{I}_j}(\eta_i)-\overline{\dagger^{2,l}_{\Re_j}(\eta_i)}\right)+\left(\dagger^{1,u}_{\mathbb{I}_j}(\eta_i)-\overline{\dagger^{1,u}_{\mathbb{I}_j}(\eta_i)}\right)\left(\dagger^{2,u}_{\mathbb{I}_j}(\eta_i)-\overline{\dagger^{2,u}_{\mathbb{I}_j}(\eta_i)}\right)}{\sqrt{\sum_{j=1}^{n}\left(\dagger^{1,l}_{\mathbb{I}_j}(\eta_i)-\overline{\dagger^{1,l}_{\mathbb{I}_j}(\eta_i)}\right)^2\sum_{j=1}^{n}\left(\dagger^{2,l}_{\mathbb{I}_j}(\eta_i)-\overline{\dagger^{2,l}_{\mathbb{I}_j}(\eta_i)}\right)^2+\sum_{j=1}^{n}\left(\dagger^{1,u}_{\mathbb{I}_j}(\eta_i)-\overline{\dagger^{1,u}_{\mathbb{I}_j}(\eta_i)}\right)^2\sum_{j=1}^{n}\left(\dagger^{2,u}_{\mathbb{I}_j}(\eta_i)-\overline{\dagger^{2,u}_{\mathbb{I}_j}(\eta_i)}\right)^2}}\right)$$

$$\mathbb{C}_{\mathbb{F}}\left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{F}},\overset{\frown}{\Upsilon}^{2}_{\mathbb{F}}\right)=$$

$$\frac{1}{|\Omega|}\sum_{\eta_i\in\Omega}\left(\frac{\sum_{j=1}^{n}\left(\dagger^{1,l}_{\mathbb{F}_j}(\eta_i)-\overline{\dagger^{1,l}_{\mathbb{F}_j}(\eta_i)}\right)\left(\dagger^{2,l}_{\mathbb{F}_j}(\eta_i)-\overline{\dagger^{2,l}_{\Re_j}(\eta_i)}\right)+\left(\dagger^{1,u}_{\mathbb{F}_j}(\eta_i)-\overline{\dagger^{1,u}_{\mathbb{F}_j}(\eta_i)}\right)\left(\dagger^{2,u}_{\mathbb{F}_j}(\eta_i)-\overline{\dagger^{2,u}_{\mathbb{F}_j}(\eta_i)}\right)}{\sqrt{\sum_{j=1}^{n}\left(\dagger^{1,l}_{\mathbb{F}_j}(\eta_i)-\overline{\dagger^{1,l}_{\mathbb{F}_j}(\eta_i)}\right)^2\sum_{j=1}^{n}\left(\dagger^{2,l}_{\mathbb{F}_j}(\eta_i)-\overline{\dagger^{2,l}_{\mathbb{F}_j}(\eta_i)}\right)^2+\sum_{j=1}^{n}\left(\dagger^{1,u}_{\mathbb{F}_j}(\eta_i)-\overline{\dagger^{1,u}_{\mathbb{F}_j}(\eta_i)}\right)^2\sum_{j=1}^{n}\left(\dagger^{2,u}_{\mathbb{F}_j}(\eta_i)-\overline{\dagger^{2,u}_{\mathbb{F}_j}(\eta_i)}\right)^2}}\right)$$

$$\mathbb{C}_{P}\left(\hat{\mu}^1,\hat{\mu}^2\right)=\frac{1}{|\Omega|}\sum_{\eta_i\in\Omega}\left(\frac{\sum_{j=1}^{n}\left(\hat{\mu}^1(\eta_i)-\overline{\hat{\mu}^1(\eta_i)}\right)\left(\hat{\mu}^2(\eta_i)-\overline{\hat{\mu}^2(\eta_i)}\right)}{\sqrt{\sum_{j=1}^{n}\left(\hat{\mu}^1(\eta_i)-\overline{\hat{\mu}^1(\eta_i)}\right)^2\sum_{j=1}^{n}\left(\hat{\mu}^2(\eta_i)-\overline{\hat{\mu}^2(\eta_i)}\right)^2}}\right)$$

and

$$\overline{\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)}=\frac{1}{|\ddot{Z}|}\sum_{j=1}^{n}\dagger^{1,l}_{\mathbb{T}_j}(\eta_i),\overline{\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)}=\frac{1}{|\ddot{Z}|}\sum_{j=1}^{n}\dagger^{1,u}_{\mathbb{T}_j}(\eta_i),$$

$$\overline{\dagger^{1,l}_{\mathbb{I}_j}(\eta_i)}=\frac{1}{|\ddot{Z}|}\sum_{j=1}^{n}\dagger^{1,l}_{\mathbb{I}_j}(\eta_i),\overline{\dagger^{1,u}_{\mathbb{I}_j}(\eta_i)}=\frac{1}{|\ddot{Z}|}\sum_{j=1}^{n}\dagger^{1,u}_{\mathbb{I}_j}(\eta_i),$$

$$\overline{\dagger^{1,l}_{\mathbb{F}_j}(\eta_i)}=\frac{1}{|\ddot{Z}|}\sum_{j=1}^{n}\dagger^{1,l}_{\mathbb{F}_j}(\eta_i),\overline{\dagger^{1,u}_{\mathbb{F}_j}(\eta_i)}=\frac{1}{|\ddot{Z}|}\sum_{j=1}^{n}\dagger^{1,u}_{\mathbb{F}_j}(\eta_i),$$

$$\overline{\mu^1(\eta_i)}=\frac{1}{|\ddot{Z}|}\sum_{j=1}^{n}\mu^1(\eta_i),\overline{\mu^2(\eta_i)}=\frac{1}{|\ddot{Z}|}\sum_{j=1}^{n}\mu^2(\eta_i).$$

Proposition 5.2. *Let* $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ *and* $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ *be two piv-NHSSs. Then the correlation coefficient of* $\mathbb{C}_{ivp-NHSs}$ *between* $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ *and* $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ *carry out the next properties:*

(i.) $\mathbb{C}_{ivp-NHSs}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}\right) = \mathbb{C}_{ivp-NHSs}\left(\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}, \overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}\right).$

(ii.) If $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} = \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ *then* $\mathbb{C}_{ivp-NHSs}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}\right) = 1.$

(iii.) $\left|\mathbb{C}_{ivp-NHSs}\left(\overset{\frown}{\Upsilon}^{1}_{\Re}, \overset{\frown}{\Upsilon}^{1}_{\Re}\right)\right| \leq 1.$

Proof. (i.) By definition (111), we have
$\mathbb{C}_{ivp-NHSs}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}\right) = \frac{1}{6}\left(\mathbb{C}_{\mathbb{T}}\left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{T}}, \overset{\frown}{\Upsilon}^{2}_{\mathbb{T}}\right) + \mathbb{C}_{\mathbb{I}}\left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{I}}, \overset{\frown}{\Upsilon}^{2}_{\mathbb{I}}\right) + \mathbb{C}_{\mathbb{F}}\left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{F}}, \overset{\frown}{\Upsilon}^{2}_{\mathbb{F}}\right)\right)\mathbb{C}_{P}\left(\hat{\mu}^1, \hat{\mu}^2\right)$ where for T-term, then $\mathbb{C}_{\mathbb{T}}\left(\overset{\frown}{\Upsilon}^{1}_{\mathbb{T}}, \overset{\frown}{\Upsilon}^{2}_{\mathbb{T}}\right) =$

$$\frac{1}{|\Omega|}\sum_{\eta_i\in\Omega}\left(\frac{\sum_{j=1}^{n}\left(\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)}\right)\left(\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)}\right)+\left(\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)}\right)\left(\dagger^{2,u}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{2,u}_{\mathbb{T}_j}(\eta_i)}\right)}{\sqrt{\sum_{j=1}^{n}\left(\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)}\right)^2\sum_{j=1}^{n}\left(\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)}\right)^2+\sum_{j=1}^{n}\left(\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)}\right)^2\sum_{j=1}^{n}\left(\dagger^{2,u}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{2,u}_{\mathbb{T}_j}(\eta_i)}\right)^2}}\right)$$

$$=\frac{1}{|\Omega|}\sum_{\eta_i\in\Omega}\left(\frac{\sum_{j=1}^{n}\left(\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)}\right)\left(\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)}\right)+\left(\dagger^{2,u}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{2,u}_{\mathbb{T}_j}(\eta_i)}\right)\left(\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)}\right)}{\sqrt{\sum_{j=1}^{n}\left(\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{2,l}_{\mathbb{T}_j}(\eta_i)}\right)^2\sum_{j=1}^{n}\left(\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,l}_{\mathbb{T}_j}(\eta_i)}\right)^2+\sum_{j=1}^{n}\left(\dagger^{2,u}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{2,u}_{\mathbb{T}_j}(\eta_i)}\right)^2\sum_{j=1}^{n}\left(\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)-\overline{\dagger^{1,u}_{\mathbb{T}_j}(\eta_i)}\right)^2}}\right)$$
$$= \mathbb{C}_{\mathbb{T}}\left(\overset{\frown}{\Upsilon}^{2}_{\mathbb{T}}, \overset{\frown}{\Upsilon}^{1}_{\mathbb{T}}\right)$$
and for I-term and F-term same then we get the point (i).

Example 5.3. Consider the two piv-NHSs $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ given as following:

$$\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1} = \left\{\left(\frac{\ddot{z}^1}{\langle[0.2,0.5],[0.1,0.3],[0.4,0.4]\rangle}, 0.4\right), \left(\frac{\ddot{z}^2}{\langle[0.3,0.6],[0.4,0.5],[0.2,0.5]\rangle}, 0.7\right)\right\}$$

and

$$\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2} = \left\{\left(\frac{\ddot{z}^1}{\langle[0.4,0.8],[0.1,0.1],[0.2,0.3]\rangle}, 0.7\right), \left(\frac{\ddot{z}^2}{\langle[0.7,0.7],[0.2,0.2],[0.1,0.3]\rangle}, 0.9\right)\right\}$$

Then the $\mathbb{C}_{ivp-NHSs}\left(\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}\right) = 0.7436$

□

5.1 Aplication on piv-NHSSs using Correlation Coefficient for piv-NHSSs

In this special part we will present a practical algorithm for selecting the ideal employee as in the fourth part of this article, here we will present a new algorithm based on matrix issues for piv-NHSSs. This algorithm is given as in the following steps:

Step 1. Analyze the information of each candidate based on piv-NHSS $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$. :

Step 2. Analyzing piv-NSHSs$\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$ second model for another expert.

Step 3. Calculate the value of Correlation Coefficient between $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$.

Step 4. Ranking the value of Correlation Coefficient between every candidates in $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$.

Step 5. Select the maximum value for Correlation Coefficient between every candidates in $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$ and $\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$.

Now to implement this algorithm effectively, we start by activating the first and second steps as follows:

$$\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}} = \left\{ \begin{array}{l} \widehat{\Upsilon}_{\hat{\mu}}(\eta_1) = \left\{ \left(\frac{\ddot{z}^1}{\langle[0.2,0.5],[0.1,0.3],[0.4,0.4]\rangle}, 0.4 \right), \left(\frac{\ddot{z}^2}{\langle[0.5,0.6],[0.3,0.5],[0.3,0.8]\rangle}, 0.7 \right), \right. \\ \left. \left(\frac{\ddot{z}^3}{\langle[0.2,0.4],[0.1,0.3],[0.5,0.6]\rangle}, 0.6 \right), \left(\frac{\ddot{z}^4}{\langle[0.3,0.6],[0.4,0.5],[0.2,0.5]\rangle}, 0.8 \right) \right\} \\ \widehat{\Upsilon}_{\hat{\mu}}(\eta_2) = \left\{ \left(\frac{\ddot{z}^1}{\langle[0.3,0.6],[0.2,0.5],[0.2,0.3]\rangle}, 0.2 \right), \left(\frac{\ddot{z}^2}{\langle[0.4,0.7],[0.4,0.8],[0.1,0.5]\rangle}, 0.5 \right), \right. \\ \left. \left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.3,0.7],[0.5,0.6]\rangle}, 0.3 \right), \left(\frac{\ddot{z}^4}{\langle[0.5,0.7],[0.8,0.6],[0.7,0.7]\rangle}, 0.6 \right) \right\} \\ \widehat{\Upsilon}_{\hat{\mu}}(\eta_3) = \left\{ \left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.7,0.9],[0.6,0.8]\rangle}, 0.3 \right), \left(\frac{\ddot{z}^2}{\langle[0.2,0.4],[0.6,0.9],[0.5,0.7]\rangle}, 0.5 \right), \right. \\ \left. \left(\frac{\ddot{z}^3}{\langle[0.4,0.7],[0.8,0.8],[0.4,0.8]\rangle}, 0.9 \right), \left(\frac{\ddot{z}^4}{\langle[0.1,0.2],[0.7,0.7],[0.3,0.3]\rangle}, 0.4 \right) \right\} \\ \widehat{\Upsilon}_{\hat{\mu}}(\eta_4) = \left\{ \left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.2,0.6],[0.2,0.2]\rangle}, 0.4 \right), \left(\frac{\ddot{z}^2}{\langle[0,0.7],[0.6,0.6],[0.3,0.3]\rangle}, 0.2 \right), \right. \\ \left. \left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.7,0.7],[0.6,0.9]\rangle}, 0.5 \right), \left(\frac{\ddot{z}^4}{\langle[0.2,0.3],[0.4,0.4],[0.3,0.7]\rangle}, 0.8 \right) \right\} \end{array} \right\}$$

and

$$\overset{\frown}{\Upsilon}^{2}_{\hat{\mu}} = \left\{ \begin{array}{l} \widehat{\Upsilon}_{\hat{\mu}}(\eta_1) = \left\{ \left(\frac{\ddot{z}^1}{\langle[0.2,0.5],[0.1,0.3],[0.4,0.4]\rangle}, 0.4 \right), \left(\frac{\ddot{z}^2}{\langle[0.5,0.6],[0.3,0.5],[0.3,0.8]\rangle}, 0.7 \right), \right. \\ \left. \left(\frac{\ddot{z}^3}{\langle[0.2,0.4],[0.1,0.3],[0.5,0.6]\rangle}, 0.6 \right), \left(\frac{\ddot{z}^4}{\langle[0.3,0.6],[0.4,0.5],[0.2,0.8]\rangle}, 0.6 \right) \right\} \\ \widehat{\Upsilon}_{\hat{\mu}}(\eta_2) = \left\{ \left(\frac{\ddot{z}^1}{\langle[0.3,0.6],[0.2,0.5],[0.1,0.3]\rangle}, 0.8 \right), \left(\frac{\ddot{z}^2}{\langle[0.4,0.4],[0.4,0.7],[0.1,0.5]\rangle}, 0.9 \right), \right. \\ \left. \left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.3,0.7],[0.1,0.6]\rangle}, 0.3 \right), \left(\frac{\ddot{z}^4}{\langle[0.2,0.7],[0.8,0.6],[0.3,0.8]\rangle}, 0.5 \right) \right\} \\ \widehat{\Upsilon}_{\hat{\mu}}(\eta_3) = \left\{ \left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.7,0.9],[0.6,0.8]\rangle}, 0.3 \right), \left(\frac{\ddot{z}^2}{\langle[0.2,0.4],[0.6,0.9],[0.5,0.7]\rangle}, 0.5 \right), \right. \\ \left. \left(\frac{\ddot{z}^3}{\langle[0.4,0.7],[0.8,0.8],[0.6,0.6]\rangle}, 0.9 \right), \left(\frac{\ddot{z}^4}{\langle[0.5,0.5],[0.7,0.7],[0.3,0.3]\rangle}, 0.5 \right) \right\} \\ \widehat{\Upsilon}_{\hat{\mu}}(\eta_4) = \left\{ \left(\frac{\ddot{z}^1}{\langle[0.1,0.8],[0.5,0.7],[0.2,0.2]\rangle}, 0.4 \right), \left(\frac{\ddot{z}^2}{\langle[0.1,0.1],[0.6,0.6],[0.7,0.8]\rangle}, 0.1 \right), \right. \\ \left. \left(\frac{\ddot{z}^3}{\langle[0.6,0.6],[0.7,0.7],[0.9,0.9]\rangle}, 0.5 \right), \left(\frac{\ddot{z}^4}{\langle[0.2,0.3],[0.4,0.8],[0.4,0.7]\rangle}, 0.6 \right) \right\} \end{array} \right\}$$

The rest of the steps can be seen in Table 2.

Table 2: Correlation Coefficient values of piv-NHSSs $\overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}, \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2}$

$\ddot{Z}^i of \overset{\frown}{\Upsilon}^{2}_{\hat{\mu}^2} / \ddot{Z}^i of \overset{\frown}{\Upsilon}^{1}_{\hat{\mu}^1}$	z^1	z^2	z^3	z^4
z^1	0.291	0.872	0.761	0.342
z^2	0.658	0.327	0.659	0.638
z^3	0.542	0.693	0.875	0.351
z^4	0.359	0.424	0.632	0.871
Sum($\ddot{Z}^i$)	1.850	2.134	2.927	2.202

As showing in Table 2 we got the best choice, is $\ddot{z}^3$.

Conclusion

In this study , through the concept possibility interval valued neutrosophic hyper soft set (abbreviated as piv-NHSS) which is combined from the hypersoft set (HSS) and interval-valued neutrosophic set under the posobolity degree and each iv-NHSS is assigned a possibility degree in the interval $[0, 1]$. Based on this concept, we presented a more flexible, expanded method for a previous concept named possibility interval valued neutrosophic hyper soft matrix (piv-NHSM) as a new generalization of piv-NHSS. In this work, we also present several algebraic operations and also all the mathematical properties associated with this model. In addition to the above, we have presented a clear algorithm based on the matrix properties of this model, which has been used to solve one of the multi-property decision-making problems. Finally, the correlation coefficient for this concept was defined and explained in detail according to an approved mechanism, with a numerical example provided to illustrate the mechanism of use. Moreover, we developed a new algorithm for solving the decision-making issue based on the proposed correlation coefficient for piv-NHSS .This concept can be further developed through work on it in future studies, where it can be linked to many other fuzzy and algebraic tools. See.[20]–[25]

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