Optimize Decision-Making in the Industrial Sector under Uncertainty: A Neutrosophic Inverse Exponential Distribution Approach

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Abstract

The most widely used distribution for risk management data for modeling longevity is the one-parameter inverse exponential distribution. Among alternative models, we suggest the neutrosophic inverse exponential (NIE) model, which generalizes the extended inverse exponential distributions and the classical structure. For the suggested model, we derive explicit formulations for the quantile functions, median, mode, cumulative distribution function, and probability density function. Data generating process of the proposed model under neutrosophic environment is discussed. To estimate the model parameters, we use the maximum likelihood approach. Using the proposed model, we run the simulation setup for randomly generated data. A genuine data set is also used to support the proposed model applicability.

Keywords: Neutrosophic probability; Exponential model; Simulation; Estimation

1. Introduction

Exponential distribution is one of the continuous probability distributions, which describes the time required to complete an event [1]. It has many applications in modelling random events where each event occurs in discrete but steady average frequency per time interval (period) e.g. waiting time between events in queue, lifecycle of mechanical device, time until radioactive particle decay etc [2]. In telecommunications, it is widely used to model the intervals of time between the arrival of data packets at a network router, assisting engineers in designing efficient systems and predicting congestion [3]. In healthcare, for example, the exponential distribution is used in estimating the time between one event and another of interest, e.g., time until a patient arrives in an emergency room, time between hospitalizations [4]. A third use case is for reliability engineering, which uses the concept to model the lifetime of machines or products to help schedule maintenance periods or assess the performance of warranties [5]. In the area of medicine, the exponential distribution is also important in survival analysis, which deals with estimating how much time they will last new coming events, such as how long it takes for a particular treatment to fail or how long it takes for a patient to recover [6]. It is also used in finance to model the time until the next major event for stock prices or an insurance claim. The exponential distribution is a powerful and flexible tool with wide-ranging applications, providing insights into systems where events occur randomly and over time in diverse sectors.

The importance of understanding inverse of probability distributions is even more apparent when modelling various real-life scenarios where we are interested in the reciprocal of the random variable like $1/_X$. The distribution of the inverse is useful in various circumstances, such as when a random variable assumes a specific distribution (e.g., exponential, normal, among others) [7]. As an illustration, such a variable with an exponential distribution has an inverse that can represent the distribution of time between rare events, such as system failures or arrivals to a queue. Linear transformations of gaussian distributions tend to be highly beneficial and useful across various domains such as finance where inverse proportionality often occurs in terms of returns or prices. The inverse distribution can also be helpful in other contexts, like in queuing theory or reliability engineering,

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where the inverse distribution can provide insight into waiting times or failure rates of systems whose outcomes are inversely related to some underlying factor [8]. Hence, the applications of those inverse distributions are applicable to various fields including telecommunications, healthcare, engineering, and economics [9]. Researchers and professionals analyze the behavior of the inverse of a distribution to gain insights into processes that exhibit reciprocal relationships, enhance predictive accuracy, and optimize decision-making under uncertainty. Inverse distributions are imperative in broadening the applications of probabilistic models and offering solutions to difficult real-life problems.

This is a significant reason why fuzzy theory and fuzzy random variables are necessary in practical problems involving uncertainty and imprecision, which are beyond the scope of the probabilistic framework [10]. While traditional probability theory is based on the assumption of clear-cut outcomes with some chance of occurrence, Fuzzy set theory allows for ambiguity and imprecision when information is available, enabling a more nuanced representation of scenarios. Fuzzy random variables generalizes this factor with both fuzziness and randomness and are useful for system characterized with uncertainty and imprecision [11]. In fields such as decision-making, engineering, and artificial intelligence, human intuition and unclear data are common elements, making this extremely valuable [12]. Examples of such applications include fuzzy random variables in supply chain management, where they can be employed to effectively represent uncertainty in demand and supply, providing realistic predictions and supporting better decision making [13]. In medicine, they are used to model the potential range of outcomes or treatment effectiveness for patients, even when we do not have exact data. Controller systems also use the fuzzy theory since it enables to process of imprecise sensor inputs and have optimal performance in a varying environment. An example of this is fuzzy logic and fuzzy random variables which translate vague concepts into mathematical terms, allowing for ranges of truth rather than true/false, thus making it possible to quantify the uncertainty about data a crucial ability in domains ranging from robotics to finance, which often requires operations on incomplete or rough data [14-16]. The neutrosophic set is a generalization of the fuzzy set, which introduced the concept of extending certainty into three parts: truth, indeterminacy (the truly unknown part that the state of a natural system may have) and falsity [17-18]. Neutrosophic sets, in comparison to classical sets that only accept true or false membership, can define an element as partially member of a set with degrees of membership that ranges from truth, falsity, and indeterminacy, which makes the neutrosophic set ideal for modeling complex, messy, and real-world systems where information is often ambiguous or partial in nature. It is used in the context of statistics by neutrosophic statistics to add new tools oriented towards working with uncertain or incomplete data [19-21]. This approach is useful in situations where traditional statistical methods fall short; it allows researchers to analyze data that is obfuscated, ambiguous, and even contradictory. It is used in many fields including decision-making, data mining and artificial intelligence where conflicting or missing information is a major issue [22]. Neutrosophic statistics is one such area that extends the scope of traditional statistics by providing tools and techniques to account for uncertainty and vagueness in data. Unlike fuzzy sets that represent uncertainty based on a single degree between 0 and 1, neutrosophic sets involve multiple degrees of freedom for each element representing truth, indeterminacy and falsehood [23]. The increased flexibility of neutrosophic sets enables it to deal with scenarios involving uncertain, inconsistent, or incomplete data, making it a more potent method for describing complex uncertainties than fuzzy set theory itself. Conventional probability distributions cannot represent vagueness, uncertainty, and incomplete information that show up in real-world problems; hence, the development of neutrosophic probability distributions is of utmost importance to cope with uncertainty, vagueness, and incomplete data [24-26]. The ability to split distributions into components of truth, indeterminacy, and falsity provides an extension of classical probability theory that is more suitable to complex systems [27]. New neutrosophic probability distributions have been introduced in the last few years, with the focus of their applications in decision-making, reliability engineering, risk analysis, and artificial intelligence. The state of indeterminacy within phenomena has led researchers to generalizing standard distributions, such as the exponential, normal, and Weibull distributions, through neutrosophic extensions [28]. In addition, for the analysis of neutrosophic distributions, advanced computation methods and software packages have been developed to incorporate neutrosophic distribution into statistical inference and machine learning algorithms [29]. With their nature, these developments eased resolving multi-criteria decision-making issues, industrial process optimization, and uncertainties in economic and environmental systems [30]. Over time, neutrosophic probability distributions have emerged as valuable tools for modeling uncertainty in situations where traditional probabilistic approaches fall short.

In this study, we introduce a new distribution called the NIE model, and it changes the game in modeling uncertainty, indeterminacy, and vagueness, especially in industrial applications. This approach reflects the more realistic scenarios where there is uncertainty, which cannot be modeled by classical distributions, through integrating neutrosophic components into the classical exponential distribution. Its benefits include improving decisions in industrial decisions with uncertainty, for instance in reliability analysis, resource allocation and risk management. This extension creates new opportunities for improving decision accuracy, system performance, and sustainability in complex industry.

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The rest of the work is organized as: In Section 2, neutrosophic extension is described. Section 3 explains quantile function of the proposed model. In section 4 the estimation procedure is demonstrated. Section 5 discusses a real application of the model and Section 6 provides a final summary of the work.

2. Proposed Neutrosophic Extension

In this section, essential statistical characteristics of the proposed NIE distribution are described in detail. The probability density function (PDF) is a crucial piece of information in probability, as it shows the probability of a continuous random variable taking on a specific value. It is therefore often used to model and understand the behavior of many different types of random phenomena in a number of fields. Neutrosophic structure merges the concept of truth and falsity into the framework of PDF, allowing for the consideration of various relationships among components, tolerance to inexactness, and other difficulties in using traditional PDF. As this extension incorporates neutrosophic components into the PDF, which enables expressing undefined, vague, or partially known data. A random variable Z is assumed to follow the NIE distribution if it has the following PDF structure.

$$f_Z(z;\theta_N) = \frac{\theta_N}{z^2} e^{-\frac{\theta_N}{z}},\ z > 0;\ 0,\ z \geq 0 \quad (1)$$

Note that the random variable is imprecise because of indeterminacy $\theta_N = [\theta_l, \theta_u]$.

The closely related function to PDF is the cumulative distribution function (CDF) which is defined as given below:

$$F_Z(z;\theta_N) = e^{-\frac{\theta_N}{z}},\ z > 0;\ 0,\ z \leq 0$$

Expression given in Eq. (2) can be easily established as:

$$F_Z(z;\theta_N) = \int_0^z f_Z(t;\theta_N)\ dt$$

$$F_Z(z;\theta_N) = \int_0^z \frac{\theta_N}{t^2} e^{-\frac{\theta_N}{t}}\ dt$$

Substituting transformation $u = \frac{\theta_N}{t}, du = -\frac{\theta_N}{t^2}\ dt$

$$F_Z(z;\theta_N) = \int_{\frac{\theta_N}{z}}^{\infty} e^{-u}\ du \quad (2)$$

Further simplification to Eq. (2) resulted the desired expression

$$F_Z(z;\theta_N) = e^{-\frac{\theta_N}{z}}, z > 0 \quad (3)$$

The CDF is an essential aspect of probability theory, being a function that indicates the probability that a random variable takes on a value less than or equal to the value of the desired point. It takes a vital role in determining the spread of data, studying cumulative probabilities, and drawing statistical predictions. The CDF neutrosophic extension brings additional richness by introducing truth, indeterminacy and falsity, which can prove convenient when data is under uncertainty or lack of completeness. The neutrosophic version of probability offers a more precise modeling of many real-life systems where the precise probability of an event is difficult to determine, such as decision making under uncertainty, risk assessment or complex imprecise phenomena modeling. The neutrosophic CDF, on the other hand, accommodates ambiguity by taking into account both known and unknown attributes of the data, offering superior decision-making and predictive capabilities in applications where traditional CDFs prove inadequate.

To clarify the difference of this model with the classical structure we sketch the PDF and CDF of the classical model in Figure 1.

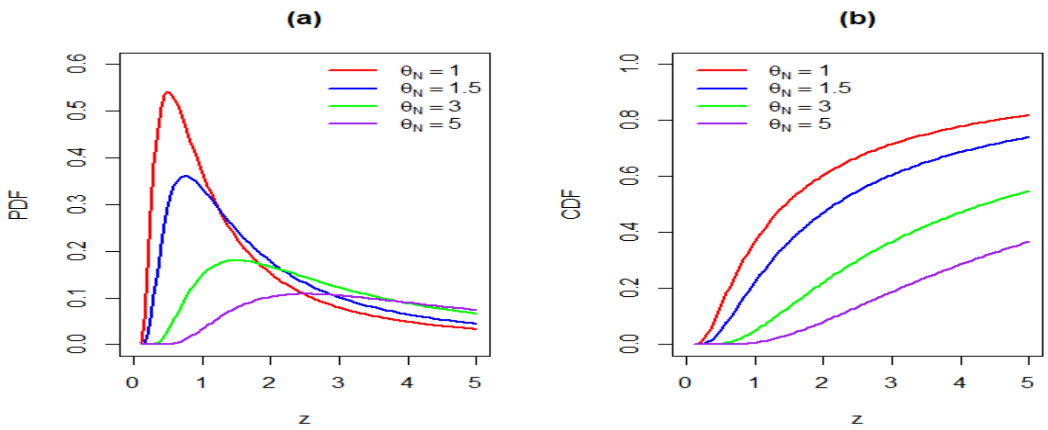

Figure 1. Classical curves of the PDF and CDF of the inverse exponential distribution.

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In Figure 1, it can be seen that that each curve is represented by precise value of distribution whereas in the neutrosophic this value is not precise number but representing an interval value. To see this difference, Figure 2 and Figure 3 are constructed respectively for PDF and CDF of the proposed model.

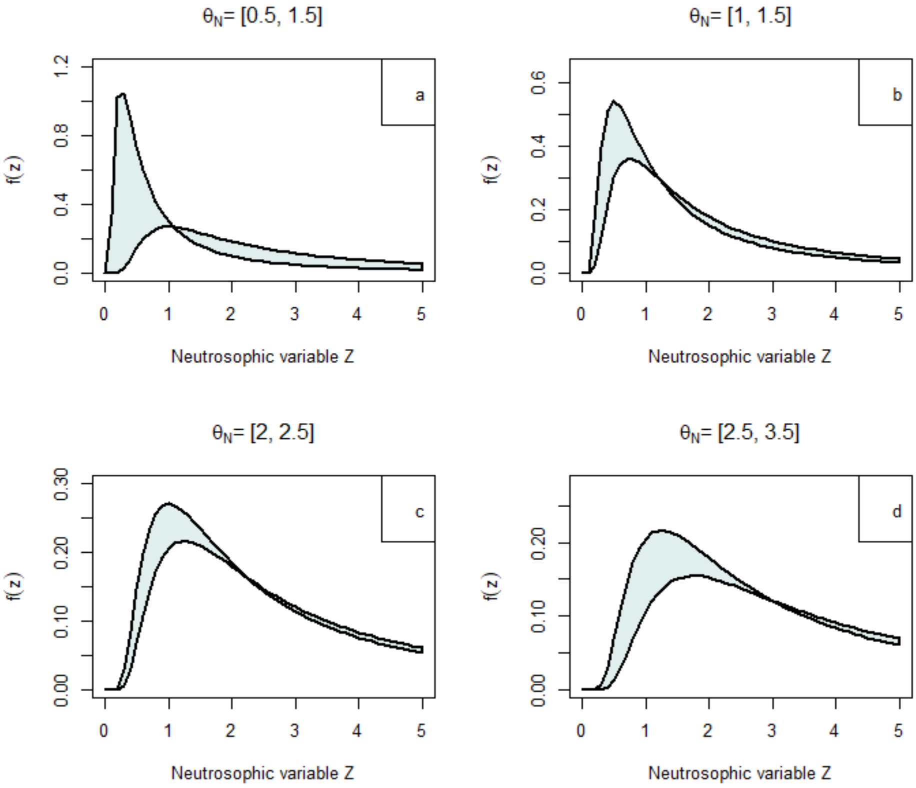

Figure 2. PDF curves of the proposed model with different parameter setting.

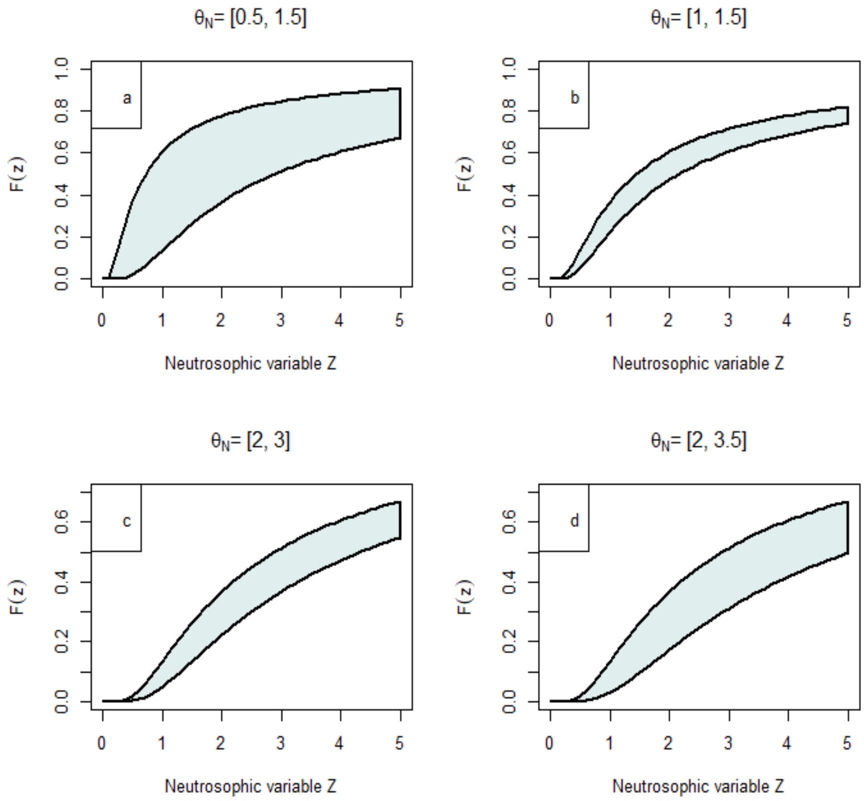

Figure 3. CDF of the proposed model with different parameter setting.

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The PDF of the neutrosophic inverse exponential distribution is shown in Figure 2, in which the probability of the random variable is given for various values. It is fatty curve because of indeterminacy in distributional parameter. The PDF exhibits an unprecedented characteristic in that the distribution models uncertain values but also reflections of truth to model both extreme absolute certainty and extreme absolute uncertainty in the context of classical exponential distributions. As we might expect, the distribution is skewed, with greater probability near the origin, meaning smaller values have higher likelihood. Figure 3 describes the cumulative probability of the random variable represented by the neutrosophic inverse exponential distribution having any value less than or equal to a particular point. While the CDF ranges from 0 to 1 monotonically, the presence of neutrosophic elements leads instead to a gradual progression, one which accounts for indeterminate and imprecise values. These together characterize the improved flexibility and accuracy of the neutrosophic inverse exponential distribution to model such uncertain systems, leading to a more complete grasp of this distribution behavior for different scenarios. These neutrosophic structures are quite useful in some domains where there is uncertainty and vagueness, including decision-making, risk analysis, reliability engineering, etc.

The survival function is another important related function in distribution theory. The survival function describes the probability that a subject or system will survive past time t. It is a key model in reliability engineering, survival analysis, and other fields where time is an event is of interest. It is used in healthcare to predict the likelihood of patients surviving, in insurance to determine life expectancy, and in manufacturing to assess the durability of a product. The survival function is a complementary function to the probability density function (PDF) and cumulative distribution function (CDF), as it provides a different point of view—aiming at the tail probabilities, which are very important to assess extreme events or long-term behavior. The capability of the model to deal with uncertainty can be increased even more when the model was extended to neutrosophic or fuzzy frameworks, which has provided it a wider application field in real problems that may be described with incomplete or imprecise data. The survival of the proposed model is defined as:

$$S_Z(z;\theta_N) = 1 - e^{-\frac{\theta_N}{z}},\ z > 0;\ 1,\ z \leq 0 \quad (4)$$

Similarly hazard function of the proposed model can be defined as:

$$h_Z(z;\theta_N) = \frac{f_Z(z;\theta_N)}{S_Z(z;\theta_N)}$$

$$h_Z(z;\theta_N) = \frac{\frac{\theta_N}{z^2} e^{-\frac{\theta_N}{z}},}{1-e^{-\frac{\theta_N}{z}}} \quad (5)$$

The cumulative hazard function of the proposed model is defined by:

$$H(z) = \int_0^z h(t)\ dt$$

$$H_Z(z;\theta_N) = \frac{\theta_N}{z},\ z > 0;\ 0,\ z \leq (6)$$

The mean is a basic and importance property of any distribution. It can be defined as:

$$E[Z] = \int_0^\infty z\, f_Z(z)\, dz \quad (6)$$

$$E[Z] = \int_0^\infty z \cdot \frac{\theta_N}{z^2} e^{-\frac{\theta_N}{z}}\, dz$$

$$E[Z] = \theta_N \int_0^\infty \frac{e^{-\frac{\theta_N}{z}}}{z}\, dz$$

$$E[Z] = \infty$$

This shows that mean does exist for the proposed model. This leads to inexistence of variance. However, we can find the other central measures tendency. The median is another position average, and it can be defined for the model as:

$$F_Z(z;\theta_N) = e^{-\frac{\theta_N}{z}}, \quad z > 0 \quad (7)$$

$$F_Z(m;\theta_N) = 0.5$$

$$e^{-\frac{\theta_N}{m}} = 0.5 \quad (8)$$

Further simplification of Eq.(8) yielded:

$$-\frac{\theta_N}{m} = \ln(0.5)$$

$$\frac{\theta_N}{m} = \ln(2)$$

$$m = \frac{\theta_N}{\ln(2)} \quad (9)$$

Thus the model of the proposed distribution can be found using expression given in Eq (9).

The model of the proposed model can be found by:

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$\frac{d}{dz} f_Z(z; \theta_N) = 0$ (10)

$$\frac{d}{dz}\left(\frac{\theta_N}{z^2} e^{-\frac{\theta_N}{z}}\right) = e^{-\frac{\theta_N}{z}}\left(-\frac{2\theta_N}{z^3} + \frac{\theta_N^2}{z^4}\right)$$

$e^{-\frac{\theta_N}{z}}\left(-\frac{2\theta_N}{z^3} + \frac{\theta_N^2}{z^4}\right) = 0$ (11)

Simplifying Eq. (11) resulted:

$$-\frac{2\theta_N}{z^3} + \frac{\theta_N^2}{z^4} = 0$$

$z = \frac{\theta_N}{2}$ (12)

which is required model value of the proposed model.

Now in the next section, we will see how the neutrosophic random samples can be generated using the quantile function of the proposed model.

3. Random Sample Generation

Random samples generation using the inverse CDF method is discussed in this part. The inverse CDF method is a commonly used method for generating random samples from a given probability distribution. The technique exploits the property of a continuous random variable where the CDF is a mapping between the range of the random variable and the interval 0,1. To produce a random sample, we first draw a uniform random number from the interval 0,1. The uniform random number is passed through the inverse of the CDF of the neutrosophic distribution, to generate the number from the distribution required. This value is a random sample from the desired distribution.

The CDF function of the proposed model is given by:

$$F_Z(z; \theta_N) = e^{-\frac{\theta_N}{z}}$$

Equating the expression to uniform model provided:

$F_Z(Q(p); \theta_N) = p$ (13)

Solving (a) yielded:

$Q(p) = \frac{\theta_N}{-\ln(p)}, 0 < p < 1$ (14)

Now write a simple program in R, we can generate random samples from the proposed for different parameter settings. If we assume that $\theta_N = [0.5, 2]$, then random samples of 30 values are given in Table 1.

Table 1: random samples generation from the proposed NIE model

Random Samples					
[6.17, 24.69]	[1.04, 4.19]	[0.86, 3.47]	[0.98, 3.92]	[0.98, 3.93]	[15.32, 61.28]
[1.95, 7.82]	[0.29, 1.19]	[3.17, 12.70]	[1.75, 7.01]	[0.25, 1.02]	[0.29, 1.19]
[0.27, 1.08]	[1.21, 4.86]	[0.41, 1.64]	[0.38, 1.55]	[1.58, 6.341	[0.42, 1.70]
[0.48, 1.94]	[0.90, 3.63]	[0.20, 0.82]	[1.74, 6.97]	[1.89, 7.59]	[3.15, 12.61]
[0.32, 1.30]	[6.50, 26.02]	[5.57, 22.28]	[0.73, 2.93]	[12.00, 48.02]	[3.85, 15.41]

Table 1 shows the thirty random samples from a proposed model with parameter fixed at $\theta_N = [0.5, 2]$. Each record takes the format of a pair of intervals with the lower bound and upper bound, to account for the uncertainties within the neutrosophic model. A specific random seed creates these values; any other random values with the same parameter settings will return a different value based on that seed. The random generated intervals can then be used to analyze the statistical properties and behaviour of the proposed modelling system, or its applicability in a real-life scenario, which involves both uncertainty and indeterminacy.

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4. Estimation Using Sampling Data

In this section, we will discuss the estimation procedure for the proposed model. The Maximum Likelihood Method (MLM) plays a crucial role in statistical inference as it provides a solid foundation for parameter estimation in probabilistic models. You find parameters that maximize the probability of observed data given a certain model with a set of parameters in a statistical method. This approach is favorable not only in terms of its consistency but also in terms of efficiency and asymptotic normality: it provides reliable estimates, which becomes more and more accurate as the size of the sample grows. It is widely used, as it is applicable in various fields including, economics, biology, engineering and numerous fields in machine learning, hence a fundamental toolbox for data analytics. It is flexible enough to address joint dependencies, very complex models that do not just involve observing outcomes (e.g., censored data, latent variables, mixtures, etc), where other approaches have failed. The Maximum Likelihood Method (MLM) for parameter estimation of neutrosophic distributions in the absence of uncertainty is modified to incorporate the uncertainty, which is a characteristic of the neutrosophic data. In contrast to classic MLM, where it is assumed that observations are precise and deterministic, the neutrosophic maximum likelihood method (NMLM) generalizes the likelihood method by including neutrosophic parameters, i.e. intervals or fuzzy sets. The likelihood function for the sample $z_1, z_2, \dots z_n$ is given by:

$$\ln L\,(\theta_N) = \sum_{i=1}^{n} \ln\left(\frac{\theta_N}{z_i^2} e^{-\frac{\theta_N}{z_i}}\right) \tag{15}$$

The log-likelihood expression is given by:

$$\ln L\,(\theta_N) = \sum_{i=1}^{n} \ln\left(\frac{\theta_N}{z_i^2} e^{-\frac{\theta_N}{z_i}}\right) \tag{16}$$

Further simplification resulted:

$$\ln L\,(\theta_N) = \sum_{i=1}^{n} \ln(\theta_N) - 2\sum_{i=1}^{n} \ln(z_i) - \sum_{i=1}^{n} \frac{\theta_N}{z_i} \tag{17}$$

Derivative of Eq (15) with respect to unknown parameter yielded:

$$\frac{\partial}{\partial\theta_N} \ln L\,(\theta_N) = \sum_{i=1}^{n} \frac{1}{\theta_N} - \sum_{i=1}^{n} \frac{1}{z_i} \tag{18}$$

Equating Eq (16) to zero provided:

$$\widehat{\theta_N} = \frac{n}{\sum_{i=1}^{n} \frac{1}{z_i}} \tag{19}$$

which is required to estimate the distribution neutrosophic parameter.

Now how we can understand the estimation procedure using a simple Monte Carlo experiment. A simple program can be written in R that generates random data from the proposed model using quantile function and then estimate of unknow parameter based on sample data can be found. In this experiment, we suppose that true value of neutrosophic parameter $\theta_N = [2, 3.5]$ and estimate the true values using random sample data for different sample size $n = 25, 50, 100, 200, 300$.The output of this experiment is given in Table 2.

Table 2: Maximum likelihood estimates of the unknow parameter at various sample sizes

Sample size	Estimated value
25	[2.45, 2.88]
50	[1.74, 2.87]
100	[2.25, 3.31]
200	[2.03, 3.50]
300	2.02, 3.31]

From the inverse exponential distribution, Table 2 indicates neutrosophic evaluations of the true parameter $\theta_N = [2, 3.5]$ for diverse, sample sizes. The intervals represent the range of estimated values, with the upper and lower bounds of the intervals representing uncertainty in the parameter estimation. This shows that as the sample size becomes larger, the estimated values become stable and lie within the intervals closer to the true intervals, showing that the neutrosophic approach is robust against the imprecision and uncertainty in parameter estimation.

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5. Real Data Application

In this, we have employed the proposed model for analyzing electricity demand data for Saudi Arabia from year 2000 to 2020. The data reflects the energy sector, whose analysis is crucial for policymakers and strategic planning to support the needs of a country's growing population and development. The dataset shows Saudi Arabia annual electricity demand in gigawatt-hours (GWh) from 2000 to 2020 taken from reported [31]. The growth rate in electricity consumption was consistent with the rapid urbanization process, industrial expansion and population growth of the country during this period. This growth can be attributed primarily to Saudia's continued development with Vision 2030 goals focusing on residential, commercial, and industrial activities, including further economic diversification and modernization. Our power generation statistics are necessary to national energy planning, as it is essential to know how much investment must be made in infrastructure and sustainable sources of energy and electricity management to address the growing demand. In addition, this knowledge helps decision makers to tackle issues like peak demand, energy security and environmental issues while ensuring long-term energy sustainability for Saudi Arabia. Consumption numbers are always fraught with uncertainty because of data collection errors, demand forecast differences, seasonal effects, and business cycle effects. Instead of exact observations, the data is converted into neutrosophic type, based on the method used in [32], to cure this vagueness. By extending the analysis to include degrees of truth and falsity, and identifying indeterminacy, we form a rigorous foundation for uncertainty in energy planning. The electricity consumption neutrosophic data is given in Table 3.

Table 3: Electricity consumption data for Saudi Arabia for the time 2001-2020

[116123.7, 116124.3]	[123574.2, 123575.8]	[130747.6, 130748.4]	[139875.1, 139876.9]	[150233.1, 150234.9]
[162587.0, 162587.0]	[175993.5, 175994.5]	[190253.1, 190254.9]	[205486.5, 205487.5]	[221761.5, 221762.5]
[239086.0, 239088.0]	[257425.5, 257426.5]	[276511.3, 276512.7]	[295870.4, 295871.6]	[315733.9, 315734.1]
[336023.1, 336024.9]	[349122.8, 349123.2]	[357652.0, 357652.0]	[360244.7, 360245.3]	[358199.0, 358201.0]

Table 3 shows that consumption data is given in interval form so classical model is not suitable for analyzing this data whereas proposed model may be adequate for such type of data. Using the MLE program written in R, estimated value of the NIE distribution can be determined which is given below:

$$\hat{\theta}_N = [206064.4, 206065.7]$$

This shows that estimated value based on Eq (17) is vague value because of existing imprecise data. Now the other statistical properties such as median and mode based on this interval information can easily be obtained.

6. Conclusion

In this study, we have proposed the neutrosophic inverse exponential (NIE) distribution as a suitable extension of the traditional inverse exponential distribution for modelling uncertain data in risk management, particularly in terms of longevity. We derive closed-form expressions of important statistical functions such as the quantile function, the median, the mode, CDF and PDF. Robust fitting to the data is assured through the maximum likelihood estimation method of parameter estimation in the proposed model. By means of simulations and use on real data, we show the applications and versatility of works with the NIE model, which allows to provide a way of generality of modeling uncertainty or imprecision data in risk management. Our results indicate the reliable estimation results can be obtained using larger sample size data. Finally, we have also shown the effectiveness and usage of the proposed model using the real data on electricity consumption data facing uncertainties. Numerical results from real data example show that proposed model is equipped with functionality to analyze imprecise data set.

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