A novel Q-neutrosophic soft under interval matrix setting and its applications

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Abstract

Decision-making theory serves as an effective framework to guide decision-makers in solving problems. One notable application of this theory is in the medical field, where it aids doctors in analyzing patient data to determine whether a patient is infected. To enhance this theory with more adaptable mathematical methods, we propose an expanded approach based on previously introduced matrixes of Q-neutrosophic soft under an Interval-valued setting (IV-Q-NSM). This represents a new finding of existing mathematical tools to address the two-dimensional uncertainty prevalent in various life domains.

This work explores several algebraic properties and matrix operations associated with IV-Q-NSM. Subsequently, we introduce a new methodology for decision-making (DM) in medical diagnosis selection problems. This approach aims to provide a more flexible and comprehensive framework for evaluating complex medical data and improving diagnostic accuracy.

Keywords: Neutrosophic set; Q- neutrosophic set; Q-neutrosophic soft matrix; interval-valued neutrosophic set; soft set; interval-valued neutrosophic soft set

1. Introduction

In today's complex world, researchers and practitioners across various scientific disciplines collaborate to develop effective decision-making strategies. While many decisions can be made quickly, some require well-structured approaches tailored to the specific problem at hand. During the decision-making process, risks and uncertainties often emerge, complicating the evaluation of options. Factors such as vagueness, uncertainty, and ambiguity can significantly affect real-life decision-making scenarios.

To navigate these complexities, experts leverage specialized theories, innovative approaches, and methodologies that enhance their understanding of circumstances and facilitate the evaluation of alternatives before arriving at the most suitable choice. This decision-making process is applicable at both macro and micro levels, extending beyond scientific research to fields such as finance, economics, game theory, and marketing.

The foundational work in this area was established by Lotfi Zadeh, who introduced the concept of fuzzy sets (FS), which serve as an extension of traditional crisp sets. In fuzzy set theory, each element is assigned a degree of membership between 0 and 1, allowing for a more nuanced representation of uncertainty. This ground-breaking concept has spurred a wealth of research across various domains, as evidenced by numerous studies [2-5].
As everyday life becomes increasingly complex, there is a growing need for new mathematical tools or for the enhancement of existing ones. In this context, Florentin Smarandache introduced neutrosophic set (NS) theory in 1998, which expands upon Zadeh's fuzzy sets by assigning three degrees of membership to each element, all of which lie within the interval [0, 1]. This allows for a richer representation of uncertainty.

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Deli further advanced this framework by introducing the concept of interval-valued neutrosophic soft sets (IV-NSS), merging aspects of neutrosophic sets and soft sets (SS). Al-Sharqi et al. integrated IV-NSS with various mathematical tools, including algebra [9,10], graph theory [11,12], complex analysis [13,14], and possibility theory [15,16]. Following these developments, Al-Quran et al. combined neutrosophic sets with additional fuzzy extensions [18,19], while Ali and Mohammed introduced innovative topological tools for fuzzy neutrosophic environments [20,21]. Palanikumar et al. explored the integration of neutrosophic sets with various mathematical frameworks to enhance the accuracy of multi-criteria decision-making (MCDM) methods [22,23]. Al-Qudah et al. addressed complex uncertainty in decision-making problems, presenting significant contributions to the field [24,25].

Moreover, the transformation of human knowledge into mathematical representations—and vice versa—preserves the meaning of information. Consequently, these mathematical tools have found applications across diverse fields, including science [28,29], engineering [30,31], and administrative sciences [32,33].

Recently, Ghazi and Al-Salihi introduced a new hyper model termed IV-Q-neutrosophic soft sets (IV-Q-NSS), which combines the features of soft sets and neutrosophic sets within an interval setting, utilizing Q-two-dimensional universal information [35,36]. In this work, we reintroduce the concept of matrices, proposing a new model called IV-Q-NS-Matrix. This model aims to harness the properties and advantages of matrix techniques to enhance decision-making processes, particularly in the context of medical diagnoses.

Additional

i. Matrix Representation: The IV-Q-NS-Matrix allows for the organization of data in a structured format, facilitating easier manipulation and analysis of complex information.
ii. Algebraic Properties: We explore various algebraic properties of the IV-Q-NS-Matrix, including operations such as addition, multiplication, and inversion, which are crucial for mathematical modelling in decision-making.
iii. Application in Medical Diagnosis: The proposed methodology can significantly aid healthcare professionals by providing a systematic approach to evaluating patient data. By analysing the fuzzy soft matrices, doctors can better assess the likelihood of patient infections and make informed decisions based on comprehensive data analysis.
iv. Future Directions: The integration of IV-Q-NS-Matrix into decision-making frameworks opens avenues for further research, including the development of algorithms that can automate the evaluation process and enhance predictive accuracy in various applications.

2. Basic definitions and background

Definition 2.1 [1]**:** A following structure

$$\mathrm{N} = \{\mathrm{v}_j \square\, \hat{P}^t(\mathrm{v}_j) \square\, \hat{P}^i(\mathrm{v}_j) \square\, \hat{P}^f(\mathrm{v}_j) \mid \mathrm{v}_j \in \mathrm{V}\}$$

called NS on universal set V. Such that $\mathrm{N} = \hat{P}^t(\mathrm{v}_j), \hat{P}^i(\mathrm{v}_j), \hat{P}^f(\mathrm{v}_j) : \mathrm{V} \to [0,1]$ and $\hat{P}^t(\mathrm{v}_j), \hat{P}^i(\mathrm{v}_j), \hat{P}^f(\mathrm{v}_j) \in [0,1]$.

Definition 2.2 [1]: A following structure

$$\mathrm{N}_{\mathfrak{Q}} = \{\mathrm{v}_j \square\, \hat{P}^t_{\mathfrak{Q}}(v \square \mathrm{q}) \square\, \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(v \square \mathrm{q}) \square\, \hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(v \square \mathrm{q}) \mid (v \square \mathrm{q}) \in \mathrm{V} \times \mathfrak{Q}\}$$

called Q-NS on Q× V. Such that $\mathrm{N}_{\mathfrak{Q}} = \hat{P}^t_{\mathfrak{Q}}(v,\mathrm{q}), \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(v,\mathrm{q}), \hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(v,\mathrm{q}) : \mathrm{V} \to [0,1]$ and $\hat{P}^t_{\mathfrak{Q}}(v,\mathrm{q}), \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(v,\mathrm{q}), \hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(v,\mathrm{q}) \in [0,1]$.

Definition 2.3 [1]: A following structure

$$IVN_Q = \{e \in \bar{\mathrm{A}} \square < \hat{P}^t_{\mathfrak{Q}}(v \square \mathrm{q})(e) \square\, \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(v \square \mathrm{q})(e) \square\, \hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(v \square \mathrm{q})(e) > \mid (v \square \mathrm{q}) \in V \times \mathfrak{Q}\}$$

Where

$$\hat{P}^t_{\mathfrak{Q}_{\bar{A}}}(v \square \mathrm{q})(\mathrm{e}) = [\hat{P}^{t \square l}_{\mathfrak{Q}_{\bar{A}}}(v \square \mathrm{q})(e) \square\, \hat{P}^{t \square u}_{\mathfrak{Q}_{\bar{A}}}(v \square \mathrm{q})(\mathrm{e})]$$

$$\hat{P}^i_{\mathfrak{Q}_{\bar{A}}}(v \square \mathrm{q})(\mathrm{e}) = [\hat{P}^{i \square l}_{\mathfrak{Q}_{\bar{A}}}(v \square \mathrm{q})(e) \square\, \hat{P}^{i \square u}_{\mathfrak{Q}_{\bar{A}}}(v \square \mathrm{q})(\mathrm{e})]$$

$$\hat{P}^f_{\mathfrak{Q}_{\bar{A}}}(v \square \mathrm{q})(\mathrm{e}) = [\hat{P}^{f \square l}_{\mathfrak{Q}_{\bar{A}}}(v \square \mathrm{q})(e) \square\, \hat{P}^{f \square u}_{\mathfrak{Q}_{\bar{A}}}(v \square \mathrm{q})(\mathrm{e})]$$

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called IV-Q-NS on Q× V. Such that $IVN_{\mathfrak{Q}} = \hat{P}^{t}_{\mathfrak{Q}}(v, q), \hat{P}^{i}_{\mathfrak{Q}}(v, q), \hat{P}^{f}_{\mathfrak{Q}}(v, q): V \to [0,1]$ and $\hat{P}^{t}_{\mathfrak{Q}}(v, q), \hat{P}^{i}_{\mathfrak{Q}}(v, q), \hat{P}^{f}_{\mathfrak{Q}}(v, q) \in [0,1]$.

Example 2.4 Let

$$\hat{\mathfrak{P}}_{Q_{\bar{A}}} =$$

$$\left\{\left(e_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(v_1, q_1)}, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(v_1, q_2)}\right.\right.$$

$$\frac{\langle[0.3,0.6],[0.2,0.7],[0.5,0.8]\rangle}{(v_2, q_1)}, \frac{\langle[0.4,0.6],[0.2,0.9],[0.5,0.7]\rangle}{(v_2, q_2)}$$

$$\left.\frac{\langle[0.1,0.5],[0.3,0.7],[0.2,0.8]\rangle}{(v_3, q_1)}, \frac{\langle[0.4,0.8],[0.4,0.6],[0.2,0.8]\rangle}{(v_3, q)}\right)$$

$$\left(e_2, \frac{\langle[0.1,0.8],[0.5,0.7],[0.3,0.4]\rangle}{(v_1, q_1)}, \frac{\langle[0.1,0.8],[0.4,0.7],[0.2,0.6]\rangle}{(v_1, q_2)}\right.$$

$$\frac{\langle[0.5,0.8],[0.4,0.9],[0.2,0.7]\rangle}{(v_2, q_1)}, \frac{\langle[0.1,0.2],[0.2,0.5],[0.4,0.7]\rangle}{(v_2, q_2)}$$

$$\left.\frac{\langle[0.1,0.4],[0.2,0.5],[0.3,0.7]\rangle}{(v, q_1)}, \frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(v_3, q_2)}\right)$$

$$\left(e_3, \frac{\langle[0.7,0.9],[0.2,0.8],[0.3,0.6]\rangle}{(v_1, q_1)}, \frac{\langle[0.4,0.7],[0.2,0.5],[0.1,0.7]\rangle}{(v_1, q_2)}\right.$$

$$\frac{\langle[0.1,0.8],[0.1,0.4],[0.3,0.6]\rangle}{(v_2, q_1)}, \frac{\langle[0.5,0.6],[0.3,0.6],[0.2,0.7]\rangle}{(v_2, q_2)}$$

$$\left.\left.\frac{\langle[0.4,0.6],[0.2,0.7],[0.3,0.6]\rangle}{(v_3, q_1)}, \frac{\langle[0.4,0.8],[0.8,0.9],[0.3,0.7]\rangle}{(v_3, q_2)}\right)\right\}$$

3. An IV-Q-NSs in Matrix Form

In this section, we will demonstrate the algebraic ability with our proposed concept by presenting our proposed concept in this thesis with a matrix system.

Definition 3.1. Let $\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}, < \hat{P}^{t}_{\mathfrak{Q}}(\hat{v}, \breve{q})(e) > |(\hat{v}, \breve{q}) \in V \times \mathfrak{Q}\} \in IV-Q-NSS(V)$.Then, the IV-Q-NSs in matrix form given as a following matrix :

$$[\hat{P}_{\mathfrak{Q}_{\bar{A}}}]_{m\times n} = \begin{bmatrix} \backslash & e_1 & e_1 & \cdots & e_m \\ (\hat{v}_1, \breve{q}_1) & \hat{P}_{\mathfrak{Q}_{\bar{A}_{1,1}}} & \hat{P}_{\mathfrak{Q}_{\bar{A}_{1,2}}} & \cdots & \hat{P}_{\mathfrak{Q}_{\bar{A}_{1,j}}} \\ (\hat{v}_2, \breve{q}_2) & \hat{P}_{\mathfrak{Q}_{\bar{A}_{2,1}}} & \hat{P}_{\mathfrak{Q}_{\bar{A}_{2,2}}} & \cdots & \hat{P}_{\mathfrak{Q}_{\bar{A}_{2,j}}} \\ \vdots & \vdots & \vdots & & \vdots \\ (\hat{v}_n, \breve{q}_j) & \hat{P}_{\mathfrak{Q}_{\bar{A}_{i,1}}} & \hat{P}_{\mathfrak{Q}_{\bar{A}_{i,2}}} & \cdots & \hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \end{bmatrix}_{m\times n}$$

Where $\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}} = \left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle$

, $i = 1,2,3, \dots, n$ number of Rows and $j = 1,2,3, \dots, m$ number of Columns.

Example 3.2. Assume that the values given in the **example 2.4** are as follows:

Now, through the **definition 3.1**, these values can be represented as a matrix as follows:

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$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}}}\right]_{6\times 3}=\begin{bmatrix} \backslash & e_1 & e_2 & e_3 \\ (\mathfrak{u}_1,\mathfrak{q}_1) & \langle[0.2,0.5]\rangle & \langle[0.2,0.2]\rangle & \langle[0.5,0.8]\rangle \\ (\mathfrak{u}_1,\mathfrak{q}_2) & \langle[0.1,0.4]\rangle & \langle[0.1,0.8]\rangle & \langle[0.4,0.7]\rangle \\ (\mathfrak{u}_2,\mathfrak{q}_1) & \langle[0.3,0.4]\rangle & \langle[0.5,0.8]\rangle & \langle[0.3,0.6]\rangle \\ (\mathfrak{u}_2,\mathfrak{q}_2) & \langle[0.6,0.8]\rangle & \langle[0.1,0.2]\rangle & \langle[0.5,0.6]\rangle \\ (\mathfrak{u}_3,\mathfrak{q}_1) & \langle[0.1,0.5]\rangle & \langle[0.2,0.5]\rangle & \langle[0.3,0.6]\rangle \\ (\mathfrak{u}_3,\mathfrak{q}_2) & \langle[0.4,0.8]\rangle & \langle[0.4,0.5]\rangle & \langle[0.4,0.8]\rangle \end{bmatrix}_{6\times 3}$$

In the above matrix, the row represents a set of $\mathcal{E}=\{e_1,e_2,e_3\}$, while the column represents a set of both $\mathfrak{U}=\{\mathfrak{u}_1,\mathfrak{u}_2,\mathfrak{u}_3\}$ and $\mathfrak{Q}=\{\mathfrak{q}_1,\mathfrak{q}_2\}$.

Definition 3.3 Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ be two IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}=\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}=\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i=1,2,\dots,r,\ j=1,2,\dots,s$. Then we have the following point:

1. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ is called zero-IV-Q-NSM if $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}=[\langle[0,0],[1,1],[1,1\rangle]_{m\times n}$.
2. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ is absolute-IV-Q-NSM if $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}=[\langle[1,1],[0,0],[0,0\rangle]_{m\times n}$.
3. An IV-Q-NSM $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ is IV-Q-NS-submatrix $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ and denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\subseteq\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ if the degrees: $\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\le\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\le\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\ge\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\ge\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$ and $\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\ge\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\ge\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$.
4. A square IV-Q-NSM $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ has a transpose by exchange of rows and columns of $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and it denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^t_{m\times n}$ such that

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^t_{m\times n}=\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]^t_{m\times n}$$
$$=\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}},\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right]\right\rangle\right]^t_{m\times n}.$$

5. A square IV-Q-NSM $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ is denoted by symmetric square IV-Q-NSM if $\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}=\hat{P}_{\mathfrak{Q}_{\bar{A}_{j,i}}}$ i.e $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}=\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^t_{m\times n}$.

Definition 3.4 Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ be two IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}=\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$, $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}=\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n}=\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i=1,2,\dots,r,\ j=1,2,\dots,s$. Then, the fundamental algebraic operations on IV-Q-NSM are defined as follows:

Addition: The addition operation of two IV-Q-NSM is defined as follow:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}+\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}=\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n}=\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Where

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$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}} - \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}} - \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}},$
$\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$
$\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}.$

Subtraction: The operation of subtraction of two IV-Q-NSMs is defined as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} - \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Where
$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \left|\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} - \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right|, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \left|\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} - \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right|,$
$\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = max\left(\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = max\left(\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right)$
$\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \left|\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} - \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right|, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \left|\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} - \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right|.$

Multiplication: The operation of multiplication for two IV-Q-NSMs are defined as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \times \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Where
$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$
$\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}} - \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}} - \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$
$\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}} - \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}} - \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$

Scalar Multiplication: The operation of scalar multiplication for two IV-Q-NSMs are defined as follows:

$$K\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle \left[K\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, K\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[K\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, K\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[K\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, K\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Where $K \in R$.

Proposition 3.5 : Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ be two IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$, $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i = 1,2,\dots,r$, $j = 1,2,\dots,s$. Then, the following point have been satisfied:

1. For both $\alpha, \beta \in R$ then $\alpha\left(\beta\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right) = \alpha.\beta\left(\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right)$.
2. For $\alpha \le \beta$ then $\alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \le \beta\left[\hat{P}_{\mathfrak{Q}_{B_{i,j}}}\right]_{m\times n}$.
3. If $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \le \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ then $\alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \le \beta\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$.
4. $\left(\alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right)^t = \alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^t$.

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5. $\left(\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^{t}\right)^{t}=\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$.

Prof (1). Take

$$\alpha\left(\beta\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right)=\alpha\left[\left\langle\left[\widehat{\beta P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\beta\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\beta\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\beta\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\beta\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\beta\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

$$=\left[\left\langle\left[\alpha\widehat{\beta P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\alpha\beta\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\alpha\beta\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\alpha\beta\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\alpha\beta\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\alpha\beta\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

$$=\alpha\beta\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}=\alpha\beta\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}.$$

Prof (3). Suppose that $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\leq\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ then $\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\leq\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\leq\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\geq\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\geq\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$ and $\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\geq\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\geq\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$.
Now for $\alpha\in R$, then
$\alpha\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\leq\widehat{\alpha P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\alpha\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\leq\alpha\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\alpha\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\geq\alpha\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\alpha\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\geq\alpha\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$ and
$\alpha\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\geq\alpha\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$, $\alpha\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\geq\alpha\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$.

Then $\alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\leq\alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$.

Prof (4). We have $\left(\left[\widehat{\alpha P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right)^{t}\in IV-Q-NSM$, then

$$\left(\left[\widehat{\alpha P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right)^{t}=\left[\left\langle\left[\widehat{\alpha P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}},\alpha\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right],\left[\alpha\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}},\alpha\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right],\left[\alpha\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}},\alpha\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right]\right\rangle\right]_{m\times n}$$

$$=\alpha\left(\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]\right)^{t}$$

$$=\alpha\left(\left[\widehat{\alpha P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right)^{t}.$$

Definition 3.6 The union of two IV-Q-NSMs denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n}=\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\cup\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ where
$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}=\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$, $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}=$
$\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n}=$
$\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}},\hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i=1,2,\ldots,r$, $j=1,2,\ldots,s$. Then,

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n}=\begin{cases}\left(max\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right],max\left[\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right)\\\left(min\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right],min\left[\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right)\\\left(min\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right],min\left[\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}},\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right)\end{cases}$$

Definition 3.7 The intersection of two IV-Q-NSMs denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n}=\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\cap\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ where

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$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$, $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i = 1,2,\dots,r, j = 1,2,\dots,s$. Then,

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{cases} \left(min\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], max\left[\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right) \\ \left(max\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], min\left[\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right) \\ \left(max\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], min\left[\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right) \end{cases}$$

Definition 3.8 The complement of two IV-Q-NSMs denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^{\boldsymbol{c}}$ where

$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^{\boldsymbol{c}} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^c, \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^c, \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^c\right\rangle\right]_{m\times n}$, $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ =for all $i = 1,2,\dots,r, j = 1,2,\dots,s$. Then,

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^{\boldsymbol{c}} = \begin{cases} \left(\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right) \\ \left(\left[1-\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, 1-\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right) \\ \left(\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right) \end{cases}$$

Definition 3.9 Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ be two IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i = 1,2,\dots,r, j = 1,2,\dots,s$. Then the OR-operation (∨) of IV-Q-NSMs denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \otimes \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ and defined as following form:

$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \vee \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n}$ such that

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle \max min\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \min max\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \min max\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Definition 3.10 Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ be two IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i = 1,2,\dots,r, j = 1,2,\dots,s$. Then the AND-operation (∧) of IV-Q-NSMs denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \odot \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ and defined as following form:

$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \wedge \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n}$ such that

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle \min max\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \max min\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \max min\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

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Example 3.11. Take two IV-Q-NSSs:

$\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}} = \left\{\left(e_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(v_1,q_1)}, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(v_1,q_2)}\right)\right\}$ and

$\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}} = \left\{\left(e_1, \frac{\langle[0.3,0.6],[0.2,0.7],[0.5,0.8]\rangle}{(v_1,q_1)}, \frac{\langle[0.4,0.6],[0.2,0.9],[0.5,0.7]\rangle}{(v_1,q_2)}\right)\right\}$ Then:both $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ it is presented as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{2\times 1} = \begin{bmatrix}[0.2,0.8],[0.1,0.7],[0.4,0.8]\\ [0.1,0.4],[0.5,0.8],[0.7,0.8]\end{bmatrix}_{2\times 1}, \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{2\times 1} = \begin{bmatrix}[0.3,0.6],[0.2,0.7],[0.5,0.8]\\ [0.4,0.6],[0.2,0.9],[0.5,0.7]\end{bmatrix}_{2\times 1}$$

Then:

1. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} + \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.3,0.9],[0.4,0.9],[0.5,0.9]\\ [0.4,0.5],[0.7,0.6],[0.8,0.9]\end{bmatrix}_{2\times 1}$
2. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} - \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.2,0.8],[0.1,0.7],[0.4,0.8]\\ [0.1,0.4],[0.5,0.8],[0.7,0.8]\end{bmatrix}_{2\times 1}$
3. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \times \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.2,0.8],[0.1,0.7],[0.4,0.8]\\ [0.1,0.4],[0.5,0.8],[0.7,0.8]\end{bmatrix}_{2\times 1}$
4. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \cup \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.3,0.8],[0.1,0.7],[0.4,0.8]\\ [0.4,0.6],[0.2,0.8],[0.5,0.7]\end{bmatrix}_{2\times 1}$
5. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \cap \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.2,0.6],[0.2,0.7],[0.5,0.8]\\ [0.1,0.4],[0.5,0.9],[0.7,0.8]\end{bmatrix}_{2\times 1}$
6. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^{c} = \begin{bmatrix}[0.4,0.8],[0.3,0.9],[0.2,0.8]\\ [0.7,0.8],[0.2,0.5],[0.1,0.4]\end{bmatrix}_{2\times 1}$
7. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \vee \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.3,0.8],[0.1,0.7],[0.4,0.8]\\ [0.4,0.6],[0.2,0.8],[0.5,0.7]\end{bmatrix}_{2\times 1}$
8. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \wedge \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.2,0.6],[0.2,0.7],[0.5,0.8]\\ [0.1,0.4],[0.5,0.9],[0.7,0.8]\end{bmatrix}_{2\times 1}$

4. Practical application of IV-Q-NSM in Medical Field

In this section, we will outline the framework for implementing our proposed model in addressing everyday situations. We will begin by presenting a case study in the medical field, illustrating how to represent relevant data using our model. Following this, we will develop an algorithm composed of sequential steps designed to analyze both the algebraic structure of our model and the data it represents.

Definition 4.1. Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ be IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$ then IV-Q-NSMs reduce to IV-Q-FSMs $\Psi_{\bar{A}_{i,j}}$ where the lower and upper given as following formalhs $\Psi^{l}_{\bar{A}_{i,j}} = \frac{1}{3}\left(\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right)$, $\Psi^{u}_{\bar{A}_{i,j}} = \frac{1}{3}\left(\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right)$.

Definition 4.2. (Comparison Matrix) Comparison matrix is define as a matrix that provides a comprehensive comparison between the membership function values of each object with other objects according to a rule that gives a value of 1 if it is larger and a value of 0 if the value is smaller.

Algorithm

Step 1. Build the IV-Q-NSM based on the IV-Q-NSS.

Step 2. Convert IV-Q-NSM to IV-Q-FSM using definition.

Step 3. Convert IV-Q-FSM to SV-Q-FSM using the following formal $\Psi_{\bar{A}_{i,j}} = \frac{\Psi^{l,t}_{\bar{A}_{i,j}} + \Psi^{u,t}_{\bar{A}_{i,j}}}{2}$

Step 4. Find **c**omparison matrix based on given definition above.

Step 5. Find the score values M_t for all $(v,q) \in V \times Q$, where $t = 1,2, \dots n$.

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Step 6. The decision taken is to pick the highest score of M_t,i.e. Decision taken= $max\{M_t\}$.

Step 7. End Algorithm.

Case Study

Now, we provide a case study related to the medical field for IV-Q-NSS strategic decision-making method.

On a cold winter day, numerous patients visited the office of a respiratory doctor to diagnose their health conditions (whether COVID-positive or not) based on the symptoms they were experiencing. To assist the doctor in organizing and analyzing patient data according to our proposed model, we requested that he select a value between 0 and 1 that reflects the severity of symptoms and their association with the disease (COVID). In this context, a ratio closer to 1 indicates more severe symptoms and a greater impact on the disease. Therefore:

Suppose that $V = \{v_1, v_2, v_3, v_4\}$ be a patient set contains four patients, $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2\}$ where $\mathfrak{q}_1$ =infected and $\mathfrak{q}_2$ =uninfected,while $\bar{A} \subseteq \mathcal{E} = \{e_1, \bar{e}_2, \bar{e}_3\}$ be a set of symptoms contains four symptoms such that $\bar{a}_1$ =Headache, $\bar{a}_2$ =Sore throat, $\bar{a}_3$ =Muscle pain

Now, after the doctor has examined each patient and set a numerical value between 0 and 1 for each of the symptoms above, our proposed model can be built in a way that is consistent with the examining doctor's report. The examining physician's opinion will be analyzed and help him make the appropriate decision about which of the examined patients are infected or not .

$$\hat{P}_{\mathfrak{Q}_{\bar{A}}} =$$

$$\left\{\left(\bar{e}_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(v_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(v_1,\mathfrak{q}_2)}\right.\right.$$

$$\frac{\langle[0.3,0.6],[0.2,0.7],[0.5,0.8]\rangle}{(v_2,\mathfrak{q}_1)}, \frac{\langle[0.4,0.6],[0.2,0.9],[0.5,0.7]\rangle}{(v_2,\mathfrak{q}_2)}$$

$$\frac{\langle[0.6,0.8],[0.4,0.5],[0.3,0.5]\rangle}{(v_3,\mathfrak{q}_1)}, \frac{\langle[0.3,0.7],[0.2,0.4],[0.1,0.8]\rangle}{(v_3,\mathfrak{q}_2)}$$

$$\left.\frac{\langle[0.1,0.5],[0.3,0.7],[0.2,0.8]\rangle}{(v_4,\mathfrak{q}_1)}, \frac{\langle[0.4,0.8],[0.4,0.6],[0.2,0.8]\rangle}{(v_4,\mathfrak{q}_2)}\right)$$

$$\left(e_2, \frac{\langle[0.1,0.8],[0.5,0.7],[0.3,0.4]\rangle}{(v_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.8],[0.4,0.7],[0.2,0.6]\rangle}{(v_1,\mathfrak{q}_2)}\right.$$

$$\frac{\langle[0.5,0.8],[0.4,0.9],[0.2,0.7]\rangle}{(v_2,\mathfrak{q}_1)}, \frac{\langle[0.1,0.2],[0.2,0.5],[0.4,0.7]\rangle}{(v_2,\mathfrak{q}_2)}$$

$$\frac{\langle[0.6,0.8],[0.4,0.5],[0.3,0.5]\rangle}{(v_3,\mathfrak{q}_1)}, \frac{\langle[0.3,0.7],[0.2,0.4],[0.1,0.8]\rangle}{(v_3,\mathfrak{q}_2)}$$

$$\left.\frac{\langle[0.1,0.4],[0.2,0.5],[0.3,0.7]\rangle}{(v_4,\mathfrak{q}_1)}, \frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(v_4,\mathfrak{q}_2)}\right)$$

$$\left(e_3, \frac{\langle[0.7,0.9],[0.2,0.8],[0.3,0.6]\rangle}{(v_1,\mathfrak{q}_1)}, \frac{\langle[0.4,0.7],[0.2,0.5],[0.1,0.7]\rangle}{(v_1,\mathfrak{q}_2)}\right.$$

$$\frac{\langle[0.1,0.8],[0.1,0.4],[0.3,0.6]\rangle}{(v_2,\mathfrak{q}_1)}, \frac{\langle[0.5,0.6],[0.3,0.6],[0.2,0.7]\rangle}{(v_2,\mathfrak{q}_2)}$$

$$\frac{\langle[0.6,0.8],[0.4,0.5],[0.3,0.5]\rangle}{(v_3,\mathfrak{q}_1)}, \frac{\langle[0.3,0.7],[0.2,0.4],[0.1,0.8]\rangle}{(v_3,\mathfrak{q}_2)}$$

$$\left.\left.\frac{\langle[0.4,0.6],[0.2,0.7],[0.3,0.6]\rangle}{(v_4,\mathfrak{q}_1)}, \frac{\langle[0.4,0.8],[0.8,0.9],[0.3,0.7]\rangle}{(v_4,\mathfrak{q}_2)}\right)\right\}$$

To approximate the working mechanism, we take the part

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$\left(e_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(v_1,q_1)}\right)$ Here, this part indicates that the strength of the symptom $\bar{a}_1$ of the first patient suspected of having the disease is strongly estimated between [0.2,0.8], the degree of non-injury rating [0.4,0.8] and the degree of neutrality in whether or not an injury is present is estimated [0.1,0.7].

Now we use the above algorithm to solve this problem by applying its steps sequentially and clearly.

Step 1. Build the IV-Q-NSM $[\hat{P}_{\mathfrak{Q}_{\bar{A}}}]_{m\times n}$ based on the IV-Q-NSS $\hat{P}_{\mathfrak{Q}_{\bar{A}}}$.

$[\hat{P}_{\mathfrak{Q}_{\bar{A}}}]_{3\times 8}$=

\	e_1	e_2	e_3
(v_1,q_1)	⟨[0.2,0.8], [0.1,0.7], [0.4,0.8]⟩	⟨[0.1,0.8], [0.5,0.7], [0.3,0.4]⟩	⟨[0.7,0.9], [0.2,0.8], [0.3,0.6]⟩
(v_1,q_2)	[0.1,0.4], [0.5,0.8], [0.7,0.8]	⟨[0.1,0.8], [0.4,0.7], [0.2,0.6]⟩	⟨[0.4,0.7], [0.2,0.5], [0.1,0.7]⟩
(v_2,q_1)	⟨[0.3,0.6], [0.2,0.7], [0.5,0.8]⟩	⟨[0.5,0.8], [0.4,0.9], [0.2,0.7]⟩	⟨[0.1,0.8], [0.1,0.4], [0.3,0.6]⟩
(v_2,q_2)	⟨[0.4,0.6], [0.2,0.9], [0.5,0.7]⟩	⟨[0.1,0.2], [0.2,0.5], [0.4,0.7]⟩	⟨[0.5,0.6], [0.3,0.6], [0.2,0.7]⟩
(v_3,q_1)	⟨[0.6,0.8], [0.4,0.5], [0.3,0.5]⟩	⟨[0.6,0.8], [0.4,0.5], [0.3,0.5]⟩	⟨[0.6,0.8], [0.4,0.5], [0.3,0.5]⟩
(v_3,q_2)	⟨[0.3,0.7], [0.2,0.4], [0.1,0.8]⟩	⟨[0.3,0.7], [0.2,0.4], [0.1,0.8]⟩	⟨[0.3,0.7], [0.2,0.4], [0.1,0.8]⟩
(v_4,q_1)	⟨[0.1,0.5], [0.3,0.7], [0.2,0.8]⟩	⟨[0.1,0.4], [0.2,0.5], [0.3,0.7]⟩	⟨[0.4,0.6], [0.2,0.7], [0.3,0.6]⟩
(v_4,q_2)	⟨[0.4,0.8], [0.4,0.6], [0.2,0.8]⟩	⟨[0.1,0.6], [0.4,0.5], [0.5,0.7]⟩	⟨[0.4,0.8], [0.8,0.9], [0.3,0.7]⟩

$_{3\times 8}$

Step 2. Convert IV-Q-NSM to IV-Q-FSM using definition.

$[\hat{P}_{\mathfrak{Q}_{\bar{A}}}]_{3\times 8}$ =

\	e_1	e_2	e_3
(v_1,q_1)	⟨[0.21,0.69]⟩	⟨[0.54,0.76]⟩	⟨[0.43,0.65]⟩
(v_1,q_2)	⟨[0.43,0.64]⟩	⟨[0.24,0.79]⟩	⟨[0.24,0.74]⟩
(v_2,q_1)	⟨[0.21,0.78]⟩	⟨[0.57,0.86]⟩	⟨[0.35,0.76]⟩
(v_2,q_2)	⟨[0.34,0.89]⟩	⟨[0.34,0.78]⟩	⟨[0.25,0.68]⟩
(v_3,q_1)	⟨[0.56,0.80]⟩	⟨[0.53,0.86]⟩	⟨[0.45,0.79]⟩
(v_3,q_2)	⟨[0.46,0.84]⟩	⟨[0.32,0.69]⟩	⟨[0.24,0.78]⟩
(v_4,q_1)	⟨[0.58,0.94]⟩	⟨[048,0.72]⟩	⟨[0.21,0.74]⟩
(v_4,q_2)	⟨[0.36,0.78]⟩	⟨[0.43,0.79]⟩	⟨[0.68,0.91]⟩

$_{3\times 8}$

Step 3. Convert IV-Q-FSM to SV-Q-FSM using the following formal $\Psi_{\bar{A}_{i,j}} = \frac{\Psi^{l,t}_{\bar{A}_{i,j}} + \Psi^{u,t}_{\bar{A}_{i,j}}}{2}$

$[\hat{P}_{\mathfrak{Q}_{\bar{A}}}]_{3\times 8}$ =

\	e_1	e_2	e_3
(v_1,q_1)	⟨0.45⟩	⟨0.65⟩	⟨0.54⟩
(v_1,q_2)	⟨0.53⟩	⟨0.52⟩	⟨0.49⟩
(v_2,q_1)	⟨0.50⟩	⟨0.72⟩	⟨0.60⟩
(v_2,q_2)	⟨0.61⟩	⟨0.56⟩	⟨0.47⟩
(v_3,q_1)	⟨0.68⟩	⟨0.70⟩	⟨0.62⟩
(v_3,q_2)	⟨0.65⟩	⟨0.51⟩	⟨0.51⟩
(v_4,q_1)	⟨0.76⟩	⟨0.6⟩	⟨0.48⟩
(v_4,q_2)	⟨0.57⟩	⟨0.61⟩	⟨0.80⟩

$_{3\times 8}$

Step 4. Find comparison matrix based on given definition above.

$[\hat{P}_{\mathfrak{Q}_{\bar{A}}}]_{3\times 8}$ =

\	e_1	e_2	e_3
(v_1,q_1)	1	5	4
(v_1,q_2)	2	1	2
(v_2,q_1)	1	7	5
(v_2,q_2)	4	2	0
(v_3,q_1)	6	6	6
(v_3,q_2)	5	0	2
(v_4,q_1)	7	3	1
(v_4,q_2)	3	4	7

$_{3\times 8}$

Step 5. Find the score values M_t for all $(v,q) \in V \times Q$, where $t = 1,2, \dots n$.

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$$\begin{bmatrix} (v_i, q_j) & M_i \\ (v_1, q_1) & M_1 = 10 \\ (v_1, q_2) & M_2 = 5 \\ (v_2, q_1) & M_3 = 13 \\ (v_2, q_2) & M_4 = 6 \\ (v_3, q_1) & M_5 = 18 \\ (v_3, q_2) & M_6 = 7 \\ (v_4, q_1) & M_7 = 11 \\ (v_4, q_2) & M_8 = 14 \end{bmatrix}$$

Step 6. The decision taken is to pick the highest score of M_t,i.e. Decision taken= $max\{M_t\}$. Here the Decision taken showing in the following table:

Table 1: Comparison of the results obtained from the above algorithm

Patients	Degree of (v, q_1)	Degree of (v, q_2)	Comparison	Result
v_1	10	5	$q_1 > q_2$	Yes
v_2	13	6	$q_1 > q_2$	Yes
v_3	18	7	$q_1 > q_2$	Yes
v_4	11	14	$q_1 < q_2$	No

By looking at Table 1. Above, which contains a comparison between the results obtained, it is clear that all patientsv_1, v_2, v_3 are infected except the patient v_4.

Based on the results obtained from Algorithm 1, we can construct the following statistical chart, which illustrates the variation in the severity of symptoms among the patients examined in the doctor's clinic.

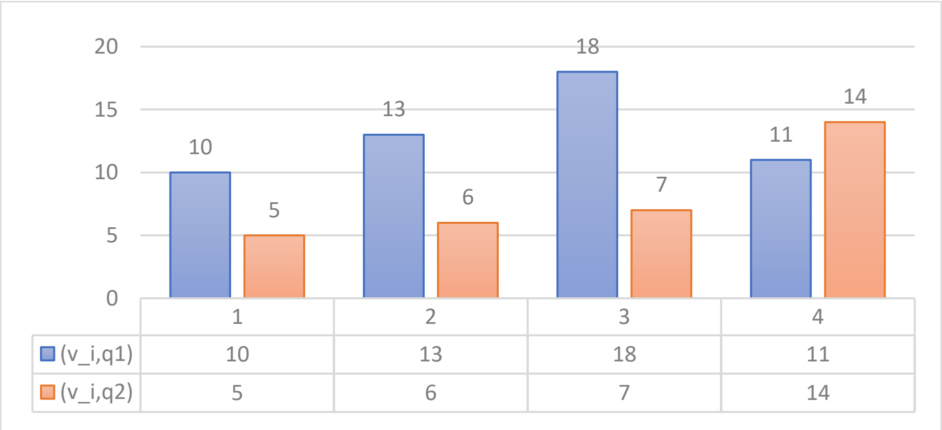

Figure 1. Statistical chart, which shows the disparity in the severity of the injury among those examined in the doctor's clinic.

5. Conclusion

In this work, we present a new framework based on the previously established concept of Interval-Valued Q-Neutrosophic Soft Sets (IV-Q-NSS) within a matrix system. First, we introduce the concept of Interval-Valued Q-Neutrosophic Soft Matrix (IV-Q-NSM) and clarify its properties. Building on this foundation, we outline several theories that demonstrate the mechanisms underlying the proposed tools. Furthermore, we highlight the significance of this concept in addressing real-life challenges by developing a multi-step algorithm designed to tackle decision-making problems, particularly in the medical field. Finally, we recommend avenues for future research, suggesting that these tools could be further enhanced through integration with various mathematical methodologies, as discussed in works [35-37].

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