Time Factor's Impact On Fuzzy Soft Expert Sets

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Abstract

In this study, I introduce time-fuzzy soft expert set (T-FSES) as an extension of fuzzy soft set. I will also define and investigate the features of its main operations (complement, union intersection, AND and OR). Finally, I'll apply this approach to decision-making difficulties.

Keywords: soft set; Fuzzy soft set;Time-fuzzy soft expert set

1 Introduction

The majority of problems in engineering, medical research, economics, and the environment are fraught with uncertainty. Molodtsov[1]] introduced the notion of soft set theory as a tool in math for coping with such uncertainty. Following Molodtsov's work,,[3] Maji et al.[4] and Maji et al.[5] researched several soft set operations and applications. Also Maji et al.[8] they presented the notion of fuzzy soft set as a more broad concept, as well as a combination of fuzzy set and soft set, and investigated its features. Roy and Maji[9] also applied this idea to handle decision-making challenges. Recently, various scholars have begun studying the properties and applications of soft set theory as in the research,[11],[21].[29] Furthermore, in 2010 Çağman et al.[26] established the notion of fuzzy parameterized fuzzy soft set and its operations.In addition, the fpfs-aggregation operator is used to create the fpfs-decision making technique, which allows for more efficient decision processes.[22] proposed the notion of soft expert sets and fuzzy soft expert sets, which allow users to get the views of all experts in one model without any procedures.Hazaymeh. A[7] discusses fuzzy parameterized fuzzy soft expert sets, which offer a membership value for each parameter in a collection of parameters and are an extension of fuzzy soft expert sets. Wang[30]showed that in many real situations, immediate sensory data is insufficient for decision making.[10] provide an overview of generalized fuzzy soft expert set. Recently, various scholars have begun studying the properties and applications of soft set theory. Some topics in algebraic structures are extended by fuzzy soft sets, neutrosophic, or even plithogenic logical sets, as in the research,[19],[14],[27] and there are also studies in fuzzy tapology and neutrosophic fuzzy topology. For more details about neutrosophic topology, see,[2].[6] Additionally, researchers introduced using a neutrosophic fuzzy soft set to solve decision-making problems, like in,[16],[28] other researchers introduced topics in complex fuzzy as,[31] we are looking to integrate time fuzzy soft set and fuzzy soft set with new concepts as in the works a[17],[23],[20],[24],[25],[15].[35] Through integrating our research with different disciplines, we may provide novel and significant subjects. For instance, we may incorporate the study of fuzzy soft sets with the findings of scholars like,[32],[33],[34].[36] These generalizations synthesized and broadened the previously established findings, resulting in more applicable outcomes.In addition, there are many works that have discussed broad applications, including:[37–42] Enriching the state with knowledge about prior actions and events can help you distinguish between situations that might otherwise look identical,

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allowing you to make accurate judgments while also learning the proper options. Furthermore, knowledge of the past can eliminate the need for unrealistic sensors, such as knowing your exact location in a maze. Using historical information as part of the state representation provides us with important information to assist us in making better judgments in situations when temporal value is not taken into account, resulting in less accurate decision-making. If we want to take the views of more than one time (period), we must perform various operations such as union, intersection, etc. For a solution to this problem, we take a collection of time intervals, generalize it into what we call a time-fuzzy soft expert set (T-FSES) that is induced by,[18] investigate some of its features, and apply this notion to a decision-making problem. It is critical to understand the history of the parameters under consideration in order to ensure the credibility of the information provided by specialists. The experts' previous experiences are gathered in the number of periods (years, months, etc.) in which they are involved in a certain decision-making circumstance, and by looking at the time component, individuals are more confident in the conclusion that they make. We must examine the influence of time on fuzzy soft set applications, not only for the present period, but also for the past and future periods (forecasting information). In this paper, we will present the notion of time-fuzzy soft expert set,Which is more effective and valuable, as we will see and the decisions made will be more precise, this means we will take the component time value of the information in our consideration when we are making decision. We will also define and investigate the attributes of its basic operations, which are complement, union and intersection. Finally, we'll apply this approach to decision-making difficulties.

2 Preliminaries

In this part, we cover several fundamental concepts in soft set theory. Molodtsov[1] defined soft sets as follows over U: Let U represent the universe set and P the set of parameters. $P(U)$ signifies the power set of U and $J \subseteq P$.

Definition 2.1. [1] Think about this mapping

$$J : M \to P(U).$$

Any A pair (J, M) is considered a *soft set* over U. In other terms, a soft set over U is a parameterized collection of subsets of the universe set U. For $\delta \in J, J(\delta)$ can be viewed as the set of δ-approximate members of the soft set (J, M).

Definition 2.2. [8] Let U be the initial universal set, and P be the set of parameters. Let I^U be the power set of all fuzzy subsets of U. Let $J \subseteq P$, and F be the mapping

$$F : A \to I^U.$$

A pair (J, P) is known as a *fuzzy soft set* over U.

Definition 2.3. [8] Regarding two fuzzy soft sets (J, M) and (K, N) over U, (J, M) is known as a fuzzy soft subset of. (K, N) if

1. $M \subset N$ and
2. $\forall \delta \in J, J(\delta)$ is fuzzy subset of $K(\delta)$.

The association is represented by $(F,\ A) \tilde{\subset} (K, N)$. In this situation, (K, N) is known as a fuzzy soft superset of. (J, M).

Definition 2.4. [8] $(J, M)^c$ represents the complement of a fuzzy soft set (J, M), which has been described by $(J, M)^c = (J^c, \rceil A)$ where $J^c : \rceil A \to P(U)$ is a mapping provided by

$$J^c(\Gamma) = c(J(\rceil\Gamma)), \forall \Gamma \in \rceil M.$$

c describes any fuzzy complement.

Definition 2.5. [8] If (J, M) and (K, N) are two fuzzy soft sets then (J, M) AND (K, N) denoted by $(J, M) \wedge (K, N)$ is defined by

$$(J, M) \wedge (K, N) = (C, M \times N)$$

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such that $C(\Gamma,\lambda) = t(J(\Gamma), K(\lambda)), \forall(\Gamma,\lambda) \in J \times N$, where t is any t-norm.

Definition 2.6. [8] If (J, M) and (K, N) are two fuzzy soft sets then (J, M) OR (K, N) denoted by $(J, M) \vee (K, N)$ is defined by

$$(J, M) \vee (K, N) = (O, M \times N)$$

such that $O(\Gamma,\lambda) = s(J(\Gamma), G(\lambda)), \forall(\Gamma,\lambda) \in M \times N$, where s is any s-norm.

Definition 2.7. [8] The union of two fuzzy soft sets (J, M) and (K, N) over a common universe U is the fuzzy soft set (H, C) where $C = M \cup N$, and $\forall\delta \in C$,

$$H(\delta) = \begin{cases} J(\delta), & if \quad \delta \in M - N, \\ G(\delta), & if \quad \delta \in N - M, \\ s(J(\delta), G(\delta)), & if \quad \delta \in M \cup N. \end{cases}$$

Where s is any s-norm.

Definition 2.8. [8] The intersection of two fuzzy soft sets (J, M) and (K, N) over a common universe U is the fuzzy soft set (H, C) where $C = M \cup N$, and $\forall\delta \in R$,

$$H(\delta) = \begin{cases} J(\delta), & if \quad \delta \in M - N, \\ G(\delta), & if \quad \delta \in N - M, \\ s(J(\delta), G(\delta)), & if \quad \delta \in M \cap N. \end{cases}$$

Definition 2.9. .[12] Let U be a set of universes, P a set of parameters, X a set of experts (agents). Let $O = \{o_1, o_2, ..., o_n\}$ be a set of opinions, $Z = E \times X \times O$ and $M \subseteq Z$. A pair $(F,\ M)$ is called a soft expert set over U, where F is a mapping given by

$$F \ : \ A \ \rightarrow \ P(U)$$

where $P(U)$ denoted the power set of U.

Definition 2.10. .[13] Let U be a set of universes, P a set of parameters, X a set of experts (agents). Let O be a set of opinions, $Z = E \times X \times O$ and $A \subseteq Z$. A pair (J, M) is called a fuzzy soft expert set over U, where F is a mapping given by

$$J : M \rightarrow I^U$$

Where I^U denotes all fuzzy subsets of U.

Definition 2.11. Let U be an initial universal set and let P be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U, let $J \subseteq P$ and T be a set of time where $T = \{t_1, t_2, ..., t_n\}$. A collection of pairs $(F, E)_t \, \forall \, t \in T$ is called a *time-fuzzy soft set* $T - FSS$ over U where J_t is a mapping given by

$$F_t : A \rightarrow I^U.$$

3 Time Fuzzy Soft Expert Set (T-FSES)

The definition of a time-fuzzy soft expert set and its fundamental characteristics are presented in this section. We must take into account the component time value of the information when making judgments since doing so will result in more accurate decisions because some or all of the parameters have a time value for prior knowledge. Additionally, we define and examine the features of its fundamental operations—union, complement, intersection, AND, and OR. Lastly, we provide speculative uses of this idea in decision-making situations.

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3.1 Main Definition

This section defines the term time-fuzzy soft expert set and outlines its fundamental characteristics.

Definition 3.1. Define U as the universe, E as a set of parameters, X as a group of experts, and $O = \{o_1, o_2, ..., o_n\}$ as a collection of viewpoints. For Z to be equal to $E \times X \times O$, $A \subseteq Z$. Let T be a collection of times where $T = \{t_1, t_2, ..., t_n\}$, and let I^U represent the power set of all fuzzy subsets of U. *Atime-fuzzy soft expert set* T-FSES over U is a set of pairings $(F, A)_t \forall t_i \in T$, where F is a mapping provided by

$$F_t : A \to I^U.$$

Example 3.2. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of universe, $E = \{e_1, e_2, e_3\}$ a set of parameters and $T = \{t_1, t_2, t_3\}$ be a set of time and $X = \{m, n, r\}$ a set of experts. Define a function

$$F_t : A \to I^U.$$

as follows:

$$F_1(e_1, m, 1) = \left\{\frac{u_1{}^{t_1}}{0.8}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.3}\right\}, F_1(e_1, n, 1) = \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.9}, \frac{u_3{}^{t_1}}{0.6}, \frac{u_4{}^{t_1}}{0.5}\right\},$$
$$F_1(e_1, r, 1) = \left\{\frac{u_1{}^{t_1}}{0.7}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.3}, \frac{u_4{}^{t_1}}{0.7}\right\}, F_1(e_2, m, 1) = \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.1}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.8}\right\},$$
$$F_1(e_2, n, 1) = \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.5}\right\}, F_1(e_2, r, 1) = \left\{\frac{u_1{}^{t_1}}{0.2}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.7}, \frac{u_4{}^{t_1}}{0.3}\right\},$$
$$F_1(e_3, m, 1) = \left\{\frac{u_1{}^{t_1}}{0.2}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.9}, \frac{u_4{}^{t_1}}{0.6}\right\}, F_1(e_3, n, 1) = \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.6}, \frac{u_4{}^{t_1}}{0.7}\right\},$$
$$F_1(e_3, r, 1) = \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.1}, \frac{u_4{}^{t_1}}{0.5}\right\}, F_2(e_1, m, 1) = \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.8}, \frac{u_3{}^{t_2}}{0.2}, \frac{u_4{}^{t_2}}{0.4}\right\},$$
$$F_2(e_1, n, 1) = \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.9}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.8}\right\}, F_2(e_1, r, 1) = \left\{\frac{u_1{}^{t_2}}{0.9}, \frac{u_2{}^{t_2}}{0.2}, \frac{u_3{}^{t_2}}{0.3}, \frac{u_4{}^{t_2}}{0.3}\right\},$$
$$F_2(e_2, m, 1) = \left\{\frac{u_1{}^{t_2}}{0.2}, \frac{u_2{}^{t_2}}{0.9}, \frac{u_3{}^{t_2}}{0.5}, \frac{u_4{}^{t_2}}{0.5}\right\}, F_2(e_2, n, 1) = \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.6}, \frac{u_3{}^{t_2}}{0.2}, \frac{u_4{}^{t_2}}{0.6}\right\},$$
$$F_2(e_2, r, 1) = \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.4}\right\}, F_2(e_3, m, 1) = \left\{\frac{u_1{}^{t_2}}{0.9}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.1}, \frac{u_4{}^{t_2}}{0.7}\right\},$$
$$F_2(e_3, n, 1) = \left\{\frac{u_1{}^{t_2}}{0.8}, \frac{u_2{}^{t_2}}{0.6}, \frac{u_3{}^{t_2}}{0.1}, \frac{u_4{}^{t_2}}{0.5}\right\}, F_2(e_3, r, 1) = \left\{\frac{u_1{}^{t_2}}{0.2}, \frac{u_2{}^{t_2}}{0.9}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.7}\right\},$$
$$F_3(e_1, m, 1) = \left\{\frac{u_1{}^{t_3}}{0.2}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.3}\right\}, F_3(e_1, n, 1) = \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.9}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.5}\right\},$$
$$F_3(e_1, r, 1) = \left\{\frac{u_1{}^{t_3}}{0.6}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.7}, \frac{u_4{}^{t_3}}{0.8}\right\}, F_3(e_2, m, 1) = \left\{\frac{u_1{}^{t_3}}{0.5}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.7}, \frac{u_4{}^{t_3}}{0.9}\right\},$$
$$F_3(e_2, n, 1) = \left\{\frac{u_1{}^{t_3}}{0.8}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.7}\right\}, F_3(e_2, r, 1) = \left\{\frac{u_1{}^{t_3}}{0.1}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.5}, \frac{u_4{}^{t_3}}{0.8}\right\},$$
$$F_3(e_3, m, 1) = \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.1}, \frac{u_4{}^{t_3}}{0.8}\right\}, F_3(e_3, n, 1) = \left\{\frac{u_1{}^{t_3}}{0.6}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.3}, \frac{u_4{}^{t_3}}{0.9}\right\},$$
$$F_3(e_3, r, 1) = \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.5}, \frac{u_4{}^{t_3}}{0.6}\right\}, F_1(e_1, m, 0) = \left\{\frac{u_1{}^{t_1}}{0.3}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.7}, \frac{u_4{}^{t_1}}{0.8}\right\},$$
$$F_1(e_1, n, 0) = \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.3}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.4}\right\}, F_1(e_1, r, 0) = \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.3}\right\},$$
$$F_1(e_2, m, 0) = \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.3}\right\}, F_1(e_2, n, 0) = \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.9}, \frac{u_4{}^{t_1}}{0.6}\right\},$$
$$F_1(e_2, r, 0) = \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.3}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.1}\right\}, F_1(e_3, m, 0) = \left\{\frac{u_1{}^{t_1}}{0.7}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.3}\right\},$$
$$F_1(e_3, n, 0) = \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.2}\right\}, F_1(e_3, r, 0) = \left\{\frac{u_1{}^{t_1}}{0.8}, \frac{u_2{}^{t_1}}{0.2}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.4}\right\},$$
$$F_2(e_1, m, 0) = \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.5}\right\}, F_2(e_1, n, 0) = \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.2}, \frac{u_3{}^{t_2}}{0.5}, \frac{u_4{}^{t_2}}{0.4}\right\},$$
$$F_2(e_1, r, 0) = \left\{\frac{u_1{}^{t_2}}{0.3}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.8}, \frac{u_4{}^{t_2}}{0.8}\right\}, F_2(e_2, m, 0) = \left\{\frac{u_1{}^{t_2}}{0.9}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.6}\right\},$$

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$$F_2\left(e_2,n,0\right)=\left\{\frac{u_1{}^{t_2}}{0.4},\frac{u_2{}^{t_2}}{0.5},\frac{u_3{}^{t_2}}{0.9},\frac{u_4{}^{t_2}}{0.4}\right\},F_2\left(e_2,r,0\right)=\left\{\frac{u_1{}^{t_2}}{0.6},\frac{u_2{}^{t_2}}{0.7},\frac{u_3{}^{t_2}}{0.4},\frac{u_4{}^{t_2}}{0.1}\right\},$$

$$F_2\left(e_3,m,0\right)=\left\{\frac{u_1{}^{t_2}}{0.2},\frac{u_2{}^{t_2}}{0.7},\frac{u_3{}^{t_2}}{0.8},\frac{u_4{}^{t_2}}{0.2}\right\},F_2\left(e_3,n,0\right)=\left\{\frac{u_1{}^{t_2}}{0.4},\frac{u_2{}^{t_2}}{0.5},\frac{u_3{}^{t_2}}{0.7},\frac{u_4{}^{t_2}}{0.6}\right\},$$

$$F_2\left(e_3,r,0\right)=\left\{\frac{u_1{}^{t_2}}{0.7},\frac{u_2{}^{t_2}}{0.1},\frac{u_3{}^{t_2}}{0.3},\frac{u_4{}^{t_2}}{0.3}\right\},F_3\left(e_1,m,0\right)=\left\{\frac{u_1{}^{t_3}}{0.6},\frac{u_2{}^{t_3}}{0.8},\frac{u_3{}^{t_3}}{0.2},\frac{u_4{}^{t_3}}{0.4}\right\},$$

$$F_3\left(e_1,n,0\right)=\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.5},\frac{u_3{}^{t_3}}{0.4},\frac{u_4{}^{t_3}}{0.6}\right\},F_3\left(e_1,r,0\right)=\left\{\frac{u_1{}^{t_3}}{0.5},\frac{u_2{}^{t_3}}{0.7},\frac{u_3{}^{t_3}}{0.2},\frac{u_4{}^{t_3}}{0.5}\right\},$$

$$F_3\left(e_2,m,0\right)=\left\{\frac{u_1{}^{t_3}}{0.3},\frac{u_2{}^{t_3}}{0.7},\frac{u_3{}^{t_3}}{0.4},\frac{u_4{}^{t_3}}{0.1}\right\},F_3\left(e_2,n,0\right)=\left\{\frac{u_1{}^{t_3}}{0.1},\frac{u_2{}^{t_3}}{0.5},\frac{u_3{}^{t_3}}{0.6},\frac{u_4{}^{t_3}}{0.4}\right\},$$

$$F_3\left(e_2,r,0\right)=\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.3},\frac{u_3{}^{t_3}}{0.3},\frac{u_4{}^{t_3}}{0.2}\right\},F_3\left(e_3,m,0\right)=\left\{\frac{u_1{}^{t_3}}{0.4},\frac{u_2{}^{t_3}}{0.6},\frac{u_3{}^{t_3}}{0.8},\frac{u_4{}^{t_3}}{0.3}\right\},$$

$$F_3\left(e_3,n,0\right)=\left\{\frac{u_1{}^{t_3}}{0.3},\frac{u_2{}^{t_3}}{0.4},\frac{u_3{}^{t_3}}{0.7},\frac{u_4{}^{t_3}}{0.2}\right\},F_3\left(e_3,r,0\right)=\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.2},\frac{u_3{}^{t_3}}{0.6},\frac{u_4{}^{t_3}}{0.5}\right\}.$$

Next, we may determine the time-fuzzy soft expert sets $(F,E)_t$, which comprise the subsequent set of approximations:

$$\begin{aligned}(F,E)_t=\Bigg\{&\left(\left(e_1,m,1\right),\left\{\frac{u_1{}^{t_1}}{0.8},\frac{u_2{}^{t_1}}{0.5},\frac{u_3{}^{t_1}}{0.4},\frac{u_4{}^{t_1}}{0.3}\right\}\right),\left(\left(e_1,n,1\right),\left\{\frac{u_1{}^{t_1}}{0.6},\frac{u_2{}^{t_1}}{0.9},\frac{u_3{}^{t_1}}{0.6},\frac{u_4{}^{t_1}}{0.5}\right\}\right)\\ &\vdots \qquad\qquad \ddots\\ &\left(\left(e_3,n,0\right),\left\{\frac{u_1{}^{t_3}}{0.3},\frac{u_2{}^{t_3}}{0.4},\frac{u_3{}^{t_3}}{0.7},\frac{u_4{}^{t_3}}{0.2}\right\}\right),\left(\left(e_3,r,0\right),\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.2},\frac{u_3{}^{t_3}}{0.6},\frac{u_4{}^{t_3}}{0.5}\right\}\right)\Bigg\}.\end{aligned}$$

Definition 3.3. For two T-FSES's $(F,A)_t$ and $(G,B)_t$ over U, $(F,A)_t$ is denoted by a T-FSES subset of $(G,B)_t$ if

1. $B\subseteq A$,
2. $\forall t\in T,\delta\in B$, $G_t\left(\delta\right)$ is time fuzzy soft expert subset of $F_t\left(\delta\right)$.

Definition 3.4. If $(F,A)_t$ is a T-FSES subset of $(G,A)_t$ and $(G,A)_t$ is a T-FSES subset of $(F,A)_t$, then two T-FSES's $(F,A)_t$ and $(G,B)_t$ over U are said to be *equal*.

Example 3.5. Think about Example 3.2, where

$$\begin{aligned}A=\Big\{&(e_1,m,1)_{t_1},(e_2,m,1)_{t_1},(e_2,n,1)_{t_3},(e_2,r,1)_{t_3},(e_2,n,0)_{t_2},(e_2,r,0)_{t_2},(e_2,r,0)_{t_3},\\&(e_3,m,0)_{t_3}\Big\},\end{aligned}$$

$$B=\Big\{(e_1,m,1)_{t_1},(e_2,m,1)_{t_1},(e_2,n,1)_{t_3},(e_2,r,1)_{t_3},(e_2,n,0)_{t_2}\Big\}.$$

Clearly $B\subset A$. Now, let $(G,B)_t$ and $(F,A)_t$ be defined as follows:

$$\begin{aligned}(F,E)_t=\Bigg\{&\left(\left(e_1,m,1\right),\left\{\frac{u_1{}^{t_1}}{0.8},\frac{u_2{}^{t_1}}{0.5},\frac{u_3{}^{t_1}}{0.4},\frac{u_4{}^{t_1}}{0.3}\right\}\right),\left(\left(e_2,m,1\right),\left\{\frac{u_1{}^{t_1}}{0.6},\frac{u_2{}^{t_1}}{0.1},\frac{u_3{}^{t_1}}{0.4},\frac{u_4{}^{t_1}}{0.8}\right\}\right),\\&\left(\left(e_2,n,1\right),\left\{\frac{u_1{}^{t_3}}{0.8},\frac{u_2{}^{t_3}}{0.6},\frac{u_3{}^{t_3}}{0.2},\frac{u_4{}^{t_3}}{0.7}\right\}\right),\left(\left(e_2,r,1\right),\left\{\frac{u_1{}^{t_3}}{0.1},\frac{u_2{}^{t_3}}{0.8},\frac{u_3{}^{t_3}}{0.5},\frac{u_4{}^{t_3}}{0.8}\right\}\right),\\&\left(\left(e_2,n,0\right),\left\{\frac{u_1{}^{t_2}}{0.4},\frac{u_2{}^{t_2}}{0.5},\frac{u_3{}^{t_2}}{0.9},\frac{u_4{}^{t_2}}{0.4}\right\}\right),\left(\left(e_2,r,0\right)\left\{\frac{u_1{}^{t_2}}{0.6},\frac{u_2{}^{t_2}}{0.7},\frac{u_3{}^{t_2}}{0.4},\frac{u_4{}^{t_2}}{0.1}\right\}\right),\end{aligned}$$

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$$\left((e_2,r,0),\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.3},\frac{u_3{}^{t_3}}{0.3},\frac{u_4{}^{t_3}}{0.2}\right\}\right),\left((e_3,m,0),\left\{\frac{u_1{}^{t_3}}{0.4},\frac{u_2{}^{t_3}}{0.6},\frac{u_3{}^{t_3}}{0.8},\frac{u_4{}^{t_3}}{0.3}\right\}\right)\Bigg\},$$

$$(G,E)_t=\Bigg\{\left((e_1,m,1),\left\{\frac{u_1{}^{t_1}}{0.6},\frac{u_2{}^{t_1}}{0.2},\frac{u_3{}^{t_1}}{0.1},\frac{u_4{}^{t_1}}{0.0}\right\}\right),\left((e_2,m,1),\left\{\frac{u_1{}^{t_1}}{0.5},\frac{u_2{}^{t_1}}{0.1},\frac{u_3{}^{t_1}}{0.4},\frac{u_4{}^{t_1}}{0.7}\right\}\right),$$
$$\left((e_2,n,1),\left\{\frac{u_1{}^{t_3}}{0.8},\frac{u_2{}^{t_3}}{0.5},\frac{u_3{}^{t_3}}{0.2},\frac{u_4{}^{t_3}}{0.6}\right\}\right),\left((e_2,r,1),\left\{\frac{u_1{}^{t_3}}{0.0},\frac{u_2{}^{t_3}}{0.6},\frac{u_3{}^{t_3}}{0.5},\frac{u_4{}^{t_3}}{0.3}\right\}\right),$$
$$\left((e_2,n,0),\left\{\frac{u_1{}^{t_2}}{0.4},\frac{u_2{}^{t_2}}{0.3},\frac{u_3{}^{t_2}}{0.7},\frac{u_4{}^{t_2}}{0.2}\right\}\right)\Bigg\}.$$

We can easily verify that $(G,E)_t \subseteq (F,E)_t$.

Definition 3.6. An *agree-TFSES* $((F,A)_t)_1$ over U is a T-FSES subset of $(F,A)_t$,and has the following definition:

$$((F,A)_t)_1=\left\{F^t{}_1(\alpha):\alpha\in E\times X\times\{1\}\right\}.$$

Definition 3.7. A *disagree-TFSES* $((F,A)_t)_0$ over U is a T-FSES subset of $(F,A)_t$, and has the following definition:

$$((F,A)_t)_0=\left\{F^t{}_0(\alpha):\alpha\in E\times X\times\{0\}\right\}.$$

Example 3.8. Think about Example 3.2. Then the agree-time fuzzy soft expert set $((F,A)_t)_1$ over U is

$$((F,A)_t)_1=\Bigg\{\left((e_1,m,1),\left\{\frac{u_1{}^{t_1}}{0.8},\frac{u_2{}^{t_1}}{0.5},\frac{u_3{}^{t_1}}{0.4},\frac{u_4{}^{t_1}}{0.3}\right\}\right),\left((e_1,n,1),\left\{\frac{u_1{}^{t_1}}{0.6},\frac{u_2{}^{t_1}}{0.9},\frac{u_3{}^{t_1}}{0.6},\frac{u_4{}^{t_1}}{0.5}\right\}\right),$$
$$\left((e_1,r,1),\left\{\frac{u_1{}^{t_1}}{0.7},\frac{u_2{}^{t_1}}{0.6},\frac{u_3{}^{t_1}}{0.3},\frac{u_4{}^{t_1}}{0.7}\right\}\right),\left((e_2,m,1),\left\{\frac{u_1{}^{t_1}}{0.6},\frac{u_2{}^{t_1}}{0.1},\frac{u_3{}^{t_1}}{0.4},\frac{u_4{}^{t_1}}{0.8}\right\}\right),$$
$$\left((e_2,n,1),\left\{\frac{u_1{}^{t_1}}{0.5},\frac{u_2{}^{t_1}}{0.7},\frac{u_3{}^{t_1}}{0.2},\frac{u_4{}^{t_1}}{0.5}\right\}\right),\left((e_2,r,1),\left\{\frac{u_1{}^{t_1}}{0.2},\frac{u_2{}^{t_1}}{0.6},\frac{u_3{}^{t_1}}{0.7},\frac{u_4{}^{t_1}}{0.3}\right\}\right),$$
$$\left((e_3,m,1),\left\{\frac{u_1{}^{t_1}}{0.2},\frac{u_2{}^{t_1}}{0.4},\frac{u_3{}^{t_1}}{0.9},\frac{u_4{}^{t_1}}{0.6}\right\}\right),\left((e_3,n,1),\left\{\frac{u_1{}^{t_1}}{0.6},\frac{u_2{}^{t_1}}{0.5},\frac{u_3{}^{t_1}}{0.6},\frac{u_4{}^{t_1}}{0.7}\right\}\right),$$
$$\left((e_3,r,1),\left\{\frac{u_1{}^{t_1}}{0.4},\frac{u_2{}^{t_1}}{0.7},\frac{u_3{}^{t_1}}{0.1},\frac{u_4{}^{t_1}}{0.5}\right\}\right),\left((e_1,m,1),\left\{\frac{u_1{}^{t_2}}{0.4},\frac{u_2{}^{t_2}}{0.8},\frac{u_3{}^{t_2}}{0.2},\frac{u_4{}^{t_2}}{0.4}\right\}\right),$$
$$\left((e_1,n,1),\left\{\frac{u_1{}^{t_2}}{0.6},\frac{u_2{}^{t_2}}{0.9},\frac{u_3{}^{t_2}}{0.4},\frac{u_4{}^{t_2}}{0.8}\right\}\right),\left((e_1,r,1),\left\{\frac{u_1{}^{t_2}}{0.9},\frac{u_2{}^{t_2}}{0.2},\frac{u_3{}^{t_2}}{0.3},\frac{u_4{}^{t_2}}{0.3}\right\}\right),$$
$$\left((e_2,m,1),\left\{\frac{u_1{}^{t_2}}{0.2},\frac{u_2{}^{t_2}}{0.9},\frac{u_3{}^{t_2}}{0.5},\frac{u_4{}^{t_2}}{0.5}\right\}\right),\left((e_2,n,1),\left\{\frac{u_1{}^{t_2}}{0.7},\frac{u_2{}^{t_2}}{0.6},\frac{u_3{}^{t_2}}{0.2},\frac{u_4{}^{t_2}}{0.6}\right\}\right),$$
$$\left((e_2,r,1),\left\{\frac{u_1{}^{t_2}}{0.5},\frac{u_2{}^{t_2}}{0.4},\frac{u_3{}^{t_2}}{0.7},\frac{u_4{}^{t_2}}{0.4}\right\}\right),\left((e_3,m,1),\left\{\frac{u_1{}^{t_2}}{0.9},\frac{u_2{}^{t_2}}{0.4},\frac{u_3{}^{t_2}}{0.1},\frac{u_4{}^{t_2}}{0.7}\right\}\right),$$
$$\left((e_3,n,1),\left\{\frac{u_1{}^{t_2}}{0.8},\frac{u_2{}^{t_2}}{0.6},\frac{u_3{}^{t_2}}{0.1},\frac{u_4{}^{t_2}}{0.5}\right\}\right),\left((e_3,r,1),\left\{\frac{u_1{}^{t_2}}{0.2},\frac{u_2{}^{t_2}}{0.9},\frac{u_3{}^{t_2}}{0.6},\frac{u_4{}^{t_2}}{0.7}\right\}\right),$$
$$\left((e_1,m,1),\left\{\frac{u_1{}^{t_3}}{0.2},\frac{u_2{}^{t_3}}{0.7},\frac{u_3{}^{t_3}}{0.8},\frac{u_4{}^{t_3}}{0.3}\right\}\right),\left((e_1,n,1),\left\{\frac{u_1{}^{t_3}}{0.4},\frac{u_2{}^{t_3}}{0.9},\frac{u_3{}^{t_3}}{0.6},\frac{u_4{}^{t_3}}{0.5}\right\}\right),$$
$$\left((e_1,r,1),\left\{\frac{u_1{}^{t_3}}{0.6},\frac{u_2{}^{t_3}}{0.5},\frac{u_3{}^{t_3}}{0.7},\frac{u_4{}^{t_3}}{0.8}\right\}\right),\left((e_2,m,1),\left\{\frac{u_1{}^{t_3}}{0.5},\frac{u_2{}^{t_3}}{0.4},\frac{u_3{}^{t_3}}{0.7},\frac{u_4{}^{t_3}}{0.9}\right\}\right),$$
$$\left((e_2,n,1),\left\{\frac{u_1{}^{t_3}}{0.8},\frac{u_2{}^{t_3}}{0.6},\frac{u_3{}^{t_3}}{0.2},\frac{u_4{}^{t_3}}{0.7}\right\}\right),\left((e_2,r,1),\left\{\frac{u_1{}^{t_3}}{0.1},\frac{u_2{}^{t_3}}{0.8},\frac{u_3{}^{t_3}}{0.5},\frac{u_4{}^{t_3}}{0.8}\right\}\right),$$
$$\left((e_3,m,1),\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.4},\frac{u_3{}^{t_3}}{0.1},\frac{u_4{}^{t_3}}{0.8}\right\}\right),\left((e_3,n,1),\left\{\frac{u_1{}^{t_3}}{0.6},\frac{u_2{}^{t_3}}{0.5},\frac{u_3{}^{t_3}}{0.3},\frac{u_4{}^{t_3}}{0.9}\right\}\right),$$
$$\left((e_3,r,1),\left\{\frac{u_1{}^{t_3}}{0.4},\frac{u_2{}^{t_3}}{0.7},\frac{u_3{}^{t_3}}{0.5},\frac{u_4{}^{t_3}}{0.6}\right\}\right)\Bigg\}.$$

and the disagree- time fuzzy soft expert set $((F,A)_t)_0$ over U is

$$((F,A)_t)_0=\Bigg\{\left((e_1,m,0),\left\{\frac{u_1{}^{t_1}}{0.3},\frac{u_2{}^{t_1}}{0.6},\frac{u_3{}^{t_1}}{0.7},\frac{u_4{}^{t_1}}{0.8}\right\}\right),\left((e_1,n,0),\left\{\frac{u_1{}^{t_1}}{0.5},\frac{u_2{}^{t_1}}{0.3},\frac{u_3{}^{t_1}}{0.5},\frac{u_4{}^{t_1}}{0.4}\right\}\right),$$

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$$\left((e_1,r,0),\left\{\frac{u_1{}^{t_1}}{0.4},\frac{u_2{}^{t_1}}{0.5},\frac{u_3{}^{t_1}}{0.8},\frac{u_4{}^{t_1}}{0.3}\right\}\right),\left((e_2,m,0),\left\{\frac{u_1{}^{t_1}}{0.5},\frac{u_2{}^{t_1}}{0.7},\frac{u_3{}^{t_1}}{0.5},\frac{u_4{}^{t_1}}{0.3}\right\}\right),$$
$$\left((e_2,n,0),\left\{\frac{u_1{}^{t_1}}{0.4},\frac{u_2{}^{t_1}}{0.4},\frac{u_3{}^{t_1}}{0.9},\frac{u_4{}^{t_1}}{0.6}\right\}\right),\left((e_2,r,0),\left\{\frac{u_1{}^{t_1}}{0.6},\frac{u_2{}^{t_1}}{0.3},\frac{u_3{}^{t_1}}{0.4},\frac{u_4{}^{t_1}}{0.1}\right\}\right),$$
$$\left((e_3,m,0),\left\{\frac{u_1{}^{t_1}}{0.7},\frac{u_2{}^{t_1}}{0.5},\frac{u_3{}^{t_1}}{0.2},\frac{u_4{}^{t_1}}{0.3}\right\}\right),\left((e_3,n,0),\left\{\frac{u_1{}^{t_1}}{0.5},\frac{u_2{}^{t_1}}{0.4},\frac{u_3{}^{t_1}}{0.5},\frac{u_4{}^{t_1}}{0.2}\right\}\right),$$
$$\left((e_3,r,0),\left\{\frac{u_1{}^{t_1}}{0.8},\frac{u_2{}^{t_1}}{0.2},\frac{u_3{}^{t_1}}{0.8},\frac{u_4{}^{t_1}}{0.4}\right\}\right),\left((e_1,m,0),\left\{\frac{u_1{}^{t_2}}{0.7},\frac{u_2{}^{t_2}}{0.4},\frac{u_3{}^{t_2}}{0.7},\frac{u_4{}^{t_2}}{0.5}\right\}\right),$$
$$\left((e_1,n,0),\left\{\frac{u_1{}^{t_2}}{0.5},\frac{u_2{}^{t_2}}{0.2},\frac{u_3{}^{t_2}}{0.5},\frac{u_4{}^{t_2}}{0.4}\right\}\right),\left((e_1,r,0),\left\{\frac{u_1{}^{t_2}}{0.3},\frac{u_2{}^{t_2}}{0.7},\frac{u_3{}^{t_2}}{0.8},\frac{u_4{}^{t_2}}{0.8}\right\}\right),$$
$$\left((e_2,m,0),\left\{\frac{u_1{}^{t_2}}{0.9},\frac{u_2{}^{t_2}}{0.3},\frac{u_3{}^{t_2}}{0.6},\frac{u_4{}^{t_2}}{0.6}\right\}\right),\left((e_2,n,0),\left\{\frac{u_1{}^{t_2}}{0.4},\frac{u_2{}^{t_2}}{0.5},\frac{u_3{}^{t_2}}{0.9},\frac{u_4{}^{t_2}}{0.4}\right\}\right),$$
$$\left((e_2,r,0)\left\{\frac{u_1{}^{t_2}}{0.6},\frac{u_2{}^{t_2}}{0.7},\frac{u_3{}^{t_2}}{0.4},\frac{u_4{}^{t_2}}{0.1}\right\}\right),\left((e_3,m,0),\left\{\frac{u_1{}^{t_2}}{0.2},\frac{u_2{}^{t_2}}{0.7},\frac{u_3{}^{t_2}}{0.8},\frac{u_4{}^{t_2}}{0.2}\right\}\right),$$
$$\left((e_3,n,0),\left\{\frac{u_1{}^{t_2}}{0.4},\frac{u_2{}^{t_2}}{0.5},\frac{u_3{}^{t_2}}{0.7},\frac{u_4{}^{t_2}}{0.6}\right\}\right),\left((e_3,r,0),\left\{\frac{u_1{}^{t_2}}{0.7},\frac{u_2{}^{t_2}}{0.1},\frac{u_3{}^{t_2}}{0.3},\frac{u_4{}^{t_2}}{0.3}\right\}\right),$$
$$\left((e_1,m,0),\left\{\frac{u_1{}^{t_3}}{0.6},\frac{u_2{}^{t_3}}{0.8},\frac{u_3{}^{t_3}}{0.2},\frac{u_4{}^{t_3}}{0.4}\right\}\right),\left((e_1,n,0),\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.5},\frac{u_3{}^{t_3}}{0.4},\frac{u_4{}^{t_3}}{0.6}\right\}\right),$$
$$\left((e_1,r,0),\left\{\frac{u_1{}^{t_3}}{0.5},\frac{u_2{}^{t_3}}{0.7},\frac{u_3{}^{t_3}}{0.2},\frac{u_4{}^{t_3}}{0.5}\right\}\right),\left((e_2,m,0),\left\{\frac{u_1{}^{t_3}}{0.3},\frac{u_2{}^{t_3}}{0.7},\frac{u_3{}^{t_3}}{0.4},\frac{u_4{}^{t_3}}{0.1}\right\}\right),$$
$$\left((e_2,n,0),\left\{\frac{u_1{}^{t_3}}{0.1},\frac{u_2{}^{t_3}}{0.5},\frac{u_3{}^{t_3}}{0.6},\frac{u_4{}^{t_3}}{0.4}\right\}\right),\left((e_2,r,0),\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.3},\frac{u_3{}^{t_3}}{0.3},\frac{u_4{}^{t_3}}{0.2}\right\}\right),$$
$$\left((e_3,m,0),\left\{\frac{u_1{}^{t_3}}{0.4},\frac{u_2{}^{t_3}}{0.6},\frac{u_3{}^{t_3}}{0.8},\frac{u_4{}^{t_3}}{0.3}\right\}\right),\left((e_3,n,0),\left\{\frac{u_1{}^{t_3}}{0.3},\frac{u_2{}^{t_3}}{0.4},\frac{u_3{}^{t_3}}{0.7},\frac{u_4{}^{t_3}}{0.2}\right\}\right),$$
$$\left((e_3,r,0),\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.2},\frac{u_3{}^{t_3}}{0.6},\frac{u_4{}^{t_3}}{0.5}\right\}\right)\Bigg\}.$$

4 Fundamental Operation

I define complement, union, and intersection of T-FSES, deduce various features, and provide some examples to illustrate our points in this section.

4.1 Complement

Definition 4.1. The complement of time fuzzy soft expert set $(F,A)_t$, denoted by $(F,A)_t^c$, is defined by $(F,A)_t^c=(F^c,\rceil A)_t$ where $F_t^c:\rceil A\to P(U)$ is a mapping given by

$$F_t^c(\alpha)=c(F_t(\rceil\alpha)),\forall\alpha\subset\rceil A,$$

where c is a time fuzzy soft expert set complement.

Example 4.2. Think about Example 3.2. I have utilized the simple fuzzy complement to

$$\widetilde{c}(F,A)_t=\Bigg\{\left((e_1,m,1),\left\{\frac{u_1{}^{t_1}}{0.2},\frac{u_2{}^{t_1}}{0.5},\frac{u_3{}^{t_1}}{0.6},\frac{u_4{}^{t_1}}{0.7}\right\}\right),\left((e_1,n,1),\left\{\frac{u_1{}^{t_1}}{0.4},\frac{u_2{}^{t_1}}{0.1},\frac{u_3{}^{t_1}}{0.4},\frac{u_4{}^{t_1}}{0.5}\right\}\right)$$
$$\left((e_1,r,1),\left\{\frac{u_1{}^{t_1}}{0.3},\frac{u_2{}^{t_1}}{0.4},\frac{u_3{}^{t_1}}{0.7},\frac{u_4{}^{t_1}}{0.3}\right\}\right),\left((e_2,m,1),\left\{\frac{u_1{}^{t_1}}{0.4},\frac{u_2{}^{t_1}}{0.9},\frac{u_3{}^{t_1}}{0.6},\frac{u_4{}^{t_1}}{0.2}\right\}\right),$$
$$\left((e_2,n,1),\left\{\frac{u_1{}^{t_1}}{0.5},\frac{u_2{}^{t_1}}{0.3},\frac{u_3{}^{t_1}}{0.8},\frac{u_4{}^{t_1}}{0.5}\right\}\right),\left((e_2,r,1),\left\{\frac{u_1{}^{t_1}}{0.8},\frac{u_2{}^{t_1}}{0.4},\frac{u_3{}^{t_1}}{0.3},\frac{u_4{}^{t_1}}{0.7}\right\}\right),$$
$$\left((e_3,m,1),\left\{\frac{u_1{}^{t_1}}{0.8},\frac{u_2{}^{t_1}}{0.6},\frac{u_3{}^{t_1}}{0.1},\frac{u_4{}^{t_1}}{0.4}\right\}\right),\left((e_3,n,1),\left\{\frac{u_1{}^{t_1}}{0.4},\frac{u_2{}^{t_1}}{0.5},\frac{u_3{}^{t_1}}{0.4},\frac{u_4{}^{t_1}}{0.3}\right\}\right),$$

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$$\left((e_3,r,1),\left\{\tfrac{u_1{}^{t_1}}{0.6},\tfrac{u_2{}^{t_1}}{0.3},\tfrac{u_3{}^{t_1}}{0.9},\tfrac{u_4{}^{t_1}}{0.5}\right\}\right),\left((e_1,m,1),\left\{\tfrac{u_1{}^{t_2}}{0.6},\tfrac{u_2{}^{t_2}}{0.2},\tfrac{u_3{}^{t_2}}{0.8},\tfrac{u_4{}^{t_2}}{0.6}\right\}\right),$$
$$\left((e_1,n,1),\left\{\tfrac{u_1{}^{t_2}}{0.4},\tfrac{u_2{}^{t_2}}{0.1},\tfrac{u_3{}^{t_2}}{0.6},\tfrac{u_4{}^{t_2}}{0.2}\right\}\right),\left((e_1,r,1),\left\{\tfrac{u_1{}^{t_2}}{0.1},\tfrac{u_2{}^{t_2}}{0.8},\tfrac{u_3{}^{t_2}}{0.7},\tfrac{u_4{}^{t_2}}{0.7}\right\}\right),$$
$$\left((e_2,m,1),\left\{\tfrac{u_1{}^{t_2}}{0.8},\tfrac{u_2{}^{t_2}}{0.1},\tfrac{u_3{}^{t_2}}{0.5},\tfrac{u_4{}^{t_2}}{0.5}\right\}\right),\left((e_2,n,1),\left\{\tfrac{u_1{}^{t_2}}{0.3},\tfrac{u_2{}^{t_2}}{0.4},\tfrac{u_3{}^{t_2}}{0.8},\tfrac{u_4{}^{t_2}}{0.4}\right\}\right),$$
$$\left((e_2,r,1),\left\{\tfrac{u_1{}^{t_2}}{0.5},\tfrac{u_2{}^{t_2}}{0.6},\tfrac{u_3{}^{t_2}}{0.3},\tfrac{u_4{}^{t_2}}{0.6}\right\}\right),\left((e_3,m,1),\left\{\tfrac{u_1{}^{t_2}}{0.1},\tfrac{u_2{}^{t_2}}{0.6},\tfrac{u_3{}^{t_2}}{0.9},\tfrac{u_4{}^{t_2}}{0.3}\right\}\right),$$
$$\left((e_3,n,1),\left\{\tfrac{u_1{}^{t_2}}{0.2},\tfrac{u_2{}^{t_2}}{0.4},\tfrac{u_3{}^{t_2}}{0.9},\tfrac{u_4{}^{t_2}}{0.5}\right\}\right),\left((e_3,r,1),\left\{\tfrac{u_1{}^{t_2}}{0.8},\tfrac{u_2{}^{t_2}}{0.1},\tfrac{u_3{}^{t_2}}{0.4},\tfrac{u_4{}^{t_2}}{0.3}\right\}\right),$$
$$\left((e_1,m,1),\left\{\tfrac{u_1{}^{t_3}}{0.8},\tfrac{u_2{}^{t_3}}{0.3},\tfrac{u_3{}^{t_3}}{0.2},\tfrac{u_4{}^{t_3}}{0.7}\right\}\right),\left((e_1,n,1),\left\{\tfrac{u_1{}^{t_3}}{0.6},\tfrac{u_2{}^{t_3}}{0.1},\tfrac{u_3{}^{t_3}}{0.4},\tfrac{u_4{}^{t_3}}{0.5}\right\}\right),$$
$$\left((e_1,r,1),\left\{\tfrac{u_1{}^{t_3}}{0.4},\tfrac{u_2{}^{t_3}}{0.5},\tfrac{u_3{}^{t_3}}{0.3},\tfrac{u_4{}^{t_3}}{0.2}\right\}\right),\left((e_2,m,1),\left\{\tfrac{u_1{}^{t_3}}{0.5},\tfrac{u_2{}^{t_3}}{0.6},\tfrac{u_3{}^{t_3}}{0.3},\tfrac{u_4{}^{t_3}}{0.1}\right\}\right),$$
$$\left((e_2,n,1),\left\{\tfrac{u_1{}^{t_3}}{0.2},\tfrac{u_2{}^{t_3}}{0.4},\tfrac{u_3{}^{t_3}}{0.8},\tfrac{u_4{}^{t_3}}{0.3}\right\}\right),\left((e_2,r,1),\left\{\tfrac{u_1{}^{t_3}}{0.9},\tfrac{u_2{}^{t_3}}{0.2},\tfrac{u_3{}^{t_3}}{0.5},\tfrac{u_4{}^{t_3}}{0.2}\right\}\right),$$
$$\left((e_3,m,1),\left\{\tfrac{u_1{}^{t_3}}{0.3},\tfrac{u_2{}^{t_3}}{0.6},\tfrac{u_3{}^{t_3}}{0.9},\tfrac{u_4{}^{t_3}}{0.2}\right\}\right),\left((e_3,n,1),\left\{\tfrac{u_1{}^{t_3}}{0.4},\tfrac{u_2{}^{t_3}}{0.5},\tfrac{u_3{}^{t_3}}{0.7},\tfrac{u_4{}^{t_3}}{0.1}\right\}\right),$$
$$\left((e_3,r,1),\left\{\tfrac{u_1{}^{t_3}}{0.6},\tfrac{u_2{}^{t_3}}{0.3},\tfrac{u_3{}^{t_3}}{0.5},\tfrac{u_4{}^{t_3}}{0.4}\right\}\right),\left((e_1,m,0),\left\{\tfrac{u_1{}^{t_1}}{0.7},\tfrac{u_2{}^{t_1}}{0.4},\tfrac{u_3{}^{t_1}}{0.3},\tfrac{u_4{}^{t_1}}{0.2}\right\}\right),$$
$$\left((e_1,n,0),\left\{\tfrac{u_1{}^{t_1}}{0.5},\tfrac{u_2{}^{t_1}}{0.7},\tfrac{u_3{}^{t_1}}{0.5},\tfrac{u_4{}^{t_1}}{0.6}\right\}\right),\left((e_1,r,0),\left\{\tfrac{u_1{}^{t_1}}{0.6},\tfrac{u_2{}^{t_1}}{0.5},\tfrac{u_3{}^{t_1}}{0.2},\tfrac{u_4{}^{t_1}}{0.7}\right\}\right),$$
$$\left((e_2,m,0),\left\{\tfrac{u_1{}^{t_1}}{0.5},\tfrac{u_2{}^{t_1}}{0.3},\tfrac{u_3{}^{t_1}}{0.5},\tfrac{u_4{}^{t_1}}{0.7}\right\}\right),\left((e_2,n,0),\left\{\tfrac{u_1{}^{t_1}}{0.6},\tfrac{u_2{}^{t_1}}{0.6},\tfrac{u_3{}^{t_1}}{0.1},\tfrac{u_4{}^{t_1}}{0.4}\right\}\right),$$
$$\left((e_2,r,0),\left\{\tfrac{u_1{}^{t_1}}{0.4},\tfrac{u_2{}^{t_1}}{0.7},\tfrac{u_3{}^{t_1}}{0.6},\tfrac{u_4{}^{t_1}}{0.9}\right\}\right),\left((e_3,m,0),\left\{\tfrac{u_1{}^{t_1}}{0.3},\tfrac{u_2{}^{t_1}}{0.5},\tfrac{u_3{}^{t_1}}{0.8},\tfrac{u_4{}^{t_1}}{0.7}\right\}\right),$$
$$\left((e_3,n,0),\left\{\tfrac{u_1{}^{t_1}}{0.5},\tfrac{u_2{}^{t_1}}{0.6},\tfrac{u_3{}^{t_1}}{0.5},\tfrac{u_4{}^{t_1}}{0.8}\right\}\right),\left((e_3,r,0),\left\{\tfrac{u_1{}^{t_1}}{0.2},\tfrac{u_2{}^{t_1}}{0.8},\tfrac{u_3{}^{t_1}}{0.2},\tfrac{u_4{}^{t_1}}{0.6}\right\}\right),$$
$$\left((e_1,m,0),\left\{\tfrac{u_1{}^{t_2}}{0.3},\tfrac{u_2{}^{t_2}}{0.6},\tfrac{u_3{}^{t_2}}{0.3},\tfrac{u_4{}^{t_2}}{0.5}\right\}\right),\left((e_1,n,0),\left\{\tfrac{u_1{}^{t_2}}{0.5},\tfrac{u_2{}^{t_2}}{0.8},\tfrac{u_3{}^{t_2}}{0.5},\tfrac{u_4{}^{t_2}}{0.6}\right\}\right),$$
$$\left((e_1,r,0),\left\{\tfrac{u_1{}^{t_2}}{0.7},\tfrac{u_2{}^{t_2}}{0.3},\tfrac{u_3{}^{t_2}}{0.2},\tfrac{u_4{}^{t_2}}{0.2}\right\}\right),\left((e_2,m,0),\left\{\tfrac{u_1{}^{t_2}}{0.1},\tfrac{u_2{}^{t_2}}{0.7},\tfrac{u_3{}^{t_2}}{0.4},\tfrac{u_4{}^{t_2}}{0.4}\right\}\right),$$
$$\left((e_2,n,0),\left\{\tfrac{u_1{}^{t_2}}{0.6},\tfrac{u_2{}^{t_2}}{0.5},\tfrac{u_3{}^{t_2}}{0.1},\tfrac{u_4{}^{t_2}}{0.6}\right\}\right),\left((e_2,r,0)\left\{\tfrac{u_1{}^{t_2}}{0.4},\tfrac{u_2{}^{t_2}}{0.3},\tfrac{u_3{}^{t_2}}{0.6},\tfrac{u_4{}^{t_2}}{0.9}\right\}\right),$$
$$\left((e_3,m,0),\left\{\tfrac{u_1{}^{t_2}}{0.8},\tfrac{u_2{}^{t_2}}{0.3},\tfrac{u_3{}^{t_2}}{0.2},\tfrac{u_4{}^{t_2}}{0.8}\right\}\right),\left((e_3,n,0),\left\{\tfrac{u_1{}^{t_2}}{0.6},\tfrac{u_2{}^{t_2}}{0.5},\tfrac{u_3{}^{t_2}}{0.3},\tfrac{u_4{}^{t_2}}{0.4}\right\}\right),$$
$$\left((e_3,r,0),\left\{\tfrac{u_1{}^{t_2}}{0.3},\tfrac{u_2{}^{t_2}}{0.9},\tfrac{u_3{}^{t_2}}{0.7},\tfrac{u_4{}^{t_2}}{0.7}\right\}\right),\left((e_1,m,0),\left\{\tfrac{u_1{}^{t_3}}{0.4},\tfrac{u_2{}^{t_3}}{0.2},\tfrac{u_3{}^{t_3}}{0.8},\tfrac{u_4{}^{t_3}}{0.6}\right\}\right),$$
$$\left((e_1,n,0),\left\{\tfrac{u_1{}^{t_3}}{0.3},\tfrac{u_2{}^{t_3}}{0.5},\tfrac{u_3{}^{t_3}}{0.6},\tfrac{u_4{}^{t_3}}{0.4}\right\}\right),\left((e_1,r,0),\left\{\tfrac{u_1{}^{t_3}}{0.5},\tfrac{u_2{}^{t_3}}{0.3},\tfrac{u_3{}^{t_3}}{0.8},\tfrac{u_4{}^{t_3}}{0.5}\right\}\right),$$
$$\left((e_2,m,0),\left\{\tfrac{u_1{}^{t_3}}{0.7},\tfrac{u_2{}^{t_3}}{0.3},\tfrac{u_3{}^{t_3}}{0.6},\tfrac{u_4{}^{t_3}}{0.9}\right\}\right),\left((e_2,n,0),\left\{\tfrac{u_1{}^{t_3}}{0.9},\tfrac{u_2{}^{t_3}}{0.5},\tfrac{u_3{}^{t_3}}{0.4},\tfrac{u_4{}^{t_3}}{0.6}\right\}\right),$$
$$\left((e_2,r,0),\left\{\tfrac{u_1{}^{t_3}}{0.3},\tfrac{u_2{}^{t_3}}{0.7},\tfrac{u_3{}^{t_3}}{0.7},\tfrac{u_4{}^{t_3}}{0.8}\right\}\right),\left((e_3,m,0),\left\{\tfrac{u_1{}^{t_3}}{0.6},\tfrac{u_2{}^{t_3}}{0.4},\tfrac{u_3{}^{t_3}}{0.2},\tfrac{u_4{}^{t_3}}{0.7}\right\}\right),$$
$$\left.\left((e_3,n,0),\left\{\tfrac{u_1{}^{t_3}}{0.7},\tfrac{u_2{}^{t_3}}{0.6},\tfrac{u_3{}^{t_3}}{0.3},\tfrac{u_4{}^{t_3}}{0.8}\right\}\right),\left((e_3,r,0),\left\{\tfrac{u_1{}^{t_3}}{0.3},\tfrac{u_2{}^{t_3}}{0.8},\tfrac{u_3{}^{t_3}}{0.4},\tfrac{u_4{}^{t_3}}{0.5}\right\}\right)\right\}.$$

Proposition 4.3. *If $(F,A)_t$ is a T-FSES over U, then $\widetilde{c}\,(\widetilde{c}(F,A)_t) = (F,A)_t$.*

Proof. From Definition 4.1 we have ${(F,A)_t}^c = (F^c, A)$, where ${F_t}^c(\alpha) = \bar{1} - F_t(\alpha), \forall \alpha \in A$. Now,
$({(F,A)_t}^c)^c = ((F_t^c)^c, A)$
Where $(F_t^c)^c(\alpha) = \bar{1} - (\bar{1} - F_t(\alpha)), \forall (\alpha) \in A$
$= F_t(\alpha), \forall \alpha \in A.$

□

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4.2 Union Operation

Definition 4.4. The T-FSES $(H,C)_t$ is the *union* of two T-FSES's $(F,A)_t$ and $(G,B)_t$ over U; it is represented as $(F,A)_t \,\widetilde{\cup}\, (G,B)_t$, such that $C = A \cup B \subset Z$. It is defined as follows.

$$H_t(\delta) = \begin{cases} F_t(\delta), & if\, \delta \in A - B, \\ G_t(\delta), & if\, \delta \in B - A, \\ F_t(\delta) \,\tilde{\cup}\, G_t(\delta), & if\, \delta \in A \cap B, \end{cases}$$

where the fuzzy soft expert union was indicated by $\tilde{\cup}$.

Example 4.5. Think about Example 3.2. Assume two time-fuzzy soft expert sets over U, $(F,A)_t$ and $(G,B)_t$, are such that

$$\begin{aligned}(F,A)_t = \Bigg\{ & \left((e_2,n,1), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.5}\right\}\right), \left((e_2,r,1), \left\{\frac{u_1{}^{t_1}}{0.2}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.7}, \frac{u_4{}^{t_1}}{0.3}\right\}\right), \\ & \left((e_2,r,1), \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.4}\right\}\right), \left((e_3,n,1), \left\{\frac{u_1{}^{t_2}}{0.8}, \frac{u_2{}^{t_2}}{0.6}, \frac{u_3{}^{t_2}}{0.1}, \frac{u_4{}^{t_2}}{0.5}\right\}\right), \\ & \left((e_3,m,1), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.1}, \frac{u_4{}^{t_3}}{0.8}\right\}\right), \left((e_1,m,0), \left\{\frac{u_1{}^{t_1}}{0.3}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.7}, \frac{u_4{}^{t_1}}{0.8}\right\}\right), \\ & \left((e_3,n,0), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.2}\right\}\right), \left((e_3,r,0), \left\{\frac{u_1{}^{t_1}}{0.8}, \frac{u_2{}^{t_1}}{0.2}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.4}\right\}\right), \\ & \left((e_1,m,0), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.5}\right\}\right), \left((e_3,m,0), \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.3}\right\}\right) \Bigg\}. \end{aligned}$$

$$\begin{aligned}(G,B)_t = \Bigg\{ & \left((e_2,n,1), \left\{\frac{u_1{}^{t_1}}{0.7}, \frac{u_2{}^{t_1}}{0.9}, \frac{u_3{}^{t_1}}{0.1}, \frac{u_4{}^{t_1}}{0.3}\right\}\right), \left((e_2,r,1), \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.1}\right\}\right), \\ & \left((e_2,r,1), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.2}, \frac{u_4{}^{t_2}}{0.6}\right\}\right), \left((e_3,n,1), \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.8}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.3}\right\}\right), \\ & \left((e_1,m,0), \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.6}, \frac{u_3{}^{t_2}}{0.8}, \frac{u_4{}^{t_2}}{0.4}\right\}\right), \left((e_1,r,0), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.7}\right\}\right), \\ & \left((e_2,m,0), \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.9}\right\}\right), \left((e_3,m,0), \left\{\frac{u_1{}^{t_3}}{0.2}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.9}, \frac{u_4{}^{t_3}}{0.1}\right\}\right) \Bigg\}. \end{aligned}$$

Then $(F,A)_t \,\widetilde{\cup}\, (G,B)_t = (H,C)_t$ where

$$\begin{aligned}(H,C)_t = \Bigg\{ & \left((e_2,n,1), \left\{\frac{u_1{}^{t_1}}{0.7}, \frac{u_2{}^{t_1}}{0.9}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.5}\right\}\right), \left((e_2,r,1), \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.3}\right\}\right), \\ & \left((e_2,r,1), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.6}\right\}\right), \left((e_3,n,1), \left\{\frac{u_1{}^{t_2}}{0.8}, \frac{u_2{}^{t_2}}{0.8}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.5}\right\}\right), \\ & \left((e_3,m,1), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.1}, \frac{u_4{}^{t_3}}{0.8}\right\}\right), \left((e_1,m,0), \left\{\frac{u_1{}^{t_1}}{0.3}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.7}, \frac{u_4{}^{t_1}}{0.8}\right\}\right), \\ & \left((e_3,n,0), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.2}\right\}\right), \left((e_3,r,0), \left\{\frac{u_1{}^{t_1}}{0.8}, \frac{u_2{}^{t_1}}{0.2}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.4}\right\}\right), \\ & \left((e_1,m,0), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.6}, \frac{u_3{}^{t_2}}{0.8}, \frac{u_4{}^{t_2}}{0.5}\right\}\right), \left((e_1,r,0), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.7}\right\}\right), \\ & \left((e_2,m,0), \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.9}\right\}\right), \left((e_3,m,0), \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.9}, \frac{u_4{}^{t_3}}{0.3}\right\}\right) \Bigg\}. \end{aligned}$$

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Proposition 4.6. *Three T-FSES's over U are denoted by $(F,A)_t$, $(G,B)_t$, and $(H,C)_t$, then*

1. $(F,A)_t \widetilde{\cup} ((G,B)_t \widetilde{\cup} (H,C)_t) = ((F,A)_t \widetilde{\cup} (G,B))_t \widetilde{\cup} (H,C)_t$,
2. $(F,A)_t \widetilde{\cup} (F,A)_t = (F,A)_t$.

Proof. 1. Our goal is proving that $(F,A)_t \widetilde{\cup} \left((G,B)_t \widetilde{\cup} (H,C)_t\right) = \left((F,A)_t \widetilde{\cup} (G,B)_t\right) \widetilde{\cup} (H,C)_t$.

Definition 4.4 allows us (where s is s-norm) to have

$$\left((G,B)_t \widetilde{\cup} (H,C)_t\right) = \begin{cases} G_t(\delta), & \text{if } \delta \in B - C \\ H_t(\delta), & \text{if } \delta \in C - B \\ s\left(G_t(\delta), H_t(\delta)\right), & \text{if } \delta \in B \cap C. \end{cases}$$

We take into account the situation when $\delta \in B \cap C$. After the insignificant examples, we have

$$(G,B)_t \widetilde{\cup} (H,C)_t = \left(s\left(G_t(\delta), H_t(\delta)\right), B \cup C\right).$$

Here, we additionally take the scenario when $\delta \in A$ into account. Since the other situations are insignificant, we have

$$\begin{aligned} (F,A)_t \widetilde{\cup} \left((G,B)_t \widetilde{\cup} (H,C)_t\right) &= \left(s\left(F_t(\delta), s\left(G_t(\delta), H_t(\delta)\right)\right), A \cup (B \cup C)\right). \\ &= \left(s\left(s\left(F_t(\delta), G_t(\delta)\right) \cup H_t(\delta)\right), (A \cup B) \cup C\right). \\ &= \left((F,A)_t \widetilde{\cup} (G,B)_t\right) \widetilde{\cup} (H,C)_t. \end{aligned}$$

2. The proof is straightforward.

□

4.3 Intersection Operation

Definition 4.7. The T-FSES $(H,C)_t$ is the *intersection* of two T-FSES's $(F,A)_t$ and $(G,B)_t$ over U, represented as $(F,A)_t \widetilde{\cap} (G,B)_t$, with $C = A \cap B \subset Z$, and defined as follows

$$H_t(\delta) = \begin{cases} F_t(\delta), & if\, \delta \in A - B, \\ G_t(\delta), & if\, \delta \in B - A, \\ F_t(\delta) \tilde{\cap}\, G_t(\delta), & if\, \delta \in A \cap B, \end{cases}$$

where $\tilde{\cap}$ represented the intersection of the fuzzy soft expert.

Example 4.8. Think About Example 4.5. We have $(F,A)_t \widetilde{\cap} (G,B)_t = (H,C)_t$ where

$$\begin{aligned} (H,C)_t = \Bigg\{ & \left((e_2,n,1), \left\{\tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.7}, \tfrac{{u_3}^{t_1}}{0.1}, \tfrac{{u_4}^{t_1}}{0.3}\right\}\right), \left((e_2,r,1), \left\{\tfrac{{u_1}^{t_1}}{0.2}, \tfrac{{u_2}^{t_1}}{0.4}, \tfrac{{u_3}^{t_1}}{0.7}, \tfrac{{u_4}^{t_1}}{0.1}\right\}\right), \\ & \left((e_2,r,1), \left\{\tfrac{{u_1}^{t_2}}{0.5}, \tfrac{{u_2}^{t_2}}{0.3}, \tfrac{{u_3}^{t_2}}{0.2}, \tfrac{{u_4}^{t_2}}{0.4}\right\}\right), \left((e_3,n,1), \left\{\tfrac{{u_1}^{t_2}}{0.5}, \tfrac{{u_2}^{t_2}}{0.6}, \tfrac{{u_3}^{t_2}}{0.1}, \tfrac{{u_4}^{t_2}}{0.3}\right\}\right), \\ & \left((e_3,m,1), \left\{\tfrac{{u_1}^{t_3}}{0.7}, \tfrac{{u_2}^{t_3}}{0.4}, \tfrac{{u_3}^{t_3}}{0.1}, \tfrac{{u_4}^{t_3}}{0.8}\right\}\right), \left((e_1,m,0), \left\{\tfrac{{u_1}^{t_1}}{0.3}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.7}, \tfrac{{u_4}^{t_1}}{0.8}\right\}\right), \\ & \left((e_3,n,0), \left\{\tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.4}, \tfrac{{u_3}^{t_1}}{0.5}, \tfrac{{u_4}^{t_1}}{0.2}\right\}\right), \left((e_3,r,0), \left\{\tfrac{{u_1}^{t_1}}{0.8}, \tfrac{{u_2}^{t_1}}{0.2}, \tfrac{{u_3}^{t_1}}{0.8}, \tfrac{{u_4}^{t_1}}{0.4}\right\}\right), \\ & \left((e_1,m,0), \left\{\tfrac{{u_1}^{t_2}}{0.5}, \tfrac{{u_2}^{t_2}}{0.4}, \tfrac{{u_3}^{t_2}}{0.7}, \tfrac{{u_4}^{t_2}}{0.4}\right\}\right), \left((e_1,r,0), \left\{\tfrac{{u_1}^{t_2}}{0.7}, \tfrac{{u_2}^{t_2}}{0.5}, \tfrac{{u_3}^{t_2}}{0.6}, \tfrac{{u_4}^{t_2}}{0.7}\right\}\right), \\ & \left((e_2,m,0), \left\{\tfrac{{u_1}^{t_2}}{0.6}, \tfrac{{u_2}^{t_2}}{0.7}, \tfrac{{u_3}^{t_2}}{0.4}, \tfrac{{u_4}^{t_2}}{0.9}\right\}\right), \left((e_3,m,0), \left\{\tfrac{{u_1}^{t_3}}{0.2}, \tfrac{{u_2}^{t_3}}{0.6}, \tfrac{{u_3}^{t_3}}{0.8}, \tfrac{{u_4}^{t_3}}{0.1}\right\}\right) \Bigg\}. \end{aligned}$$

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Proposition 4.9. *Three T-FSES's over U are denoted by $(F,A)_t$, $(G,B)_t$, and $(H,C)_t$, then*

1. $(F,A)_t \widetilde{\cap} ((G,B)_t \widetilde{\cap} (H,C)_t) = ((F,A)_t \widetilde{\cap} (G,B))_t \widetilde{\cap} (H,C)_t$,
2. $(F,A)_t \widetilde{\cap} (F,A)_t = (F,A)_t$.

Proof. 1. Our goal is proving that $(F,A)_t \widetilde{\cap} \left((G,B)_t \widetilde{\cap} (H,C)_t\right) = \left((F,A)_t \widetilde{\cap} (G,B)_t\right) \widetilde{\cap} (H,C)_t$.

By employing Definition 4.7 (where $\widetilde{\mathfrak{t}}$ is $\widetilde{\mathfrak{t}}$-norm) we have

$$\left((G,B)_t \widetilde{\cap} (H,C)_t\right) = \begin{cases} G_t(\delta), & \text{if } \delta \in B-C \\ H_t(\delta), & \text{if } \delta \in C-B \\ \widetilde{\mathfrak{t}}(G_t(\delta), H_t(\delta)), & \text{if } \delta \in B \cap C. \end{cases}$$

We take into account the situation when $\delta \in B \cap C$. After the insignificant examples, we have

$(G,B)_t \widetilde{\cap} (H,C)_t = \left(\widetilde{\mathfrak{t}}(G_t(\delta), H_t(\delta)), B \cup C\right)$.

We also consider here the case when $\delta \in A$. The other cases are trivial, then we have

$$\begin{aligned} (F,A)_t \widetilde{\cap} \left((G,B)_t \widetilde{\cap} (H,C)_t\right) &= \left(\widetilde{\mathfrak{t}}\left(F_t(\delta), \widetilde{\mathfrak{t}}(G_t(\delta), H_t(\delta))\right), A \cup (B \cup C)\right). \\ &= \left(\widetilde{\mathfrak{t}}\left(\widetilde{\mathfrak{t}}(F_t(\delta), G_t(\delta)) \cap H_t(\delta)\right), (A \cup B) \cup C\right). \\ &= \left((F,A)_t \widetilde{\cap} (G,B)_t\right) \widetilde{\cap} (H,C)_t. \end{aligned}$$

2. The proof is straightforward.

□

Proposition 4.10. *Assume that there are three T-FSESs over U: $(F,A)_t$, $(G,B)_t$, and $(H,C)_t$, then*

1. $(F,A)_t \widetilde{\cup} \left((G,B)_t \widetilde{\cap} (H,C)_t\right) = \left((F,A)_t \widetilde{\cup} (G,B)_t\right) \widetilde{\cap} \left((F,A)_t \widetilde{\cup} (H,C)_t\right)$,
2. $(F,A)_t \widetilde{\cap} \left((G,B)_t \widetilde{\cup} (H,C)_t\right) = \left((F,A)_t \widetilde{\cap} (G,B)_t\right) \widetilde{\cup} \left((F,A)_t \widetilde{\cap} (H,C)_t\right)$.

Proof. 1. Our goal is to prove that
$(F,A)_t \widetilde{\cup} \left((G,B)_t \widetilde{\cap} (H,C)\right)_t = \left((F,A)_t \widetilde{\cup} (G,B)_t\right) \widetilde{\cap} \left((F,A)_t \widetilde{\cup} (H,C)_t\right)$,

By Applying Definitions 4.4 and 4.7 we have

$$\left((G,B)_t \widetilde{\cap} (H,C)_t\right) = \begin{cases} G_t(\delta), & \text{if } \delta \in B-C \\ H_t(\delta), & \text{if } \delta \in C-B \\ \widetilde{\mathfrak{t}}(G_t(\delta), H_t(\delta)), & \text{if } \delta \in B \cap C. \end{cases}$$

Since the other situations are trivial, we investigate the case when $\delta \in B \cap C$. In this case, we obtain

$(G,B)_t \widetilde{\cap} (H,C)_t = \left(\widetilde{\mathfrak{t}}(G_t(\delta), H_t(\delta)), B \cup C\right)$.

We also consider here the case when $\delta \in A$. The other cases are trivial, then we have

$$\begin{aligned} (F,A)_t \widetilde{\cup} \left((G,B)_t \widetilde{\cap} (H,C)_t\right) &= \left(s\left(F_t(\delta), \widetilde{\mathfrak{t}}(G_t(\delta), H_t(\delta))\right), A \cup (B \cup C)\right). \\ &= \widetilde{\mathfrak{t}}(s(F_t(\delta), G_t(\delta)), s(F_t(\delta) \cup H_t(\delta))). \\ &= \left((F,A)_t \widetilde{\cup} (G,B)_t\right) \widetilde{\cap} \left((F,A)_t \widetilde{\cup} (H,C)_t\right). \end{aligned}$$

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2. Our goal is proving that
$(F,A)_t \widetilde{\cap} \left((G,B)_t \widetilde{\cup} (H,C)\right)_t = \left((F,A)_t \widetilde{\cap} (G,B)_t\right) \widetilde{\cup} \left((F,A)_t \widetilde{\cap} (H,C)_t\right),$

By applying definitions 4.4 and 4.7 we have

$$\left((G,B)_t \widetilde{\cup} (H,C)_t\right) = \begin{cases} G_t(\delta), & \text{if } \delta \in B - C \\ H_t(\delta), & \text{if } \delta \in C - B \\ s\left(G_t(\delta), H_t(\delta)\right), & \text{if } \delta \in B \cap C. \end{cases}$$

We consider the case when $\delta \in B \cap C$. The other cases are trivial, then we have

$(G,B)_t \widetilde{\cup} (H,C)_t = \left(s\left(G_t(\delta), H_t(\delta)\right), B \cup C\right).$

Here, we additionally take the situation when $\delta \in A$ into account. Since the other situations are trivial, we have

$$\begin{aligned} (F,A)_t \widetilde{\cap} \left((G,B)_t \widetilde{\cup} (H,C)_t\right) &= \left(\widetilde{\mathfrak{t}}\left(F_t(\delta), s\left(G_t(\delta), H_t(\delta)\right)\right), A \cup (B \cup C)\right). \\ &= s\left(\widetilde{\mathfrak{t}}\left(F_t(\delta), G_t(\delta)\right), \widetilde{\mathfrak{t}}\left(F_t(\delta) \cup H_t(\delta)\right)\right). \\ &= \left((F,A)_t \widetilde{\cap} (G,B)_t\right) \widetilde{\cup} \left((F,A)_t \widetilde{\cap} (H,C)_t\right). \end{aligned}$$

□

5 AND and OR Operations

We define the AND and OR operations for T-FSES, as well as their characteristics and several instances, in this section.

Definition 5.1. With two T-FSES's over U, $(F,A)_t$ and $(G,B)_t$, the expression $(F,A)_t$ AND $(G,B)_t$ is denoted by $(F,A)_t \wedge (G,B)_t$, it is defined with

$$(F,A)_t \wedge (G,B)_t = (H, A \times B)_t$$

such that $H(\alpha,\beta)_t = F(\alpha)_t \widetilde{\bigcap} G(\beta)_t, \forall (\alpha,\beta) \in A \times B$, where $\widetilde{\bigcap}$ is time-fuzzy soft expert intersection.

Example 5.2. Think about Example 3.2. Assuming two time-fuzzy soft expert sets over U, let $(F,A)_t$ and $(G,B)_t$ be such that

$$\begin{aligned} (F,A)_t = \Bigg\{ &\left((e_2,n,1), \left\{\tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.7}, \tfrac{{u_3}^{t_1}}{0.2}, \tfrac{{u_4}^{t_1}}{0.5}\right\}\right), \left((e_1,m,1), \left\{\tfrac{{u_1}^{t_2}}{0.4}, \tfrac{{u_2}^{t_2}}{0.8}, \tfrac{{u_3}^{t_2}}{0.2}, \tfrac{{u_4}^{t_2}}{0.4}\right\}\right), \\ &\left((e_3,r,1), \left\{\tfrac{{u_1}^{t_3}}{0.4}, \tfrac{{u_2}^{t_3}}{0.7}, \tfrac{{u_3}^{t_3}}{0.5}, \tfrac{{u_4}^{t_3}}{0.6}\right\}\right), \left((e_1,m,0), \left\{\tfrac{{u_1}^{t_1}}{0.3}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.7}, \tfrac{{u_4}^{t_1}}{0.8}\right\}\right) \Bigg\}. \end{aligned}$$

$$(G,B)_t = \Bigg\{ \left((e_3,n,1), \left\{\tfrac{{u_1}^{t_2}}{0.4}, \tfrac{{u_2}^{t_2}}{0.9}, \tfrac{{u_3}^{t_2}}{0.3}, \tfrac{{u_4}^{t_2}}{0.1}\right\}\right), \left((e_2,r,0), \left\{\tfrac{{u_1}^{t_3}}{0.5}, \tfrac{{u_2}^{t_3}}{0.8}, \tfrac{{u_3}^{t_3}}{0.2}, \tfrac{{u_4}^{t_3}}{0.6}\right\}\right) \Bigg\}.$$

Then $(F,A) \wedge (G,B) = (H, A \times B)$ where

$$\begin{aligned} (H, A \times B) = \Bigg\{ &\left(\left((e_2,n,1)_{t_1}, (e_3,n,1)_{t_2}\right), \left\{\tfrac{{u_1}^{t_{1,2}}}{0.4}, \tfrac{{u_2}^{t_{1,2}}}{0.7}, \tfrac{{u_3}^{t_{1,2}}}{0.2}, \tfrac{{u_4}^{t_{1,2}}}{0.1}\right\}\right), \\ &\left(\left((e_2,n,1)_{t_1}, (e_2,r,0)_{t_3}\right), \left\{\tfrac{{u_1}^{t_{1,3}}}{0.5}, \tfrac{{u_2}^{t_{1,3}}}{0.7}, \tfrac{{u_3}^{t_{1,3}}}{0.2}, \tfrac{{u_4}^{t_{1,3}}}{0.5}\right\}\right), \end{aligned}$$

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$$\left(\left((e_1,m,1)_{t_2},(e_3,n,1)_{t_2}\right),\left\{\frac{{u_1}^{t_{2,2}}}{0.4},\frac{{u_2}^{t_{2,2}}}{0.8},\frac{{u_3}^{t_{2,2}}}{0.2},\frac{{u_4}^{t_{2,2}}}{0.1}\right\}\right),$$

$$\left(\left((e_1,m,1)_{t_2},(e_2,r,0)_{t_3}\right),\left\{\frac{{u_1}^{t_{2,3}}}{0.4},\frac{{u_2}^{t_{2,3}}}{0.8},\frac{{u_3}^{t_{2,3}}}{0.2},\frac{{u_4}^{t_{2,3}}}{0.4}\right\}\right),$$

$$\left(\left((e_3,r,1)_{t_3},(e_3,n,1)_{t_2}\right),\left\{\frac{{u_1}^{t_{3,2}}}{0.4},\frac{{u_2}^{t_{3,2}}}{0.7},\frac{{u_3}^{t_{3,2}}}{0.3},\frac{{u_4}^{t_{3,2}}}{0.1}\right\}\right),$$

$$\left(\left((e_3,r,1)_{t_3},(e_2,r,0)_{t_3}\right),\left\{\frac{{u_1}^{t_{3,3}}}{0.4},\frac{{u_2}^{t_{3,3}}}{0.7},\frac{{u_3}^{t_{3,3}}}{0.2},\frac{{u_4}^{t_{3,3}}}{0.6}\right\}\right),$$

$$\left(\left((e_1,m,0)_{t_1},(e_3,n,1)_{t_2}\right),\left\{\frac{{u_1}^{t_{1,2}}}{0.3},\frac{{u_2}^{t_{1,2}}}{0.6},\frac{{u_3}^{t_{1,2}}}{0.3},\frac{{u_4}^{t_{1,2}}}{0.1}\right\}\right),$$

$$\left(\left((e_1,m,0)_{t_1},(e_2,r,0)_{t_3}\right),\left\{\frac{{u_1}^{t_{1,3}}}{0.3},\frac{{u_2}^{t_{1,3}}}{0.6},\frac{{u_3}^{t_{1,3}}}{0.2},\frac{{u_4}^{t_{1,3}}}{0.6}\right\}\right)\Bigg\}.$$

Assuming that there are two T-FSES's over U, $(F,A)_t$ and $(G,B)_t$, then $(F,A)_t$ OR $(G,B)_t$ is represented as $(F,A)_t \vee (G,B)_t$.

Definition 5.3. Assuming that there are two T-FSES's over U, $(F,A)_t$ and $(G,B)_t$, then $(F,A)_t$ OR $(G,B)_t$ is represented as $(F,A)_t \vee (G,B)_t$, is defined by

$$(F,A)_t \vee (G,B)_t = (H, A\times B)_t$$

such that $H(\alpha,\beta)_t = F(\alpha)_t \widetilde{\bigcup} G(\beta)_t, \forall (\alpha,\beta)\in A\times B$, $\widetilde{\bigcup}$ represents the union of time-fuzzy soft expert.

Example 5.4. Think about Example 5.2 we have U then $(F,A)_t$ OR $(G,B)_t$ represented by $(H,C)_t = (F,A)_t \bigvee (G,B)_t$ where

$$(H,C)_t=\Bigg\{\left(\left((e_2,n,1)_{t_1},(e_3,n,1)_{t_2}\right),\left\{\frac{{u_1}^{t_{1,2}}}{0.5},\frac{{u_2}^{t_{1,2}}}{0.9},\frac{{u_3}^{t_{1,2}}}{0.3},\frac{{u_4}^{t_{1,2}}}{0.5}\right\}\right),$$

$$\left(\left((e_2,n,1)_{t_1},(e_2,r,0)_{t_3}\right),\left\{\frac{{u_1}^{t_{1,3}}}{0.5},\frac{{u_2}^{t_{1,3}}}{0.8},\frac{{u_3}^{t_{1,3}}}{0.2},\frac{{u_4}^{t_{1,3}}}{0.6}\right\}\right),$$

$$\left(\left((e_1,m,1)_{t_2},(e_3,n,1)_{t_2}\right),\left\{\frac{{u_1}^{t_{2,2}}}{0.4},\frac{{u_2}^{t_{2,2}}}{0.9},\frac{{u_3}^{t_{2,2}}}{0.3},\frac{{u_4}^{t_{2,2}}}{0.4}\right\}\right),$$

$$\left(\left((e_1,m,1)_{t_2},(e_2,r,0)_{t_3}\right),\left\{\frac{{u_1}^{t_{2,3}}}{0.5},\frac{{u_2}^{t_{2,3}}}{0.8},\frac{{u_3}^{t_{2,3}}}{0.2},\frac{{u_4}^{t_{2,3}}}{0.6}\right\}\right),$$

$$\left(\left((e_3,r,1)_{t_3},(e_3,n,1)_{t_2}\right),\left\{\frac{{u_1}^{t_{3,2}}}{0.4},\frac{{u_2}^{t_{3,2}}}{0.9},\frac{{u_3}^{t_{3,2}}}{0.5},\frac{{u_4}^{t_{3,2}}}{0.6}\right\}\right),$$

$$\left(\left((e_3,r,1)_{t_3},(e_2,r,0)_{t_3}\right),\left\{\frac{{u_1}^{t_{3,3}}}{0.5},\frac{{u_2}^{t_{3,3}}}{0.8},\frac{{u_3}^{t_{3,3}}}{0.5},\frac{{u_4}^{t_{3,3}}}{0.6}\right\}\right),$$

$$\left(\left((e_1,m,0)_{t_1},(e_3,n,1)_{t_2}\right),\left\{\frac{{u_1}^{t_{1,2}}}{0.4},\frac{{u_2}^{t_{1,2}}}{0.9},\frac{{u_3}^{t_{1,2}}}{0.7},\frac{{u_4}^{t_{1,2}}}{0.8}\right\}\right),$$

$$\left(\left((e_1,m,0)_{t_1},(e_2,r,0)_{t_3}\right),\left\{\frac{{u_1}^{t_{1,3}}}{0.5},\frac{{u_2}^{t_{1,3}}}{0.8},\frac{{u_3}^{t_{1,3}}}{0.7},\frac{{u_4}^{t_{1,3}}}{0.8}\right\}\right)\Bigg\}.$$

Proposition 5.5. *let (F,A) and (G,B) are time-fuzzy soft expert sets over U, then*

1. $((F,A)_t \wedge (G,B)_t)^c = (F,A)_t^c \vee (G,B)_t^c$
2. $((F,A)_t \vee (G,B)_t)^c = (F,A)_t^c \wedge (G,B)_t^c$

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Proof. 1. Suppose that $(F, A)_t \wedge (G, B)_t = (O, A \times B)_t$.
Therefore, $((F, A)_t \wedge (G, B)_t)^c = (O_t, A \times B)^c = (O_t^c, (A \times B))$. Now,
$((F, A)_t \vee (G, B)_t)^c = ((F_t^c, A) \vee (G_t^c, B))$
$= (J_t, (A \times B))$, where $J_t(x, y) = t\,(c(F_t\,(\alpha)), c(G_t\,(\beta)))$.
Now, take $(\alpha, \beta) \in (A \times B)$.
Then,
$O_t^c(\alpha, \beta) = \bar{1} - O_t(\alpha, \beta),$
$= \bar{1} - [F_t\,(\alpha) \bigcup G_t\,(\beta)]$
$= [\bar{1} - F_t\,(\alpha)] \bigcap [\bar{1} - G_t\,(\beta)]$
$= t\,(c(F_t\,(\alpha)), c(G_t\,(\beta)))$
$= J_t(\alpha, \beta)$
Therefore O_t^c and J_t are the same. Hence, proved.

2. Suppose that $(F, A)_t \vee (G, B)_t = (O, A \times B)_t$.
Therefore, $((F, A)_t \vee (G, B)_t)^c = (O_t, A \times B)^c = (O_t^c, (A \times B))$. Now,
$((F, A)_t \wedge (G, B)_t)^c = ((F_t^c, A) \wedge (G_t^c, B))$
$= (J_t, (A \times B))$, where $J_t(x, y) = s\,(c(F_t\,(\alpha)), c(G_t\,(\beta)))$.
Now, take $(\alpha, \beta) \in (A \times B)$.
Then,
$O_t^c(\alpha, \beta) = \bar{1} - O_t(\alpha, \beta),$
$= \bar{1} - [F_t\,(\alpha) \bigcap G_t\,(\beta)]$
$= [\bar{1} - F_t\,(\alpha)] \bigcup [\bar{1} - G_t\,(\beta)]$
$= s\,(c(F_t\,(\alpha)), c(G_t\,(\beta)))$
$= J_t(\alpha, \beta)$
Therefore O_t^c and J_t are the same. Hence, proved.

□

Proposition 5.6. *let (F, A), (G, B) and (H, C) be time-fuzzy soft expert sets over U, then*

1. $(F, A)_t \wedge ((G, B)_t \wedge (H, C)_t) = ((F, A)_t \wedge (G, B)_t) \wedge (H, C)_t,$
2. $(F, A)_t \vee ((G, B)_t \vee (H, C)_t) = ((F, A)_t \vee (G, B)_t) \vee (H, C)_t,$
3. $(F, A)_t \vee ((G, B)_t \wedge (H, C)_t) = ((F, A)_t \vee (G, B)_t) \wedge ((F, A)_t \vee (H, C)_t)$,
4. $(F, A)_t \wedge ((G, B)_t \vee (H, C)_t) = ((F, A)_t \wedge (G, B)_t) \vee ((F, A)_t \wedge (H, C)_t)$.

Proof. We give the proofs of 1 and 2.

1. Suppose that $(G, B)_t \wedge (H, C)_t = \widetilde{\mathfrak{t}}\,(G_t\,(\alpha)\,, H_t\,(\beta)), \forall\,(\alpha, \beta) \in B \times C.$

   Then
   $(F, A)_t \wedge ((G, B)_t \wedge (H, C)_t) = \widetilde{\mathfrak{t}}\left(F_t\,(\gamma)\,, \widetilde{\mathfrak{t}}\,(G_t\,(\alpha)\,, H_t\,(\beta))\right), \forall\,(\gamma, (\alpha, \beta)) \in A \times (B \times C).$

   $= \widetilde{\mathfrak{t}}\left(\widetilde{\mathfrak{t}}\,(F_t\,(\gamma)\,, G_t\,(\alpha))\,, H_t\,(\beta)\right), \forall((\gamma, \alpha)\,, \beta) \in (A \times B) \times C.$

   $= ((F, A)_t \wedge (G, B)_t) \wedge (H, C)_t.$

2. Suppose that $(G, B)_t \vee (H, C)_t = s\,(G_t\,(\alpha)\,, H_t\,(\beta)), \forall\,(\alpha, \beta) \in B \times C.$

   Then
   $(F, A)_t \vee ((G, B)_t \wedge (H, C)_t) = s\,(F_t\,(\gamma)\,, s\,(G_t\,(\alpha)\,, H_t\,(\beta))), \forall\,(\gamma, (\alpha, \beta)) \in A \times (B \times C).$

   $= s\,(s\,(F_t\,(\gamma)\,, G_t\,(\alpha))\,, H_t\,(\beta)), \forall((\gamma, \alpha)\,, \beta) \in (A \times B) \times C.$

   $= ((F, A)_t \vee (G, B)_t) \vee (H, C)_t.$

□

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6 An application of time fuzzy soft expert set with two opinions in decision making

This section presents a theoretical application of the time fuzzy soft expert set theory to a decision-making issue, indicating the viability and generalizability of this approach to problems in many domains with uncertainty. Two opinions $\{\text{agree}, \text{disagree}\}$ make up the application.

Example 6.1. Measuring the effectiveness of governments just through opinion surveys is insufficient. Instead, performance metrics that encompass all of the activities and duties listed below should be used to gauge government success. Suppose that the company of strategic studies consider a collection of experts to carry out an analysis using particular metrics to assess the performance of the governments of four nations throughout the course of four prior time periods; these metrics are listed below. Consider a collection of governments $U = \{u_1,\ u_2,\ u_3, u_4\}$, Five parameters could be present. Assume that the decision parameters $E = \{e_1, e_2, e_3\}$ are used to evaluate the performance of governments. For i = 1, 2, 3, the parameters $e_i\ (i = 1, 2, 3)$ stand for the achievement of the anti-corruption campaign, efficient use of natural resources in management, Investments and savings. And let $T = \{t_1, t_2, t_3, t_4\}$. $X = \{m, n, r\}$ be the members of a committee. The findings of the research will make it evident which government best fits the aforementioned requirements.

$$
\begin{aligned}
(F,Z)_t = \Bigg\{ & \left((e_1,m,1),\left\{\tfrac{{u_1}^{t_1}}{0.6},\tfrac{{u_2}^{t_1}}{0.3},\tfrac{{u_3}^{t_1}}{0.5},\tfrac{{u_4}^{t_1}}{0.6}\right\}\right),\left((e_1,n,1),\left\{\tfrac{{u_1}^{t_1}}{0.7},\tfrac{{u_2}^{t_1}}{0.5},\tfrac{{u_3}^{t_1}}{0.4},\tfrac{{u_4}^{t_1}}{0.7}\right\}\right),\\
& \left((e_2,m,1),\left\{\tfrac{{u_1}^{t_1}}{0.7},\tfrac{{u_2}^{t_1}}{0.4},\tfrac{{u_3}^{t_1}}{0.2},\tfrac{{u_4}^{t_1}}{0.7}\right\}\right),\left((e_2,n,1),\left\{\tfrac{{u_1}^{t_1}}{0.5},\tfrac{{u_2}^{t_1}}{0.6},\tfrac{{u_3}^{t_1}}{0.3},\tfrac{{u_4}^{t_1}}{0.5}\right\}\right),\\
& \left((e_3,m,1),\left\{\tfrac{{u_1}^{t_1}}{0.4},\tfrac{{u_2}^{t_1}}{0.6},\tfrac{{u_3}^{t_1}}{0.7},\tfrac{{u_4}^{t_1}}{0.3}\right\}\right),\left((e_3,n,1),\left\{\tfrac{{u_1}^{t_1}}{0.6},\tfrac{{u_2}^{t_1}}{0.5},\tfrac{{u_3}^{t_1}}{0.9},\tfrac{{u_4}^{t_1}}{0.5}\right\}\right),\\
& \left((e_1,m,1),\left\{\tfrac{{u_1}^{t_2}}{0.4},\tfrac{{u_2}^{t_2}}{0.8},\tfrac{{u_3}^{t_2}}{0.2},\tfrac{{u_4}^{t_2}}{0.4}\right\}\right),\left((e_1,n,1),\left\{\tfrac{{u_1}^{t_2}}{0.6},\tfrac{{u_2}^{t_2}}{0.9},\tfrac{{u_3}^{t_2}}{0.4},\tfrac{{u_4}^{t_2}}{0.8}\right\}\right),\\
& \left((e_2,m,1),\left\{\tfrac{{u_1}^{t_2}}{0.4},\tfrac{{u_2}^{t_2}}{0.9},\tfrac{{u_3}^{t_2}}{0.7},\tfrac{{u_4}^{t_2}}{0.5}\right\}\right),\left((e_2,n,1),\left\{\tfrac{{u_1}^{t_2}}{0.6},\tfrac{{u_2}^{t_2}}{0.8},\tfrac{{u_3}^{t_2}}{0.5},\tfrac{{u_4}^{t_2}}{0.6}\right\}\right),\\
& \left((e_3,m,1),\left\{\tfrac{{u_1}^{t_2}}{0.6},\tfrac{{u_2}^{t_2}}{0.4},\tfrac{{u_3}^{t_2}}{0.1},\tfrac{{u_4}^{t_2}}{0.8}\right\}\right),\left((e_3,n,1),\left\{\tfrac{{u_1}^{t_2}}{0.8},\tfrac{{u_2}^{t_2}}{0.6},\tfrac{{u_3}^{t_2}}{0.3},\tfrac{{u_4}^{t_2}}{0.7}\right\}\right),\\
& \left((e_1,m,1),\left\{\tfrac{{u_1}^{t_3}}{0.2},\tfrac{{u_2}^{t_3}}{0.7},\tfrac{{u_3}^{t_3}}{0.8},\tfrac{{u_4}^{t_3}}{0.3}\right\}\right),\left((e_1,n,1),\left\{\tfrac{{u_1}^{t_3}}{0.4},\tfrac{{u_2}^{t_3}}{0.9},\tfrac{{u_3}^{t_3}}{0.6},\tfrac{{u_4}^{t_3}}{0.5}\right\}\right),\\
& \left((e_2,m,1),\left\{\tfrac{{u_1}^{t_3}}{0.5},\tfrac{{u_2}^{t_3}}{0.4},\tfrac{{u_3}^{t_3}}{0.7},\tfrac{{u_4}^{t_3}}{0.9}\right\}\right),\left((e_2,n,1),\left\{\tfrac{{u_1}^{t_3}}{0.8},\tfrac{{u_2}^{t_3}}{0.6},\tfrac{{u_3}^{t_3}}{0.2},\tfrac{{u_4}^{t_3}}{0.7}\right\}\right),\\
& \left((e_3,m,1),\left\{\tfrac{{u_1}^{t_3}}{0.7},\tfrac{{u_2}^{t_3}}{0.6},\tfrac{{u_3}^{t_3}}{0.3},\tfrac{{u_4}^{t_3}}{0.7}\right\}\right),\left((e_3,n,1),\left\{\tfrac{{u_1}^{t_3}}{0.6},\tfrac{{u_2}^{t_3}}{0.5},\tfrac{{u_3}^{t_3}}{0.3},\tfrac{{u_4}^{t_3}}{0.9}\right\}\right),\\
& \left((e_1,m,1),\left\{\tfrac{{u_1}^{t_4}}{0.2},\tfrac{{u_2}^{t_4}}{0.9},\tfrac{{u_3}^{t_4}}{0.6},\tfrac{{u_4}^{t_4}}{0.7}\right\}\right),\left((e_1,n,1),\left\{\tfrac{{u_1}^{t_4}}{0.7},\tfrac{{u_2}^{t_4}}{0.6},\tfrac{{u_3}^{t_4}}{0.8},\tfrac{{u_4}^{t_4}}{0.4}\right\}\right),\\
& \left((e_2,m,1),\left\{\tfrac{{u_1}^{t_4}}{0.8},\tfrac{{u_2}^{t_4}}{0.6},\tfrac{{u_3}^{t_4}}{0.2},\tfrac{{u_4}^{t_4}}{0.7}\right\}\right),\left((e_2,n,1),\left\{\tfrac{{u_1}^{t_4}}{0.7},\tfrac{{u_2}^{t_4}}{0.4},\tfrac{{u_3}^{t_4}}{0.5},\tfrac{{u_4}^{t_4}}{0.8}\right\}\right),\\
& \left((e_3,m,1),\left\{\tfrac{{u_1}^{t_4}}{0.7},\tfrac{{u_2}^{t_4}}{0.5},\tfrac{{u_3}^{t_4}}{0.3},\tfrac{{u_4}^{t_4}}{0.9}\right\}\right),\left((e_3,n,1),\left\{\tfrac{{u_1}^{t_4}}{0.6},\tfrac{{u_2}^{t_4}}{0.8},\tfrac{{u_3}^{t_4}}{0.4},\tfrac{{u_4}^{t_4}}{0.5}\right\}\right),\\
& \left((e_1,m,0),\left\{\tfrac{{u_1}^{t_1}}{0.5},\tfrac{{u_2}^{t_1}}{0.6},\tfrac{{u_3}^{t_1}}{0.4},\tfrac{{u_4}^{t_1}}{0.3}\right\}\right),\left((e_1,n,0),\left\{\tfrac{{u_1}^{t_1}}{0.4},\tfrac{{u_2}^{t_1}}{0.5},\tfrac{{u_3}^{t_1}}{0.7},\tfrac{{u_4}^{t_1}}{0.2}\right\}\right),\\
& \left((e_2,m,0),\left\{\tfrac{{u_1}^{t_1}}{0.4},\tfrac{{u_2}^{t_1}}{0.6},\tfrac{{u_3}^{t_1}}{0.7},\tfrac{{u_4}^{t_1}}{0.4}\right\}\right),\left((e_2,n,0),\left\{\tfrac{{u_1}^{t_1}}{0.4},\tfrac{{u_2}^{t_1}}{0.3},\tfrac{{u_3}^{t_1}}{0.6},\tfrac{{u_4}^{t_1}}{0.6}\right\}\right),\\
& \left((e_3,m,0),\left\{\tfrac{{u_1}^{t_1}}{0.7},\tfrac{{u_2}^{t_1}}{0.5},\tfrac{{u_3}^{t_1}}{0.3},\tfrac{{u_4}^{t_1}}{0.4}\right\}\right),\left((e_3,n,0),\left\{\tfrac{{u_1}^{t_1}}{0.4},\tfrac{{u_2}^{t_1}}{0.6},\tfrac{{u_3}^{t_1}}{0.2},\tfrac{{u_4}^{t_1}}{0.6}\right\}\right),\\
& \left((e_1,m,0),\left\{\tfrac{{u_1}^{t_2}}{0.7},\tfrac{{u_2}^{t_2}}{0.4},\tfrac{{u_3}^{t_2}}{0.7},\tfrac{{u_4}^{t_2}}{0.5}\right\}\right),\left((e_1,n,0),\left\{\tfrac{{u_1}^{t_2}}{0.5},\tfrac{{u_2}^{t_2}}{0.2},\tfrac{{u_3}^{t_2}}{0.5},\tfrac{{u_4}^{t_2}}{0.4}\right\}\right),\\
& \left((e_2,m,0),\left\{\tfrac{{u_1}^{t_2}}{0.2},\tfrac{{u_2}^{t_2}}{0.3},\tfrac{{u_3}^{t_2}}{0.4},\tfrac{{u_4}^{t_2}}{0.6}\right\}\right),\left((e_2,n,0),\left\{\tfrac{{u_1}^{t_2}}{0.5},\tfrac{{u_2}^{t_2}}{0.2},\tfrac{{u_3}^{t_2}}{0.7},\tfrac{{u_4}^{t_2}}{0.3}\right\}\right),\\
& \left((e_3,m,0),\left\{\tfrac{{u_1}^{t_2}}{0.3},\tfrac{{u_2}^{t_2}}{0.5},\tfrac{{u_3}^{t_2}}{0.8},\tfrac{{u_4}^{t_2}}{0.3}\right\}\right),\left((e_3,n,0),\left\{\tfrac{{u_1}^{t_2}}{0.5},\tfrac{{u_2}^{t_2}}{0.4},\tfrac{{u_3}^{t_2}}{0.7},\tfrac{{u_4}^{t_2}}{0.2}\right\}\right),\\
& \left((e_1,m,0),\left\{\tfrac{{u_1}^{t_3}}{0.7},\tfrac{{u_2}^{t_3}}{0.2},\tfrac{{u_3}^{t_3}}{0.1},\tfrac{{u_4}^{t_3}}{0.6}\right\}\right),\left((e_1,n,0),\left\{\tfrac{{u_1}^{t_3}}{0.3},\tfrac{{u_2}^{t_3}}{0.4},\tfrac{{u_3}^{t_3}}{0.6},\tfrac{{u_4}^{t_3}}{0.4}\right\}\right),\\
& \left((e_2,m,0),\left\{\tfrac{{u_1}^{t_3}}{0.3},\tfrac{{u_2}^{t_3}}{0.7},\tfrac{{u_3}^{t_3}}{0.4},\tfrac{{u_4}^{t_3}}{0.1}\right\}\right),\left((e_2,n,0),\left\{\tfrac{{u_1}^{t_3}}{0.1},\tfrac{{u_2}^{t_3}}{0.5},\tfrac{{u_3}^{t_3}}{0.6},\tfrac{{u_4}^{t_3}}{0.5}\right\}\right),
\end{aligned}
$$

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$$\left((e_3,m,0),\left\{\frac{u_1{}^{t_3}}{0.4},\frac{u_2{}^{t_3}}{0.6},\frac{u_3{}^{t_3}}{0.8},\frac{u_4{}^{t_3}}{0.3}\right\}\right),\left((e_3,n,0),\left\{\frac{u_1{}^{t_3}}{0.3},\frac{u_2{}^{t_3}}{0.4},\frac{u_3{}^{t_3}}{0.7},\frac{u_4{}^{t_3}}{0.2}\right\}\right),$$
$$\left((e_1,m,0),\left\{\frac{u_1{}^{t_4}}{0.6},\frac{u_2{}^{t_4}}{0.1},\frac{u_3{}^{t_4}}{0.3},\frac{u_4{}^{t_4}}{0.4}\right\}\right),\left((e_1,n,0),\left\{\frac{u_1{}^{t_4}}{0.4},\frac{u_2{}^{t_4}}{0.4},\frac{u_3{}^{t_4}}{0.3},\frac{u_4{}^{t_4}}{0.7}\right\}\right),$$
$$\left((e_2,m,0),\left\{\frac{u_1{}^{t_4}}{0.3},\frac{u_2{}^{t_4}}{0.3},\frac{u_3{}^{t_4}}{0.7},\frac{u_4{}^{t_4}}{0.2}\right\}\right),\left((e_2,n,0),\left\{\frac{u_1{}^{t_4}}{0.4},\frac{u_2{}^{t_4}}{0.6},\frac{u_3{}^{t_4}}{0.6},\frac{u_4{}^{t_4}}{0.3}\right\}\right),$$
$$\left((e_3,m,0),\left\{\frac{u_1{}^{t_4}}{0.6},\frac{u_2{}^{t_4}}{0.4},\frac{u_3{}^{t_4}}{0.7},\frac{u_4{}^{t_4}}{0.2}\right\}\right),\left((e_3,n,0),\left\{\frac{u_1{}^{t_4}}{0.5},\frac{u_2{}^{t_4}}{0.3},\frac{u_3{}^{t_4}}{0.7},\frac{u_4{}^{t_4}}{0.5}\right\}\right)\Bigg\}.$$

The committee may use the following techniques for evaluating their selection and determine which option is best for the decision.

1. Input the T-FSES (F, Z).
2. Find an agree-time fuzzy soft expert set and a disagree-time fuzzy soft expert set, Tables (1, 2).
3. Construct $F(Z)$'s tabular representation using the tables in Tables (3, 4), where $F(Z)$ outlined as follows:
$$F(z)=\left\{\frac{u}{\sum_{i=1}^{n} t_i F_{t_i}(z) \big/ n \sum_{i=1}^{n} F_{t_i}(z)} : u\in U, z\in Z\right\} \tag{1}$$
where $n=|T|$.
4. Apply Alkhazaleh algorithm on fuzzy soft expert (F, Z).
   - Construct $c_j=\sum_i u_{ij}$ for agree-FSES, (Table 3).
   - Construct $k_j=\sum_i u_{ij}$ for disagree-FSES, (Table 4).
   - Construct $s_j=c_j-k_j$, (Table 5).
   - Construct m, for which $\mathrm{s}_m=\max s_j$.
     In such case, s_m is the best option object. The restaurant chain may use its choice to select any one of the values if m has many values.

The agree-time fuzzy soft expert set and disagree-time fuzzy soft expert set are shown in Tables 1 and 2, respectively.

Table 1: Agree time fuzzy soft expert set

U	u_1	u_2	u_3	u_4
$(e_1,m)_{t_1}$	0.6	0.3	0.5	0.6
$(e_1,m)_{t_2}$	0.4	0.8	0.2	0.4
$(e_1,m)_{t_3}$	0.2	0.7	0.8	0.3
$(e_1,m)_{t_4}$	0.2	0.9	0.6	0.7
$(e_1,n)_{t_1}$	0.7	0.5	0.4	0.7
$(e_1,n)_{t_2}$	0.6	0.9	0.4	0.8
$(e_1,n)_{t_3}$	0.4	0.9	0.6	0.5
$(e_1,n)_{t_4}$	0.7	0.6	0.8	0.4
$(e_2,m)_{t_1}$	0.7	0.4	0.2	0.7
$(e_2,m)_{t_2}$	0.4	0.9	0.7	0.5
$(e_2,m)_{t_3}$	0.5	0.4	0.7	0.9
$(e_2,m)_{t_4}$	0.8	0.6	0.2	0.7
$(e_2,n)_{t_1}$	0.5	0.6	0.3	0.5

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Table 1 – continued

U	u_1	u_2	u_3	u_4
$(e_2, n)_{t_2}$	0.6	0.8	0.5	0.6
$(e_2, n)_{t_3}$	0.8	0.6	0.2	0.7
$(e_2, n)_{t_4}$	0.7	0.4	0.5	0.8
$(e_3, m)_{t_1}$	0.4	0.6	0.7	0.3
$(e_3, m)_{t_2}$	0.6	0.4	0.1	0.8
$(e_3, m)_{t_3}$	0.7	0.6	0.3	0.7
$(e_3, m)_{t_4}$	0.7	0.5	0.3	0.9
$(e_3, n)_{t_1}$	0.6	0.5	0.9	0.5
$(e_3, n)_{t_2}$	0.8	0.6	0.3	0.7
$(e_3, n)_{t_3}$	0.6	0.5	0.3	0.9
$(e_3, n)_{t_4}$	0.6	0.8	0.4	0.8

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Table 2: Disagree-time fuzzy soft expert set

U	u_1	u_2	u_3	u_4
$(e_1, m)_{t_1}$	0.5	0.6	0.4	0.3
$(e_1, m)_{t_2}$	0.7	0.4	0.7	0.5
$(e_1, m)_{t_3}$	0.7	0.2	0.1	0.6
$(e_1, m)_{t_4}$	0.6	0.1	0.3	0.4
$(e_1, n)_{t_1}$	0.4	0.5	0.7	0.2
$(e_1, n)_{t_2}$	0.5	0.2	0.5	0.4
$(e_1, n)_{t_3}$	0.3	0.4	0.6	0.5
$(e_1, n)_{t_4}$	0.4	0.4	0.3	0.7
$(e_2, m)_{t_1}$	0.4	0.6	0.7	0.4
$(e_2, m)_{t_2}$	0.2	0.3	0.4	0.6
$(e_2, m)_{t_3}$	0.3	0.7	0.4	0.1
$(e_2, m)_{t_4}$	0.3	0.3	0.7	0.2
$(e_2, n)_{t_1}$	0.4	0.3	0.6	0.6
$(e_2, n)_{t_2}$	0.5	0.2	0.7	0.3
$(e_2, n)_{t_3}$	0.1	0.5	0.6	0.5
$(e_2, n)_{t_4}$	0.4	0.6	0.6	0.3
$(e_3, m)_{t_1}$	0.7	0.5	0.3	0.4
$(e_3, m)_{t_2}$	0.3	0.5	0.8	0.3
$(e_3, m)_{t_3}$	0.4	0.6	0.8	0.3
$(e_3, m)_{t_4}$	0.6	0.4	0.7	0.2
$(e_3, n)_{t_1}$	0.4	0.6	0.2	0.6
$(e_3, n)_{t_2}$	0.5	0.4	0.7	0.2
$(e_3, n)_{t_3}$	0.3	0.4	0.7	0.2
$(e_3, n)_{t_4}$	0.5	0.3	0.7	0.5

Next by using relation (1) we calculate $F(Z)$ to convert the agree time fuzzy soft expert set to agree fuzzy soft expert set,to illustrate this step we calculate $F(e_1)$ for u_1 as show below.

$$F(e_1) = \left\{ \frac{u_1}{\sum_{i=1}^{4} t_i F_{t_i}(e) \big/ 4 \sum_{i=1}^{4} F_{t_i}(e)} : u \in U, e \in E \right\}$$
$$= \left\{ \frac{u_1}{((1*0.5)+(2*0.7)+(3*0.7)+(4*0.6)) \big/ 4(0.5+0.7+0.7+0.6)} \right\}$$
$$= \left\{ \frac{u_1}{6.4 \big/ 10} \right\}$$
$$= \frac{u_1}{0.64}.$$

then we compute c_j, where c_j represent the column sum of an object u_i. The results are shown in Table 3.

Table 3: Converting agree-TFSES to agree-FSES

U	u_1	u_2	u_3	u_4
(e_1, m)	0.50	0.70	0.67	0.63
(e_1, n)	0.61	0.63	0.70	0.56
(e_2, m)	0.64	0.63	0.62	0.64
(e_2, n)	0.66	0.58	0.65	0.67
(e_3, m)	0.67	0.61	0.53	0.70
(e_3, n)	0.80	0.66	0.52	0.67

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Table 3 – continued

U	u_1	u_2	u_3	u_4
$c_j = \sum_i u_{ij}$	$c_1 = 3.88$	$c_2 = 3.81$	$c_3 = 3.69$	$c_4 = 3.87$

Likewise, by using relation (1), we calculate $F(Z)$ to convert the disagree time fuzzy soft expert set to disagree fuzzy soft expert set, as we explained in converting the agree time fuzzy soft expert set to agree fuzzy soft expert set, then we compute k_j, where k_j represent the column sum of an object u_i. The results are shown in Table 4.

Table 4: Converting disagree-TFSES to disagree-FSES

U	u_1	u_2	u_3	u_4
(e_1, m)	0.58	0.46	0.55	0.65
(e_1, n)	0.60	0.61	0.55	0.73
(e_2, m)	0.60	0.59	0.62	0.51
(e_2, n)	0.58	0.75	0.62	0.57
(e_3, m)	0.61	0.61	0.68	0.56
(e_3, n)	0.63	0.55	0.70	0.60
$k_j = \sum_i u_{ij}$	$k_1 = 3.60$	$k_2 = 3.57$	$k_3 = 3.72$	$k_4 = 3.62$

The most suitable option is selected using the greatest choice value of s_j, which is obtained by subtracting the agree-FSES's (c_j) from the disagree-FSES's k_j (Table 5).

Table 5: Choice value of FSES

$c_j = \sum_i u_{ij}$	$k_j = \sum_i u_{ij}$	$s_j = c_j - k_j$
$c_1 = 3.88$	$k_1 = 3.60$	$s_1 = 0.28$
$c_2 = 3.81$	$k_2 = 3.57$	$s_2 = 0.24$
$c_3 = 3.69$	$k_3 = 3.72$	$s_3 = -0.3$
$c_4 = 3.87$	$k_4 = 3.62$	$s_4 = 0.25$

Then $\max s_j = s_1$, scored by u_1 the decision is in favour of selecting u_1.

7 Conclusion

In this work, I have examined some of the features of the time fuzzy soft expert set and proposed the idea. The time-fuzzy soft set has been used to describe the complement, union, and intersection operations. A speculative implementation of this theory to address decision-making dilemmas was provided.For future studies of these results, these tools can be developed by linking them with other concepts that can be found in the following works see[43]–.[48]

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