Decision Making in the Case of Confirmed Data Neutrosophic Linear Models to Choose the Advertising Medium

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Abstract

In light of the great development witnessed by our contemporary world, it has become necessary to focus on scientific methods and use the quantitative method to reach more accurate decisions, appropriate to the surrounding circumstances and factors. The process of decision-making and choosing the optimal alternative depends on the type and quality of data that describes the issue for which the decision is to be made. Regarding it, in this chapter we present a study of the issue of determining the ideal advertising medium to display a company's products. This issue is considered one of the issues of decision-making in the case of confirmed data, so we build the appropriate mathematical model and through the optimal solution to it we can make the ideal decision through which the company achieves its goal from the campaign. Informative, we will divide this study into two parts. In the first section, we will develop a general formula for this issue, and the data will be classical values. We will obtain a linear mathematical model. In the second section, we will formulate the issue from the perspective of neutrosophic science, meaning we will take the data as neutrosophic values, obtaining a linear neutrosophic model.

Keywords: Decision making in the case of confirmed data; Linear models; Neutrosophic science; Neutrosophic linear models; The issue of choosing the advertising medium

1. Introduction

In the world of rapid changes, there has become an urgent need to make a rational decision based on quantitative methods that limit the level of risk, especially if the decisions are critical and the decision issues are huge and complex. Decision making theory is one of the important theories provided by operations research. The scientific basis for this theory depends on the data of the case under study. This data forms the basic description of the problem, from which we can determine the type of solution for the case. In decision theory, we have three types of data: certain data, uncertain data, and random data that repeat according to a certain probability distribution law. In the case of confirmed data, we can formulate the problem in the form of a mathematical model with the optimal solution that enables us to choose the appropriate alternative. Classic studies of decision theory provided appropriate rules for choosing the optimal decision for each type of data. After the emergence of neutrosophic logic, many studies were presented in most fields using the concepts of this logic. This is evidenced by what was presented by (Smarandache et al., 2023), and as for the theory of decision-making, (Jdid et al., 2022) presented a neutrosophic study of the theory of decision-making in the case of uncertain data, and (Jdid et al.,2023) also presented a neutrosophic study of the theory of decision-making. In case of risk. In this chapter, we present a neutrosophical study of the issue of choosing the appropriate advertising medium, which is a question of making a decision (the case of confirmed data), meaning that the decision maker can determine the results of each of the available alternatives with certainty. Using the information provided by (Bukajh et al., 1998), we provide a general formulation of this issue. The optimal solution to the mathematical model represented in it helps decision makers choose the appropriate decision that achieves the company's goal from the advertising campaign. Using the information provided by (Jdid et al., 2022), we will reformulate the problem using the concepts of neutrosophic logic and thus obtain a linear neutrosophic model. We use the simple neutrosophic algorithm presented by (Jdid et al., 2022) or the modified simple neutrosophic algorithm presented by (Jdid et al., 2024) to obtain the ideal neutrosophic solution that enables us to make the appropriate decision that achieves the company's goal from the advertising campaign.

2. Discussion

1- Mathematical background:

The science of operations research, with its various methods, has provided optimal solutions to many issues and helped make decisions based on scientific foundations. Its basic essence is to build a model and then use appropriate algorithms to obtain the optimal solution to it, through which the ideal and appropriate decision for the nature of the issue under study is determined. The most important methods of operations research are the linear programming method because we find that many real-world problems can be represented by a linear model and also because there are many algorithms that help us obtain the optimal solution. Given this importance, we have reformulated the linear models and the most important algorithms used to find the optimal solution using the concepts of neutrosophic logic. In this chapter, we present a neutrosophical study of one of the decision-making issues in the case of confirmed data, which can be formulated through a linear model whose optimal solution gives us the appropriate decision, which is the issue of choosing the advertising medium.

1-1-   A model for choosing the advertising medium. See (Bukajh et al., 1998).
The text of the issue as stated in the reference:

An advertising company wants to plan a media campaign using three different means: television, radio, and magazines. This campaign aims to reach the largest possible number of potential customers. The results of the market study give the following table:

Table 1: Issue data

advertising medium Information	TV		Radio	Magazines
	During the day	During the evening		
Cost of advertising unit	40000$	75000$	30000$	15000$
The number of customers that can be reached for the advertising unit	400000	900000	500000	200000
The number of female customers that can be reached by the advertising unit	300000	400000	200000	100000

Assuming that the company does not want to spend more than 800000$ on advertising, and that it wants the following:

a.   The number of views from women should reach at least two million.
b.   The cost of advertising on TV is set at 500000$.
c.   To purchase at least three advertising units on TV during the day and two units in the evening.
d.   The number of advertising units on radio and magazines should range between 5 and 10 units for each.

We assume that the number of advertising units purchased on TV during the day and in the evening, on the radio, and on magazines, respectively, is: x_1, x_2, x_3, x_4 so the number of potential customers estimated in thousands is:

$$400x_1 + 900x_2 + 500x_3 + 200x_4$$

Since the company does not want to spend more than 800000$ on advertising, we get the following constraint:

$$40000x_1 + 75000x_2 + 30000x_3 + 15000x_4 \leq 800000$$

The restriction on the number of female views that can be achieved through the media campaign is:

$$300000x_1 + 400000x_2 + 200000x_3 + 100000x_4 \geq 2000000$$

The cost restriction for TV advertising is:

$$40000x_1 + 75000x_2 \leq 500000$$

The restriction number of units in the TV:

$$x_1 \geq 3$$

$$x_2 \geq 2$$

The restriction on the number of advertising units on radio is:

$$5 \leq x_3 \leq 10$$

The restriction on the number of advertising units on magazines is:

$$5 \leq x_4 \leq 10$$

Mathematical model:
Find:

$$MaxZ = 400000x_1 + 900000x_2 + 500000x_3 + 200000x_4$$

Constraints:

$$40000x_1 + 75000x_2 + 30000x_3 + 15000x_4 \leq 800000$$

$$300000x_1 + 400000x_2 + 200000x_3 + 100000x_4 \geq 2000000$$

$$40000x_1 + 75000x_2 \leq 500000$$

$$x_1 \geq 3$$

$$x_2 \geq 2$$

$$x_3 \geq 5$$

$$x_3 \leq 10$$

$$x_4 \geq 5$$

$$x_4 \leq 10$$

$$x_1, x_2, x_3, x_4 \geq 0$$

1-2-   A study of neutrosophic linear models, see (Jdid et al., 2022).

Neutrosophic linear models are linear models that include in their composition a set of neutrosophic variables, which can undergo change as a result of the work environment, and the presence of neutrosophic values guarantees that the facility that operates according to these models has a safe workflow, in which the variables in the objective function are neutrosophic values, i.e. $c_j \pm \varepsilon_j$ Where ε_j is the indeterminacy and it can be $\varepsilon_j \in [\lambda_1, \lambda_2]$or $\varepsilon_j \in \{\lambda_1, \lambda_2\}$ where $j = 1,2, \dots, n$

Also, the values that express the available possibilities are neutrosophic values. This means that $b_i \pm \delta_i$ where δ_i is indeterminacy and it is possible to be

$\delta_i \in [\mu_1, \mu_2]$or $\delta_i \in \{\mu_1, \mu_2\}$ where $i = 1,2, \dots, m$, and therefore the neutrosophic linear model is written in the following general form:

The general neutrosophic formula for the linear mathematical model is given in abbreviated form as follows:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Max \ or \ Min$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} b_i \pm \delta_i \quad ; \quad i = 1,2, \dots, m$$

$$x_j \geq 0$$

Where $c_j + \varepsilon_j$, $b_i \pm \delta_i$, a_{ij}, $j = 1,2, \dots, n$, $i = 1,2, \dots, m$ are constants having set or interval values according to the nature of the given problem, x_j are decision variables.

Future research direction:

In this chapter, we will develop a general formulation of one of the important issues that we encounter in our daily lives, which is the issue of choosing the appropriate advertising medium through which we achieve access to the largest possible number of customers to display a company's products. This is in two cases:

1-3-   Model data are classical values

Formulation of the issue:

A company wants to establish an advertising campaign to display its products using **n** media means. This campaign aims to reach the largest possible number of customers. The team responsible for collecting information about each of the means provided the following data:

Table 2: Data for the classic problem

advertising medium Information	A_1	A_2	...	A_n
Cost of advertising unit	c_1	c_2	...	c_n
The number of customers that can be reached for the advertising unit	b_1	b_2	...	b_n
The number of female customers that can be reached by the advertising unit	w_1	w_2	...	w_n
The number of male customers that can be reached for the advertising unit	m_1	m_2	...	m_n

Assuming that the company does not want to spend more than d\$ on advertising, and that it wants the following:
a.   The number of female views must reach at least S.
b.   The number of male views should reach at most K.
c.   The cost of advertising through the advertising medium A_r should not exceed the amount d_r\$.
d.   The number of advertising units in the advertising medium A_t should range between E and F unit.

Assuming that the number of advertising units purchased in each advertising method $A_1, A_2, \dots, A_n$ is $x_1, x_2, \dots, x_n$, then the number of potential customers is:

$$b_1 x_1 + b_2 x_2 + \cdots + b_n x_n$$

Since the company does not want to spend more than d dollars on advertising, we get the following constraint:

$$c_1 x_1 + c_2 x_2 + \cdots + c_n x_n \leq d$$

The restriction on the number of female views that can be achieved through the media campaign is:

$$w_1 x_1 + w_2 x_2 + \cdots + w_n x_n \geq S$$

The restriction on the number of male views that can be achieved through the media campaign is:

$$m_1 x_1 + m_2 x_2 + \cdots + m_n x_n \leq K$$

The cost entry for advertising using the advertising medium A_r is:

$$c_r x_r \leq d_r$$

The restriction on the number of advertising units in the advertising medium A_t is:

$$E \leq x_t \leq F$$

Mathematical model:

Find:

$$MaxZ = b_1 x_1 + b_2 x_2 + \cdots + b_n x_n$$

Constraints:

$$c_1 x_1 + c_2 x_2 + \cdots c_n x_n \leq d$$

$$w_1 x_1 + w_2 x_2 + \cdots w_n x_n \geq S$$

$$m_1 x_1 + m_2 x_2 + \cdots m_n x_n \leq K$$

$$c_r x_r \leq d_r$$

$$E \leq x_t \leq F$$

$$x_1, x_2, \dots, x_n \geq 0$$

Example 1:

We rephrase the example in reference (Bukajh et al., 1998) as follows:

A company that produces women's bags and shoes wants to establish a media campaign to display its products using three media means: TV, radio, and road signs. This campaign aims to reach the largest possible number of customers. The team responsible for collecting information about each of the three means provided the following data:

Table 3: Classic example data

advertising medium Information	TV		Radio	Road signs
	During the day	During the evening		
Cost of advertising unit	40000$	75000$	30000$	15000$
The number of customers that can be reached for the advertising unit	400000	900000	500000	200000
The number of female customers that can be reached by the advertising unit	300000	400000	200000	100000

Assuming that the company does not want to spend more than 800000$ on advertising, and that it wants the following:

a. The number of views from women should reach at least two million.
b. The cost of advertising on TV is set at 500000$.
c. To purchase at least three advertising units on TV during the day and two units in the evening.
d. The number of advertising units on radio and road billboards should range between 5 and 10 units for each.

We assume that the number of advertising units purchased on TV during the day and in the evening, on the radio, and on road signs, respectively, is: x_1, x_2, x_3, x_4 so the number of potential customers estimated in thousands is:

$$400x_1 + 900x_2 + 500x_3 + 200x_4$$

Since the company does not want to spend more than 800000$ on advertising, we get the following constraint:

$$40000x_1 + 75000x_2 + 30000x_3 + 15000x_4 \leq 800000$$

The restriction on the number of female views that can be achieved through the media campaign is:

$$300000x_1 + 400000x_2 + 200000x_3 + 100000x_4 \geq 2000000$$

The cost restriction for TV advertising is:

$$40000x_1 + 75000x_2 \leq 500000$$

The restriction number of units in the TV:

$$x_1 \geq 3$$
$$x_2 \geq 2$$

The restriction on the number of advertising units on radio is:

$$5 \leq x_3 \leq 10$$

The restriction on the number of advertising units on road signs is:

$$5 \leq x_4 \leq 10$$

Mathematical model:

Find:

$$MaxZ = 400000x_1 + 900000x_2 + 500000x_3 + 200000x_4$$

Constraints:

$$40000x_1 + 75000x_2 + 30000x_3 + 15000x_4 \leq 800000$$

$$300000x_1 + 400000x_2 + 200000x_3 + 100000x_4 \geq 2000000$$

$$40000x_1 + 75000x_2 \leq 500000$$

$$x_1 \geq 3$$

$$x_2 \geq 2$$

$$x_3 \geq 5$$

$$x_3 \leq 10$$

$$x_4 \geq 5$$

$$x_4 \leq 10$$

$$x_1, x_2, x_3, x_4 \geq 0$$

The model we obtained is a linear model. We use the direct simplex algorithm or one of its modifications to find the optimal solution through which the company achieves the goal of the media campaign.

1-4-   Model data are neutrosophic values:

Based on the study presented in the paper (Jdid et al., 2022) of the linear neutrosophic models, we can develop the following general formula for the neutrosophic media selection model.

Formulation of the issue:

A company wants to establish an advertising campaign to display its products using n media means. This campaign aims to reach the largest possible number of customers. The team responsible for collecting information about each of the means provided the following data:

Table 4: Issue data Neutrosophic values

advertising medium Information	A_1	A_2	...	A_n
Cost of advertising unit	$C_{1N} = c_1 \pm \varepsilon_1$	$C_{2N} = c_2 \pm \varepsilon_2$	...	$C_{nN} = c_n \pm \varepsilon_n$
The number of customers that can be reached for the advertising unit	$B_{1N} = b_1 \pm \delta_1$	$B_{2N} = b_2 \pm \delta_2$	...	$B_{nN} = b_n \pm \delta_n$
The number of female customers that can be reached by the advertising unit	$W_{1N} = w_1 \pm \gamma_1$	$W_{2N} = w_2 \pm \gamma_2$	...	$W_{nN} = w_n \pm \gamma_n$
The number of male customers that can be reached for the advertising unit	$M_{1N} = m_1 \pm \beta_1$	$M_{2N} = m_2 \pm \beta_2$	...	$M_{nN} = m_n \pm \beta_n$

Where $\varepsilon_j, \delta_j, \gamma_j, \beta_j$ is the indeterminacy of the cost of the advertising unit, the number of possible customers, the number of female customers, and the number of male customers. It may be a field or a group and expresses the increase or decrease to which the data of the issue may be exposed.

Assuming that the company does not want to spend more than $d\$$ on advertising, and that it wants the following:
a.   The number of female views must reach at least S.
b.   The number of male views should reach at most K.
c.   The cost of advertising through the advertising medium A_r should not exceed the amount $d_r\$$.
d.   The number of advertising units in the advertising medium A_t should range between E and F unit.

Assuming that the number of advertising units purchased in each advertising method $A_1, A_2, \ldots, A_n$ is $x_1, x_2, \ldots, x_n$, then the number of potential customers is:

$$B_{1N}x_1 + B_{2N}x_2 + \cdots + B_{nN}x_n$$

Since the company does not want to spend more than d dollars on advertising, we get the following constraint:

$$C_{1N}x_1 + C_{2N}x_2 + \cdots + C_{nN}x_n \leq d$$

The restriction on the number of female views that can be achieved through the media campaign:

$$W_{1N}x_1 + W_{2N}x_2 + \cdots + W_{nN}x_n \geq S$$

The restriction on the number of male views that can be achieved through the media campaign is:

$$M_{1N}x_1 + M_{2N}x_2 + \cdots + M_{nN}x_n \leq K$$

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The cost entry for advertising using the advertising medium A_r is:

$$C_{rN}x_r \leq d_r$$

The restriction on the number of advertising units in the advertising medium A_t is:

$$E \leq x_t \leq F$$

Mathematical model:

Find:

$$MaxZ_N = B_{1N}x_1 + B_{2N}x_2 + \cdots + B_{nN}x_n$$

Constraints:

$$C_{1N}x_1 + C_{2N}x_2 + \cdots + C_{nN}x_n \leq d$$

$$W_{1N}x_1 + W_{2N}x_2 + \cdots + W_{nN}x_n \geq S$$

$$M_{1N}x_1 + M_{2N}x_2 + \cdots + M_{nN}x_n \leq K$$

$$C_{rN}x_r \leq d_r$$

$$E \leq x_t \leq F$$

$$x_1, x_2, \ldots, x_n \geq 0$$

Example 2:

Neutrosophic formulation of the example in reference (Bukajh et al., 1998) using the information in reference (Jdid et al., 2022):

A company that produces women's bags and shoes wants to establish a media campaign to display its products using three media means: TV, radio, and road signs. This campaign aims to reach the largest possible number of customers. The team responsible for collecting information about each of the three means provided the following data:

Table 5: Example data Neutrosophic values

advertising medium Information	TV		Radio	Road signs
	During the day	During the evening		
Cost of advertising unit	$40000 + \varepsilon_1$\$	$75000 + \varepsilon_2$\$	$30000 + \varepsilon_3$\$	$15000 + \varepsilon_4$\$
The number of customers that can be reached for the advertising unit	400000	900000	500000	200000
The number of female customers that can be reached by the advertising unit	300000	400000	200000	100000

Assuming that the company does not want to spend more than 800000\$ on advertising, and that it wants the following:

a. The number of views from women should reach at least two million
b. The cost of advertising on TV is set at 500000\$
c. To purchase at least three advertising units on TV during the day and two units in the evening.
d. The number of advertising units on radio and road billboards should range between 5 and 10 units for each.

We assume that the number of advertising units purchased on TV during the day and in the evening, on the radio, and on road signs, respectively, is: x_1, x_2, x_3, x_4 so the number of potential customers estimated in thousands is:

$$400x_1 + 900x_2 + 500x_3 + 200x_4$$

Since the company does not want to spend more than 800000\$ on advertising, we get the following constraint:

$$(40000 \pm \varepsilon_1)x_1 + (75000 \pm \varepsilon_2)x_2 + (30000 \pm \varepsilon_3)x_3 + (15000 \pm \varepsilon_4)x_4 \leq 800000$$

The restriction on the number of female views that can be achieved through the media campaign is:

$$300000x_1 + 400000x_2 + 200000x_3 + 100000x_4 \geq 2000000$$

The cost restriction for TV advertising is:

$$(40000 + \varepsilon_1)x_1 + (75000 + \varepsilon_2)x_2 \leq 500000$$

The restriction number of units in the TV:

$$x_1 \geq 3$$

$$x_2 \geq 2$$

The restriction on the number of advertising units on radio is:

$$5 \leq x_3 \leq 10$$

The restriction on the number of advertising units on road signs is:

$5 \leq x_4 \leq 10$

Taking $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = [0,5000]$,we get the following form:

Mathematical model:
Find:

$$MaxZ = 400000x_1 + 900000x_2 + 500000x_3 + 200000x_4$$

Constraints:

$$[40000,45000]x_1 + [75000,80000]x_2 + [30000,35000]x_3 + [15,20000]x_4 \leq 800000$$

$$300000x_1 + 400000x_2 + 200000x_3 + 100000x_4 \geq 2000000$$

$$(40000 + \varepsilon_1)x_1 + (75000 + \varepsilon_2)x_2 \leq 500000$$

$$x_1 \geq 3$$

$$x_2 \geq 2$$

$$x_3 \geq 5$$

$$x_3 \leq 10$$

$$x_4 \geq 5$$

$$x_4 \leq 10$$

$$x_1, x_2, x_3, x_4 \geq 0$$

The model we obtained is a linear neutrosophic model. We use the direct simplex algorithm (see (Jdid et al.2022), or the modified neutrosophic simplex algorithm (see (Jdid et al.2024), to find the optimal solution through which the company achieves the goal of the media campaign.

3. Conclusion and results

Through the previous study, we presented a general formulation of an important issue that we encounter in daily life and which many companies need to market their products. We presented the issue using classical data and obtained a linear model through which the company achieves the optimal goal of the advertising campaign. In order for the study to be more realistic and appropriate for all circumstances, we by formulating the issue from the perspective of neutrosophic science, we obtained a neutrosophic linear model whose optimal solution is neutrosophic values that enjoy a margin of freedom, take into account all circumstances, and achieve the company's goal of the advertising campaign.

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