Time Fuzzy Soft Sets and its application in design-making

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Abstract

In this study, we define time-fuzzy soft set (T-FSS) as an extension of fuzzy soft set. We will also define and investigate the features of its main operations (complement, union intersection, "AND" and "OR"). Finally, we'll apply this approach to decision-making difficulties.

Keywords: Soft set; Fuzzy soft set; Time-fuzzy soft set

1 Introduction

The majority of problems in engineering, medical research, economics, and the environment are fraught with uncertainty. Molodtsov[1]] introduced the notion of soft set theory as a mathematical tool for coping with such uncertainty. Following Molodtsov's work,,[3] Maji et al.[4] and Maji et al.[5] researched several soft set operations and applications. Also Maji et al.[8] they presented the notion of fuzzy soft set as a more broad concept, as well as a combination of fuzzy set and soft set, and investigated its features. Roy and Maji[9] also applied this idea to handle decision-making challenges. Recently, various scholars have begun studying the properties and applications of soft set theory as in the research,[11],[15].[23] Furthermore, in 2010 Çağman et al.[20] established the notion of fuzzy parameterized fuzzy soft set (*fpfs*) and its operations.In addition, the fpfs-aggregation operator is used to create the fpfs-decision making technique, which allows for more efficient decision processes. Alkhazaleh and Salleh[16] proposed the notion of soft expert sets and fuzzy soft expert sets, which allow users to get the views of all experts in one model without any procedures.Hazaymeh. A[7] discusses fuzzy parameterized fuzzy soft expert sets, which offer a membership value for each parameter in a collection of parameters and are an extension of fuzzy soft expert sets. Wang[24]showed that in many real situations, immediate sensory data is insufficient for decision making.[10] provide an overview of generalized fuzzy soft expert set. Recently, various scholars have begun studying the properties and applications of soft set theory. Some topics in algebraic structures are extended by fuzzy soft sets, neutrosophic, or even plithogenic logical sets, as in the research,[14],[12],[21] and there are also studies in fuzzy tapology and neutrosophic fuzzy topology. For more details about neutrosophic topology, see,[2].[6] Additionally, researchers introduced using a neutrosophic fuzzy soft set to solve decision-making problems, like in,[13],[22] other researchers introduced topics in complex fuzzy as,[25] we are looking to integrate time fuzzy soft set and fuzzy soft set with new concepts as in the works of,[17],[18].[19] Enriching the state with knowledge about prior actions and events can help you distinguish between situations that might otherwise look identical, allowing you to make accurate judgments while also learning the proper options. Furthermore, knowledge of the past can eliminate the need for unrealistic sensors, such as knowing your exact location in a maze. Using historical information as part of the state representation provides us with important information to assist us make better judgments in situations when temporal value is not taken into account, resulting in less accurate decision-making. If we want to take the views of more than one time (period), we must perform various operations such as union, intersection, etc. For a solution to this

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problem, we take a collection of time (periods), generalize it into what we call a time-fuzzy soft set (T-FSS), investigate some of its features, and apply this notion to a decision-making problem. It is critical to understand the history of the parameters under consideration in order to ensure the credibility of the information provided by specialists. The experts' previous experiences are gathered in the number of periods (years, months, etc.) in which they are involved in a certain decision-making circumstance, and by looking at the time component, individuals are more confident in the conclusion that they make. We must examine the influence of time on fuzzy soft set applications, not only for the present period, but also for the past and future periods (forecasting information), as shown in Figure 1.

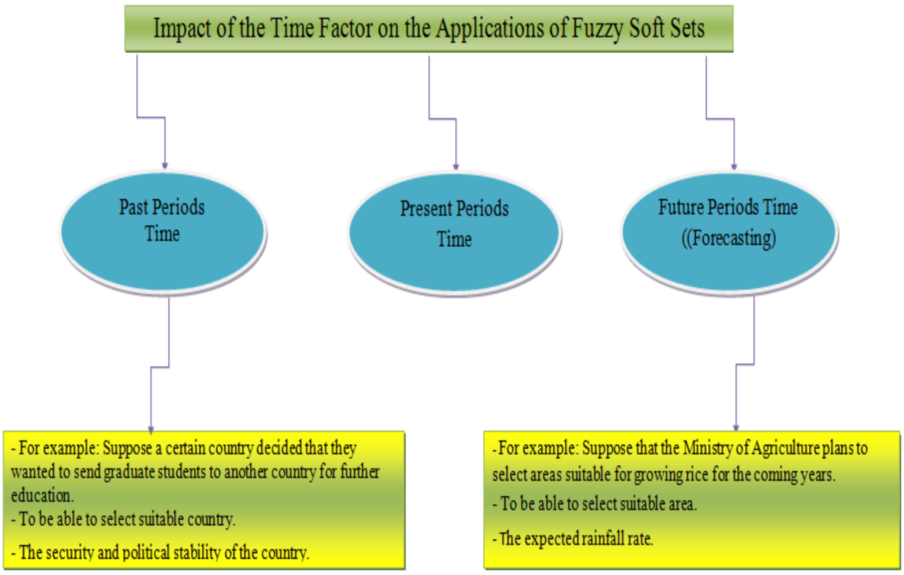

Figure 1: Impact of the Time Factor on the Applications of Fuzzy Soft Sets

In this paper, we will present the notion of time-fuzzy soft set,Which is more effective and valuable, as we will see and the decisions made will be more precise, this means we will take the component time value of the information in our consideration when we are making decision. We will also define and investigate the attributes of its basic operations, which are complement, union and intersection. Finally, we'll apply this approach to decision-making difficulties.

2 Preliminaries

In this part, we cover several fundamental concepts in soft set theory. Molodtsov[1] defined soft sets over U as follows: Let U be a universe set and E set of parameters, $P(U)$ denotes the power set of U and $A \subseteq E$.

Definition 2.1. [1] consider this mapping

$$F : A \to P(U).$$

Any A pair (F, A) is considered a *soft set* over U. In other terms, a soft set over U is a parameterized collection of subsets of the universe set U. For $\varepsilon \in A, F(\varepsilon)$ can be viewed as the set of ε-approximate members of the soft set (F, A).

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Definition 2.2. [8] Let U be the initial universal set, and E be the set of parameters. Let I^U be the power set of all fuzzy subsets of U. Let $A \subseteq E$, and F be the mapping

$$F : A \to I^U.$$

A pair (F, E) is known as a *fuzzy soft set* over U.

Definition 2.3. [8] Regarding two fuzzy soft sets (F, A) and (G, B) over U, (F, A) is known as a fuzzy soft subset of. (G, B) if

1. $A \subset B$ and
2. $\forall \varepsilon \in A, F(\varepsilon)$ is fuzzy subset of $G(\varepsilon)$.

The relationship is represented by $(F,\ A) \tilde{\subset} (G, B)$. In this situation, (G, B) is known as a fuzzy, soft superset of. (F, A).

Definition 2.4. [8] The complement of a fuzzy soft set (F, A) is denoted by $(F, A)^c$ And has been defined by $(F, A)^c = (F^c, \rceil A)$ where $F^c : \rceil A \to P(U)$ is a mapping provided by

$$F^c(\alpha) = c(F(\rceil\alpha)), \forall \alpha \in \rceil A.$$

c describes any fuzzy complement.

Definition 2.5. [8] If (F, A) and (G, B) are two fuzzy soft sets then "(F, A) AND (G, B)" denoted by $(F,\ A) \wedge (G, B)$ is defined by

$$(F, A) \wedge (G, B) = (H, A \times B)$$

such that $H(\alpha, \beta) = t(F(\alpha), G(\beta)), \forall (\alpha, \beta) \in A \times B$, where t is any t-norm.

Definition 2.6. [8] If (F, A) and (G, B) are two fuzzy soft sets then "(F, A) OR (G, B)" denoted by $(F, A) \vee (G, B)$ is defined by

$$(F, A) \vee (G, B) = (O, A \times B)$$

such that $O(\alpha, \beta) = s(F(\alpha), G(\beta)), \forall (\alpha, \beta) \in A \times B$, where s is any s-norm.

Definition 2.7. [8] The union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if \quad \varepsilon \in A - B, \\ G(\varepsilon), & if \quad \varepsilon \in B - A, \\ s(F(\varepsilon), G(\varepsilon)), & if \quad \varepsilon \in A \cap B. \end{cases}$$

Where s is any s-norm.

Definition 2.8. [8] The intersection of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if \quad \varepsilon \in A - B, \\ G(\varepsilon), & if \quad \varepsilon \in B - A, \\ s(F(\varepsilon), G(\varepsilon)), & if \quad \varepsilon \in A \cap B. \end{cases}$$

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3 Time-Fuzzy Soft Set

In this part, we define a time-fuzzy soft set and discuss its fundamental properties. Some or all of the factors contain a time value for previous data, which implies we must consider the component time value of the information while making judgments, since this will result in more precise decisions.

Definition 3.1. Let U be the initial universal set, and let E be the set of parameters. Let I^U be the power set of all fuzzy subsets of U. let $A \subseteq E$ and T be a set of time where $T = \{t_1, t_2, ..., t_n\}$. A collection of pairs $(F, E)_t \, \forall \, t \in T$ is called a *time-fuzzy soft set* $\{T - FSS\}$ over U where F is a mapping provided by

$$F_t : A \to I^U.$$

Example 1. *Let* $U = \{u_1, u_2, u_3, u_4\}$ *be a set of universe,* $E = \{e_1, e_2, e_3\}$ *a set of parameters and* $T = \{t_1, t_2, t_3,\}$ *be a set of time. Define a function*

$$F_t : A \to I^U.$$

as shown below:

$$F_1(e_1) = \left\{ \frac{{u_1}^{t_1}}{0.6}, \frac{{u_2}^{t_1}}{0.3}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.4} \right\}, F_1(e_2) = \left\{ \frac{{u_1}^{t_1}}{0.5}, \frac{{u_2}^{t_1}}{0.3}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.7} \right\},$$

$$F_1(e_3) = \left\{ \frac{{u_1}^{t_1}}{0.3}, \frac{{u_2}^{t_1}}{0.6}, \frac{{u_3}^{t_1}}{0.8}, \frac{{u_4}^{t_1}}{0.9} \right\}, F_2(e_1) = \left\{ \frac{{u_1}^{t_2}}{0.7}, \frac{{u_2}^{t_2}}{0.4}, \frac{{u_3}^{t_2}}{0.1}, \frac{{u_4}^{t_2}}{0.3} \right\},$$

$$F_2(e_2) = \left\{ \frac{{u_1}^{t_2}}{0.5}, \frac{{u_2}^{t_2}}{0.3}, \frac{{u_3}^{t_2}}{0.2}, \frac{{u_4}^{t_2}}{0.7} \right\}, F_2(e_3) = \left\{ \frac{{u_1}^{t_2}}{0.7}, \frac{{u_2}^{t_2}}{0.8}, \frac{{u_3}^{t_2}}{0.6}, \frac{{u_4}^{t_2}}{0.4} \right\},$$

$$F_3(e_1) = \left\{ \frac{{u_1}^{t_3}}{0.7}, \frac{{u_2}^{t_3}}{0.8}, \frac{{u_3}^{t_3}}{0.6}, \frac{{u_4}^{t_3}}{0.4} \right\}, F_3(e_2) = \left\{ \frac{{u_1}^{t_3}}{0.7}, \frac{{u_2}^{t_3}}{0.5}, \frac{{u_3}^{t_3}}{0.6}, \frac{{u_4}^{t_3}}{0.7} \right\},$$

$$F_3(e_3) = \left\{ \frac{{u_1}^{t_3}}{0.3}, \frac{{u_2}^{t_3}}{0.6}, \frac{{u_3}^{t_3}}{0.8}, \frac{{u_4}^{t_3}}{0.9} \right\}.$$

The time-fuzzy soft sets $(F, E)_t$ *are made up of the approximations shown below:*

$$\begin{aligned}(F, E)_t = \Bigg\{ & \left(e_1, \left\{ \frac{{u_1}^{t_1}}{0.6}, \frac{{u_2}^{t_1}}{0.3}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.4} \right\}\right), \left(e_2, \left\{ \frac{{u_1}^{t_1}}{0.5}, \frac{{u_2}^{t_1}}{0.3}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.7} \right\}\right), \\ & \left(e_3, \left\{ \frac{{u_1}^{t_1}}{0.3}, \frac{{u_2}^{t_1}}{0.6}, \frac{{u_3}^{t_1}}{0.8}, \frac{{u_4}^{t_1}}{0.9} \right\}\right), \left(e_1, \left\{ \frac{{u_1}^{t_2}}{0.7}, \frac{{u_2}^{t_2}}{0.4}, \frac{{u_3}^{t_2}}{0.1}, \frac{{u_4}^{t_2}}{0.3} \right\}\right), \\ & \left(e_2, \left\{ \frac{{u_1}^{t_2}}{0.5}, \frac{{u_2}^{t_2}}{0.3}, \frac{{u_3}^{t_2}}{0.2}, \frac{{u_4}^{t_2}}{0.7} \right\}\right), \left(e_3, \left\{ \frac{{u_1}^{t_2}}{0.7}, \frac{{u_2}^{t_2}}{0.8}, \frac{{u_3}^{t_2}}{0.6}, \frac{{u_4}^{t_2}}{0.4} \right\}\right), \\ & \left(e_1, \left\{ \frac{{u_1}^{t_3}}{0.7}, \frac{{u_2}^{t_3}}{0.8}, \frac{{u_3}^{t_3}}{0.6}, \frac{{u_4}^{t_3}}{0.4} \right\}\right), \left(e_2, \left\{ \frac{{u_1}^{t_3}}{0.7}, \frac{{u_2}^{t_3}}{0.5}, \frac{{u_3}^{t_3}}{0.6}, \frac{{u_4}^{t_3}}{0.7} \right\}\right), \\ & \left(e_3, \left\{ \frac{{u_1}^{t_3}}{0.3}, \frac{{u_2}^{t_3}}{0.6}, \frac{{u_3}^{t_3}}{0.8}, \frac{{u_4}^{t_3}}{0.9} \right\}\right) \Bigg\}.\end{aligned}$$

Definition 3.2. For two T-FSSs $(F, A)_t$ and $(G, B)_t$ over U, $(F, A)_t$ is called a T-FSS subset of $(G, B)_t$ if

1. $A \subseteq B$,
2. $\forall t \in T, \epsilon \in A,\ F_t(\epsilon)$ is fuzzy soft subset of $G_t(\epsilon)$.

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Definition 3.3. Two T-FSSs $(F,A)_t$ and $(G,B)_t$ over U, are said to be *equal* if $(F,A)_t$ is a T-FSS subset of $(G,A)_t$ and $(G,A)_t$ is a T-FSS subset of $(F,A)_t$.

Example 2. *Consider Example 1, assuming that the*

$$(F,E)_t = \left\{ \left(e_1, \left\{\frac{u_1{}^{t_1}}{0.8}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.3}, \frac{u_4{}^{t_1}}{0.7}\right\}\right), \left(e_2, \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.9}\right\}\right), \right.$$

$$\left(e_2, \left\{\frac{u_1{}^{t_2}}{0.8}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.5}, \frac{u_4{}^{t_2}}{0.7}\right\}\right), \left(e_3, \left\{\frac{u_1{}^{t_2}}{0.8}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.8}\right\}\right),$$

$$\left.\left(e_1, \left\{\frac{u_1{}^{t_3}}{0.5}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.7}\right\}\right), \left(e_3, \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.7}\right\}\right) \right\}.$$

$$(G,E)_t = \left\{ \left(e_2, \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.3}, \frac{u_3{}^{t_1}}{0.3}, \frac{u_4{}^{t_1}}{0.5}\right\}\right), \left(e_3, \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{2}}{0.6}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.7}\right\}\right), \right.$$

$$\left.\left(e_1, \left\{\frac{u_1{}^{t_3}}{0.5}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.3}\right\}\right) \right\}.$$

Therefore $(G,E)_t \subseteq (F,E)_t$.

Definition 3.4. A time fuzzy soft set $(F,A)_t$ over U is stated to be semi-null. T-FSS is represented by $T_{\sim}\varphi$, if $\forall t \in T$, $F_t(e) = \Phi$ for at least one e.

Definition 3.5. A time fuzzy soft set $(F,A)_t$ over U is said to be null T-FSS denoted by T_φ, if $\forall t \in T$, $F_t(e) = \Phi\ \forall e$.

Definition 3.6. A time fuzzy soft set $(F,A)_t$ over U is said to be semi-absolute T-FSS denoted by $T_{\sim}A$, if $\forall t \in T$, $F_t(e) = \bar{1}$ for at least one e.

Definition 3.7. A time fuzzy soft set $(F,A)_t$ over U is said to be absolute T-FSS denoted by T_A, if $\forall t \in T$, $F_t(e) = \bar{1}\ \forall e$.

Example 3. *Consider Example 1. Let*

$$(F,A)_t = \left\{ \left(e_1, \{\Phi\}^{t_1}\right), \left(e_2, \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.3}, \frac{u_4{}^{t_1}}{0.6}\right\}\right), \right.$$

$$\left(e_3, \left\{\frac{u_1{}^{t_1}}{0.2}, \frac{u_2{}^{t_1}}{0.3}, \frac{u_3{}^{t_1}}{0.6}, \frac{u_4{}^{t_1}}{0.5}\right\}\right), \left(e_1, \{\Phi\}^{t_2}\right),$$

$$\left(e_2, \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.8}, \frac{u_4{}^{t_2}}{0.3}\right\}\right), \left(e_3, \left\{\frac{u_1{}^{t_2}}{0.3}, \frac{u_2{}^{t_2}}{0.2}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.6}\right\}\right),$$

$$\left(e_1, \{\Phi\}^{t_3}\right), \left(e_2, \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.3}\right\}\right),$$

$$\left.\left(e_3, \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.1}\right\}\right) \right\}.$$

Then $(F,A)_t = T_{\sim}\varphi$.

Let

$$(F,A)_t = \left\{ \left(e_1, \{\Phi\}^{t_1}\right), \left(e_2, \{\Phi\}^{t_1}\right), \left(e_3, \{\Phi\}^{t_1}\right), \right.$$

$$\left(e_1, \{\Phi\}^{t_2}\right), \left(e_2, \{\Phi\}^{t_2}\right), \left(e_3, \{\Phi\}^{t_2}\right),$$

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$$\left(e_1, \{\Phi\}^{t_3}\right), \left(e_2, \{\Phi\}^{t_3}\right), \left(e_3, \{\Phi\}^{t_3}\right) \Bigg\}.$$

Then $(F, A)_t = T_\varphi$.

$$(F,A)_t = \Bigg\{ \left(e_1, \left\{\frac{u_1{}^{t_1}}{1}, \frac{u_2{}^{t_1}}{1}, \frac{u_3{}^{t_1}}{1}, \frac{u_4{}^{t_1}}{1}\right\}\right), \left(e_2, \left\{\frac{u_1{}^{t_1}}{1}, \frac{u_2{}^{t_1}}{1}, \frac{u_3{}^{t_1}}{1}, \frac{u_4{}^{t_1}}{1}\right\}\right),$$
$$\left(e_3, \left\{\frac{u_1{}^{t_1}}{1}, \frac{u_2{}^{t_1}}{1}, \frac{u_3{}^{t_1}}{1}, \frac{u_4{}^{t_1}}{1}\right\}\right), \left(e_1, \left\{\frac{u_1{}^{t_2}}{1}, \frac{u_2{}^{t_2}}{1}, \frac{u_3{}^{t_2}}{1}, \frac{u_4{}^{t_2}}{1}\right\}\right),$$
$$\left(e_2, \left\{\frac{u_1{}^{t_2}}{1}, \frac{u_2{}^{t_2}}{1}, \frac{u_3{}^{t_2}}{1}, \frac{u_4{}^{t_2}}{1}\right\}\right), \left(e_3, \left\{\frac{u_1{}^{t_2}}{1}, \frac{u_2{}^{t_2}}{1}, \frac{u_3{}^{t_2}}{1}, \frac{u_4{}^{t_2}}{1}\right\}\right),$$
$$\left(e_1, \left\{\frac{u_1{}^{t_3}}{0.5}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.1}, \frac{u_4{}^{t_3}}{0.6}\right\}\right), \left(e_2, \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.3}\right\}\right),$$
$$\left(e_3, \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.1}\right\}\right) \Bigg\}.$$

Then $(F, A)_t = T_\sim A$.

Let

$$(F,A)_t = \Bigg\{ \left(e_1, \left\{\frac{u_1{}^{t_1}}{1}, \frac{u_2{}^{t_1}}{1}, \frac{u_3{}^{t_1}}{1}, \frac{u_4{}^{t_1}}{1}\right\}\right), \left(e_2, \left\{\frac{u_1{}^{t_1}}{1}, \frac{u_2{}^{t_1}}{1}, \frac{u_3{}^{t_1}}{1}, \frac{u_4{}^{t_1}}{1}\right\}\right),$$
$$\left(e_2, \left\{\frac{u_1{}^{t_2}}{1}, \frac{u_2{}^{t_2}}{1}, \frac{u_3{}^{t_2}}{1}, \frac{u_4{}^{t_2}}{1}\right\}\right), \left(e_3, \left\{\frac{u_1{}^{t_2}}{1}, \frac{u_2{}^{t_2}}{1}, \frac{u_3{}^{t_2}}{1}, \frac{u_4{}^{t_2}}{1}\right\}\right),$$
$$\left(e_1, \left\{\frac{u_1{}^{t_3}}{1}, \frac{u_2{}^{t_3}}{1}, \frac{u_3{}^{t_3}}{1}, \frac{u_4{}^{t_3}}{1}\right\}\right), \left(e_3, \left\{\frac{u_1{}^{t_3}}{1}, \frac{u_2{}^{t_3}}{1}, \frac{u_3{}^{t_3}}{1}, \frac{u_4{}^{t_3}}{1}\right\}\right) \Bigg\}.$$

Then $(F, A)_t = T_A$.

Definition 3.8. The *complement* of T-FSS $(F, A)_t$ is denoted by $\widetilde{c}(F, A)_t$ $\forall t \in T$ where $\widetilde{c}$ is a fuzzy soft complement.

Example 4. *Consider Example 1. Using the simple fuzzy complement, we get*

$$\widetilde{c}(F,A)_t = \Bigg\{ \left(e_1, \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.6}\right\}\right), \left(e_2, \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.3}\right\}\right),$$
$$\left(e_3, \left\{\frac{u_1{}^{t_1}}{0.7}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.1}\right\}\right), \left(e_1, \left\{\frac{u_1{}^{t_2}}{0.3}, \frac{u_2{}^{t_2}}{0.6}, \frac{u_3{}^{t_2}}{0.9}, \frac{u_4{}^{t_2}}{0.7}\right\}\right),$$
$$\left(e_2, \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.8}, \frac{u_4{}^{t_2}}{0.3}\right\}\right), \left(e_3, \left\{\frac{u_1{}^{t_2}}{0.3}, \frac{u_2{}^{t_2}}{0.2}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.6}\right\}\right),$$
$$\left(e_1, \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.2}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.6}\right\}\right), \left(e_2, \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.3}\right\}\right),$$
$$\left(e_3, \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.1}\right\}\right) \Bigg\}.$$

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Proposition 3.9. *If $(F,A)_t$ is a T-FSS over U, then*

1. $\widetilde{c}\left(\widetilde{c}(F,A)_t\right) = (F,A)_t$,
2. $\widetilde{c}(T_{\sim}\varphi) = (T_{\sim}A)$,
3. $\widetilde{c}(T_{\varphi}) = (T_A)$,
4. $\widetilde{c}(T_{\sim}A) = (T_{\sim}\varphi)$,
5. $\widetilde{c}(T_A) = (T_{\varphi})$.

Proof. The proof is straightforward. □

4 Union and intersection

In this section, we define the union and intersection of T-FSSs, explain their features, and provide some instances.

Definition 4.1. The *union* of two T-FSSs $(F,A)_t$ and $(G,B)_t$ over U, is the T-FSSs $(H,C)_t$, denoted by $(F,A)_t \,\widetilde{\cup}\, (G,B)_t$, such that $C = A \cup B \subset E$ and defined as follows

$$H_t(\epsilon) = \begin{cases} F_t(\epsilon), & if\, \epsilon \in A - B, \\ G_t(\epsilon), & if\, \epsilon \in B - A, \\ F_t(\epsilon) \,\tilde{\cup}\, G_t(\epsilon), & if\, \epsilon \in A \cap B, \end{cases}$$

where $\tilde{\cup}$ denoted the fuzzy soft union.

Example 5. *Consider Example 1. Suppose $(F,A)_t$ and $(G,B)_t$ are two time-fuzzy soft sets over U such that*

$$(F,A)_t = \left\{ \left(e_1, \left\{\frac{{u_1}^{t_1}}{0.6}, \frac{{u_2}^{t_1}}{0.3}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.4}\right\}\right), \left(e_2, \left\{\frac{{u_1}^{t_1}}{0.5}, \frac{{u_2}^{t_1}}{0.3}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.7}\right\}\right), \right.$$
$$\left. \left(e_2, \left\{\frac{{u_1}^{t_2}}{0.5}, \frac{{u_2}^{t_2}}{0.3}, \frac{{u_3}^{t_2}}{0.2}, \frac{{u_4}^{t_2}}{0.7}\right\}\right), \left(e_3, \left\{\frac{{u_1}^{t_3}}{0.3}, \frac{{u_2}^{t_3}}{0.6}, \frac{{u_3}^{t_3}}{0.8}, \frac{{u_4}^{t_3}}{0.9}\right\}\right) \right\}.$$

$$(G,B)_t = \left\{ \left(e_1, \left\{\frac{{u_1}^{t_1}}{0.4}, \frac{{u_2}^{t_1}}{0.5}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.7}\right\}\right), \left(e_3, \left\{\frac{{u_1}^{t_2}}{0.5}, \frac{{u_2}^{t_2}}{0.6}, \frac{{u_3}^{t_2}}{0.9}, \frac{{u_4}^{t_2}}{0.4}\right\}\right), \right.$$
$$\left. \left(e_3, \left\{\frac{{u_1}^{t_3}}{0.5}, \frac{{u_2}^{t_3}}{0.8}, \frac{{u_3}^{t_3}}{0.6}, \frac{{u_4}^{t_3}}{0.9}\right\}\right) \right\}.$$

$$(H,C)_t = \left\{ \left(e_1, \left\{\frac{{u_1}^{t_1}}{0.6}, \frac{{u_2}^{t_1}}{0.5}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.7}\right\}\right), \left(e_2, \left\{\frac{{u_1}^{t_1}}{0.5}, \frac{{u_2}^{t_1}}{0.3}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.7}\right\}\right), \right.$$
$$\left(e_2, \left\{\frac{{u_1}^{t_2}}{0.5}, \frac{{u_2}^{t_2}}{0.3}, \frac{{u_3}^{t_2}}{0.2}, \frac{{u_4}^{t_2}}{0.7}\right\}\right), \left(e_3, \left\{\frac{{u_1}^{t_2}}{0.5}, \frac{{u_2}^{t_2}}{0.6}, \frac{{u_3}^{t_2}}{0.9}, \frac{{u_4}^{t_2}}{0.4}\right\}\right),$$
$$\left. \left(e_3, \left\{\frac{{u_1}^{t_3}}{0.5}, \frac{{u_2}^{t_3}}{0.8}, \frac{{u_3}^{t_3}}{0.8}, \frac{{u_4}^{t_3}}{0.9}\right\}\right) \right\}.$$

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Proposition 4.2. *If $(F,A)_t$, $(G,B)_t$ and $(H,C)_t$ are three T-FSSs over U, then*

1. $(F,A)_t \widetilde{\cup} ((G,B)_t \widetilde{\cup} (H,C)_t) = ((F,A)_t \widetilde{\cup} (G,B))_t \widetilde{\cup} (H,C)_t$,
2. $(F,A)_t \widetilde{\cup} (F,A)_t = (F,A)_t$.

Proof. The proof is straightforward. □

Definition 4.3. The *intersection* of two T-FSSs $(F,A)_t$ and $(G,B)_t$ over U, is the T-FSSs $(H,C)_t$, denoted by $(F,A)_t \widetilde{\cap} (G,B)_t$, such that $C = A \cup B \subset E$ and defined as follows

$$H_t(\epsilon) = \begin{cases} F_t(\epsilon), & if\, \epsilon \in A - B, \\ G_t(\epsilon), & if\, \epsilon \in B - A, \\ F_t(\epsilon) \tilde{\cap} G_t(\epsilon), & if\, \epsilon \in A \cap B, \end{cases}$$

where $\tilde{\cap}$ denoted the fuzzy soft intersection.

Example 6. *Consider Example 1. Let*

$$(H,C)_t = \left\{ \left(e_1, \left\{ \frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.3}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.4} \right\}\right), \left(e_2, \left\{ \frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.3}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.7} \right\}\right), \right.$$
$$\left(e_2, \left\{ \frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.2}, \frac{u_4{}^{t_2}}{0.7} \right\}\right), \left(e_3, \left\{ \frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.6}, \frac{u_3{}^{t_2}}{0.9}, \frac{u_4{}^{t_2}}{0.4} \right\}\right),$$
$$\left. \left(e_3, \left\{ \frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.9} \right\}\right) \right\}.$$

Proposition 4.4. *If $(F,A)_t$, $(G,B)_t$ and $(H,C)_t$ are three T-FSSs over U, then*

1. $(F,A)_t \widetilde{\cap} ((G,B)_t \widetilde{\cap} (H,C)_t) = ((F,A)_t \widetilde{\cap} (G,B))_t \widetilde{\cap} (H,C)_t$,
2. $(F,A)_t \widetilde{\cap} (F,A)_t = (F,A)_t$.

Proof. The proof is straightforward. □

Proposition 4.5. *If $(F,A)_t$, $(G,B)_t$ and $(H,C)_t$ are three T-FSSs over U, then*

1. $(F,A)_t \widetilde{\cup} \left((G,B)_t \widetilde{\cap} (H,C)_t\right) = \left((F,A)_t \widetilde{\cup} (G,B)_t\right) \widetilde{\cap} \left((F,A)_t \widetilde{\cup} (H,C)_t\right)$,
2. $(F,A)_t \widetilde{\cap} \left((G,B)_t \widetilde{\cup} (H,C)_t\right) = \left((F,A)_t \widetilde{\cap} (G,B)_t\right) \widetilde{\cup} \left((F,A)_t \widetilde{\cap} (H,C)_t\right)$.

Proof. The proof is straightforward □

Proposition 4.6. *If $(F,A)_t$ and $(G,B)_t$ are two T-FSSs over U, then*

1. $\left((F,A)_t \widetilde{\cup} (G,B)_t\right)^c = (F,A)_t^c \widetilde{\cap} (G,B)_t^c$,
2. $\left((F,A)_t \widetilde{\cap} (G,B)_t\right)^c = (F,A)_t^c \widetilde{\cup} (G,B)_t^c$.

Proof. The proof is straightforward □

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5 AND and OR operations

In this section, we define the AND and OR operations for TFSSs, deduce their features, and provide examples.

Definition 5.1. If $(F,A)_t$ and $(G,B)_t$ are two T-FSS over U then "$(F,A)_t$ AND $(G,B)_t$" denoted by $(F,A)_t \wedge (G,B)_t$, is defined by

$$(F,A)_t \wedge (G,B)_t = (H, A\times B)_t$$

such that $H(\alpha,\beta)_t = F(\alpha)_t \widetilde{\bigcap} G(\beta)_t, \forall(\alpha,\beta)\in A\times B$, where $\widetilde{\bigcap}$ is time-fuzzy intersection.

Example 7. *Consider Example 1. Let Suppose $(F,A)_t$ and $(G,B)_t$ are two time-fuzzy soft sets over U such that*

$$(F,A)_t = \left\{ \left(e_1, \left\{\frac{{u_1}^{t_1}}{0.6}, \frac{{u_2}^{t_1}}{0.3}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.4}\right\}\right), \left(e_2, \left\{\frac{{u_1}^{t_1}}{0.5}, \frac{{u_2}^{t_1}}{0.3}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.7}\right\}\right),\right.$$

$$\left.\left(e_2, \left\{\frac{{u_1}^{t_2}}{0.4}, \frac{{u_2}^{t_2}}{0.6}, \frac{{u_3}^{t_2}}{0.8}, \frac{{u_4}^{t_2}}{0.3}\right\}\right), \left(e_3, \left\{\frac{{u_1}^{t_3}}{0.3}, \frac{{u_2}^{t_3}}{0.6}, \frac{{u_3}^{t_3}}{0.8}, \frac{{u_4}^{t_3}}{0.9}\right\}\right) \right\}.$$

$$(G,B)_t = \left\{ \left(e_1, \left\{\frac{{u_1}^{t_1}}{0.8}, \frac{{u_2}^{t_1}}{0.5}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.3}\right\}\right), \left(e_3, \left\{\frac{{u_1}^{t_2}}{0.4}, \frac{{u_2}^{t_2}}{0.8}, \frac{{u_3}^{t_2}}{0.5}, \frac{{u_4}^{t_2}}{0.7}\right\}\right),\right.$$

$$\left.\left(e_3, \left\{\frac{{u_1}^{t_3}}{0.1}, \frac{{u_2}^{t_3}}{0.6}, \frac{{u_3}^{t_3}}{0.7}, \frac{{u_4}^{t_3}}{0.9}\right\}\right) \right\}.$$

Then $(F,A)\wedge(G,B) = (H, A\times B)$

$$\left\{ \left(\left(e_1^{t_1}, e_1^{t_1}\right), \left\{\frac{{u_1}^{t_{1,1}}}{0.6}, \frac{{u_2}^{t_{1,1}}}{0.3}, \frac{{u_3}^{t_{1,1}}}{0.2}, \frac{{u_4}^{t_{1,1}}}{0.3}\right\}\right), \left(\left(e_1^{t_1}, e_3^{t_2}\right), \left\{\frac{{u_1}^{t_{1,2}}}{0.4}, \frac{{u_2}^{t_{1,2}}}{0.3}, \frac{{u_3}^{t_{1,2}}}{0.2}, \frac{{u_4}^{t_{1,2}}}{0.4}\right\}\right),\right.$$

$$\left(\left(e_1^{t_1}, e_3^{t_3}\right), \left\{\frac{{u_1}^{t_{1,3}}}{0.1}, \frac{{u_2}^{t_{1,3}}}{0.3}, \frac{{u_3}^{t_{1,3}}}{0.2}, \frac{{u_4}^{t_{1,3}}}{0.4}\right\}\right), \left(\left(e_2^{t_1}, e_1^{t_1}\right), \left\{\frac{{u_1}^{t_{1,1}}}{0.5}, \frac{{u_2}^{t_{1,1}}}{0.3}, \frac{{u_3}^{t_{1,1}}}{0.2}, \frac{{u_4}^{t_{1,1}}}{0.3}\right\}\right),$$

$$\left(\left(e_2^{t_1}, e_3^{t_2}\right), \left\{\frac{{u_1}^{t_{1,2}}}{0.4}, \frac{{u_2}^{t_{1,2}}}{0.3}, \frac{{u_3}^{t_{1,2}}}{0.2}, \frac{{u_4}^{t_{1,2}}}{0.7}\right\}\right), \left(\left(e_2^{t_1}, e_3^{t_3}\right), \left\{\frac{{u_1}^{t_{1,3}}}{0.1}, \frac{{u_2}^{t_{1,3}}}{0.3}, \frac{{u_3}^{t_{1,3}}}{0.2}, \frac{{u_4}^{t_{1,3}}}{0.7}\right\}\right),$$

$$\left(\left(e_2^{t_2}, e_1^{t_1}\right), \left\{\frac{{u_1}^{t_{2,1}}}{0.4}, \frac{{u_2}^{t_{2,1}}}{0.5}, \frac{{u_3}^{t_{2,1}}}{0.2}, \frac{{u_4}^{t_{2,1}}}{0.3}\right\}\right), \left(\left(e_2^{t_2}, e_3^{t_2}\right), \left\{\frac{{u_1}^{t_{2,2}}}{0.4}, \frac{{u_2}^{t_{2,2}}}{0.6}, \frac{{u_3}^{t_{2,2}}}{0.5}, \frac{{u_4}^{t_{2,2}}}{0.3}\right\}\right),$$

$$\left(\left(e_2^{t_2}, e_3^{t_3}\right), \left\{\frac{{u_1}^{t_{2,3}}}{0.1}, \frac{{u_2}^{t_{2,3}}}{0.6}, \frac{{u_3}^{t_{2,3}}}{0.7}, \frac{{u_4}^{t_{2,3}}}{0.3}\right\}\right), \left(\left(e_3^{t_3}, e_1^{t_1}\right), \left\{\frac{{u_1}^{t_{3,1}}}{0.3}, \frac{{u_2}^{t_{3,1}}}{0.5}, \frac{{u_3}^{t_{3,1}}}{0.2}, \frac{{u_4}^{t_{3,1}}}{0.3}\right\}\right),$$

$$\left.\left(\left(e_3^{t_3}, e_3^{t_2}\right), \left\{\frac{{u_1}^{t_{3,2}}}{0.3}, \frac{{u_2}^{t_{3,2}}}{0.6}, \frac{{u_3}^{t_{3,2}}}{0.5}, \frac{{u_4}^{t_{3,2}}}{0.7}\right\}\right), \left(\left(e_3^{t_3}, e_3^{t_3}\right), \left\{\frac{{u_1}^{t_{3,3}}}{0.1}, \frac{{u_2}^{t_{3,3}}}{0.6}, \frac{{u_3}^{t_{3,3}}}{0.7}, \frac{{u_4}^{t_{3,3}}}{0.9}\right\}\right) \right\},$$

Definition 5.2. If $(F,A)_t$ and $(G,B)_t$ are two T-FSS over U then "$(F,A)_t$ OR $(G,B)_t$" denoted by $(F,A)_t \vee (G,B)_t$, is defined by

$$(F,A)_t \vee (G,B)_t = (H, A\times B)_t$$

such that $H(\alpha,\beta)_t = F(\alpha)_t \widetilde{\bigcup} G(\beta)_t, \forall(\alpha,\beta)\in A\times B$, where $\widetilde{\bigcup}$ is time-fuzzy union.

Example 8. *Consider Example 7 we have U then "$(F,A)_t$ OR $(G,B)_t$" denoted by $(H,C)_t = (F,A)_t \bigvee (G,B)_t$ where $(H,C)_t$*

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$$\left\{\left(\left(e_1^{t_1},e_1^{t_1}\right),\left\{\frac{u_1{}^{t_{1,1}}}{0.8},\frac{u_2{}^{t_{1,1}}}{0.5},\frac{u_3{}^{t_{1,1}}}{0.2},\frac{u_4{}^{t_{1,1}}}{0.4}\right\}\right),\left(\left(e_1^{t_1},e_3^{t_2}\right),\left\{\frac{u_1{}^{t_{1,2}}}{0.6},\frac{u_2{}^{t_{1,2}}}{0.8},\frac{u_3{}^{t_{1,2}}}{0.5},\frac{u_4{}^{t_{1,2}}}{0.7}\right\}\right),\right.$$

$$\left(\left(e_1^{t_1},e_3^{t_3}\right),\left\{\frac{u_1{}^{t_{1,3}}}{0.6},\frac{u_2{}^{t_{1,3}}}{0.6},\frac{u_3{}^{t_{1,3}}}{0.7},\frac{u_4{}^{t_{1,3}}}{0.9}\right\}\right),\left(\left(e_2^{t_1},e_1^{t_1}\right),\left\{\frac{u_1{}^{t_{1,1}}}{0.8},\frac{u_2{}^{t_{1,1}}}{0.5},\frac{u_3{}^{t_{1,1}}}{0.2},\frac{u_4{}^{t_{1,1}}}{0.7}\right\}\right),$$

$$\left(\left(e_2^{t_1},e_3^{t_2}\right),\left\{\frac{u_1{}^{t_{1,2}}}{0.5},\frac{u_2{}^{t_{1,2}}}{0.8},\frac{u_3{}^{t_{1,2}}}{0.5},\frac{u_4{}^{t_{1,2}}}{0.7}\right\}\right),\left(\left(e_2^{t_1},e_3^{t_3}\right),\left\{\frac{u_1{}^{t_{1,3}}}{0.5},\frac{u_2{}^{t_{1,3}}}{0.6},\frac{u_3{}^{t_{1,3}}}{0.7},\frac{u_4{}^{t_{1,3}}}{0.9}\right\}\right),$$

$$\left(\left(e_2^{t_2},e_1^{t_1}\right),\left\{\frac{u_1{}^{t_{2,1}}}{0.8},\frac{u_2{}^{t_{2,1}}}{0.6},\frac{u_3{}^{t_{2,1}}}{0.8},\frac{u_4{}^{t_{2,1}}}{0.3}\right\}\right),\left(\left(e_2^{t_2},e_3^{t_2}\right),\left\{\frac{u_1{}^{t_{2,2}}}{0.4},\frac{u_2{}^{t_{2,2}}}{0.8},\frac{u_3{}^{t_{2,2}}}{0.8},\frac{u_4{}^{t_{2,2}}}{0.7}\right\}\right),$$

$$\left(\left(e_2^{t_2},e_3^{t_3}\right),\left\{\frac{u_1{}^{t_{2,3}}}{0.4},\frac{u_2{}^{t_{2,3}}}{0.6},\frac{u_3{}^{t_{2,3}}}{0.8},\frac{u_4{}^{t_{2,3}}}{0.9}\right\}\right),\left(\left(e_3^{t_3},e_1^{t_1}\right),\left\{\frac{u_1{}^{t_{3,1}}}{0.8},\frac{u_2{}^{t_{3,1}}}{0.6},\frac{u_3{}^{t_{3,1}}}{0.8},\frac{u_4{}^{t_{3,1}}}{0.9}\right\}\right),$$

$$\left.\left(\left(e_3^{t_3},e_3^{t_2}\right),\left\{\frac{u_1{}^{t_{3,2}}}{0.4},\frac{u_2{}^{t_{3,2}}}{0.8},\frac{u_3{}^{t_{3,2}}}{0.8},\frac{u_4{}^{t_{3,2}}}{0.9}\right\}\right),\left(\left(e_3^{t_3},e_3^{t_3}\right),\left\{\frac{u_1{}^{t_{3,3}}}{0.1},\frac{u_2{}^{t_{3,3}}}{0.6},\frac{u_3{}^{t_{3,3}}}{0.8},\frac{u_4{}^{t_{3,3}}}{0.9}\right\}\right)\right\},$$

6 An application of Time-Fuzzy fuzzy soft set in decision making

In this section, we present an application of time-fuzzy soft set theory in a decision making problem. Assume that one of the broadcasting channels want to ask professionals to review their show through a debate of a contentious subject and get their feedback on the situation. The creators of the show utilized the following criteria to assess their findings. The four options are as follows:$U = \{u_1,\ u_2,\ u_3, u_4\}$,. assume e there are five parameters $E = \{e_1, e_2, e_3, e_4, e_5,\}$, Select the specialists for the programs. For i = 1, 2, 3, 4, 5 the parameters e_i $(i = 1, 2, 3, 4, 5)$ stand for "this criterion is employed to discriminate.", "this criterion is independent of the other ones. ", "This criterion measures a single thing. ", "The universal criterion ", "Criteria that are crucial for some stakeholders". $T = \{t_1, t_2, t_3, t_4, t_5\}$be a collection of prior time periods. Using such facts, we can determine the best option for the decision. Following a serious deliberation, the committee creates the following time-fuzzy soft set.

$$(F,E)_t=\left\{\left(e_1,\left\{\frac{u_1{}^{t_1}}{0.6},\frac{u_2{}^{t_1}}{0.3},\frac{u_3{}^{t_1}}{0.2},\frac{u_4{}^{t_1}}{0.4}\right\}\right),\left(e_2,\left\{\frac{u_1{}^{t_1}}{0.5},\frac{u_2{}^{t_1}}{0.3},\frac{u_3{}^{t_1}}{0.2},\frac{u_4{}^{t_1}}{0.7}\right\}\right),\right.$$

$$\left(e_3,\left\{\frac{u_1{}^{t_1}}{0.3},\frac{u_2{}^{t_1}}{0.6},\frac{u_3{}^{t_1}}{0.8},\frac{u_4{}^{t_1}}{0.9}\right\}\right),\left(e_4,\left\{\frac{u_1{}^{t_1}}{0.5},\frac{u_2{}^{t_1}}{0.4},\frac{u_3{}^{t_1}}{0.6},\frac{u_4{}^{t_1}}{0.8}\right\}\right),$$

$$\left(e_5,\left\{\frac{u_1{}^{t_1}}{0.9},\frac{u_2{}^{t_1}}{0.2},\frac{u_3{}^{t_1}}{0.4},\frac{u_4{}^{t_1}}{0.8}\right\}\right),\left(e_1,\left\{\frac{u_1{}^{t_2}}{0.7},\frac{u_2{}^{t_2}}{0.4},\frac{u_3{}^{t_2}}{0.1},\frac{u_4{}^{t_2}}{0.3}\right\}\right),$$

$$\left(e_2,\left\{\frac{u_1{}^{t_2}}{0.3},\frac{u_2{}^{t_2}}{0.1},\frac{u_3{}^{t_2}}{0.2},\frac{u_4{}^{t_2}}{0.6}\right\}\right),\left(e_3,\left\{\frac{u_1{}^{t_2}}{0.7},\frac{u_2{}^{t_2}}{0.8},\frac{u_3{}^{t_2}}{0.6},\frac{u_4{}^{t_2}}{0.4}\right\}\right),$$

$$\left(e_4,\left\{\frac{u_1{}^{t_2}}{0.9},\frac{u_2{}^{t_2}}{0.3},\frac{u_3{}^{t_2}}{0.4},\frac{u_4{}^{t_2}}{0.7}\right\}\right),\left(e_5,\left\{\frac{u_1{}^{t_2}}{0.2},\frac{u_2{}^{t_2}}{0.7},\frac{u_3{}^{t_2}}{0.8},\frac{u_4{}^{t_2}}{0.5}\right\}\right),$$

$$\left(e_1,\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.8},\frac{u_3{}^{t_3}}{0.6},\frac{u_4{}^{t_3}}{0.4}\right\}\right),\left(e_2,\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.5},\frac{u_3{}^{t_3}}{0.6},\frac{u_4{}^{t_3}}{0.4}\right\}\right),$$

$$\left(e_3,\left\{\frac{u_1{}^{t_3}}{0.6},\frac{u_2{}^{t_3}}{0.4},\frac{u_3{}^{t_3}}{0.5},\frac{u_4{}^{t_3}}{0.7}\right\}\right),\left(e_4,\left\{\frac{u_1{}^{t_3}}{0.6},\frac{u_2{}^{t_3}}{0.7},\frac{u_3{}^{t_3}}{0.5},\frac{u_4{}^{t_3}}{0.3}\right\}\right),$$

$$\left(e_5,\left\{\frac{u_1{}^{t_3}}{0.7},\frac{u_2{}^{t_3}}{0.2},\frac{u_3{}^{t_3}}{0.6},\frac{u_4{}^{t_3}}{0.3}\right\}\right),\left(e_1,\left\{\frac{u_1{}^{t_4}}{0.4},\frac{u_2{}^{t_4}}{0.5},\frac{u_3{}^{t_4}}{0.3},\frac{u_4{}^{t_4}}{0.1}\right\}\right),$$

$$\left(e_2,\left\{\frac{u_1{}^{t_4}}{0.5},\frac{u_2{}^{t_4}}{0.3},\frac{u_3{}^{t_4}}{0.4},\frac{u_4{}^{t_4}}{0.5}\right\}\right),\left(e_3,\left\{\frac{u_1{}^{t_4}}{0.3},\frac{u_2{}^{t_4}}{0.5},\frac{u_3{}^{t_4}}{0.2},\frac{u_4{}^{t_4}}{0.1}\right\}\right),$$

$$\left(e_4,\left\{\frac{u_1{}^{t_4}}{0.5},\frac{u_2{}^{t_4}}{0.6},\frac{u_3{}^{t_4}}{0.6},\frac{u_4{}^{t_4}}{0.7}\right\}\right),\left(e_5,\left\{\frac{u_1{}^{t_4}}{0.6},\frac{u_2{}^{t_4}}{0.4},\frac{u_3{}^{t_4}}{0.5},\frac{u_4{}^{t_4}}{0.1}\right\}\right),$$

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$$\left(e_1, \left\{\frac{u_1{}^{t_5}}{0.7}, \frac{u_2{}^{t_5}}{0.9}, \frac{u_3{}^{t_5}}{0.4}, \frac{u_4{}^{t_5}}{0.6}\right\}\right), \left(e_2, \left\{\frac{u_1{}^{t_5}}{0.3}, \frac{u_2{}^{t_5}}{0.5}, \frac{u_3{}^{t_5}}{0.7}, \frac{u_4{}^{t_5}}{0.9}\right\}\right),$$

$$\left(e_3, \left\{\frac{u_1{}^{t_5}}{0.2}, \frac{u_2{}^{t_5}}{0.6}, \frac{u_3{}^{t_5}}{0.8}, \frac{u_4{}^{t_5}}{0.9}\right\}\right), \left(e_4, \left\{\frac{u_1{}^{t_5}}{0.1}, \frac{u_2{}^{t_5}}{0.3}, \frac{u_3{}^{t_5}}{0.6}, \frac{u_4{}^{t_5}}{0.4}\right\}\right),$$

$$\left(e_5, \left\{\frac{u_1{}^{t_5}}{0.3}, \frac{u_2{}^{t_5}}{0.5}, \frac{u_3{}^{t_5}}{0.8}, \frac{u_4{}^{t_5}}{0.6}\right\}\right) \Bigg\}.$$

6.1 Algorthim

1. Find the tabular form of $(F, E)_t$ as shown in Table 1.
2. Table 2 shows a tabular representation of $F(E)$, which is defined as follows.:

$$F(e) = \left\{ \frac{u}{\sum_{i=1}^{n} t_i F_t(e) \setminus n \sum_{i=1}^{n} F_t(e)} : u \in U, e \in E \right\}$$

where $n = |T|$.

3. Apply Roy & Maji's method to $F(E)$.

Table 1: A tabular depiction of $(F, E)_t$

U	u_1	u_2	u_3	u_4
(e_1, t_1)	0.6	0.3	0.2	0.4
(e_1, t_2)	0.7	0.4	0.1	0.3
(e_1, t_3)	0.7	0.8	0.6	0.4
(e_1, t_4)	0.4	0.5	0.3	0.1
(e_1, t_5)	0.7	0.9	0.4	0.6
(e_2, t_1)	0.5	0.3	0.2	0.7
(e_2, t_2)	0.3	0.1	0.2	0.6
(e_2, t_3)	0.7	0.5	0.6	0.4
(e_2, t_4)	0.5	0.3	0.4	0.5
(e_2, t_5)	0.3	0.5	0.7	0.9
(e_3, t_1)	0.3	0.6	0.8	0.9
(e_3, t_2)	0.7	0.8	0.6	0.4
(e_3, t_3)	0.6	0.4	0.5	0.7
(e_3, t_4)	0.3	0.5	0.2	0.1
(e_3, t_5)	0.2	0.6	0.8	0.9
(e_4, t_1)	0.5	0.4	0.6	0.8
(e_4, t_2)	0.9	0.3	0.4	0.7
(e_4, t_3)	0.6	0.7	0.5	0.3
(e_4, t_4)	0.5	0.6	0.6	0.7
(e_4, t_5)	0.1	0.3	0.6	0.4
(e_5, t_1)	0.9	0.2	0.4	0.8
(e_5, t_2)	0.2	0.7	0.8	0.5
(e_5, t_3)	0.7	0.2	0.6	0.3
(e_5, t_4)	0.6	0.4	0.5	0.1
(e_5, t_5)	0.3	0.5	0.8	0.6

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Table 2: A tabular depiction of $F\left(E\right)$

U	u_1	u_2	u_3	u_4
e_1	0.59	0.68	0.67	0.62
e_2	0.58	0.67	0.71	0.61
e_3	0.54	0.57	0.57	0.58
e_4	0.50	0.60	0.61	0.52
e_5	0.54	0.63	0.63	0.53

By using Roy and Maji's Algorithm we get u_3 the best choices.

Table 3: Comparison table

U	u_1	u_2	u_3	u_4
u_1	5	0	0	1
u_2	5	5	3	4
u_3	5	4	5	4
u_4	4	1	1	5

Table 4: score table

U	r_i	t_j	S_i
u_1	6	19	-13
u_2	17	10	7
u_3	18	9	9
u_4	11	14	-3

From the above score table, it is clear that the maximum score is 9, scored by u_3 the decision is in favour of selecting u_3.

7 Conclusion

I have presented the idea of a time-fuzzy soft set and examined some of its characteristics in this study. On the time-fuzzy soft set, the complement, union, and intersection operations have been specified. An example of how this theory may be used to solve a decision-making dilemma is provided[27]–.[32]

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