Utilization of Jaccard Index Measures on Multiple Attribute Group Decision Making under Neutrosophic Environment

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Abstract

In this paper, we introduce the concept of Jaccard index measures under the neutrosophic environment to make the right decision in multiple attributes. Here, we insinuate two Jaccard index measures based on distance and the included weighted Jaccard of two vectors between the neutrosophic environment. Then, we determine the Multiple Attribute group decision-making method (in short MAGDM) based on the Jaccard index measures under the neutrosophic environment and also we compare the applications of the proposed MAGDM method in the neutrosophic environment. Finally, certain descriptive examples are on hand to verify the residential handle and to express its practicality and effectiveness.

Keywords: Jaccard index measure, Neutrosophic vague set, MAGDM.

1.INTRODUCTION

In 1999, Smarandache [28] presents another part of the theory known as neutrosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with various ideational spectra. The neutrosophic set is the generalization of the classic set, fuzzy sets [35], interval-valued fuzzy set [29], intuitionistic fuzzy set [5], interval-valued intuitionistic fuzzy set [4], paraconsistent set, dialetheist set, paradoxical set, and tautological set. A neutrosophic set has three basic components such that truth-membership, indeterminacy-membership, and falsity-membership, and they are independent [28], for more informations on the neutrosophic theory we refere the readers to [36-39].

Vague sets have been presented by Gau and Buehrar in 1993 as an extension of the fuzzy set theory [20]. It is considered as an effective tool to deal with uncertainty since it gives more data when contrasted with fuzzy sets [30]. A vague set is defined by a truth-membership function t_v and a false-membership function f_v [17,18].

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Shawkat Alkhazaleh [27] in 2015 presented the idea of the neutrosophic vague set as a combination of neutrosophic set and vague set. Neutrosophic vague theory is an effective tool to process incomplete, indeterminate and inconsistent information. In 2019, Hashim et al.[21] developed a new generalized mathematical model called interval neutrosophic vague sets which are a combination of vague sets and interval neutrosophic sets and a generalization of interval neutrosophic vague sets. Al-Quran and Hassan [1,2, 3] in 2018 presented and gave more application on neutrosophic vague soft under decision making.

In 2013, Ye [33] introduces the Multi-attribute decision-making method using the correlation coefficient under a single-valued neutrosophic environment. It is one of the most significant angles in the executive's science which can deliver significant financial benefits in an assortment of fields, such as manufacturing domain[17], disaster assessment, company investment management. For decision-making problems in engineering practice, the decision information is generally incomplete and indeterminate [34]. To apply them to multi-criteria decision-making problems with simplified neutrosophic information. Recently, Chakraborty et al. developed a multi-criteria decision-making problem for different used in the bipolar neutrosophic domain[12]. Furthermore, Abdel basset develop multi-criteria decision-making problem under a hybrid neutrosophic set[24].

The similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily selected as well. In fact, the degree of similarity between the objects under study plays an important role. In vector space, especially the Jaccard similarity measures [9,10,11,13] are often used in information retrieval, citation analysis, and automatic classification. Ye [32] proposed the Jaccard , Dice, and cosine similarity measures between trapezoidal intuitionistic fuzzy numbers (TIFNs) that are treated as continuous and applied them to multicriteria group decision-making problems.In 2014 Ye [32] developed three vector similarity measures between single valued neutrosophic sets as a generalization of the Jaccard , Dice, and cosine similarity measures between two vectors. Furthermore, in 2016, Mehmet and Deli developed a multi-criteria decision making for bipolar neutrosophic sets based on Jaccard vector similarity measures and applied to a numerical examplein order to confirm the practicality and accuracy of the proposed method [23]. In the paper, we using Jaccard index measures which are more efficiency, further this method will give better result.

1.1. Motivation

A significant issue then arises if one considers a neutrosophic vague number: what will be a Jaccard index neutrosophic vague measures and a weighted Jaccard index neutrosophic vague measures? How should we utilize a Jaccard index neutrosophic vague measure in MAGDM? In light of this point of view, we built up the subject of this exploration article. We succeeded in producing an illustration example.

1.2. Novelties

Various works have just been distributed right now setting. Analysts have just built up a few definitions and applications in different fields. In any case, many interesting outcomes are as yet obscure. Our work aimed to create thoughts for those obscure viewpoints:

(i)Introduction of a Jaccard index measures of neutrosophic vague set and its definition.

(ii) Application in a Jaccard index measures in MAGDM.

1.3. The structure of the paper

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The paper is organized as follows: In section 1, we have discussed the introduction and literature review. In section 2, contains the preliminaries section. In section 3, the concept of the neutrosophic vague set, a Jaccard index measures and its properties. In section 4, we introduce the algorithm to solve a Multiple Attribute Group Decision-Making (MAGDM). The practical problem is considered in section 5. The compression of the result has been done with two more research in section 6. The conclusions are written in section 7.

2. PRELIMINARIES

Definition.2.1 [27]

A Vague set V on the universe of discourse X written as A={ < x, $t_A(x)$, 1-$f_A(x)$ > | x ∈ X }. is characterized by a truth-membership function t_v, and a false- membership function f_v, as follows:

t_v: U → [0,1], f_v: U→ [0,1], and $t_v + f_v \leq 1$

Definition.2.2 [27]

Let A and B be vague sets of the form A = { < x, $t_A(x)$, 1-$f_A(x)$ > | x ∈ X } and

B = { < x, t_B (x), 1-f_B (x) > |x ∈ X }.Then

i. A ⊆ if and only if $t_A(x) \leq t_B(x)$ and $1-f_A(x) \leq 1-f_B(x)$.
ii. A=B if and only if A ⊆ B and B ⊆A.
iii. A^c = {< x, 1-$f_A(x)$,$t_A(x)$ > | x ∈ X}.
iv. A∪B ={< x, max ($t_A(x)$, $t_B(x)$), max (1-$f_A(x)$, 1-$f_B(x)$) > | x ∈ X}.
v. A∩B ={< x, min ($t_A(x)$,t_B (x)), min (1-$f_A(x)$, 1-$f_B(x)$) > | x ∈ X}.

Definition.2.3 [27]

A neutrosophic set A on the universe of discourse X is defined as

A ={< x, $T_A(x)$, $I_A(x)$, $F_A(x)$ >, x ∈ X}where T, I, F: X→ $]^{-}0,1^{+}[$ and $^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$.

Definition.2.4 [27]

A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as A_{NV} = { <x; $T_{ANV}(x)$, $I_{ANV}(x)$, $F_{ANV}(x)$ >; x ∈ X } whose truth-membership, indeterminacy-membership and false-membership functions is defined as:

$T_{ANV}(x) = [T^-, T^+]$, $I_{ANV}(x) = [I^-, I^+]$, $F_{ANV} = [F^-, F^+]$ where

1) $T^+ = 1-F^-$

2) $F^+=1- T^-$ and

3) $^{-}0 \leq T^- + I^- + F^- \leq 2^+$.

Definition.2.5 [25]

Let X be a universe of discourse. A bipolar neutrosophic set A_{BNS} in X is defined as an object of the form

$$A_{BNS}= \{ <x; T^+(x), I^+(x),F^+(x), T^-(x) ,\ I^-(x) ,\ F^-(x) >; x \in X \}$$

Where $T^+, F^+, I^+ : X \to [1,0]$ and $T^-, F^-, I^- : X \to [-1,0]$

3. Jaccard Index Measure of Neutrosophic Vague Sets.

Definition.3.1

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Let A_{NV} ={ < x; $T_{ANV}(x)$, $I_{ANV}(x)$, $F_{ANV}(x)$ >; x ∈ X} and B_{NV} ={< x; $T_{BNV}(x)$, $I_{BNV}(x)$, $F_{BNV}(x)$ >; x ∈ X }two neutrosophic vague set in X. Then the two Jaccard index measure of A_{NV} and B_{NV} are proposed based on distance and the included weighted Jaccard index measure of two vectors, respectively as follows:

Jaccard index measure based on distance

$$J_{NV}(A_{NV}, B_{NV})= \sum_{i=1}^{n}\left(\frac{[T^{+}_{ANV}(x_i).T^{+}_{BNV}(x_i)]+[T^{-}_{ANV}(x_i).T^{-}_{BNV}(x_i)]+[I^{+}_{ANV}(x_i).I^{+}_{BNV}(x_i)]+[I^{-}_{ANV}(x_i).I^{-}_{BNV}(x_i)] + [F^{+}_{ANV}(x_i).F^{+}_{BNV}(x_i)]+[F^{-}_{ANV}(x_i).F^{-}_{BNV}(x_i)]}{\left(\begin{array}{c}[T^{+}_{ANV}(x_i)]^2+[T^{+}_{BNV}(x_i)]^2+[T^{-}_{ANV}(x_i)]^2+[T^{-}_{BNV}(x_i)]^2+[I^{+}_{ANV}(x_i)]^2+[I^{+}_{BNV}(x_i)]^2+[I^{-}_{ANV}(x_i)]^2+[I^{-}_{BNV}(x_i)]^2 \\ +[F^{+}_{ANV}(x_i)]^2+[F^{+}_{BNV}(x_i)]^2+[F^{-}_{ANV}(x_i)]^2+[F^{-}_{BNV}(x_i)]^2\end{array}\right) - \left(\begin{array}{c}[T^{+}_{ANV}(x_i).T^{+}_{BNV}(x_i)]+[T^{-}_{ANV}(x_i).T^{-}_{BNV}(x_i)]+[I^{+}_{ANV}(x_i).I^{+}_{BNV}(x_i)]+[I^{-}_{ANV}(x_i).I^{-}_{BNV}(x_i)] \\ +[F^{+}_{ANV}(x_i).F^{+}_{BNV}(x_i)]+[F^{-}_{ANV}(x_i).F^{-}_{BNV}(x_i)]\end{array}\right)}\right) \quad (5)$$

Weighted Jaccard index measure based on two vectors

$$WJ_{NV}(A_{NV}, B_{NV}) = \sum_{i=1}^{n}(w_i)\left(\frac{[T^{+}_{ANV}(x_i).T^{+}_{BNV}(x_i)]+[T^{-}_{ANV}(x_i).T^{-}_{BNV}(x_i)]+[I^{+}_{ANV}(x_i).I^{+}_{BNV}(x_i)]+[I^{-}_{ANV}(x_i).I^{-}_{BNV}(x_i)] + [F^{+}_{ANV}(x_i).F^{+}_{BNV}(x_i)]+[F^{-}_{ANV}(x_i).F^{-}_{BNV}(x_i)]}{\left(\begin{array}{c}[T^{+}_{ANV}(x_i)]^2+[T^{+}_{BNV}(x_i)]^2+[T^{-}_{ANV}(x_i)]^2+[T^{-}_{BNV}(x_i)]^2+[I^{+}_{ANV}(x_i)]^2+[I^{+}_{BNV}(x_i)]^2+[I^{-}_{ANV}(x_i)]^2+[I^{-}_{BNV}(x_i)]^2 \\ +[F^{+}_{ANV}(x_i)]^2+[F^{+}_{BNV}(x_i)]^2+[F^{-}_{ANV}(x_i)]^2+[F^{-}_{BNV}(x_i)]^2\end{array}\right) - \left(\begin{array}{c}[T^{+}_{ANV}(x_i).T^{+}_{BNV}(x_i)]+[T^{-}_{ANV}(x_i).T^{-}_{BNV}(x_i)]+[I^{+}_{ANV}(x_i).I^{+}_{BNV}(x_i)]+[I^{-}_{ANV}(x_i).I^{-}_{BNV}(x_i)] \\ +[F^{+}_{ANV}(x_i).F^{+}_{BNV}(x_i)]+[F^{-}_{ANV}(x_i).F^{-}_{BNV}(x_i)]\end{array}\right)}\right) \quad (6)$$

According to the above definition 3.1, the two Jaccard index measures $J(A_{NV}, B_{NV})$ for NVs satisfy the following properties (p1)–(p3):

(p1) $0 \le J_{NV}(A_{NV}, B_{NV}) \le 1$;

(p2) $J_{NV}(A_{NV}, B_{NV}) = J_{NV}(B_{NV}, A_{NV})$;

(p3) If $A_{NV} = B_{NV}$, then $J_{NV}(A_{NV}, B_{NV}) = 1$.

Proof. Firstly, we prove the properties(p1)-(p3) of $J(A_{NV}, B_{NV})$

(p1) It is clear that $J_{NV}(A_{NV}, B_{NV}) \ge 0$

We have to proof $J_{NV}(A_{NV}, B_{NV}) \le 1$ By the inequality

$$2ab \le a^2 + b^2.$$

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$$\sum_{i=1}^{n}\left(\begin{array}{c}[T^{+}_{ANV}(x_i).T^{+}_{BNV}(x_i)]+[T^{-}_{ANV}(x_i).T^{-}_{BNV}(x_i)]+[I^{+}_{ANV}(x_i).I^{+}_{BNV}(x_i)]+[I^{-}_{ANV}(x_i).I^{-}_{BNV}(x_i)]\\+[F^{+}_{ANV}(x_i).F^{+}_{BNV}(x_i)]+[F^{-}_{ANV}(x_i).F^{-}_{BNV}(x_i)]\end{array}\right)\leq$$

$$\sum_{i=1}^{n}\left(\begin{array}{c}\left(\begin{array}{c}[T^{+}_{ANV}(x_i)]^2+[T^{+}_{BNV}(x_i)]^2+[T^{-}_{ANV}(x_i)]^2+[T^{-}_{BNV}(x_i)]^2+[I^{+}_{ANV}(x_i)]^2+[I^{+}_{BNV}(x_i)]^2\\ [I^{-}_{ANV}(x_i)]^2+[I^{-}_{BNV}(x_i)]^2+[F^{+}_{ANV}(x_i)]^2+[F^{+}_{BNV}(x_i)]^2+[F^{-}_{ANV}(x_i)]^2+[F^{-}_{BNV}(x_i)]^2\end{array}\right)\\-\left(\begin{array}{c}[T^{+}_{ANV}(x_i).T^{+}_{BNV}(x_i)]+[T^{-}_{ANV}(x_i).T^{-}_{BNV}(x_i)]+[I^{+}_{ANV}(x_i).I^{+}_{BNV}(x_i)]+[I^{-}_{ANV}(x_i).I^{-}_{BNV}(x_i)]\\+[F^{+}_{ANV}(x_i).F^{+}_{BNV}(x_i)]+[F^{-}_{ANV}(x_i).F^{-}_{BNV}(x_i)]\end{array}\right)\end{array}\right)$$

$$\sum_{i=1}^{n}\left(\frac{\begin{array}{c}[T^{+}_{ANV}(x_i).T^{+}_{BNV}(x_i)]+[T^{-}_{ANV}(x_i).T^{-}_{BNV}(x_i)]+[I^{+}_{ANV}(x_i).I^{+}_{BNV}(x_i)]+[I^{-}_{ANV}(x_i).I^{-}_{BNV}(x_i)]\\+[F^{+}_{ANV}(x_i).F^{+}_{BNV}(x_i)]+[F^{-}_{ANV}(x_i).F^{-}_{BNV}(x_i)]\end{array}}{\begin{array}{c}\left(\begin{array}{c}[T^{+}_{ANV}(x_i)]^2+[T^{+}_{BNV}(x_i)]^2+[T^{-}_{ANV}(x_i)]^2+[T^{-}_{BNV}(x_i)]^2+[I^{+}_{ANV}(x_i)]^2+[I^{+}_{BNV}(x_i)]^2\\+[I^{-}_{ANV}(x_i)]^2+[I^{-}_{BNV}(x_i)]^2+[F^{+}_{ANV}(x_i)]^2+[F^{+}_{BNV}(x_i)]^2+[F^{-}_{ANV}(x_i)]^2+[F^{-}_{BNV}(x_i)]^2\end{array}\right)\\-\left(\begin{array}{c}[T^{+}_{ANV}(x_i).T^{+}_{BNV}(x_i)]+[T^{-}_{ANV}(x_i).T^{-}_{BNV}(x_i)]+[I^{+}_{ANV}(x_i).I^{+}_{BNV}(x_i)]+[I^{-}_{ANV}(x_i).I^{-}_{BNV}(x_i)]\\+[F^{+}_{ANV}(x_i).F^{+}_{BNV}(x_i)]+[F^{-}_{ANV}(x_i).F^{-}_{BNV}(x_i)]\end{array}\right)\end{array}}\right)\leq 1$$

$\therefore J_{NV}(A_{NV}, B_{NV}) \leq 1$

Hence, $0 \leq J_{NV}(A_{NV}, B_{NV}) \leq 1$ holds.

(p2) It is clear that

$J_{NV}(A_{NV}, B_{NV}) = J_{NV}(B_{NV}, A_{NV})$,

$\therefore$ It is true.

(p3) if $A_{NV} = B_{NV}$,

$(T^{+}_{ANV}(x_i), T^{-}_{ANV}(x_i); I^{+}_{ANV}(x_i), I^{-}_{ANV}(x_i); F^{+}_{ANV}(x_i), F^{-}_{ANV}(x_i)) =$

$(T^{+}_{BNV}(x_i), T^{-}_{BNV}(x_i); I^{+}_{BNV}(x_i), I^{-}_{BNV}(x_i); F^{+}_{BNV}(x_i), F^{-}_{BNV}(x_i))$

where i= 1,2,3……..,n.

Here, A_{NV} and B_{NV} considered as two vectors so ,

$\|A_{NV}\| = \|B_{NV}\|$ where

$$\|A_{NV}\| = \sqrt{[T^{+}_{ANV}(x_i)]^2+[T^{-}_{ANV}(x_i)]^2+[I^{+}_{ANV}(x_i)]^2+[I^{-}_{ANV}(x_i)]^2+[F^{+}_{ANV}(x_i)]^2+[F^{-}_{ANV}(x_i)]^2}$$

$$\|B_{NV}\| = \sqrt{[T^{+}_{BNV}(x_i)]^2+[T^{-}_{BNV}(x_i)]^2+[I^{+}_{BNV}(x_i)]^2+[I^{-}_{BNV}(x_i)]^2+[F^{+}_{BNV}(x_i)]^2+[F^{-}_{BNV}(x_i)]^2}$$

And there exist $\frac{A_{NV}.B_{NV}}{||A_{NV}||.||B_{NV}||}$

$$=\frac{\begin{array}{c}[T^{+}_{ANV}(x_i).T^{+}_{BNV}(x_i)]+[T^{-}_{ANV}(x_i).T^{-}_{BNV}(x_i)]+[I^{+}_{ANV}(x_i).I^{+}_{BNV}(x_i)]+[I^{-}_{ANV}(x_i).I^{-}_{BNV}(x_i)]\\+[F^{+}_{ANV}(x_i).F^{+}_{BNV}(x_i)]+[F^{-}_{ANV}(x_i).F^{-}_{BNV}(x_i)]\end{array}}{\sqrt{[T^{+}_{ANV}(x_i)]^2+[T^{-}_{ANV}(x_i)]^2+[I^{+}_{ANV}(x_i)]^2+[I^{-}_{ANV}(x_i)]^2+[F^{+}_{ANV}(x_i)]^2+[F^{-}_{ANV}(x_i)]^2}\,.\,\sqrt{[T^{+}_{BNV}(x_i)]^2+[T^{-}_{BNV}(x_i)]^2+[I^{+}_{BNV}(x_i)]^2+[I^{-}_{BNV}(x_i)]^2+[F^{+}_{BNV}(x_i)]^2+[F^{-}_{BNV}(x_i)]^2}}$$

$= 1.$

Hence,

$J_{NV}(A_{NV}, B_{NV}) = 1.$

Thus, we have proved.

If we consider the weighted Jaccard index measure between A_{NV} and B_{NV} are proposed, respectively, as follow:

$WJ_{NV}(A_{NV}, B_{NV})$

$$=\sum_{i=1}^{n}(w_i)\left(\frac{\begin{matrix}[T^+_{ANV}(x_i).T^+_{BNV}(x_i)]+[T^-_{ANV}(x_i).T^-_{BNV}(x_i)]+[I^+_{ANV}(x_i).I^+_{BNV}(x_i)]+[I^-_{ANV}(x_i).I^-_{BNV}(x_i)]\\+[F^+_{ANV}(x_i).F^+_{BNV}(x_i)]+[F^-_{ANV}(x_i).F^-_{BNV}(x_i)]\end{matrix}}{\begin{matrix}\left(\begin{matrix}[T^+_{ANV}(x_i)]^2+[T^+_{BNV}(x_i)]^2+[T^-_{ANV}(x_i)]^2+[T^-_{BNV}(x_i)]^2+[I^+_{ANV}(x_i)]^2+[I^+_{BNV}(x_i)]^2\\ [I^-_{ANV}(x_i)]^2+[I^-_{BNV}(x_i)]^2+[F^+_{ANV}(x_i)]^2+[F^+_{BNV}(x_i)]^2+[F^-_{ANV}(x_i)]^2+[F^-_{BNV}(x_i)]^2\end{matrix}\right)\\-\left(\begin{matrix}[T^+_{ANV}(x_i).T^+_{BNV}(x_i)]+[T^-_{ANV}(x_i).T^-_{BNV}(x_i)]+[I^+_{ANV}(x_i).I^+_{BNV}(x_i)]+[I^-_{ANV}(x_i).I^-_{BNV}(x_i)]\\+[F^+_{ANV}(x_i).F^+_{BNV}(x_i)]+[F^-_{ANV}(x_i).F^-_{BNV}(x_i)]\end{matrix}\right)\end{matrix}}\right)$$

Where $w_i \in [0,1]$, and $\sum_{i=1}^{n} w_i = 1$ for i= 1,2,….,n.

It is obvious that the two weighted Jaccard index measures $WJ(A_{NV}, B_{NV})$ also statisfy the following properties (p1)-(p3):

(p1) $0 \leq WJ_{NV}(A_{NV}, B_{NV}) \leq 1$;

(p2) $WJ_{NV}(A_{NV}, B_{NV}) = WJ_{NV}(B_{NV}, A_{NV})$;

(p3) If $A_{NV} = B_{NV}$, then $WJ_{NV}(A_{NV}, B_{NV}) = 1$.

We can easily prove the properties (p1)-(p3) for $WJ_{NV}(A_{NV}, B_{NV})$ by a similar proof process.

4. MAGDM Method Based on the Jaccard Index Measures

For an MAGDM problem, let G= { g_1, g_2, ………g_m} be a set of *m* alternatives and A= {A_1, A_2………A_n} be a set of *n* attributes. The weight vector of the attributes A_j (j = 1, 2, ... , n) is $\omega_A = (\omega_{A1}, \omega_{A2}, \dots, \omega_{An})^T$, satisfying $\omega_{Aj} \in [0, 1]$, and $\sum_{j=1}^{n} \omega_{Aj} = 1$ for j = 1, 2, ... , n. Assume that EX = {EX_1, EX_2, ... , EX_y} is a group of specialists and their corresponding weight vector is $\omega_E = (\omega_{E1}, \omega_{E2}, \dots, \omega_{Ey})^T$, satisfying $\omega_{Ek} \in [0, 1]$, and $\sum_{k=1}^{y} \omega_{Ek} = 1$. Each specialist can dole out the truth-degree, falsity-degree, and indeterminacy-degree to each attribute A_j (j = 1, 2, ... , n) on the choices g_i (i = 1, 2, ... , m) according to the neutrosophic environment respectively. Therefore, we can established in NVs decision matrix

$D^k = \left(d_{i,j}^k\right)_{m \times n} = [D_1^k, D_2^k, \dots, D_m^k]^T$, is an NVs for T_{NV}, I_{NV}, $F_{NV} \in [0,1]$

Then, we apply the Jaccard index measures of neutrosophic vague set (in short NVs) to solve MAGDM problems

4.1. Algorithm to solve MAGDM problem.

Step 1: We establish the Ns, BNs, NVs matrix $G_H^* = (g_{i,j}^*)_{4\times 3}$ G_i^* (i=1,2,3,4) as follows:

$$G_H^k = \left(g_{i,j}^k\right)_{m \times n} = [G_1^k, G_2^k, \dots, G_m^k]^T$$

Step 2: Calculate the weighted Jaccard index measures values by Eq.(2,4,6) using H.

Step 3: Calculate the overall weighted Jaccard index measure values considering the corresponding weight of each expert to evaluate the alternatives G_i (i = 1, 2, ... , m), as follows:

$$J_H(D^k, G_i)=\sum_{k=1}^{y} \omega_{Ek}\, J_H(D^k{}_i, G_i) \quad (7)$$

$$WJ_H(D^k, G_i)=\sum_{k=1}^{y} \omega_{Ek}\, J_H(D^k{}_i, G_i) \quad (8)$$

Where $\omega_{Ek} \in [0,1]$ and $\sum_{k=1}^{y} \omega_{Ek} = 1$.

Step 4: Rank all alternatives according to the value of $WJ_H(D^k, G_i)$ or $J_H(D^k, G_i)$ and

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select the better choice. The greater value of a Jaccard index measure, is the better alternative.

Step 5: End.

5. Practical example

Let us consider the decision making problem. There is a speculation organization, which needs to put an aggregate of cash in the best choices. There is a board with four potential alternatives to invest the money. (1) G_1 is a motor company; (2) G_2 is a pump company; (3) G_3 is an arms company; (4) G_4 is a furniture company. The investment company must make a decision according to three attributes given below: (1) A_1 is the growth analysis; (2) A_2 is the risk analysis; (3) A_3 is the environmental impact analysis. Then, the weight vector of the attributes is given by are 0.35,0.25 and 0.40 . Thus, when the four possible alternatives with respect to the above three attributes are evaluated by the expert, we can get the accompanying the neutrosophic vague decision matrix:

$$D^1=\begin{bmatrix} D_1^1 \\ D_2^1 \\ D_3^1 \\ D_4^1 \end{bmatrix}=\begin{bmatrix} [<0.4,0.2>,<0.2,0.7>,<0.3,0.4>],[<0.3,0.2>,<0.1,0.1>,<0.2,0.3>],[<0.4,0.3>,<0.2,0.2>,<0.4,0.5>] \\ [<0.5,0.6>,<0.7,0.2>,<0.9,0.1>],[<0.4,0.5>,<0.2,0.2>,<0.4,0.3>],[<0.3,0.1>,<0.4,0.5>,<0.8,0.1>] \\ [<0.6,0.4>,<0.2,0.2>,<0.4,0.5>],[<0.5,0.6>,<0.7,0.7>,<0.8,0.4>],[<0.3,0.5>,<0.4,0.4>,<0.4,0.3>] \\ [<0.3,0.2>,<0.4,0.4>,<0.5,0.7>],[<0.4,0.3>,<0.4,0.4>,<0.8,0.9>],[<0.4,0.8>,<0.3,0.3>,<0.4,0.5>] \end{bmatrix},$$

$$D^2=\begin{bmatrix} D_1^2 \\ D_2^2 \\ D_3^2 \\ D_4^2 \end{bmatrix}=\begin{bmatrix} [<0.3,0.4>,<0.5,0.8>,<0.2,0.4>],[<0.2,0.3>,<0.3,0.4>,<0.5,0.1>],[<0.6,0.4>,<0.3,0.4>,<0.5,0.4>] \\ [<0.4,0.3>,<0.5,0.6>,<0.6,0.8>],[<0.5,0.1>,<0.4,0.3>,<0.6,0.7>],[<0.5,0.9>,<0.2,0.3>,<0.6,0.7>] \\ [<0.1,0.3>,<0.4,0.6>,<0.8,0.5>],[<0.4,0.3>,<0.2,0.3>,<0.4,0.3>],[<0.5,0.6>,<0.3,0.5>,<0.8,0.6>] \\ [<0.6,0.1>,<0.6,0.5>,<0.6,0.3>],[<0.6,0.5>,<0.1,0.3>,<0.5,0.2>],[<0.3,0.6>,<0.2,0.1>,<0.3,0.6>] \end{bmatrix},$$

$$D^3=\begin{bmatrix} D_1^3 \\ D_2^3 \\ D_3^3 \\ D_4^3 \end{bmatrix}=\begin{bmatrix} [<0.3,0.1>,<0.1,0.6>,<0.2,0.3>],[<0.4,0.6>,<0.2,0.4>,<0.4,0.5>],[<0.5,0.7>,<0.1,0.5>,<0.3,0.4>] \\ [<0.3,0.7>,<0.6,0.5>,<0.6,0.3>],[<0.3,0.4>,<0.1,0.1>,<0.3,0.4>],[<0.4,0.3>,<0.5,0.6>,<0.7,0.2>] \\ [<0.5,0.3>,<0.4,0.5>,<0.6,0.4>],[<0.4,0.5>,<0.5,0.6>,<0.6,0.5>],[<0.4,0.2>,<0.6,0.5>,<0.2,0.4>] \\ [<0.4,0.3>,<0.5,0.3>,<0.4,0.6>],[<0.5,0.2>,<0.5,0.3>,<0.5,0.6>],[<0.5,0.6>,<0.4,0.2>,<0.3,0.2>] \end{bmatrix}$$

Then, the developed MAGDM approach can be applied to this decision- making problem using the following steps:

Step 1: we can compute the Jaccard index measures G_i (i=1,2,3,4) by using by Eq.(5) as follows:

$$G^*=\begin{bmatrix} G_1^* \\ G_2^* \\ D_3^* \\ D_4^* \end{bmatrix}=\begin{bmatrix} [<0.5,0.6>,<0.1,0.6>,<0.2,0.3>],[<0.4,0.6>,<0.1,0.1>,<0.2,0.1>],[<0.6,0.7>,<0.1,0.2>,<0.3,0.4>] \\ [<0.5,0.7>,<0.5,0.2>,<0.6,0.1>],[<0.5,0.6>,<0.1,0.1>,<0.3,0.3>],[<0.5,0.9>,<0.2,0.3>,<0.6,0.1>] \\ [<0.5,0.3>,<0.2,0.2>,<0.4,0.4>],[<0.4,0.5>,<0.2,0.3>,<0.4,0.3>],[<0.5,0.6>,<0.3,0.4>,<0.2,0.3>] \\ [<0.6,0.3>,<0.4,0.3>,<0.4,0.3>],[<0.6,0.5>,<0.1,0.3>,<0.5,0.2>],[<0.5,0.8>,<0.2,0.1>,<0.3,0.2>] \end{bmatrix}$$

Step 2: We calculate the Jaccard index measures values dependent on the distance between D_i^k

Equation (1) as follows:

$J_{NVs}(D^1, G_i)$ = { $J_{NVs}(D_1^1, G_1^*)$, $J_{NVs}(D_2^1, G_2^*)$, $J_{NVs}(D_3^1, G_3^*)$, $J_{NVs}(D_4^1, G_4^*)$} ={0.7844, 0.7088, 0.7409, 0.9409 }

$J_{NVs}(D^2, G_i)$ = { $J_{NVs}(D_1^2, G_1^*)$, $J_{NVs}(D_2^2, G_2^*)$, $J_{NVs}(D_3^2, G_3^*)$, $J_{NVs}(D_4^2, G_4^*)$}={ 0.9550, 0.9847, 0.9122, 0.9688}

$J_{NVs}(D^3, G_i)$ = { $J_{NVs}(D_1^3, G_1^*)$, $J_{NVs}(D_2^3, G_2^*)$, $J_{NVs}(D_3^3, G_3^*)$, $J_{NVs}(D_4^3, G_4^*)$}={ 0.9245, 0.9066, 0.9455, 0.9813}

Similarly, we can calculate the weighted Jaccard index measures values dependent on the two vectors between by equation (6) as follows:

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$WJ_{NVs}(D^1, G_i)$ = { $WJ_{NVs}(D_1^1, G_1^*)$, $WJ_{NVs}(D_2^1, G_2^*)$, $WJ_{NVs}(D_3^1, G_3^*)$, $WJ_{NVs}(D_4^1, G_4^*)$}={ 0.9732, 0.9644, 0.9673,0.9432}

$WJ_{NVs}(D^2, G_i)$ = { $WJ_{NVs}(D_1^2, G_1^*)$, $WJ_{NVs}(D_2^2, G_2^*)$, $WJ_{NVs}(D_3^2, G_3^*)$, $WJ_{NVs}(D_4^2, G_4^*)$}={0.8468, 0.8615, 0.9750, 0.9584}

$WJ_{NVs}(D^3, G_i)$ = { $WJ_{NVs}(D_1^3, G_1^*)$, $WJ_{NVs}(D_2^3, G_2^*)$, $WJ_{NVs}(D_3^3, G_3^*)$, $WJ_{NVs}(D_4^3, G_4^*)$}={0.7807, 0.8500, 0.9827, 0.8366}.

Step 3: Considering the relating weight $\omega_E =(0.37,0.33,0.3)^T$ of the specialists to assess the alternatives G_i (i = 1, 2, 3, 4), we can calculate the overall weighted Jaccard index measure values depends on distance by Equation (7) as follows:

$J_{NVs}(D^k, G_1)$=0.37 x $J_{NVs}(D_1^1, G_1^*)$ + 0.33 x $J_{NVs}(D_1^2, G_1^*)$ +0.3 x $J_{NVs}(D_1^3, G_1^*)$ =0.8064

$J_{NVs}(D^k, G_2)$=0.37 x $J_{NVs}(D_2^1, G_2^*)$ + 0.33 x $J_{NVs}(D_2^2, G_2^*)$ +0.3 x $J_{NVs}(D_2^3, G_2^*)$ =0.8568

$J_{NVs}(D^k, G_3)$=0.37 x $J_{NVs}(D_3^1, G_3^*)$ + 0.33 x $J_{NVs}(D_3^2, G_3^*)$ +0.3 x $J_{NVs}(D_3^3, G_3^*)$ =0.9331

$J_{NVs}(D^k, G_4)$=0.37 x $J_{NVs}(D_4^1, G_4^*)$ + 0.33 x $J_{NVs}(D_4^2, G_4^*)$ +0.3 x $J_{NVs}(D_4^3, G_4^*)$ =0.8597

Similarly, we can calculate the overall weighted Jaccard index measures values based on the two vectors between by equation (8) as follows:

$WJ_{NVs}(D^k, G_1)$= 0.8737 $WJ_{NVs}(D^k,G_2)$=0.8961 $WJ_{NVs}(D^k, G_3)$= 0.9744 $WJ_{NVs}(D^k,G_4)$= 0.9162

Step 4: According to the above values of $J_{NVs}(D^k, G_i)$ and (i=1,2,3,4), the distance value of both the Jaccard index measure and the weighted Jaccard index measure values based on two vectors, the ranking orders: $G_3 > G_4 > G_2 > G_1$ are same. As indicated by the most extreme value of Jaccard index measures, the alternative G_3 is the better decision.

6. Related Comparison

Further comparison, table 6.1 show the MAGDM results based on the Jaccard index measures of NVs proposed in this paper and the neutrosophic set and bipolar neutrosphic set were proposed Jaccard index in the relevant paper [23][32]. Here, we utilizing Enq (8) for the two neutrosophic set and bipolar neutrosophic set respectively.

MAGDM METHOD	JACCARD INDEX	RANKING ORDER	THE BEST ALTERNATIVE
$J_{NVS}(D^K,G_I)$	0.8064,0.8568,0.9331,0.8597	$G_3 > G_4 > G_2 > G_1$	G_3
$WJ_{NVS}(D^K,G_I)$	0.8737,0.8961,0.9744,0.9162	$G_3 > G_4 > G_2 > G_1$	G_3
$WJ_{NS}(D^K,G_I)$	0.8003,0.7961,0.8447,0.8147	$G_3 > G_4 > G_2 > G_1$	G_3
$WJ_{BNS}(D^K,G_I)$	0.8700,0.7456,0.8940,0.8957	$G_3 > G_4 > G_2 > G_1$	G_3

Table 6.1. Decision results based on neutrosophic environment MAGDM method

Obviously, from the result of table 6.1, ranking orders and best alternatives based on the new method based on this paper is consistent with the result provided by Mehmet and Irfan [23]. Compared with the [23,32] the calculation

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process of the Jaccard index for MAGDM proposed in this paper is relatively compared to neutrosophic set and bipolar neutrosophic set based on the Jaccard index for MAGDM in [23, 32]. The above comparisons demonstrate that thispaper present a new concept for solving decision-making problems is more efficient under a neutrosophic environment.

7 .Conclusion

In this paper we develop MAGDM method and gave its application under theneutrosophic environment and also to show the exhibit effectiveness of theproposed method, we utilized an illustration example. There are many similarity measures utilized inthe decision-making problem but we have utilized a Jaccard similarity measure toshow that the proposed method can effectively solve decision-making problems with NVs information. Furthermore, researchers can be extended to study some new correlation coefficients between NVs and their MAGDM.

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