A Novel Comparison between the Ordinary Estimators and Robust Estimators for the Parameters of Some Binary Mixed Models

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Abstract

A condensed study will be done to compare the ordinary estimators. In particular, the maximum likelihood estimator and the robust estimator, to estimate the parameters of the mixed model of order one, namely BARMA (1, 1). Simulation experiments will be applied for varieties of BARMA (1, 1) based on using small, moderate, and large sample sizes, where some new results were obtained. MAPE was used as a statistical criterion for comparison.

Keywords: Binary system; Mathematical model; Estimator; Numerical comparison

1. Introduction

One of the most used methods for estimating the parameters of time series models is the least squares method, which is denoted by the symbol (LS) when the distribution of errors is unknown, and the method of maximum likelihood method, which is denoted by the symbol (ML) when the distribution of errors is known, as well as the method of moments, which is denoted by the symbol (MOM), which is called the Yule method-Walker, too.

The estimators of these methods may be efficient, consistent and appropriate if the time series conditions are met. One of them is that the distribution is often normal as well as under the condition of stationary and inevitability, but when the conditions differ as a result of the presence of a certain factor that may be external or contingent on the time series, it is necessary to look for another suitable estimation method that can deal with the time series in which the required conditions are not met, and the estimates resulting from this method must presumably, which is the result of the difference in the data format, even if it is small, this The change is usually due to the presence of Outliers (anomalous) in the data that appear in the errors, which directly affects the assumed distribution of these errors.

The aim of this research is to make a comparison between ordinary method and robust method to estimate the parameters of the binary mixed model with lower ranks BARMA (1, 1) Using the theoretical method (statistical theory) and the experimental method (simulation) to estimate the parameters of randomly generated time series in the absence of electrolytes and then in the case of intercalation of electrolytes and compare them using the mean absolute percentage error (MAPE). This criterion was used because it is one of the most accurate criteria for comparing estimation methods in time series, and the Mean Square Error MSE was not used for comparison because it squares the error per view and then finding the average for the sum of these squares, which gives large weights to large errors compared to small errors. Therefore, this criterion is inaccurate, it does not facilitate comparison, especially for Time Series models, and it is not suitable in practice for making comparisons between different methods of estimation (Daniel, 2004).

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2. Main Discussion

2.1 Auto-regression model-first-order binary mixed moving circles

BARMA (1,1) Models

The BARMA model (1,1) is according to the following formula:

$(I-\Phi_1 B)y_t=(I-\Theta_1 B)a_t$ …………………..(1)

First: the model is stable if the roots of the equation $|I-\Phi_1 B|=0$ are outside the unit circle or if the characteristic values in Φ_1 are inside the unit circle.

Second: the model can be written in terms of random errors and according to the following formula:

$y_t=\sum_{s=0}^{\infty}\Psi_s a_{t-s}$ …………(2)

The weights Ψ_s are obtained from the equality of the coefficients B^j in the following matrix equation:

$$(I-\Phi_1 B)(I+\Psi_1 B+\Psi_2 B^2+\ldots)=(I-\Theta_1 B)$$

Thus:

$\Psi_i=\Phi_1\Psi_{j-1}=\Phi_1^{j-1}(\Phi_1-\Theta_1),\quad j\geq 1$ ………..(3)

The model is reversible if the roots of the equation$|I-\Theta_1 B|=0$ are outside the unit circle, or if the characteristic values in Θ_1 are inside the unit circle.

Third: the covariance matrix can also be derived as follows:

$$\boldsymbol{E}[\boldsymbol{y_t}(\boldsymbol{y_t}-\boldsymbol{y'}_{t-1}\Phi'_1)]=\boldsymbol{E}[\boldsymbol{y'}_{t-k}(\boldsymbol{a'}_t-\boldsymbol{a'}_{t-1}\Theta'_1)]$$

We note that:

$\boldsymbol{E}[\boldsymbol{y_t}(\boldsymbol{a'}_{t-1}\boldsymbol{\theta_1})]=\boldsymbol{E}[(\Phi_1\boldsymbol{y}_{t-1}+a_t-\Theta_1 \mathrm{a}_{t-1})(\mathrm{a'}_{t-1}\Theta'_1)]$

$=\Phi_1\Sigma\Theta'_1-\Theta_1\Sigma\Theta'_1$ …(4)

Thus, we get the following:

$\Gamma(0)-\Gamma'(1)\Sigma=\Sigma-(\Phi_1-\Theta_1)\Sigma\Theta_1\ ,\quad \boldsymbol{k=0}$

$\Gamma(1)-\Gamma(0)\Phi'_1=-\Sigma\Theta'_1,\quad \boldsymbol{k=1}$

$\Gamma(k)-\Gamma'(k-1)\Phi'_1=0,\ \ k\geq 2$ ……..(5)

$$\Gamma(k)=\begin{cases}\Gamma(1)\Sigma+\Sigma-(\Phi-\Theta)\Sigma\Theta & ,\mathrm{k}=0\\ \Gamma(0)\Phi'_1-\Sigma\Theta_1 & ,\mathrm{k}=1\\ \Gamma(k-1)\Phi'_1 & ,k\geq 2\end{cases}$$

Mixed two-variable model BARMA (1,1):

$$(I-\Phi_1 B)\mathrm{y}_t=(I-\Theta_1\boldsymbol{B})\boldsymbol{\varepsilon_t}$$

Where:

$\Phi_1=\begin{bmatrix}\phi_{11} & \phi_{12}\\ \phi_{21} & \phi_{22}\end{bmatrix}$ $\Theta_1=\begin{bmatrix}\theta_{11} & \theta_{12}\\ \theta_{21} & \theta_{22}\end{bmatrix}$

And that the covariance and covariance matrix of the vector $\begin{bmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\end{bmatrix}$ is:

$$\Sigma=\begin{bmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{21} & \sigma_{22}\end{bmatrix}$$

Assuming that the vector distribution $\begin{bmatrix}y_{1t}\\ y_{2t}\end{bmatrix}$ is a bivariate Normal distribution, it is obvious that the logarithm of the sample's probability function is $(y_1,y_2,\ldots,y_n)$ can be written according to the following formula:

$\ln L(\Phi_1,\theta_1,\Sigma|y)=cons\ tant-\frac{n}{2}\ln|\Sigma|-\frac{1}{2}\sum_{t=1}^{n}\varepsilon_t^T\Sigma^{-1}\varepsilon_t$ ………..(6)

Which can be written according to the following formula:

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$\ln L = cons\ tant - \frac{n}{2}\ln|\Sigma| - \frac{1}{2}tr\Sigma^{-1}S(\Phi_1, \Theta_1)$ …..(7)

Where:

$$S(\Phi_1, \Theta_1) = \sum_{t=1}^{n} \varepsilon_t \varepsilon_t^T$$

It is obvious that the random error ε_t is expressed according to the following formula:

$$\varepsilon_t = y_t - \Phi_1 y_{t-1} + \Theta_1 \varepsilon_{t-1}$$

Thus, the maximal potentials of the parameter matrices Φ_1, Θ_1, Σ can be calculated from the maximization of the logarithm of the maximization function.

The two-variable auto-regression model BARMA (1,0) is a special case of the general formula in the first, where the formula of the model is as follows:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \quad \text{…..(8)}$$

The logarithm of the sample probability function is given by the following formula:

$\ln L(\Phi_1, \Sigma|y) = cons\ tant - \frac{n}{2}|\Sigma| - \frac{1}{2}\sum_{t=1}^{n} \varepsilon_t^T \Sigma^{-1} \varepsilon_t$ ………..(9)

Using the properties of Matrices, the logarithm of the possible function of the sample can be written $(y_1, y_2, \dots, y_n)$ according to the following formula:

$\ln L = cons\ tant - \frac{n}{2}|\Sigma| - \frac{1}{2}tr\Sigma^{-1}S(\Phi_1)$ ………..(10)

It is clear that:

$$S(\Phi_1) = \sum_{t=1}^{n} \varepsilon_t \varepsilon_t^T$$

The limit of the random error is expressed by the following formula:

$$\varepsilon_t = y_t - \Phi_1 y_{t-1}$$

Also, the mixed two-variable model BARMA (0,1) is also a special case of the general case in the first, as the model can be written according to the following formula:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} - \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix} \quad \text{…(11)}$$

After simplification, the logarithm of the possibility function for a sample of size n can be written as:

$\ln L = cons\ tant - \frac{n}{2}ln|\Sigma| - \frac{1}{2}tr\Sigma^{-1}S(\theta_1)$ ………..(12)

Where:

$$S(\theta_1) = \sum_{t=2}^{n} \varepsilon_t \varepsilon_t^1$$

However, ε_t as a vector differs from the first and Second cases in that it is expressed by the following formula:

$$\varepsilon_t = y_t + \Theta_1 \varepsilon_{t-1}$$

2.2 Estimation of model parameters by the heuristic method

The stray values of the time series can inversely affect both the least squares estimators (LS) and the coefficient of the estimators M of the auto-regression parameters. The focus here is on obtaining robust estimators of the first-order auto-regression coefficient. So, the observations are $y_t = x_t + v_t$ with two models changing, **the first** is the innovation outliers (IO) with $(v_t = 0)$, x_t is a Possibly non-Gaussian abnormal probability and **the second** is a model of added Additive Effects Outliers (AO) with a nonzero v_t and a very high probability and a possibly quite

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large small fraction of time and X_t are normal, and the general classification of M-potentials is assumed, which have the properties of the mean squared error of both models (IO) and (AO) by the Gaussian method.

In this paper, we will study the problems of obtaining robust estimators of auto-regression of lower ranks, which is the transformation relative to the time series of outliers.

When proposing robust procedures for estimating the parameters of time series, it requires characterizing the time series contaminated with outliers with appropriate probability models.

Due to the difficulty of formulating complete probabilistic models (Martin, 1980), it seems mandatory to start with generating models for the designation of simple outliers, which can create real data containing outliers, and it has been proven in practice, that the behavior of outliers often follows one of the following forms:

a- The first possible behavior of the occurrence of outliers is that the chance of their occurrence is usually associated with the remainder of the sample vocabulary, except for the case of Initial Jump, which is known as the initial abrupt change.

b- The second possible behavior, known as a large error outlier, which may be due to various reasons, such as a registration error.

c- The third possible behavior, defined as different types of outliers with behavior unrelated to the behavior of the rest of the sample vocabulary. This type may be due to the insufficient use of the recording medium.

From the foregoing, the above possible types of behavior can be realized with suitable models, the first type of behavior can be obtained with the model of Innovation Outlier (IO), if the values of the observations are equal to the values of the essence of the process (Zch , 1979), i.e.

$y_i = \hat{x}_t$ ……(13)

If the distribution of errors (IO) is symmetric, then the model of outliers is called the Innovation model (IO), similar to the distribution t or another normal distribution (G), since:

$$G(p, \sigma_1, \sigma_2) = (1-p)N(0, {\sigma^2}_1) + pN((0, {\sigma^2}_2)$$

So that ${\sigma^2}_2 > {\sigma^2}_1$ and that the value of p is usually small.

In other words, if the values of (ε_t) white noise satisfy the condition (iid) of a random variable with a symmetric distribution (G) with a mean of zero and a measurement parameter (σ), then the values of the random variable are called Innovation. As for the second and third types of behavior, the appropriate model is known as the additive or aggregate additive outlier (AO) model, since:

$Y_t = X_t + V_t$ …..(14)

So V_t is a random variable whose distribution is independent of X_t and whose marginal distribution (when p is relatively small) is:

$$P(V_t = 0) = 1 - \gamma$$

This has been proven by the experiment in the field of time series, the range of γ is achieved between (0.25, 0.01) (Stochinger and Duter, 1987) and the normal V_t distribution can be mixed:

$CND(p, \sigma_3) = (1-\gamma)\delta_0 + \gamma N(p, {\sigma^2}_3)$ …..(15)

δ_0 denotes the degenerated distribution whose mass is concentrated at the center of gravity.

This type of outliers can occur, if the hypothesis of independence from V_t is dropped. he referred to this type of outliers for the first time (Fox 1972), as he proposed two types of outliers, those that affect viewing only when they occur and which were then treated with renewable or innovation outliers (Type I) and those that affect views in general, which were known as additive or aggregate outliers (type II), moreover, Fox in the same year also made two proposals to determine the type of outliers, **the first** is based on the idea of examining then choose the model in which the stray viewing is more extreme, and **the second** by choosing the model when there is an opportunity to reveal the extent of the viewing effect Stray into her subsequent views.

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3. The experimental side

3.1 Simulation model formulation

Simulation is a process of simulating or imitating real reality, that is, finding an exact copy of any system or model without taking that system or model itself, especially since some of these problems and statistical theories are difficult to prove mathematically, which prompted researchers to translate them into experimental communities and then draw a number of random samples from them to arrive at optimal solutions to such problems.

Therefore, in order to achieve the main goal of this research, a simulation model was formulated to compare the usual and robust methods for estimating the parameters of the mixed two-variable model BARMA(1,1) of experiments.

A: the stage of data generation for the purpose of estimating the parameters of the BARMA model (1,1)

$$\Phi(B)y_t = \Theta(B)\varepsilon_t$$

$$\varepsilon_T = \Theta^{-1}(B)\phi(B)y_t$$

$$\varepsilon'_t\varepsilon_t = y'_t\phi'(\mathrm{B})\theta^{-1}(B)\theta^{-1}(B)\phi(B)y_t$$

Since from the last equation, $(\varepsilon'_t\varepsilon_t)$ was derived for the parameters of the mixed model (ϕ, θ) and their estimates were reached.

Since the vector ε_t is distributed naturally bivariate.

$$\varepsilon_t \sim N_2\left[\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\right]$$

B: determining the parameters of the model, since the number of variables in the Model m=2 has been determined.

C: define default parameters that achieve stability and reversibility according to the following configuration (variety) :

$$\Phi = \begin{bmatrix}0.8 & -0.2\\0.2 & -0.6\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}0.6 & 0.4\\0.5 & -0.6\end{bmatrix}$$

$$\Phi = \begin{bmatrix}0.8 & 0.1\\0.1 & 0.2\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}-0.6 & 0.3\\-0.3 & -0.8\end{bmatrix}$$

$$\Phi = \begin{bmatrix}0.8 & -0.1\\-0.2 & 0.2\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}-0.2 & 0.6\\-0.6 & 0.2\end{bmatrix}$$

$$\Phi = \begin{bmatrix}0.8 & -0.6\\0.5 & 0.6\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}-0.2 & 0.1\\-0.5 & 0.2\end{bmatrix}$$

$$\Phi = \begin{bmatrix}0.8 & -0.6\\0.2 & 0.8\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}-0.8 & 0.4\\0.3 & 0.8\end{bmatrix}$$

$$\Phi = \begin{bmatrix}0.8 & 0.2\\-0.2 & 0.6\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}-0.6 & 0.2\\0.2 & 0.6\end{bmatrix}$$

$$\Phi = \begin{bmatrix}0.6 & 0.4\\0.3 & 0.2\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}0.3 & 0.1\\0.2 & 0.5\end{bmatrix}$$

$$\Phi = \begin{bmatrix}0.9 & 0.1\\0.2 & 0.7\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}0.5 & 0.2\\0.2 & 0.7\end{bmatrix}$$

$$\Phi = \begin{bmatrix}0.6 & -0.4\\-0.3 & 0.2\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}-0.3 & 0.1\\0.2 & 0.5\end{bmatrix}$$

$$\Phi = \begin{bmatrix}-0.8 & -0.6\\-0.5 & 0.4\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}-0.2 & 0.1\\-0.5 & 0.3\end{bmatrix}$$

$$\Phi = \begin{bmatrix}-0.8 & 0.6\\0.6 & 0.8\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}-0.8 & 0.6\\0.6 & -0.8\end{bmatrix}$$

$$\Phi = \begin{bmatrix}-0.8 & 0.2\\0.2 & -0.8\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}-0.8 & 0.2\\0.2 & -0.8\end{bmatrix}$$

$$\Phi = \begin{bmatrix}-0.9 & 0.1\\0.2 & 0.7\end{bmatrix} \ldots.., \quad ,\ldots\ldots \quad \Theta = \begin{bmatrix}0.8 & 0.4\\0.6 & -0.5\end{bmatrix}$$

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D: determining the sample size (n). Three sample sizes were determined (100, 50, 25). The parameters of the mixed two-variable model were estimated according to the ordinary and robust methods. Outliers were interpolated by 10%. The absolute value of the average relative errors MAPE was adopted for all methods. The ordinary method was better than the robust method before the interpolation of outliers, but in the case of interpolation of outliers by 10%, the robust method was the best and as shown in Table (1) for the sample size (25). (See (Safawi, 2005) for samples 50 and 100).

4. Conclusions

I- In the case of a small sample size, the maximum possible estimate using the standard of average relative errors (MAPE) is the preferred estimate by 87.5% in the case of no intercalation of outliers, while the robust method is superior when outliers are present by 95%.

II- In the case of an average sample size, the maximum possible estimate using the standard of average relative errors (MAPE) is the preferred estimate by 95% in the case of non-interference of outliers, while the robust method is superior when outliers are present by 93%.

III- In the case of a large sample size, the maximum possible estimate using the standard of average relative errors (MAPE) is the preferred estimate by 97% in the absence of outliers, while the robust method is superior when outliers are present by 97%.

Table 1: The values of the average absolute relative MAPE errors of the parameters of the BARMA model (1, 1) estimated when the sample size is (25)

before polluting					**After polluting**		
lineup number	**estimation method** / **Default values of parameters**	**ML**	**Robust**	**Best**	**ML**	**Robust**	**Best**
1	$\phi_{11} = 0.8$	0.00191551	0.0794192	ML	0.00050000	0.000306387	RO
	$\phi_{12} = -0.2$	0.00245530	0.0020000	RO	0.00200000	0.000000200	RO
	$\phi_{21} = 0.2$	0.00165431	0.0020000	ML	0.00200000	0.000000200	RO
	$\phi_{22} = -0.6$	0.00221439	0.0020000	RO	0.00533333	0.000000200	RO
	$\phi_{11} = 0.6$	0.00174262	7.7386600	ML	0.00173600	0.000773866	RO
	$\phi_{12} = 0.4$	0.00276808	1.8020000	ML	0.00269300	0.000180200	RO
	$\phi_{21} = 0.5$	0.00237627	1.4420000	ML	0.00231680	0.000144200	RO
	$\phi_{22} = -0.6$	0.00218199	1.5308200	ML	0.0024620	0.000153082	RO
2	$\phi_{11} = 0.8$	0.001942910	0.0816621	ML	0.0005000	0.00027991	RO
	$\phi_{12} = 0.1$	0.002479630	0.0020000	RO	0.0020000	0.00000020	RO
	$\phi_{21} = 0.1$	0.001736420	0.0020000	RO	0.0020000	0.00000020	RO
	$\phi_{22} = 0.2$	0.002167800	0.0020000	RO	0.0053333	0.00000020	RO
	$\phi_{11} = -0.6$	0.001761250	98.3582000	ML	0.0017360	0.00974395	ML
	$\phi_{12} = 0.3$	0.002716140	1.8020000	ML	0.0026930	0.00018770	RO
	$\phi_{21} = -0.3$	0.002328190	1.8420000	ML	0.0023168	0.00015020	RO
	$\phi_{22} = -0.8$	0.002132190	0.3077610	ML	0.0024620	0.000033175	RO

before polluting					**After polluting**		
lineup number	**estimation method** / **Default values of parameters**	**ML**	**Robust**	**Best**	**ML**	**Robust**	**Best**
3	$\phi_{11} = 0.8$	0.00191844	0.8367640	ML	0.00050000	0.000074563	RO
	$\phi_{12} = -0.1$	0.00249507	0.0020000	RO	0.00200000	0.000000200	RO
	$\phi_{21} = -0.2$	0.00179517	0.0020000	ML	0.00200000	0.000000200	RO
	$\phi_{22} = 0.2$	0.00212376	0.0020000	RO	0.00533333	0.000000200	RO

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	$\phi_{11} = -0.2$	0.00178199	81.9702000	ML	0.00570565	0.008203560	ML
	$\phi_{12} = 0.6$	0.00273501	1.802000	ML	0.00921596	0.000187700	RO
	$\phi_{21} = -0.6$	0.00238797	1.442000	ML	0.00244678	0.000150200	RO
	$\phi_{22} = 0.2$	0.00215055	8.945280	ML	0.00547731	0.000904132	RO
4	$\phi_{11} = 0.8$	0.00195391	0.0823780	ML	0.00166667	0.000235597	RO
	$\phi_{12} = -0.6$	0.00273353	0.0020000	RO	0.10666770	0.000000200	RO
	$\phi_{21} = 0.5$	0.00172039	0.0020000	ML	0.00666770	0.000000200	RO
	$\phi_{22} = 0.6$	0.00224935	0.0020000	RO	0.10666770	0.000000200	RO
	$\phi_{11} = -0.2$	0.00177402	15.4597000	ML	0.00200000	0.001541920	RO
	$\phi_{12} = 0.1$	0.00272224	1.8020000	ML	0.00533333	0.000187700	RO
	$\phi_{21} = -0.5$	0.00235209	1.4420000	ML	0.00269300	0.000150200	RO
	$\phi_{22} = 0.2$	0.00215510	1.2216700	ML	0.00246200	0.000133549	RO

before polluting					**After polluting**		
lineup number	**estimation method** / **Default values of parameters**	**ML**	**Robust**	**Best**	**ML**	**Robust**	**Best**
5	$\phi_{11} = 0.8$	0.00195735	0.0811064	ML	0.00050000	0.000310016	RO
	$\phi_{12} = -0.6$	0.00236050	0.0020000	RO	0.00020000	0.000000200	RO
	$\phi_{21} = 0.2$	0.00179132	0.0020000	ML	0.00020000	0.000000200	RO
	$\phi_{22} = -0.8$	0.00219714	0.0020000	RO	0.00533330	0.000000200	RO
	$\phi_{11} = -0.8$	0.00172990	49.6713000	ML	0.00213891	0.004970960	ML
	$\phi_{12} = 0.4$	0.00271187	1.802000	ML	0.00239570	0.000187700	RO
	$\phi_{21} = 0.3$	0.00232818	1.442000	ML	0.00183331	0.000150200	RO
	$\phi_{22} = 0.8$	0.00210534	0.236148	ML	0.00226380	0.000018466	RO
6	$\phi_{11} = 0.8$	0.00194614	0.0818381	ML	0.0005000	0.000391503	RO
	$\phi_{12} = 0.2$	0.00253378	0.0020000	RO	0.0020000	0.000000200	RO
	$\phi_{21} = -0.2$	0.00152255	0.0020000	ML	0.0020000	0.000000200	RO
	$\phi_{22} = 0.6$	0.00222693	0.0020000	RO	0.0005333	0.000000200	RO
	$\phi_{11} = -0.6$	0.00180898	1.5479300	ML	0.0017360	0.000155020	RO
	$\phi_{12} = 0.2$	0.00268366	1.8020000	ML	0.0026930	0.000187700	RO
	$\phi_{21} = 0.2$	0.00239610	1.4420000	ML	0.0023168	0.000150200	RO
	$\phi_{22} = 0.6$	0.00216408	0.4944350	ML	0.0024620	0.000057406	RO

before polluting					**After polluting**		
lineup number	**estimation method** / **Default values of parameters**	**ML**	**Robust**	**Best**	**ML**	**Robust**	**Best**
7	$\phi_{11} = 0.6$	0.00182011	0.0277795	ML	0.00022222	0.000000670	RO
	$\phi_{12} = 0.4$	0.00175529	18.2857000	ML	0.00200000	0.001691860	RO
	$\phi_{21} = 0.3$	0.00158952	9.33511000	ML	0.00200000	0.000947070	RO
	$\phi_{22} = 0.2$	0.00156523	0.18340000	ML	0.00085714	0.000054450	RO
	$\phi_{11} = 0.3$	0.00185794	19.47450000	ML	0.00168320	0.002040110	ML
	$\phi_{12} = 0.1$	0.00418934	48.74230000	ML	0.00338600	0.000488670	RO
	$\phi_{21} = 0.2$	0.00276094	48.74230000	ML	0.00279200	0.000488670	RO
	$\phi_{22} = 0.5$	0.00196439	0.246852000	ML	0.00160400	0.000018926	RO
	$\phi_{11} = 0.9$	0.001969780	0.0277795	ML	0.00022222	0.000000067	RO
	$\phi_{12} = 0.1$	0.000704745	18.2857000	ML	0.00200000	0.001691860	RO

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8	$\phi_{21} = 0.2$	0.001634340	9.3351100	ML	0.00200000	0.000947570	RO
	$\phi_{22} = 0.7$	0.001809310	0.1834000	ML	0.00085714	0.000054450	ROM
	$\phi_{11} = 0.5$	0.001753060	19.4745000	ML	0.00168320	0.002040110	L
	$\phi_{12} = 0.2$	0.003484420	48.7423000	ML	0.00338600	0.000488670	RO
	$\phi_{21} = 0.2$	0.003125470	48.7423000	ML	0.00279200	0.000488670	RO
	$\phi_{22} = 0.7$	0.001840750	0.2468520	ML	0.00160400	0.000018926	RO

before polluting					**After polluting**		
lineup number	**estimation method** **Default values of parameters**	**ML**	**Robust**	**Best**	**ML**	**Robust**	**Best**
9	$\phi_{11} = 0.6$	0.00183650	0.07494880	ML	0.00133333	0.00026086	RO
	$\phi_{12} = 0.4$	0.00226427	12.14130010	ML	0.00200000	0.00193030	RO
	$\phi_{21} = -0.3$	0.00226835	18.20250000	ML	0.00200000	0.00178275	RO
	$\phi_{22} = 0.2$	0.00153823	2.67203000	ML	0.00800000	0.000133702	RO
	$\phi_{11} = -0.3$	0.00218242	1.66585000	ML	0.00165187	0.000166585	RO
	$\phi_{12} = 0.1$	0.00440359	8.36564000	ML	0.00484763	0.000836564	RO
	$\phi_{21} = 0.2$	0.00271048	4.18382000	ML	0.00147781	0.000418380	RO
	$\phi_{22} = 0.5$	0.00198422	34.47560000	ML	0.00249695	0.000344756	RO
10	$\phi_{11} = -0.8$	0.00216153	0.4701300	ML	0.00450000	0.000572389	RO
	$\phi_{12} = -0.6$	0.00220332	1.9932900	ML	0.00200000	0.000209329	RO
	$\phi_{21} = -0.5$	0.00218370	5.2971500	ML	0.00200000	0.000679715	RO
	$\phi_{22} = 0.4$	0.00180604	1.0726100	ML	0.00300000	0.000266666	RO
	$\phi_{11} = -0.2$	0.00223749	8.7182500	ML	0.01199520	0.000870834	RO
	$\phi_{12} = 0.1$	0.00441582	1.4358000	ML	0.01001520	0.000161169	RO
	$\phi_{21} = -0.5$	0.00168816	0.2895600	ML	0.00759807	0.000304720	RO
	$\phi_{22} = 0.3$	0.00203284	39.608400	ML	0.00600506	0.004069800	RO

before polluting					**After polluting**		
lineup number	**estimation method** **Default values of parameters**	**ML**	**Robust**	**Best**	**ML**	**Robust**	**Best**
11	$\phi_{11} = -0.8$	0.00200798	0.275581	ML	0.0045000	0.00095766	RO
	$\phi_{12} = 0.6$	0.00188508	0.950183	ML	0.0020000	0.00007830	RO
	$\phi_{21} = 0.6$	0.00190857	1.336290	ML	0.0020000	0.000143004	RO
	$\phi_{22} = -0.8$	0.00209497	6.274660	ML	0.0045000	0.000329157	RO
	$\phi_{11} = -0.8$	0.00217061	0.205902	ML	0.0021980	0.000021845	RO
	$\phi_{12} = 0.6$	0.00249157	1.372900	ML	0.0024620	0.000141910	RO
	$\phi_{21} = 0.6$	0.00225630	1.372900	ML	0.0022640	0.000144191	RO
	$\phi_{22} = -0.8$	0.00209207	8.237870	ML	0.002346	0.000844106	RO
12	$\phi_{11} = -0.8$	0.00206821	0.0507938	ML	0.0045000	0.000300210	RO
	$\phi_{12} = 0.2$	0.00162002	12.0031000	ML	0.0020000	0.001050310	RO
	$\phi_{21} = 0.2$	0.00164455	7.4499100	ML	0.0020000	0.000735616	RO
	$\phi_{22} = -0.8$	0.00211576	0.3514030	ML	0.0045000	0.000017863	RO
	$\phi_{11} = -0.8$	0.00220751	4.7931400	ML	0.0021980	0.000478334	RO
	$\phi_{12} = 0.2$	0.00360058	14.4777000	ML	0.0033860	0.001067310	RO
	$\phi_{21} = 0.2$	0.00293582	14.4777000	ML	0.0027920	0.001607310	RO
	$\phi_{22} = -0.8$	0.00209540	0.3059830	ML	0.0023465	0.000067775	RO

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before polluting					**After polluting**		
lineup number	**estimation method** / **Default values of parameters**	**ML**	**Robust**	**Best**	**ML**	**Robust**	**Best**
13	$\phi_{11} = -0.9$	0.00207900	0.0503912	ML	0.004222220	0.000179170	RO
	$\phi_{12} = 0.1$	0.00103921	6.3309000	ML	0.002000000	0.001099760	RO
	$\phi_{21} = 0.2$	0.00203168	2.7594100	ML	0.002000000	0.000444691	RO
	$\phi_{22} = 0.7$	0.00187348	0.3486920	ML	0.000857143	0.000020147	RO
	$\phi_{11} = 0.8$	0.00179222	0.6476790	ML	0.001802000	0.000070287	RO
	$\phi_{12} = 0.4$	0.00280789	1.7994800	ML	0.002693000	0.000184471	RO
	$\phi_{21} = 0.6$	0.00231321	1.2003200	ML	0.002264000	0.000123048	RO
	$\phi_{22} = -0.5$	0.00219796	4.7930100	ML	0.002554400	0.000482457	RO

In the second experiment, if the sample size was small (n=25), the maximum possible method outperformed by 87.5% for all parameter values before contamination. Also, the robust method outperformed all methods after contamination by 95% and in all methods the criterion of average absolute relative errors was used.

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