Quadripartitioned Neutrosophic Probability Distributions

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Abstract

Quadripartitioned neutrosophic set is an extension of neutrosophic set and n-valued neutrosophic logic for solving real-world issues. In order to demonstrate the validity of the suggested idea, this paper's major goal is to provide several quadripentapartition neutrosophic probability distributions with numerical examples. Neutosophic probability has up till now been obtained from traditional statistical distributions, with less contributions to the statistical distribution's creation. With the help of numerical examples, we introduced the quadripartition neutrosophic binomial distribution, the quadripartitioned Poisson distribution, and the quadripartitioned Poisson distribution as a limiting case of the neutrosophic binomial distribution. We also proposed the quadripartitioned exponential distribution and the quadripartitioned uniform distribution. This paper paves the door for addressing problems that adhere to the classical distributions while still include inaccurately stated data.

Keywords: Neutrosophic probability distributions; neutrosophic binomial distribution; quadripartitioned Poisson probability; quadripartitioned uniform probability; quadripartitioned exponential probability

1. Introduction

The traditional probability distributions are limited to the set of data. The traditional distribution parameters are always provided with a value that has been set. However, neutrosophic probability distributions rely on ambiguity. Indeterminacy is a subject of neutrosophic probability (NP) study; however the two concepts are not the same. The classical and imprecise probabilities are generalised by NP. In the form of neutrosophic probability rules, several conventional probability rules are modified. A specific instance of the neutrosophic measure is NP. It is a prediction of an event's occurrence along with a prediction of potential uncertainty and a prediction that the event will not take place. While NP deals with unfair, flawed such things and processes, classical probability deals with fair dice, coins, roulettes, spinners, decks of cards, and random works. Prof. Florentin Smarandache originally suggested neutrosophic statistics in 1995. It is a novel area of philosophy that has been described as a generalisation of fuzzy logic [1] and intuitionistic fuzzy logic [2]. Neutrosophic set's core ideas, as introduced by Smarandache in [3]. The primary objective of neutrosophic logic is to characterise any logical justification for a given assertion in a three-dimensional neutrosophic space where each dimension stands for truth (Tr), falsehood (Fal), and indeterminacy (Ind), respectively. Different investigations have expanded the traditional distributions that are based on neutrosophic logic, such as the neutrsphic binomial distribution [3] and neutrosophic normal distribution [4], the neutrosophic multinominal distribution [5], the neutrosophic Poisson, neutrosophic exponential and neutrosophic uniform distribution [6], the neutrosophic gamma distribution [7], the neutrosophic Weibull distribution [8] and the neutrosophic beta distributions [9].

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[10] As a limiting instance of the binomial distribution, the neutrosphic poisson distribution was proposed. Its application also defined the neutrosphic uniform distribution and exponential distribution also an appendicitis dataset derived from information extraction based on evolutionary learning, fuzzy and neutrosophic probability were implemented. [12] Added the concepts of classical probability and imprecise probability to the concept of neutrosophic probability in order to represent the state of bosons that do not adhere to Pauli's exclusion principle, Schordinger's cat theory, and Heisenberg's uncertainty principle of a particle's behaviour.

The quadripartitioned single valued neutrosophic set, introduced by Rajashi Chatterjee [23], is a new set defined by the indeterminacy component of neutrosophic sets, which is divided into two parts: contradiction (both true and false) and unknown (neither true nor false). Therefore, we established some probability distribution in the neutrosophic environment in order to concentrate on this type of new quadripartitioned neutrosphic set in this study. The remainder of the essay is structured as follows. Basic definitions have been provided in section 3 for your convenience. Section 4 proposes a quadripartitioned neutrosophic binomial distribution (QNBD) and applies it. Quadripartitioned neutrosophic exponential distribution (QNED) and uniform distribution (QNUD) were defined in sections 6 and 7 with illustrative examples to demonstrate the validity of the proposed concepts, respectively. Section 5 proposed quadripartitioned neutrosophic Poisson distribution (QNPD) as a limiting case of QNBD with a numerical example. The proposed work's conclusion and future course are detailed in section 8.

2. Review of Literature

The authors [1] proposed and defined the neutrosphic probability as a generalisation of the classical and imprecise probabilities, [6] experimented with a few NP distributions to address the problem that they occasionally contain inaccurate information, [11] implemented fuzzy and neutrosphic probability-based categorization knowledge extraction based on evolutionary learning on appendicitis dataset, and [13] established neutrosphic probability-based knowledge extraction. The reliability discounting factors of the real-time evidence based neutrosophic probability were computed by the author [14]. Additionally, the author [15] introduced some pent partitioned nuetroosphic probability distributions. The author [16] introduced a novel similarity measure, the single-valued pent partitioned neutrosophic exponential similarity measure. The quadripartitioned single-valued neutrosophic rough sets were optimised in [17], the quadripartitioned single-valued neutrosophic Pythagorean set was proposed in [18], the quadripartitioned neutrosophic soft topological space was introduced in [19], the quadripartitioned neutrosophic soft topological space was extended in [21], and the single-valued and quadripartitioned single-value In [22], the author introduced the idea of score and accuracy function for quadripartitioned single-valued neutrosophic numbers and specified ranking techniques between them. The author of [24] described the theoretical terms, operations, and attributes of simple bipolar quadripartitioned single valued neutrosophic sets along with examples of such sets. There are quadripartitioned neutrosophic rough sets, soft sets, topological sets, and Pythagorean sets as of the aforementioned research and findings, but there hasn't been any contribution to probability theory to fill this void. In this paper, a quadripartitioned neutrosophic probability distribution has been proposed along with its associated applications.

3. Preliminaries

For a better understanding of the suggested notion, we have described the fundamental ideas of neutrosophic statistical distribution in this part.

Definition 3.1: [3]

"Assume that w be a random variable, which represents the number of success when events performs more than or equal to one times. Then, the corresponding probability distribution x is call a neutrosophic binomial distribution.

(i) Neutrosophic Binomial Random Variable: The number of successes greater than or equal to one times is represented by the random variable x.

(ii) Neutrosophic Binomial Probability Distribution [NBPD] [3]

The neutrosophic binomial probability distribution of x is represented by NBPD.

(iii) Indeterminacy: The outcome of an experiments success or failure is uncertain.

(iv) Indeterminacy Threshold: Outcome of an event are indeterminate form, where $th \in \{0,1,2,......m\}$, m is the sample size. Consider,

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P(S)= The chance of a particular event outcome in the case of success.

P(F)=The chance of a particular event outcome in the case of failure, for both S and F different from indeterminacy.

P(I)= Then chance of a particular event outcome in the case of an indeterminacy.

Let $x \in \{0,1,2,....m\}, NP = \{Tr_x, Ind_x, Fal_x\}$ with"

Tr_x : Chances of 'x' success and the value of (n-x) represents the number of failures and indeterminacy, such that the number of indeterminacy is less than or equal to the indeterminacy threshold. Where, n represents the population size.

Fal_x : Chances of 'y' success, with $y \neq x$ and the value of (n-y) represents the number of failures and indeterminacy, and it is less than the indeterminacy threshold.

Ind_x : Chances of 'z' indeterminacy, where 'z' is strictly greater than the indeterminacy threshold.

$$Tr_x + Ind_x + Fal_x = \left(P(S) + P(I) + P(F)\right)^m$$

"For complete probability we have $P(S) + P(I) + P(F) = 1$;

If the probability was incomplete then we have $0 \leq P(S) + P(I) + P(F) \prec 1$;

For the paraconsistent probability we have $1 \prec P(S) + P(I) + P(F) \leq 3$;

Now

$$T_x = \frac{n!}{x!(n-x)!}.P(S)^x.\sum_{k=0}^{th} C_{n-x}^k P(I)^k P(F)^{n-x-k}$$

$$T_x = \frac{n!}{x!}P(S)^x \sum_{k=0}^{th} \frac{P(I)^k P(F)^{n-x-k}}{k!(n-x-k)!} \qquad \ldots\ldots\ldots\ldots (1)$$

Eqn(1) implies the Truth membership function of Neutrosophic Binomial Distribution.

$$I_x = \sum_{z=th+1}^{n} \frac{n!}{z!(n-z)!}.P(I)^z.\left[\sum_{k=0}^{th} C_{n-z}^k P(S)^z P(F)^{n-z-k}\right]$$

$$I_x = \sum_{z=th+1}^{n} \frac{n!}{z!(n-z)!}.P(I)^z.\left[\sum_{k=0}^{n-z} \frac{(n-z)!}{k!(n-z-k)!} P(S)^k P(F)^{n-z-k}\right]$$

$$I_x = \sum_{z=th+1}^{n} \frac{n!}{z!}.P(I)^z.\left[\sum_{k=0}^{n-z} \frac{P(S)^k P(F)^{n-z-k}}{k!(n-z-k)!}\right] \qquad \ldots\ldots\ldots\ldots (2)$$

Eqn(2) implies the Indeterminacy of Neutrosophic Binomial Distribution.

$$F_x = \sum_{\substack{y=0 \\ y\neq x}}^{n} T_y = \sum_{\substack{y=0 \\ y\neq x}}^{n} \frac{n!}{y!}P(S)^y \left[\sum_{k=0}^{th} \frac{P(I)^k P(F)^{n-y-k}}{k!(n-y-k)!}\right] \qquad \ldots\ldots\ldots\ldots (3)$$

Eqn(3) implies the Falsity of Neutrosophic Binomial Distribution."

Where C_u^v means combinations of 'u' elements taken by groups of v elements: $C_u^v = \frac{u!}{v!(u-v)!}$

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4. Proposed Quadripartitioned Neutrosophic Binomial Distribution (QNBD)

In this section the definition of QNBD is proposed.

Definition 3.1: If an experiment's success rate is greater than or equal to one and the quadripartition neutrosophic random variable 'x' is assumed to be non-negative, it is said to follow the quadripartition neutrosophic binomial distribution.

(i) “Quadripartitioned Neutrosophic Binomial Random Variable: is the random variable x represents the number of success is more than or equal to one time.

(ii) Quadripartitioned Neutrosophic Binomial Probability Distribution: The quadripartitioned neutrosophic probability distribution of x with quadripartitioned neutrosophic probability density function.

(iii) Contradiction: it is a contradiction part of success and failure in which the event results cannot be contained.

(iv)Ignorance: it is an ignorance part of success and failure in which the event results cannot be confined.

(v) C.G. Threshold: represents the number of event whose outcome is imprecise. In this research C.G is $th \in \{0,1,2,......m\}$. Alternatively, C.G. is the number of events whose outcomes belong to contradiction, ignorance events.

Let

P(S)= The Scope of a particular event in which the output will be fully successful.

P(C)= Then Scope of a particular event in which the output will be contradiction.

P(G)=The Scope of a particular event in which the output will be ignored.

P(F)=The Scope of a particular event in which the output will be failure, for both S and F, except the indeterminacy (I).

Assume that $x \in \{0,1,2,......m\}$, where m represents the sample size, $NP = \{Tr_x, C_x, G_x, Fal_x\}$ with

Tr_x : Chances of x success, and (n-x) is the number of failures, contradiction and ignorance such that the events summation of contradiction and ignorance is less than or equal to C.G. Threshold. It is well known that ‘n’ represents the population size.

Fal_x : Chances of z success, with $z \neq x$, and $(m-z)$ is the number of failures and contradiction, which the summation of ignorance events is less than the C.G. threshold.”

C_x : Chances of ‘u’ contradiction, where ‘u’ is strictly greater than C.G. threshold.

G_x : Chances of ‘v’ ignorance, where ‘v’ is strictly greater than C.G. threshold.

$$Tr_x + C_x + G_x + Fal_x = \left(P(S) + P(C) + P(G) + P(F)\right)^m .$$

“For complete probability we have $P(S)+P(C)+P(G)+P(F)=1$;

If the probability was incomplete then we have $0 \le P(S)+P(C)+P(G)+P(F) \prec 1$;

For the paraconsistent probability we have $1 \prec P(S)+P(C)+P(G)+P(F) \le 4$;

$$Tr_x = \frac{n!}{x!(n-x)!}.P(S)^x.\sum_{k=0}^{th} C_{n-x}^{k}\left(P(C)+P(G)\right)^k P(F)^{n-x-k}$$

$$Tr_x = \frac{n!}{x!}P(S)^x \sum_{k=0}^{th} \frac{\left(P(C)+P(G)\right)^k P(F)^{n-x-k}}{k!(n-x-k)!} \quad \text{.....................} (4)$$

Eqn(4) implies the Truth membership function of QNBD

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$$Fal_x = \sum_{\substack{y=0 \\ y\neq x}}^{n} Tr_y = \sum_{\substack{y=0 \\ y\neq x}}^{n} \frac{n!}{y!} P(S)^y \left[\sum_{k=0}^{th} \frac{P(S)^k P(F)^{n-y-k}}{k!(n-y-k)!} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Eqn(5) implies the Falsity membership function of QNBD

$$C_x = \sum_{z=th+1}^{n} \frac{n!}{n!(n-u)!} .P(C)^u . \left[\sum_{k=0}^{th} C_{n-u}^{k} P(S)^u P(F)^{n-u-k} \right]$$

$$C_x = \sum_{z=th+1}^{n} \frac{n!}{n!(n-u)!} .P(C)^u . \left[\sum_{k=0}^{n-u} \frac{(n-u)!}{k!(n-u-k)!} P(S)^k P(F)^{n-u-k} \right]$$

$$C_x = \sum_{z=th+1}^{n} \frac{n!}{u!} .P(C)^u . \left[\sum_{k=0}^{n-u} \frac{P(S)^k P(F)^{n-u-k}}{k!(n-u-k)!} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Eqn(6) implies the Contradiction function of QNBD

$$G_x = \sum_{z=th+1}^{n} \frac{n!}{n!(n-v)!} .P(G)^v . \left[\sum_{k=0}^{th} C_{n-v}^{k} P(S)^v P(F)^{n-v-k} \right]$$

$$G_x = \sum_{z=th+1}^{n} \frac{n!}{n!(n-v)!} .P(G)^v . \left[\sum_{k=0}^{n-v} \frac{(n-v)!}{k!(n-v-k)!} P(S)^k P(F)^{n-v-k} \right]$$

$$G_x = \sum_{z=th+1}^{n} \frac{n!}{v!} .P(G)^v . \left[\sum_{k=0}^{n-v} \frac{P(S)^k P(F)^{n-v-k}}{k!(n-v-k)!} \right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Eqn(7) implies the Ignorance membership function of QNBD."

4.1 Application for QNBD

In this section, we applied the proposed concept to a specific hospital patient who was battling a specific disease.

Problem 4.1. [15]

Eight patients with a particular disease are present in a particular hospital; monitoring of the cases showed that 60% of patients die and 20% make a full recovery, thanks to medication, skilled medical staff, the contradiction of oxygen availability occurring 8% of the time, and ignorance occurring 5% of the time. Here, using QNBD, the likelihood that two of the three randomly chosen patients would recover with C.G. threshold 3 has been calculated.

Solution:

When there are four possible outcomes, one of the most significant probability models is the QNBD model.

Here patients those who have recovered from the disease are considered as a success.

Patients those who have died is considered as a failure.

Availability of oxygen occurs considered into two categories (Contradiction and ignorance)

that is indeterminacy.

Here in given problem

$P(S) = 0.2$; $P(F) = 0.6$; $P(C) = 0.08$; $P(G) = 0.05$;

Parameters are taken into account in the following way:

Number of patients $m = 8$;

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C.G. Threshold $k=3$;

From Eqn (4) truth membership function for NQBD is given by

$$T_x = \frac{n!}{x!}P(S)^x \sum_{k=0}^{th} \frac{\left(P(C)+P(G)\right)^k P(F)^{n-x-k}}{k!\left(n-x-k\right)!}$$

$Tr_x = 0.02111$

From Eqn(6) & Eqn (7) contradiction and ignorance membership function is given as follows

$$C_x = \sum_{z=th+1}^{n} \frac{n!}{u!}.P(C)^u.\left[\sum_{k=0}^{n-u} \frac{P(S)^k P(F)^{n-u-k}}{k!\left(n-u-k\right)!}\right]$$

$C_x = 0.000130$

$$G_x = \sum_{z=th+1}^{n} \frac{n!}{v!}.P(G)^v.\left[\sum_{k=0}^{n-v} \frac{P(S)^k P(F)^{n-v-k}}{k!\left(n-v-k\right)!}\right]$$

$G_x = 0.00172011$

From Eqn(4) falsity membership function for NQBD is given by

$$Fal_x = \sum_{\substack{y=0\\ y\neq x}}^{n} Tr_y = \sum_{\substack{y=0\\ y\neq x}}^{n} \frac{n!}{y!}P(S)^y \left[\sum_{k=0}^{th} \frac{P(S)^k P(F)^{n-y-k}}{k!\left(n-y-k\right)!}\right]$$

$Fal_x = 0.01866928$

5. Proposed Quadripartitioned Neutrosophic Poisson distribution (QNPD) - Limiting case of Quadripartitioned Neutrosophic Binomial Distribution (QNBD)

“In this section, we proposed QNPD as a limiting case of QNBD under single valued neutrosophic environment.

The QNBD is a law of success, indeterminacy and failure and in a series of n independent trials is defined in Equation (4)-(7)

Quadripartitioned Neutrosophic Binomial Distribution under limiting case of quadripartition neutrosophic Poisson distribution when

n is indefinitely large (i.e.,) $n \to \infty$

P(S), P(C), P(G), P(F) is very small when P(C), P(I), P(F), P(S) tends to zero.

$np = \lambda$ (a finite quantity)

By the definition of QNBD truth membership of is given in Equ(4) as follows

$$T_x = \frac{n!}{x!}P(S)^x \sum_{k=0}^{th} \frac{\left(P(C)+P(G)\right)^k P(F)^{n-x-k}}{k!\left(n-x-k\right)!}$$

Using Eqn (4), truth membership function of QNPD can be defined as follows:

$$Tr_x = \lambda(S)^x \sum_{k=0}^{th} \frac{n\left(n-1\right)\left(n-2\right)..........\left(n-k-x-1\right)}{x!k!}\left(\frac{\lambda(F)}{n}\right)^k \left(1-\frac{\left(\lambda(C)+\lambda(G)\right)}{n}\right)^{n-x-k}$$

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$$Tr_x = \lambda(S)^x \sum_{k=0}^{th} n^k \frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\ldots\ldots\left(1-\frac{(k+x-1)}{n}\right)\right]}{x!k!}\left(\frac{\lambda(F)}{n}\right)^k\left(1-\frac{(\lambda(C)+\lambda(G))}{n}\right)^{n-x-k}$$

$$= \lambda(S)^x \sum_{k=0}^{th} \frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\ldots\ldots\left(1-\frac{(k+x-1)}{n}\right)\right]}{x!k!}(\lambda(F))^k\left(1-\frac{(\lambda(C)+\lambda(G))}{n}\right)^{n-x-k}$$

Since $\left[n\to\infty \,\&\, \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-x-k} = e^{-\lambda}\right]$

$$Tr_x = \lambda(S)^x \sum_{k=0}^{th} \frac{e^{-(\lambda(C)+\lambda(G))k}\lambda(F)^k}{k!} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

which is the truth membership of a QNPD.

Using Eqn (6) Contradiction membership function of QNPD can be defined as follows:"

$$C_x = \left(\frac{\lambda(C)+\lambda(G)}{2}\right)^u \sum_{k=0}^{th} \frac{n(n-1)(n-2)\ldots\ldots(n-k-u-1)}{x!k!}\left(\frac{\lambda(S)}{n}\right)^k\left(1-\frac{\lambda(F)}{n}\right)^{n-u-k}$$

Since $\left[n\to\infty \,\&\, \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-u-k} = e^{-\lambda}\right]$

$$C_x = \sum_{z=th+1}^{n}\left(\frac{\lambda(C)+\lambda(G)}{2}\right)^u \sum_{k=0}^{n-z} \frac{e^{-\lambda(F)k}\lambda(S)^k}{k!} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Using Eqn (7) Contradiction membership function of QNPD can be defined as follows:

$$G_x = (\lambda(C)+\lambda(G))^u \sum_{k=0}^{th} \frac{n(n-1)(n-2)\ldots\ldots(n-k-u-1)}{x!k!}\left(\frac{\lambda(S)}{n}\right)^k\left(1-\frac{\lambda(F)}{n}\right)^{n-u-k}$$

Since $\left[n\to\infty \,\&\, \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-u-k} = e^{-\lambda}\right]$

$$G_x = \sum_{z=th+1}^{n}\left(\frac{\lambda(C)+\lambda(G)}{2}\right)^u \sum_{k=0}^{n-v} \frac{e^{-\lambda(F)k}\lambda(S)^k}{k!} \qquad \ldots\ldots\ldots\ldots(10)$$

Using Eqn (5) falsity membership function of QNPD can be defined as follows.

$$Fal_x = \lambda(S)^y \sum_{k=0}^{th} n^k \frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\ldots\ldots\left(1-\frac{(k+y-1)}{n}\right)\right]}{x!k!}\left(\frac{\lambda(F)}{n}\right)^k\left(1-\frac{(\lambda(C)+\lambda(G))}{n}\right)^{n-y-k}$$

Since $\left[n\to\infty \,\&\, \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-y-k} = e^{-\lambda}\right]$

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$$Fal_x = \sum_{\substack{y=0 \\ y\neq x}}^{n} \lambda(S)^y \sum_{k=0}^{th} \frac{e^{-(\lambda(C)+\lambda(G))k} \lambda(F)^k}{k!} \quad \text{.............................(11)}$$

Therefore the QNPD can be defined as

$$Tr_x = \lambda(S)^x \sum_{k=0}^{th} \frac{e^{-(\lambda(C)+\lambda(G))k} \lambda(F)^k}{k!} ; \quad \text{............................(12)}$$

$$C_x = \sum_{z=th+1}^{n} \left(\frac{\lambda(C)+\lambda(G)}{2}\right)^u \sum_{k=0}^{n-z} \frac{e^{-\lambda(F)k} \lambda(S)^k}{k!} \quad \text{............................ (13)}$$

$$G_x = \sum_{z=th+1}^{n} \left(\frac{\lambda(C)+\lambda(G)}{2}\right)^u \sum_{k=0}^{n-v} \frac{e^{-\lambda(F)k} \lambda(S)^k}{k!} \quad \text{.......................... (14)}$$

$$Fal_x = \sum_{\substack{y=0 \\ y\neq x}}^{n} \lambda(S)^y \sum_{k=0}^{th} \frac{e^{-(\lambda(C)+\lambda(G))k} \lambda(F)^k}{k!} \quad \text{.............................. (15)}$$

5.1 Application for QNPD

Proposed QNPD has been applied to a specific toll plaza to find the probability of a certain number of cars passing within a given period of time.

Problem 5.1 [15]

The number of cars passing through the toll plaza at the rate $\lambda_{QN} = [6,8]$ cars per minute. Calculate the probability of a three cars crosses through a particular minute with C.G. threshold 2

Solution:

Here in this problem, we are applying QNPD. Since QNPD are often used to understand independent events that occur at a constant rate with in a given interval of time.

Here the number of cars crossing through a particular minute is considering as QNPD.

Since number of cars arrival is not specified.

Assume ‘x’ be the number of cars passing within minutes.

Car passing through the toll plaza at the rate of 6 per minute is considered as success.

Car passing through the toll plaza at the rate of 8 per minute is considered as a failure.

Car passing through the toll plaza at the rate between [6,8] per minute is the average of $\left(\frac{6+8}{2} = 7\right)$ is considered into two categories (Contradiction and ignorance) that is indeterminacy.

Here in given problem the parameters x=3; P(S) = 6; P(F) =8; P(C) & P(G) = 7; k=2

Therefore Using Eqn (12) we get the result as follows.

$$QNPD(x=3) = Tr_x = \lambda(S)^x \sum_{k=0}^{th} \frac{e^{-(\lambda(C)+\lambda(G))k} \lambda(F)^k}{k!} = 0.08923$$

Using Eqn (13) & (14) we are getting contradiction and ignorance membership function as follows:

$$QNPD(x=3) = C_x = G_x = Ind_x = \lambda(S)^x \sum_{k=0}^{th} \frac{e^{-\left(\frac{\lambda(C)+\lambda(G)}{2}\right)k} \lambda(F)^k}{k!} = 0.005212$$

Using Eqn (15) we get the result as follows.

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$$QNPD(x=3) = Fal_x = \sum_{\substack{y=0 \\ y\neq x}}^{n} \lambda(S)^y \sum_{k=0}^{th} \frac{e^{-(\lambda(C)+\lambda(G))k} \lambda(F)^k}{k!} = 0.02863$$

6. Proposed Quadripartitioned Neutrosophic Exponential Distribution (QNED)

In this part, we presented the density function of QNED for the membership functions of Truth, Contradiction, Ignorance, and Falsity. Additionally suggested were the NED mean, variance, and their distribution function.

Definition 6.1. QNED

A generalisation of the neutrosophic exponential distribution, the quadripartitioned neutrosophic exponential distribution (QNED) can handle any type of data, including non-specific data. We may describe the density function of the QNED as follows:

Table 1: Density function for QNED

Membership Function	Density Function
Truth Membership	$f_{QN}(Tr_x(x)) = \lambda_{QN}(Tr_x(x))e^{-(\lambda_{QN}(Tr_x(x)))x}$
Contradiction	$f_{QN}(C_x(x)) = \lambda_{QN}(C_x(x))e^{-(\lambda_{QN}(C_x(x)))x}$
Ignorance	$f_{QN}(G_x(x)) = \lambda_{QN}(G_x(x))e^{-(\lambda_{QN}(G_x(x)))x}$
Falsity Membership	$f_{QN}(Fal_x(x)) = \lambda_{QN}(Fal_x(x))e^{-(\lambda_{QN}(Fal_x(x)))x}$

Where 'x' is a Neutrosophic random variable

$\lambda_{QN}(Tr_x(x))$ - Truth membership function's distribution parameter.

$\lambda_{QN}(C_x(x))$ - Contradiction membership function's distribution parameter.

$\lambda_{QN}(I_x(x))$ - Ignorance membership function's distribution parameter.

$\lambda_{QN}(Fal_x(x))$ - Falsity membership function's distribution parameter.

Table 2: Mean, Variance and distribution function for QNED

Membership Function	Mean $E(x)$	Variance $Var(x)$	Distribution Function for NED: $N(F(Tr_x, C_x, G_x, Fal_x)) = QNP(X \le x)$
Truth Membership	$E(Tr_x) = \frac{1}{\lambda_{QN}(Tr_x(x))}$	$Var(Tr_x) = \frac{1}{(\lambda_{QN}(Tr_x(x)))^2}$	$\left(1-e^{-(\lambda_{QN}(Tr_x(x)))x}\right)$
Contradiction	$E(C_x) = \frac{1}{\lambda_{QN}(C_x(x))}$	$Var(C_x) = \frac{1}{(\lambda_{QN}(C_x(x)))^2}$	$\left(1-e^{-(\lambda_{QN}(C_x(x)))x}\right)$
Ignorance	$E(G_x) = \frac{1}{\lambda_{QN}(G_x(x))}$	$Var(G_x) = \frac{1}{(\lambda_{QN}(G_x(x)))^2}$	$\left(1-e^{-(\lambda_{QN}(G_x(x)))x}\right)$

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Falsity Membership	$E\left(Fal_x\right)=\dfrac{1}{\lambda_{QN}\left(Fal_x\left(x\right)\right)}$	$Var\left(F_x\right)=\dfrac{1}{\left(\lambda_{Neu}\left(F_x\left(x\right)\right)\right)^2}$	$\left(1-e^{-\left(\lambda_{QN}\left(Fal_x(x)\right)\right)x}\right)$

6.1 Application of QNED

In this section, we applied the proposed concept in booking a auto through a online booking app.

Problem 5.1: [15]

With an average of [0.89,5] minutes, the time needed to cancel a vehicle service in a certain auto online booking app follows an exponential distribution. Let's create a density function to reflect the amount of time needed to end a taxi service, and then, using QNED, determine the likelihood of ending an auto service in under a minute.

Solution: When independent, random events occur at a constant rate acroos time, the interval between them has an exponential distribution. Therefore,the average here is an interval values, Now we try to solve this problem in the proposed concept of QNED with an average of [0.89,5] minutes.

We get the following by using Table 2

$$E\left(Tr_x\right)=\frac{1}{\lambda_{QN}\left(Tr_x\left(x\right)\right)}=[0.89,5]$$

$$\Rightarrow \lambda_{QN}\left(Tr_x\left(x\right)\right)=\frac{1}{[0.89,5]}=\left[0.2,1.124\right]$$

Now, the probability to terminate the auto service in less than one minute:

$$QNP\left(X\leq x\right)=\left(1-e^{-\left(\lambda_{QN}\left(Tr_x(x)\right)\right)x}\right)$$

$$QNP\left(X\leq 1\right)=\left(1-e^{-[0.89,5]1}\right)$$

For $\lambda_{QN}\left(Tr_x(x)\right)=0.89$

$$QNP\left(X\leq 1\right)=\left(1-e^{-0.89}\right)=0.5893$$

For $\lambda_{QN}\left(Tr_x(x)\right)=5$

$$QNP\left(X\leq 1\right)=\left(1-e^{-5}\right)=0.9933$$

As a result, the likelihood that the taxi service will end in under a minute is between [0.5893, 0.9933].

Applying the paradox of the quadripartitioned neutrosophic exponential distribution

$C\in\left[0,0.2\right]$, ignorance $G\in\left[0,0.02\right]$ and $\dfrac{\left[0,0.2\right]+\left[0,0.02\right]}{2}=0.11$; average become $\left[0.8,5\right]$.

So we get

$$\lambda_{QN}\left(C_x\left(x\right)\right)=\lambda_{QN}\left(G_x\left(x\right)\right)=\lambda_{QN}\left(Ind_x\left(x\right)\right)=\frac{1}{[0.8,5]}=\left[0.2,1.25\right]$$

The likelihood that the vehicle service will end in less than a minute is as follows:

$$QNP\left(X\leq x\right)=\left(1-e^{-\left(\lambda_{QN}\left(C_x(x)\right)\right)x}\right)=\left(1-e^{-\left(\lambda_{QN}\left(G_x(x)\right)\right)x}\right)$$

$$QNP\left(X\leq 1\right)=\left(1-e^{-[0.8,5]1}\right)$$

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For $\lambda_{QN}(x) = 0.8$

$$QNP(X \le 1) = \left(1 - e^{-0.8}\right) = 0.551$$

For $\lambda_{QN}(x) = 5$

$$QNP(X \le 1) = \left(1 - e^{-5}\right) = 0.9933$$

As a result, the chance that the taxi service will end in under a minute is between [0.551, 0.9933].

Now, applying the quadripartitioned neutrosophic exponential distribution with falsity

$$E(Fal_x) = \frac{1}{\lambda_{QN}(Fal_x(x))} = [0.79, 4.5]$$

$$\Rightarrow \lambda_{QN}(Fal_x(x)) = \frac{1}{[0.79, 4.5]} = [0.222, 1.26]$$

Now, the probability to terminate the auto service in less than one minute:

$$QNP(X \le x) = \left(1 - e^{-\left(\lambda_{QN}(Fal_x(x))\right)x}\right)$$

$$QNP(X \le 1) = \left(1 - e^{-[0.79, 4.5]1}\right)$$

For $\lambda_{QN}(Tr_x(x)) = 0.79$

$$QNP(X \le 1) = \left(1 - e^{-0.89}\right) = 0.546$$

For $\lambda_{QN}(Tr_x(x)) = 4.5$

$$QNP(X \le 1) = \left(1 - e^{-4.5}\right) = 0.983$$

Therefore, the probability of terminating the taxi service in less than one minute is within the range [0.546, 0.983].

7. Proposed Quadripartitioned Neutrosophic Uniform Distribution (QNUD)

In this part, we suggested the density function of QNUD for the functions of Truth membership, contradiction, ignorance, and Falsity. further suggested QNUD mean and variance

Definition 7.1

The QNUD of a continuous X is a conventional uniform distribution, but the distribution parameters an or b, or both, are uncertain. For instance, 'a' or 'b' or both are sets with two or more elements, and we may broaden the concept to include 'truth membership function, contradiction, ignorance, and falsity' (QNUD). The following is the suggested definition of QNUD:

$$f(Tr_x) = \begin{cases} K, & a(Tr_x) < b(Tr_x) \\ 0 & otherwise \end{cases}$$

Since the total probability always unity

$$\int_{a(Tr_x)}^{b(Tr_x)} f(Tr_x)dx = 1$$

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$$\Rightarrow \int_{a(Tr_x)}^{b(Tr_x)} K dx = 1$$

$$[K]_{a(Tr_x)}^{b(Tr_x)} = 1$$

$$K = \frac{1}{b(Tr_x) - a(Tr_x)}$$

Density function of QNUD for Truth Membership function is given by

$$f_{QN}(Tr_x) = \frac{1}{b(Tr_x) - a(Tr_x)} \quad \text{for} \quad a(Tr_x) < b(Tr_x) \quad \text{.................(15)}$$

Similarly we propose QNUD for Contradiction

$$f_{QN}(C_x) = \frac{1}{b(C_x) - a(C_x)} \quad \text{for} \quad a(C_x) < b(C_x) \quad \text{.......................(16)}$$

Similarly we propose QNUD for Ignorance

$$f_{QN}(G_x) = \frac{1}{b(G_x) - a(G_x)} \quad \text{for} \quad a(G_x) < b(G_x) \quad \text{.......................(17)}$$

Also we propose QNUD for Falsity function

$$f_{QN}(Fal_x) = \frac{1}{b(Fal_x) - a(Fal_x)} \quad \text{for} \quad a(Fal_x) < b(Fal_x) \quad \text{.............(18)}$$

Table 3: Mean, Variance of QNUD

Membership Function	Mean $E(x)$	Variance $Var(x)$
Truth Membership	$E(Tr_x) = \int_{a(Tr_x)}^{b(Tr_x)} f(Tr_x) dx = \frac{b(Tr_x) - a(Tr_x)}{2}$	$Var(Tr_x) = \frac{(b(Tr_x) - a(Tr_x))^2}{12}$
Contradiction	$E(C_x) = \int_{a(T_x)}^{b(T_x)} f(C_x) dx = \frac{b(C_x) - a(C_x)}{2}$	$Var(C_x) = \frac{(b(C_x) - a(C_x))^2}{12}$
Ignorance	$E(G_x) = \int_{a(T_x)}^{b(T_x)} f(G_x) dx = \frac{b(G_x) - a(G_x)}{2}$	$Var(G_x) = \frac{(b(G_x) - a(G_x))^2}{12}$
Falsity Membership	$E(Fal_x) = \int_{a(T_x)}^{b(T_x)} f(Fal_x) dx = \frac{b(Fal_x) - a(Fal_x)}{2}$	$Var(Fal_x) = \frac{(b(Fal_x) - a(Fal_x))^2}{12}$

7.1 Application of QNUD:

The proposed idea has been used in this part to analyze how long passengers had to wait at a specific bus stop for a bus.

Problem 7.1: [15]

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Assume that 'x' is a variable that denotes the amount of time a passenger must wait for a bus, without knowing when the bus will arrive.

(i) The bus will come either in the next five minutes [0,5] or in the next fifteen to twenty minutes [15,20], according to a different passenger.

(ii) the bus arrives after 7 minutes [0,7], or will arrive after 15 to 20 minutes [15,20].

Solution:

A QNUD will be used whenever all outcomes in a sample space are equally likely. Therefore, in application problem we are using QNUD.

First part of problem that is from (i) we know that $b=[0,5]$, $a=[15,20]$;

Second section of the problem that in from (ii) we know that $b=[0,7]$, $a=[15,20]$;

Then the probability density function for truth membership function using Eqn(15) in the interval of [0,5] we get

$$f_{QN}(Tr_x)=\frac{1}{b(Tr_x)-a(Tr_x)}=\frac{1}{[15,20]-[0,3]}=\frac{1}{[12,17]}=[0.058,0.083]$$

Then the probability density function for truth membership function using Eqn(15) in the interval of [0,7] we get

$$f_{QN}(Tr_x)=\frac{1}{b(Tr_x)-a(Tr_x)}=\frac{1}{[15,20]-[0,7]}=\frac{1}{[8,13]}=[0.076,0.125]$$

Then the probability density function for Contradiction and ignorance using Eqn(16) & Eqn(17) in the interval of [0,5] we get

The passenger arrival time is either from now to 3 minutes [0,3] with contradiction $C\in[0,0.2]$, ignorance $G\in[0,0.04]$ or will arrive after 15 to 20 minutes then $b=[0,3]+\frac{0.2+0.02}{2}=[0,3]+[0,0.12]=[0,3.12]$

$$f_{QN}(G_x)=f_{QN}(C_x)=f_{QN}(Ind_x)\frac{1}{[15,20]-[0,3.12]}=\frac{1}{[11.88,20]}=[0.05,0.084]$$

Then the probability density function

$$f_{QN}(C_x)=f_{QN}(G_x)=f_{QN}(Ind_x)=[0.05,0.084]$$

The passenger arrival time is after 7 minutes along with contradiction $C\in[0,0.2]$, ignorance $G\in[0,0.04]$ or will arrive after 15 to 20 minutes then $b=[0,7]+\frac{0.2+0.02}{2}=[0,7]+[0,0.12]=[0,7.12]$

$$f_{QN}(G_x)=f_{QN}(C_x)=f_{QN}(Ind_x)=\frac{1}{[15,20]-[0,7.12]}=\frac{1}{[7.88,20]}=[0.05,0.126]$$

Then the probability density function

$$f_{QN}(C_x)=f_{QN}(G_x)=f_{QN}(Ind_x)=[0.05,0.126]$$

Then the probability density function for falsity membership function using Eqn(18) in the interval of [0,5] we get

$$f_{QN}(Fal_x)=\frac{1}{b(Fal_x)-a(Fal_x)}=\frac{1}{[15,20]-[0,3]}=[0.035,0.076]$$

Then the probability density function for truth membership function using Eqn(18) in the interval of [0,7] we get

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$$f_{QN}\left(Fal_x\right)=\frac{1}{b\left(Fal_x\right)-a\left(Fal_x\right)}=\frac{1}{[15,20]-[0,7]}=[0.038,0.067]$$

8. Conclusion

A quadripartitioned neutrosophic probability has the fundamental advantage that it has been investigated under four logic-valued functions, namely the truth, contradiction, ignorance, and membership functions. While quadripartitioned neutrosophic probability deals with indeterminate data where the data has been presented fewer than two terminologies, classical probability always only deals with determinate data. In order to provide a more precise answer with illustrative examples, we extended the neutrosophic probability distributions in this study by introducing quadripartitioned neutrosophic probability distributions (QNBD, QNPD, QNED, and QNUD). The work that has been described may eventually be expanded using several specialized neutrosophic environment types.

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