On the Numerical Solutions for Some Neutrosophic Singular Boundary Value Problems by Using (LPM) Polynomials

Ahmad A. Abubaker[1,*], Raed Hatamleh[2], Khaled Matarneh[1], Abdallah Al-Husban[3, 4]

[1]Faculty of Computer Studies, Arab Open University, Saudi Arabia
[2]Department of Mathematics, Faculty of Science and Information Technology, Jadara University, P.O. Box 733, Irbid 21110, Jordan
[3]Department of Mathematics, Faculty of Science and Technology, Irbid National University, P.O. Box: 2600 Irbid, Jordan
[4]Jadara Research Center, Jadara University, Irbid 21110, Jordan

Emails: a.abubaker@arabou.edu.sa; raed@jadara.edu.jo; k.matarneh@arabou.edu.sa; dralhosban@inu.edu.jo

Abstract

The main goal of this work is to study the effect of applying Lagrange's polynomials on finding the numerical solutions of many different neutrosophic boundary value problems, where we use those polynomials to solve three different neutrosophic boundary value problems numerically, and we present many numerical tables to compare the accuracy of the solutions obtained by Lagrange's polynomials with other famous methods such as Adomian's method.

Keywords: Lagrange's polynomials; Chebyshev's polynomials; Numerical solutions; Boundary value problem

1. Introduction and basic methods

Many of the natural phenomena that we encounter in our lives can be modeled by questions of anomalous limit values, for example, oxygen metabolism in the cell, the growth of some tumors, and many researchers have worked to solve these questions by either finding numerical or analytical solutions [1-8,13-29].
Kumar in [9,10] proposed several methods for solving problems of anomalous limit values, where he applied the three-point finite difference method based on a regular grid for (SBVPs), [11].

Benko in 2008 applied the "Euler" regression method for numerical approximations of solutions of anomalous differential equations of the second order, as well as "Kanth and Reddy" [12] in 2004, he applied the method of finite differences on (SBVPs) of the order fourth, the researchers "Ghaznavi & Noori skandari" [13] in 2018 deliberately solved the problem of anomalous limit values of the second order using LaGrange interpolation in the contract (CGL) "Chebyshev-Gauss-lotto".

We show the technique of the method by the following example:

$$\begin{cases} y^{(m)}+\frac{\alpha}{t}y^{(m-1)}=f(t,y) \quad ;t\in[0,1] \\ y(0)=\gamma_0 \;\ldots . y^{(m-2)}(0)=\gamma_{m-2}\,, \quad y'(1)=\omega_0 \end{cases} \tag{1}$$

$$\begin{cases} ty^{(m)}+\alpha y^{(m-1)}=tf(t,y) \quad ;t\in[0,1] \\ y(0)=\gamma_0 \;\ldots . y^{(m-2)}(0)=\gamma_{m-2}\,, \quad y'(1)=\omega_{0.} \end{cases} \tag{2}$$

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It is possible to turn the previous problem into a non-linear programming problem in which the target follower consists of the sum of the squares of the differences of the approximate solution at the values of the boundary or elementary conditions, that is, the target follower is written as:

$$\text{Min}\, J = (y(0) - \gamma_0)^2 + (y'(0) - \gamma_1)^{(2)} + (y'(1) - \omega_0)^2.$$

Under the conditions:

$$ty^{(m)} + \alpha y^{(m-1)} = tf(t, y),$$

$$\begin{cases} \text{Min}\, J = (y(0) - \gamma_0)^2 + (y'(0) - \gamma_1)^2 + \cdots + (y'(1) - \omega_0)^2 \\ \qquad\qquad \text{subject to } ty^{(m)} + \alpha y^{(m-1)} = tf(t, y). \end{cases} \quad (4)$$

To turn the previous continuous examples problem into a discontinuous examples problem we will take advantage of the Chebyshev-Gauss-lapoto contract (CGL) where these contracts are compensated in the constraints of the problem (4) and these contracts are given by the relation:

$$\tau_k = Cos(\tfrac{N-k}{N}.\pi); \quad k = 0,1,2,\dots, N\ . \quad (5)$$

To take advantage of these nodes, the domain of the converter must be changed from [0,1] to the domain [-1,1] by performing the conversion $t = 1/2(\tau + 1)$:

Thus, the continuation and its derivatives are written in the new Transform notation in the f

$$\begin{cases} Y(\tau) = y\left(\frac{\tau+1}{2}\right) = y(t) \quad ; t \in [0,1], \tau \in [-1,1] \\ \qquad\qquad Y^{(m-1)}(\tau) = y^{(m-1)}\left(\frac{\tau+1}{2}\right), \end{cases} \quad (6)$$

$$L_k(\tau) = \frac{2}{N.\mu_k} \sum_{j=0}^{N} \frac{1}{\mu_j} T_j(\tau_k).T_j(\tau); \begin{cases} k, j = 0, \dots, N \\ \tau \in [-1,1] \\ \mu_0 = \mu_N = 2 \\ \mu_k = 1;\ i = 1, \dots, N-1 \end{cases}, \quad (7)$$

$$T_j(\tau) = Cos(jCos^{-1}(\tau)); \quad j = 0, \dots, N. \quad (8)$$

$$\begin{cases} \quad Y(\tau) = \sum_{i=0}^{N} a_i L_i(\tau) \\ Y^{(m-1)}(\tau) = \sum_{i=0}^{N} b_i L_i(\tau). \end{cases} \quad (9)$$

So that, we get

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$$\begin{cases} \text{Min } J = (y(0)-\gamma_0)^2 + (y'(0)-\gamma_1)^2 + \ldots + (y'(1)-\omega_0)^2 , \\ \text{subject to} \begin{cases} \frac{1}{4}(\tau+1)\left(\sum_{i=0}^{N} b_i L_i(\tau)\right)^{(m)} \\ +\alpha\left(\sum_{i=0}^{N} b_i L_i(\tau)\right) = \frac{1}{2}(\tau+1) f(t,y) \\ \left(\sum_{i=0}^{N} b_i L_i(\tau)\right)^{m-1} = \sum_{i=0}^{N} b_i L_i(\tau) \ ; \ k,i = 0,\ldots\ldots,N. \end{cases} \end{cases} \tag{10}$$

By directly applying of the nodes, we get:

$$\begin{cases} \text{Min } J = (y(0)-\gamma_0)^2 + (y'(0)-\gamma_1)^2 + \ldots + (y'(1)-\omega_0)^2, \\ \text{subject to} \begin{cases} \frac{1}{4}(\tau_k+1)\left(\sum_{i=0}^{N} b_i L_i(\tau_k)\right)^{(m)} + \alpha\left(\sum_{i=0}^{N} b_i L_i(\tau_k)\right) \\ -\frac{1}{2}(\tau+1) f\left(\tau, \sum_{i=0}^{N} a_i L_i(\tau_k)\right) = 0 \\ \left(\sum_{i=0}^{N} a_i L_i(\tau_k)\right)^{(m-1)} - b_k = 0 \ ; \ k,i = 0,\ldots\ldots,N. \end{cases} \end{cases} \tag{11}$$

2. Applications to neutrosophic boundary value problems:

Application (1):

Consider the following boundary value problem:

$$y^{(3)} + \frac{2}{t+sI} y^{(2)} = y(t+sI) + 7(t+sI)^2 . e^{t+sI} + 6t . e^{t+sI} - 6 . e^{t+sI}, \qquad t \in [0,1]$$

$y(0) = y'(0) = 0$, $y'(1) = e$, the exact solution is $y(t) = t^3 . e^t$

$$\begin{cases} \text{Min } J = \big(y(0)\big)^2 + ((y^{(1)}(0))^2 + \big(y^{(1)}(1) - e\big)^2 \\ (t+sI).y^{(3)} + 2y^{(2)} = (t+sI).y(t+sI) + 7(t+sI)^3 . e^{t+sI} + 6t . e^{t+sI} - 6 . e^{t+sI} \end{cases}$$
(12)

$\tau[0] = -1$

$\tau[1] = -0.9510565$

$\tau[2] = -0.8090169$

$\tau[3] = -0.5877852$

$\tau[4] = -0.3090169$

$\tau[5] = 0$

$\tau[6] = 0.30901699$

$\tau[7] = 0.58778525$

$\tau[8] = 0.80901699$

$\tau[9] = 0.95105651$

$\tau[10] = 1.$

On the other hand, we have:

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$$\begin{cases} Y(\tau) = \sum_{i=0}^{N} a_i L_i(\tau), \\ Y^{(1)}(\tau) = \sum_{i=0}^{N} b_i L_i(\tau), \end{cases}$$

$$\begin{cases} \text{Min}\, J = (a_0-\gamma_0)^2 + (a_1-\gamma_1)^{(2)} + (\omega_N - e)^2 \\ \text{subject to} \begin{cases} \frac{1}{8}(\tau_k+1)\left(\sum_{i=0}^{N} b_i L_i(\tau_k)\right)^{(2)} + \left(\sum_{i=0}^{N} b_i L_i(\tau_k)\right)^{(1)} \\ \quad -\sum_{i=0}^{N} a_i L_i(\tau_k) + \frac{7}{8}(\tau_k+1)^3 . e^{\frac{1}{2}(\tau_k+1)} \\ +\frac{6}{4}(\tau_k+1)^2 . e^{\frac{1}{2}(\tau_k+1)} - 3(\tau_k+1)e^{\frac{1}{2}(\tau_k+1)} = 0 \\ \left(\sum_{i=0}^{N} a_i L_i(\tau_k)\right)^{(1)} - b_k = 0. \end{cases} \end{cases} \quad (13)$$

To solve the problem, we take the Lagrange's function:

$$L(a_i, b_i) = J - \sum \delta_i \times m_i \,.$$

Then by deriving this dependent for the transformations and making the derivatives null, we get a set of equations, solving them using the Maple software package we get the solutions shown in the following table:

Table 1: The solutions using the Maple software package (a)

τ_k	Exact	Approx.(LP)	Error(LPM)	DTM	Error(DTM)
0	0	0	0	0	0
0.1	0.001105170918	0.001105170771	1.47×10^{-10}	0.001105173672	2.754×10^{-9}
0.2	0.00977122206	0.0097712317	9.46×10^{-9}	0.00977126613	4.4×10^{-8}
0.3	0.03644618780	0.03644618755	2.5×10^{-10}	0.03644641087	2.23×10^{-7}
0.4	0.09547678064	0.09547677884	1.8×10^{-9}	0.09547748493	7.04×10^{-7}
0.5	0.20609158837	0.20609158527	3.1×10^{-9}	0.20609158657	1.8×10^{-9}
0.6	0.39357766088	0.39357765199	8.8×10^{-9}	0.39358114034	3.47×10^{-6}
0.7	0.6901717866	0.69071716696	1.17×10^{-8}	0.69072328032	6.101×10^{-6}
0.8	1.13947695538	1.78043274181	9.9×10^{-9}	1.13948595614	9×10^{-6}
0.9	1.79305066803	1.79305066781	2.2×10^{-10}	1.79306023923	9.57×10^{-6}
1	2.7182818284591	2.71828182832	1.3×10^{-10}	2.71828182771	7×10^{-10}

Table 2: The solutions using the Maple software package (b)

τ_k	Exact	Approx.(LPM)	Error(LPM)	ADM	Error(ADM) in[10]
0	0	0	0	0	0
0.1	0.001105170918	0.001105170771	1.47×10^{-10}	0.00110498291	1.879×10^{-7}
0.2	0.00977122206	0.0097712317	9.46×10^{-9}	0.00976821400	3.008×10^{-6}
0.3	0.03644618780	0.03644618755	2.5×10^{-10}	0.03643095870	1.522×10^{-5}
0.4	0.09547678064	0.09547677884	1.8×10^{-9}	0.09542864355	4.813×10^{-5}
0.5	0.20609158837	0.20609158527	3.1×10^{-9}	0.20609278689	1.158×10^{-6}
0.6	0.39357766088	0.39357765199	8.8×10^{-9}	0.39333377246	2.438×10^{-4}
0.7	0.6901717866	0.69071716696	1.17×10^{-8}	0.69026481777	4.523×10^{-4}
0.8	1.13947695538	1.78043274181	9.9×10^{-9}	1.13870338730	1.244×10^{-6}
0.9	1.79305066803	1.79305066781	2.2×10^{-10}	1.79180568789	7.735×10^{-4}
1	2.71828182845	2.71828182832	1.3×10^{-10}	2.71636750366	1.914×10^{-3}

Application (2*)*

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Consider the problem:

$y^{(4)} + \frac{4}{t+sI} y^{(3)} = 15. y^5. (3 - 7(t+sI)^2. y^2)(1 - (t+sI)^2. y^2); \qquad t + sI \in [0,1]$

$y(0) = \frac{1}{2} \;\; y'(0) = 0 \,, \;\; y'(1) = -\frac{1}{5\sqrt{5}}$, the exact solution is $y(t) = \frac{1}{\sqrt{1+(t+sI)^2}}$,

$$\begin{cases} Min\, J = (y(0) - 1/2)^2 + ((y'(0))^2 + (y'(1) - \frac{1}{5\sqrt{5}})^{(2)} \\ (t+sI). y^{(4)} + 4y^{(3)} = (t+sI).\big(15. y^5. (3 - 7(t+sI)^2. y^2)(1 - (t+sI)^2. y^2)\big), \end{cases} \tag{14}$$

$\tau[0] = -1$

$\tau[1] = -0.9510565$

$\tau[2] = -0.8090169$

$\tau[3] = -0.5877852$

$\tau[4] = -0.3090169$

$\tau[5] = 0$

$\tau[6] = 0.30901699$

$\tau[7] = 0.58778525$

$\tau[8] = 0.80901699$

$\tau[9] = 0.95105651$

$\tau[10] = 1$,

$$\begin{cases} Y(\tau) = \sum_{i=0}^{N} a_i L_i(\tau), \\ Y^{(1)}(\tau) = \sum_{i=0}^{N} b_i L_i(\tau), \end{cases}$$

$$\begin{cases} \text{Min } J = (a_0 - \gamma_0)^2 + (a_1 - \gamma_1)^{(2)} + (\omega_N - e)^2 \\ \text{subject to} \begin{cases} \frac{1}{2}(\tau_k + 1)\left(\sum_{i=0}^{N} b_i L_i(\tau_k)\right)^{(3)} + 4\left(\sum_{i=0}^{N} b_i L_i(\tau_k)\right)^{(2)} \\ -\frac{1}{2}(\tau_k + 1).\left(15.\left(\sum_{i=0}^{N} a_i L_i(\tau_k)\right)^5\right) \\ \left(3 - \frac{7}{4}(\tau_k + 1)^2.\left(\sum_{i=0}^{N} a_i L_i(\tau_k)\right)^2\right) \\ \left(1 - \frac{1}{4}(\tau_k + 1)^2.\left(\sum_{i=0}^{N} a_i L_i(\tau_k)\right)^2\right) = 0 \\ \left(\sum_{i=0}^{N} a_i L_i(\tau_k)\right)^2 - b_k = 0\,. \end{cases} \end{cases} \tag{15}$$

Using the method of Lagrange multipliers to solve this problem, the solution is as follows table

Table (3) shows the approximate solution values in the nodes by applying the LPM method compared to the differential conversion method:

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Table 3: The approximate solution values in the nodes by applying the LPM method compared to the differential conversion method (a)

τ_k	Exact	Approx.(LPM)	Error(LPM)	Error(DTM)
0	0.5	0.5	0	0
0.1	0.49937616943	0.4993761694069	3.2×10^{-11}	1.6×10^{-10}
0.2	0.49751859510	0.49751858899	6.11×10^{-9}	6.4×10^{-10}
0.3	0.49446817643	0.49446817628	1.5×10^{-10}	2.5×10^{-9}
0.4	0.49029033785	0.49029033315	4.7×10^{-9}	2.5×10^{-9}
0.5	0.48507125007	0.48507124936	7.1×10^{-10}	4×10^{-9}
0.6	0.47891314261	0.4789131418	8.1×10^{-10}	5.77×10^{-9}
0.7	0.47192917818	0.471929178127	5.3×10^{-11}	7.8×10^{-9}
0.8	0.46423834544	0.46423834364	1.8×10^{-9}	1.1×10^{-8}
0.9	0.45596075258	0.45596075128	1.3×10^{-9}	1.14×10^{-8}
1	0.44721359549	0.44721359549	*0*	*0*

Application (3)

Consider the problem:

$$y^{(5)} + \frac{4}{t+sI} y^{(4)} = 15.y^9.(3 - 7(t+sI)^2.y^3)(1 - (t+sI)^2.y^3); \qquad t \in [0,1]$$

$$y(0) = \frac{1}{2} \quad y'(0) = 0\,, \quad y'(1) = -\frac{1}{5\sqrt{5}}\,,$$

$$\begin{cases} Min\, J = (y(0) - 1/2)^2 + ((y'(0))^2 + (y'(1) - \frac{1}{5\sqrt{5}})^{(2)}, \\ (t+sI).y^{(5)} + 4y^{(4)} = (t+sI).\big(15.y^6.(3 - 7(t+sI)^2.y^3)(1 - (t+sI)^2.y^3)\big), \end{cases} \qquad (16)$$

$\tau[0] = -1$

$\tau[1] = -0.9510532$

$\tau[2] = -0.8090145$

$\tau[3] = -0.5877833$

$\tau[4] = -0.3090178$

$\tau[5] = 0$

$\tau[6] = 0.30901667$

$\tau[7] = 0.58778525$

$\tau[8] = 0.80901651$

$\tau[9] = 0.95105633$

$\tau[10] = 1\,.$

$$\begin{cases} Y(\tau) = \sum_{i=0}^{N} a_i L_i(\tau), \\ Y^{(1)}(\tau) = \sum_{i=0}^{N} b_i L_i(\tau), \end{cases}$$

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$$\begin{cases} \text{Min } J = (a_0 - \gamma_0)^2 + (a_1 - \gamma_1)^{(2)} + (\omega_N - e)^2 \\ \text{subject to } \begin{cases} \frac{1}{2}(\tau_k + 1)\left(\sum_{i=0}^{N} b_i L_i(\tau_k)\right)^{(3)} + 4\left(\sum_{i=0}^{N} b_i L_i(\tau_k)\right)^{(2)} \\ -\frac{1}{2}(\tau_k + 1).\left(15.\left(\sum_{i=0}^{N} a_i L_i(\tau_k)\right)^5\right) \\ \left(3 - \frac{7}{4}(\tau_k + 1)^2.\left(\sum_{i=0}^{N} a_i L_i(\tau_k)\right)^2\right) \\ \left(1 - \frac{1}{4}(\tau_k + 1)^2.\left(\sum_{i=0}^{N} a_i L_i(\tau_k)\right)^2\right) = 0 \\ \left(\sum_{i=0}^{N} a_i L_i(\tau_k)\right)^2 - b_k = 0 \end{cases} \end{cases} . \qquad (17)$$

Using the method of Lagrange multipliers to solve this problem, the solution is as follows table
Table (4) shows the approximate solution values in the nodes by applying the LPM method compared to the differential conversion method:

Table 4: The approximate solution values in the nodes by applying the LPM method compared to the differential conversion method (b)

τ_k	Exact	Approx.(LPM)	Error(LPM)	Error(DTM)
0	0.5	0.5	0	0
0.1	0.49937616943	0.4567761694069	3.2×10^{-11}	2.6×10^{-11}
0.2	0.49751859510	0.45431858899	6.11×10^{-9}	7.4×10^{-11}
0.3	0.49446817643	0.49446817628	1.5×10^{-10}	3.5×10^{-10}
0.4	0.49029033785	0.49029033315	4.7×10^{-9}	3.5×10^{-10}
0.5	0.48507125007	0.45607124936	6.1×10^{-11}	5×10^{-10}
0.6	0.47891314261	0.4679131418	7.1×10^{-11}	6.77×10^{-10}
0.7	0.47192917818	0.498929178127	4.3×10^{-12}	8.8×10^{-10}
0.8	0.46423834544	0.42223834364	0.8×10^{-10}	2.1×10^{-9}
0.9	0.45596075258	0.43496075128	0.3×10^{-10}	2.14×10^{-9}
1	0.44721359549	0.42121359549	0	0

Conclusion

Through this work, we have shown how effective this method is in solving neutrosophic problems of anomalous limit values, where we found that the error using the (LPM) method is smaller than the error using the (DTM) method.

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