Refined Neutrosophic Rings II

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Abstract

This paper is the continuation of the work started in the paper titled "Refined Neutrosophic Rings I". In the present paper, we study refined neutrosophic ideals and refined neutrosophic homomorphisms along their elementary properties. It is shown that if $R = \mathbb{Z}(I_1, I_2)$ is a refined neutrosophic ring of integers and $J = n\mathbb{Z}(I_1, I_2)$ is a refined neutrosophic ideal of R, then $R/J \cong \mathbb{Z}_n(I_1, I_2)$.

Keywords: Neutrosophy, refined neutrosophic ring, refined neutrosophic ideal, refined neutrosophic ring homomorphism.

1 Preliminaries

In this section, we only state some useful definitions, examples and results. For full details about refined neutrosophic rings, the readers should see.[9]

Definition 1.1 ([9]). Let $(R, +, .)$ be any ring. The abstract system $(R(I_1, I_2), +, .)$ is called a refined neutrosophic ring generated by R, I_1, I_2. $(R(I_1, I_2), +, .)$ is called a commutative refined neutrosophic ring if for all $x, y \in R(I_1, I_2)$, we have $xy = yx$. If there exists an element $e = (1, 0, 0) \in R(I_1, I_2)$ such that $ex = xe = x$ for all $x \in R(I_1, I_2)$, then we say that $(R(I_1, I_2), +, .)$ is a refined neutrosophic ring with unity.

Definition 1.2 ([9]). Let $(R(I_1, I_2), +, .)$ be a refined neutrosophic ring and let $n \in \mathbb{Z}^+$.

(i) If for the least positive integer n such that $nx = 0$ for all $x \in R(I_1, I_2)$, we call $(R(I_1, I_2), +, .)$ a refined neutrosophic ring of characteristic n and n is called the characteristic of $(R(I_1, I_2), +, .)$.

(ii) $(R(I_1, I_2), +, .)$ is call a refined neutrosophic ring of characteristic zero if for all $x \in R(I_1, I_2)$, $nx = 0$ is possible only if $n = 0$.

Example 1.3 ([9]). (i) $\mathbb{Z}(I_1, I_2), \mathbb{Q}(I_1, I_2), \mathbb{R}(I_1, I_2), \mathbb{C}(I_1, I_2)$ are commutative refined neutrosophic rings with unity of characteristics zero.

(ii) Let $\mathbb{Z}_2(I_1, I_2) = \{(0, 0, 0), (1, 0, 0), (0, I_1, 0), (0, 0, I_2), (0, I_1, I_2), (1, I_1, 0), (1, 0, I_2), (1, I_1, I_2)\}$. Then $(\mathbb{Z}_2(I_1, I_2), +, .)$ is a commutative refined neutrosophic ring of integers modulo 2 of characteristic 2. Generally for a positive integer $n \geq 2$, $(\mathbb{Z}_n(I_1, I_2), +, .)$ is a finite commutative refined neutrosophic ring of integers modulo n of characteristic n.

Example 1.4 ([9]). Let $M^{\mathbb{R}}_{n\times n}(I_1, I_2) = \left\{ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} : a_{ij} \in \mathbb{R}(I_1, I_2) \right\}$ be a refined neutrosophic set of all $n \times n$ matrix. Then $(M^{\mathbb{R}}_{n\times n}(I_1, I_2), +, .)$ is a non-commutative refined neutrosophic ring under matrix multiplication.

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Theorem 1.5 ([9])**.** *Let $(R(I_1, I_2), +, .)$ be any refined neutrosophic ring. Then $(R(I_1, I_2), +, .)$ is a ring.*

Definition 1.6 ([9])**.** Let $(R(I_1, I_2), +, .)$ be a refined neutrosophic ring and let $J(I_1, I_2)$ be a nonempty subset of $R(I_1, I_2)$. $J(I_1, I_2)$ is called a refined neutrosophic subring of $R(I_1, I_2)$ if $(J(I_1, I_2), +, .)$ is itself a refined neutrosophic ring. It is essential that $J(I_1, I_2)$ contains a proper subset which is a ring. Otherwise, $J(I_1, I_2)$ will be called a pseudo refined neutrosophic subring of $R(I_1, I_2)$.

Example 1.7 ([9])**.** Let $(R(I_1, I_2), +, .) = (\mathbb{Z}(I_1, I_2), +)$ be the refined neutrosophic ring of integers. The set $J(I_1, I_2) = n\mathbb{Z}(I_1, I_2))$ for all positive integer n is a refined neutrosophic subring of $R(I_1, I_2)$.

Example 1.8 ([9])**.** Let $(R(I_1, I_2), +, .) = (\mathbb{Z}_6(I_1, I_2), +)$ be the refined neutrosophic ring of integers modulo 6. The set

$$\begin{aligned} J(I_1, I_2) &= \{(0,0,0), (0, I_1, 0), (0, 0, I_2), (0, I_1, I_2), \\ & (0, 2I_1, 0), (0, 0, 2I_2), (0, 2I_1, 2I_2), \\ & (0, 3I_1, 0), (0, 0, 3I_2), (0, 3I_1, 3I_2), \\ & (0, 4I_1, 0), (0, 0, 4I_2), (0, 4I_1, 4I_2), \\ & (0, 5I_1, 0), (0, 0, 5I_2), (0, 5I_1, 5I_2)\}. \end{aligned}$$

is a refined neutrosophic subring of $R(I_1, I_2)$.

Definition 1.9 ([9])**.** Let R be a non-empty set and let $+$ and . be two binary operations on R such that:

(i) $(R, +)$ is an abelian group.

(ii) $(R, .)$ is a semigroup.

(iii) There exists $x, y, z \in R$ such that

$$x(y + z) = xy + xz, (y + z)x = yx + zx.$$

(iv) R contains elements of the form (x, yI_1, zI_2) with $x, y, z \in \mathbb{R}$ such that $y, z \neq 0$ for at least one value.

Then $(R, +, .)$ is called a pseudo refined neutrosophic ring.

Example 1.10 ([9])**.** Let R be a set given by

$$R = \{(0,0,0), (0, 2I_1, 0), (0, 0, 2I_2), (0, 4I_1, 0), (0, 0, 4I_2), (0, 6I_1, 0), (0, 0, 6I_2)\}.$$

Then $(R, +, .)$ is a pseudo refined neutrosophic ring where $+$ and . are addition and multiplication modulo 8.

Example 1.11 ([9])**.** Let $R(I_1, I_2) = \mathbb{Z}_{12}(I_1, I_2)$ be a refined neutrosophic ring of integers modulo 12 and let T be a subset of $\mathbb{Z}_{12}(I_1, I_2)$ given by

$$\begin{aligned} T &= \{(0,0,0), (0, 2I_1, 0), (0, 0, 2I_2), (0, 4I_1, 0), (0, 0, 4I_2), (0, 4I_1, 0), (0, 0, 4I_2), \\ & (0, 6I_1, 0), (0, 0, 6I_2)(0, 8I_1, 0), (0, 0, 8I_2), (0, 10I_1, 0), (0, 0, 10I_2)\}. \end{aligned}$$

It is clear that $(T, +, .)$ is a pseudo refined neutrosophic ring.

2 Main Results

In this section except if otherwise stated, all refined neutrosophic rings $R(I_1, I_2)$ will be assumed to be commutative refined neutrosophic rings with unity.

Definition 2.1. Let $R(I_1, I_2)$ be a refined neutrosophic ring.

(i) An element $x \in R(I_1, I_2)$ is called an idempotent element if $x^2 = x$.

(ii) A nonzero element $x \in R(I_1, I_2)$ is called a zero divisor if there exists a nonzero element $y \in R(I_1, I_2)$ such that $xy = 0$.

(ii) A nonzero element $x \in R(I_1, I_2)$ is said to be invertible if there exists an element $y \in R(I_1, I_2)$ such that $xy = 1$.

Example 2.2. Consider the refined neutrosophic rings $\mathbb{Z}_2(I_1, I_2)$ and $\mathbb{Z}_3(I_1, I_2)$ of integers modulo 2 and 3 respectively. The element $x = (1, I_1, I_2)$ is idempotent in $\mathbb{Z}_2(I_1, I_2)$ and the element $x = (1, 0, I_2)$ is invertible in $\mathbb{Z}_3(I_1, I_2)$. The elements $x = (0, I_1, 0)$ and $y = (1, I_1, 0)$ are zero divisors in $\mathbb{Z}_2(I_1, I_2)$ because $xy = (0, 0, 0)$.

Definition 2.3. Let $R(I_1, I_2)$ be a refined neutrosophic ring. Then $R(I_1, I_2)$ is called a refined neutrosophic integral domain if it has no zero divisors.

Theorem 2.4. *$\mathbb{Z}_n(I_1, I_2)$ is not a refined neutrosophic integral domain for all n.*

Proof. For nonzero integers α, β, let $x = (0, \alpha I_1, 0)$ and $y = (0, \beta(1 - I_1), 0)$ be arbitrary elements in $\mathbb{Z}_n(I_1, I_2)$. It is clear that x and y are zero divisors since $xy = (0, 0, 0)\ \forall\ \alpha, \beta \in \mathbb{Z}^+$ and therefore, $\mathbb{Z}_n(I_1, I_2)$ is not a refined neutrosophic integral domain for all n. □

Corollary 2.5. *Let $R(I_1, I_2)$ be a refined neutrosophic ring where R is an integral domain. Then $R(I_1, I_2)$ is not necessarily a refined neutrosophic integral domain.*

Theorem 2.6. *If $R = \mathbb{Z}_n$ is a ring of integers modulo n, then $R(I_1, I_2)$ is a finite refined neutrosophic ring of order n^3.*

Definition 2.7. Let F be a field. A refined neutrosophic field is a set $F(I_1, I_2)$ generated by F, I_1, I_2 defined by

$$F(I_1, I_2) = \{(x, yI_1, zI_2) : x, y, z \in F\}.$$

Example 2.8. (i) $\mathbb{Q}(I_1, I_2), \mathbb{R}(I_1, I_2)$ and $\mathbb{C}(I_1, I_2)$ of rational, real and complex numbers are examples of refined neutrosophic fields.

(ii) $\mathbb{Z}_p(I_1, I_2)$ for a prime p is a refined neutrosophic field.

It is worthy of noting that refined neutrosophic fields are not fields in the classical sense since not every element of refined neutrosophic fields is invertible.

Definition 2.9. Let $R(I_1, I_2)$ be a refined neutrosophic ring and let J be a nonempty subset of $R(I_1, I_2)$. Then J is called a refined neutrosophic ideal of $R(I_1, I_2)$ if the following conditions hold:

(i) J is a refined neutrosophic subring of $R(I_1, I_2)$.

(ii) For every $x \in J$ and $r \in R(I_1, I_2)$, we have $xr \in J$.

If J is a pseudo refined neutrosophic subring of $R(I_1, I_2)$, and, for every $x \in J$ and $r \in R(I_1, I_2)$, we have $xr \in J$, then J is called a pseudo refined neutrosophic ideal of $R(I_1, I_2)$.

Example 2.10. In the refined neutrosophic ring $\mathbb{Z}_4(I_1, I_2)$ of integers modulo 4, the set $J = \{(0, 0, 0), (2, 0, 0), (0, 2I_1, 0), (0, 0, 2I_2), (2, 2I_1, 2I_2)\}$ is a refined neutrosophic ideal.

Example 2.11. Consider

$$\begin{aligned}\mathbb{Z}_3(I_1, I_2) = \{&(0,0,0),(1,0,0),(2,0,0),(0,0,I_2),(0,0,2I_2),(0,I_1,0),\\&(0,I_1,I_2),(0,I_1,2I_2),(0,2I_2,0),(0,2I_1,I_1),(0,2I_1,2I_2),\\&(1,0,I_2),(1,0,2I_2),(1,I_1,0),(1,I_1,I_2),(1,I_1,2I_2),(1,2I_2,0),\\&(1,2I_1,I_1),(1,2I_1,2I_2),(2,0,I_2),(2,0,2I_2),(2,I_1,0),\\&(2,I_1,I_2),(2,I_1,2I_2),(2,2I_2,0),(2,2I_1,I_1),(2,2I_1,2I_2)\}\end{aligned}$$

the refined neutrosophic ring of integers modulo 3. The set

$$J = \{(0, 0, 0), (0, I_1, 0), (0, 0, I_2), (0, 2I_1, 0), (0, 0, 2I_2)\}$$

is a pseudo refined neutrosophic ideal. Consider the set

$$K = \{(0, 0, 0), (2, 0, 0), (0, 2I_1, 0), (0, 0, 2I_2), (2, 2I_1, 2I_2)\}.$$

It can easily be shown that K is not a refined neutrosophic ideal of $\mathbb{Z}_3(I_1, I_2)$ and J is the only pseudo refined neutrosophic ideal.

Theorem 2.12. *Let $\{J_k(I_1, I_2)\}_1^n$ be a family of refined neutrosophic ideals (pseudo refined neutrosophic ideals) of a refined neutrosophic ring $R(I_1, I_2)$. Then $\bigcap_1^n J_k(I_1, I_2)\}$ is a refined neutrosophic ideal (pseudo refined neutrosophic ideal) of $R(I_1, I_2)$.*

Definition 2.13. Let $J(I_1, I_2)$ and $K(I_1, I_2)$ be any two refined neutrosophic ideals (pseudo refined neutrosophic ideals) of a refined neutrosophic ring $R(I_1, I_2)$. We define the sum $J(I_1, I_2) \oplus K(I_1, I_2)$ by the set

$$J(I_1, I_2) \oplus K(I_1, I_2) = \{x + y : x \in J(I_1, I_2), y \in K(I_1, I_2)\}$$

which can easily be shown to be a refined neutrosophic ideal (pseudo refined neutrosophic ideal) of $R(I_1, I_2)$

Theorem 2.14. *Let $J(I_1, I_2)$ be any refined neutrosophic ideal of a refined neutrosophic ring $R(I_1, I_2)$ and let $K(I_1, I_2)$ be any pseudo refined neutrosophic ideal of $R(I_1, I_2)$. Then:*

(i) $J(I_1, I_2) \oplus J(I_1, I_2) = J(I_1, I_2)$.

(ii) $K(I_1, I_2) \oplus K(I_1, I_2) = K(I_1, I_2)$.

(iii) $J(I_1, I_2) \oplus K(I_1, I_2)$ *is a pseudo refined neutrosophic ideal of* $R(I_1, I_2)$.

(iv) $x + J = J \; \forall \; x \in J$.

Definition 2.15. Let J be a refined neutrosophic ideal of the refined neutrosophic ring $R(I_1, I_2)$. The set $R(I_1, I_2)/J$ is defined by

$$R(I_1, I_2)/J = \{r + J : r \in R(I_1, I_2)\}.$$

If $\bar{x} = r_1 + J$ and $\bar{y} = r_2 + J$ are two arbitrary elements of $R(I_1, I_2)/J$ and $\oplus, \odot$ are two binary operations on $R(I_1, I_2)/J$ defined by

$$\begin{aligned} \bar{x} \oplus \bar{y} &= (x + y) + J, \\ \bar{x} \odot \bar{y} &= (xy) + J. \end{aligned}$$

It can be shown that $(R(I_1, I_2)/J, \oplus, \odot)$ is a refined neutrosophic ring with the additive identity J. $(R(I_1, I_2)/J, \oplus, \odot)$ is called a refined quotient neutrosophic ring.

Example 2.16. (i) Let $R = \mathbb{Z}(I_1, I_2)$ be a refined neutrosophic ring of integers and let $J = 2\mathbb{Z}(I_1, I_2)$. It is clear that J is a refined neutrosophic ideal of R. Now, R/J is obtained as follows:

$$\begin{aligned} R/J = &\{J, (1, 0, 0) + J, (0, I_1, 0) + J, (0, 0, I_2) + J, (0, I_1, I_2) + J, \\ &(1, I_1, 0) + J, (1, 0, I_2) + J, (1, I_1, I_2) + J\} \end{aligned}$$

which is a refined neutrosophic ring of order 8.

(ii) Let $S = \mathbb{Z}(I_1, I_2)$ be a refined neutrosophic ring of integers and let $K = 3\mathbb{Z}(I_1, I_2)$. It is also clear that K is a refined neutrosophic ideal of S. Now, S/K is obtained as follows:

$$\begin{aligned} S/K = &\{K, (1, 0, 0) + K, (2, 0, 0) + K, (0, I_1, 0) + K, (0, 2I_1, 0) + K, (0, 0, I_2) + K, \\ &(0, 0, 2I_2) + K, (0, 2I_1, I_2) + K, (0, 2I_1, 2I_2) + K, (0, I_1, I_2) + K, (0, I_1, 2_2) + K, \\ &(1, 0, I_2) + K, (1, I_1, 0) + K, (1, I_1, I_2) + K, (1, 2I_1, 0) + K, (1, 0, 2I_2) + K, \\ &(1, 2I_1, 2I_2) + K, (1, 2I_1, I_2) + K, (1, I_1, 2_2) + K, (2, 0, I_2) + K, \\ &(2, 0, 2I_2) + K, (2, I_1, 0) + K, (2, I_1, I_2) + K, (2, I_1, 2I_2) + K, (2, 2I_1, 0) + K, \\ &(2, 2I_1, I_2) + K, (2, 2I_2, 2I_2) + K\} \end{aligned}$$

which is a refined neutrosophic ring of order 27.

These two examples lead to the following general result:

Theorem 2.17. *Let $R = \mathbb{Z}(I_1, I_2)$ be a refined neutrosophic ring of integers and let $J = n\mathbb{Z}(I_1, I_2)$ be a refined neutrosophic ideal of R. Then*

$$R/J \cong \mathbb{Z}_n(I_1, I_2).$$

Definition 2.18. Let $(R(I_1, I_2), +, .)$ and and $(S(I_1, I_2), +, .)$ be two refined neutrosophic rings. The mapping $\phi : (R(I_1, I_2), +, .) \rightarrow (S(I_1, I_2), +, .)$ is called a refined neutrosophic ring homomorphism if the following conditions hold:

(i) $\phi(x+y) = \phi(x) + \phi(y)$.

(ii) $\phi(x.y) = \phi(x).\phi(y)$.

(iii) $\phi(I_k) = I_k \ \forall x, y \in R(I_1, I_2)$ and $k = 1, 2$.

The image of ϕ denoted by $Im\phi$ is defined by the set

$$Im\phi = \{y \in S(I_1, I_2) : y = \phi(x) \text{ for some } x \in R(I_1, I_2)\}.$$

The kernel of ϕ denoted by $Ker\phi$ is defined by the set

$$Ker\phi = \{x \in R(I_1, I_2) : \phi(x) = (0, 0, 0)\}.$$

Epimorphism, monomorphism, isomorphism, endomorphism and automorphism of ϕ are similarly defined as in the classical cases.

Example 2.19. Let $R_1(I_1, I_2)$ and $R_2(I_1, I_2)$ be two refined neutrosophic rings. Let $\phi : R_1(I_1, I_2) \times R_2(I_1, I_2) \to R_1(I_1, I_2)$ be a mapping defined by $\phi(x, y) = x$ and let $\psi : R_1(I_1, I_2) \times R_2(I_1, I_2) \to R_2(I_1, I_2)$ be a mapping defined by $\psi(x, y) = y$ for all $(x, y) \in R_1(I_1, I_2) \times R_2(I_1, I_2)$. Then ϕ and ψ are refined neutrosophic ring homomorphisms.

Example 2.20. Let $\phi : \mathbb{Z}_2(I_1, I_2) \times \mathbb{Z}_2(I_1, I_2) \to \mathbb{Z}_2(I_1, I_2)$ be a refined neutrosophic ring homomorphism defined by $\phi(x, y) = x$ for all $x, y \in \mathbb{Z}_2(I_1, I_2)$. Then

(i)

$$\begin{aligned} Im\phi &= \{(0,0,0), (1,0,0), (0,I_1,0), (0,0,I_2), \\ &\quad (0,I_1,I_2), (1,I_1,0), (1,0,I_2), (1,I_1,I_2)\} \end{aligned}$$

which is a refined neutrosophic subring.

(ii) Also,

$$\begin{aligned} Ker\phi &= \{((0,0,0),(0,0,0)), ((0,0,0),(1,0,0)), ((0,0,0),(0,I_1,0)), \\ &\quad ((0,0,0),(0,I_1,I_2)), ((0,0,0),(0,0,I_2)), ((0,0,0),(1,I_1,0)), \\ &\quad ((0,0,0),(1,0,I_2)), ((0,0,0),(1,I_1,I_2))\} \end{aligned}$$

which is just a subring, not a refined neutrosophic subring and equally not a refined neutrosophic ideal.

This example leads to the following general results:

Theorem 2.21. *Let $\phi : R_1(I_1, I_2) \to R_2(I_1, I_2)$ be a refined neutrosophic ring homomorphism. Then*

(i) $Im\phi$ is a refined neutrosophic subring $R_2(I_1, I_2)$.

(ii) $Ker\phi$ is a subring of R_1.

(iii) $Ker\phi$ is not a refined neutrosophic subring of R_1.

(iv) $Ker\phi$ is not a refined neutrosophic ideal of R_1.

Theorem 2.22. *Let $R = R(I_1, I_2)$ be a refined neutrosophic rings and let $J = J(I_1, I_2)$ be a refined neutrosophic ideal. Then the mapping $\phi : R \to R/J$ defined by $\phi(r) = r + J \ \forall r \in R$ is not a refined neutrosophic ring homomorphism.*

Proof. It is clear that $\phi(r + s) = (r + s) + J = (r + J) + (s + J) = \phi(r) + \phi(s)$ and $\phi(rs) = (rs) + J = (r + J)(s + J) = \phi(r)\phi(s)$. But then, $\phi(I_k) \neq I_k$ for $k = 1, 2$ and so, ϕ is not a refined neutrosophic ring homomorphism. □

This is different from what is obtainable in the classical rings and consequently, classical isomorphism theorems cannot hold in refined neutrosophic rings.

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