On a Two-Fold Algebra Based on the Standard Fuzzy Number Theoretical System

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Abstract

This paper is dedicated to studying the algebraic structure generated from the fusion of two-fold algebras with the standard fuzzy number theoretical system, where the novel fuzzy algebraic structure will be defined with a well-defined binary operation, and then its substructures will be studied concerning the corresponding operation such as hyper/under ideals, and two-fold fuzzy homomorphisms. Also, many examples will be illustrated to clarify the validity of our work.

Keywords: two-fold algebra; fuzzy number theoretical system; two-fold fuzzy algebra; two-fold fuzzy homomorphism.

1. Introduction and preliminaries

The concept of two-fold algebra is considered one of the modern concepts in logical algebraic structures, as it was first introduced in 2024 by the American mathematician and philosopher Smarandache [7].
The concept of two-fold algebra is mainly based on combining a classical algebraic structure and fuzzy or neutrosophic coefficients symbolizing different degrees of belonging or truth [7].
Fuzzy algebraic structures have been studied throughout history by many researchers, for example, the fuzzy group [1-3], the normal fuzzy groups [4], and even intuitionistic fuzzy groups [5-6].
In [8], Abobala formulated the theory of numbers through fuzzy logic, where he introduced concepts related to congruencies, Diophantine equations, and even division using fuzzy functions, which he called the fuzzy number theoretical systems, where many diverse fuzzy number theoretical systems were defined, the most important of which is the standard system.
In this paper, we take advantage of the standard fuzzy number theoretical system by combining it with two-fold algebra, in order to obtain a new fuzzy algebraic structure, we called the two-fold fuzzy number theoretical algebra.
This algebraic structure has many interesting substructures associated with it, of which we mention for example two-fold homomorphisms, two-fold hyper ideals, and two-fold under ideals.
We now recall some important definitions for us in this study.

Definition: [7]

Let U be a universe of disclosure and a non-empty neutrosophic set A ⊂ U,
$A(T, I, F) = \{x(TA(x), IA(x), FA(x)), (TA(x), IA(x), FA(x)) \in [0,1]3, x \in U\}$. Where $TA(x), IA(x), FA(x)$ are degrees of truth-membership, indeterminacy-membership and falsehood-membership of the generic element x with respect to the set A.

Definition: [8]

Let $\mathbb{Z}$ be the ring of integers, $\mu: \mathbb{Z} \to]0,1]$ be a membership function, we say that $(\mathbb{Z}, \mu)$ is a fuzzy number
theoretical system.

Definition: [8]

Consider the following membership function: $\mu: \mathbb{Z} \to]0,1]$ such that:
$(x) = \left\{\frac{1}{|x|}\right.$ if x is not zero, and (0) =1.
Then $(\mathbb{Z}, \mu)$ is called the standard fuzzy number theoretical system (SFNTS)

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2. Main Concepts and Discussion

Definition:

Let $\mu: \mathbb{Z} \to]0,1]$; $\mu(x) = \begin{cases} \frac{1}{|x|} ; & x \neq 0 \\ 1 ; & x = 0 \end{cases}$, we define the two-fold algebra of standard fuzzy number theoretical system $\Delta_s = \{x_{\mu(t)};\ t, x \in \mathbb{Z}\}$ with the following binary operation:

$*$: $\Delta_s \times \Delta_s \to \Delta_s$ such that:

$x_{\mu(t)} * y_{\mu(s)} = (x + y)_{\mu(t)\mu(s)};\ x, y \in \mathbb{Z}.$

Example:

Consider $= 3$, $y = -2$, then $x_{\mu(x)} * y_{\mu(y)} = (3 - 2)_{\mu(3).\mu(-2)} = 1_{\frac{1}{6}}$

Consider $x = 5$, $y = 0$, $t = 2$, $s = 1$, then:

$x_{\mu(t)} * y_{\mu(s)} = (5 + 0)_{\mu(2)\mu(1)} = 5_{\frac{1}{2}}$

Theorem:

Let Δ_s be the two-fold algebra of the standard fuzzy number theoretical system, then:

1] * is a well-defined operation.

2] * is commutative.

3] * is associative.

4] * has an identity

5] * is anti- inverse operation.

Proof:

1] Let $x_{\mu(t)} = x'_{\mu(t')}$, $y_{\mu(s)} = y'_{\mu(s')}$, then:

$x_{\mu(t)} * y_{\mu(s)} = (x + y)_{\mu(t)\mu(s)} = (x + y)_{\frac{1}{|ts|}}$.

$x'_{\mu(t')} * y'_{\mu(s')} = (x' + y')_{\mu(t')\mu(s')} = (x' + y')_{\frac{1}{|t's'|}}$, hence:

$x_{\mu(t)} * y_{\mu(s)} = x'_{\mu(t')} * y'_{\mu(s')}$.

2] $x_{\mu(t)} * y_{\mu(s)} = (x + y)_{\frac{1}{|ts|}} = (y + x)_{\frac{1}{|st|}} = y_{\mu(s)} * x_{\mu(t)}$.

3] $x_{\mu(t)} * (y_{\mu(s)} * z_{\mu(l)}) = x_{\mu(t)} * (y + z)_{\frac{1}{|sl|}} = (x + y + z)_{\frac{1}{|tsl|}} = (x + y)_{\frac{1}{|tsl|}} * z_{\frac{1}{|l|}} = (x_{\mu(t)} * y_{\mu(s)}) * z_{\mu(l)}$.

4] Consider $h = O_{\mu(o)} = O_1$, then:

$x_{\mu(t)} * O_{\mu(o)} = (x + o)_{\frac{1}{|t|}} = x_{\frac{1}{|t|}} = x_{\mu(t)}$, hence $h = O_{\mu(o)}$ is an identity.

5] If $y_{\mu(s)}$ is the inverse of $x_{\mu(t)}$, then:

$x_{\mu(t)} * y_{\mu(s)} = O_{\mu(o)}$, hence $(x + y)_{\frac{1}{|ts|}} = O_1$, thus y=-x and $|s| = \frac{1}{|t|} \notin]0,1]$, thus * is anti-inverse operation.

Remark:

We denote the two-fold algebra of the standard fuzzy number theoretical system by (TFA_S).

Definition:

Let P be an ideal of the ring $\mathbb{Z}$, we define the corresponding hyper-ideal of Δ_s as follows:

$\Delta^{(P)} = \{x_{\mu(t)};\ x \in P , t \in \mathbb{Z}\}$.

The corresponding under-ideal of Δ_s is defined as follows:

$\Delta_{(P)} = \{x_{\mu(t)};\ t \in P , x \in \mathbb{Z}\}$.

Theorem:

Let P be an ideal of $\mathbb{Z}$, then:

1] $\Delta^{(P)}$ is closed with respect to *.

2] $\Delta_{(P)}$ is closed with respect to *.

3] $O_{\mu(s)} \in \Delta^{(P)}$ for all $s \in \mathbb{Z}$

Proof:

1] Let $x_{\mu(t)}, y_{\mu(s)} \in \Delta^{(P)}$, then $(x, y) \in P$, $(s, t) \in \mathbb{Z}$, thus:

$x_{\mu(t)} * y_{\mu(s)} = (x + y)_{\frac{1}{|ts|}} \in \Delta^{(P)}$, that is because $x + y \in P$

2] Let $x_{\mu(t)}, y_{\mu(s)} \in \Delta_{(P)}$, then $(x, y) \in \mathbb{Z}$, $(t, s) \in P$, hence:

$x_{\mu(t)} * y_{\mu(s)} = (x + y)_{\frac{1}{|ts|}} = (x + y)_{\mu(ts)}$; $x + y \in \mathbb{Z}$, $ts \in P$, hence

$x_{\mu(t)} * y_{\mu(s)} \in \Delta_{(P)}$.

3] It holds directly from the definition.

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Theorem:
Let P,Q be two ideals of $\mathbb{Z}$, then:
1] If $P \subseteq Q$, we get $\Delta^{(P)} \subseteq \Delta^{(Q)}$
2] If $P \subseteq Q$, we get $\Delta_{(P)} \subseteq \Delta_{(Q)}$.
3] $\Delta^{(P)} \cap \Delta^{(Q)} = \Delta^{(P\cap Q)}$
4] $\Delta_{(P)} \cap \Delta_{(Q)} = \Delta_{(P\cap Q)}$
Proof:
1] Let $x_{\mu(t)} \in \Delta^{(P)}$, then $x \in P \subseteq Q$, and $t \in \mathbb{Z}$, thus
$x_{\mu(t)} \in \Delta^{(Q)}$ and $\Delta^{(P)} \subseteq \Delta^{(Q)}$.
2] It can be proved by a similar argument.
3] Let $x_{\mu(t)} \in \Delta^{(P\cap Q)}$, then $x \in P \cap Q$ and $t \in \mathbb{Z}$, thus:
$x_{\mu(t)} \in \Delta^{(P)} \cap \Delta^{(Q)}$.
Conversely, if $x_{\mu(t)} \in \Delta^{(P)} \cap \Delta^{(Q)}$, then $x \in P$, $x \in Q$, $t \in \mathbb{Z}$, hence $x_{\mu(t)} \in \Delta^{(P\cap Q)}$.
4] It can be proved by a similar argument.
Definition:
Let P,Q be two ideals of $\mathbb{Z}$, then:
$\Delta^{(P)}_{(Q)} = \{x_{\mu(t)};\ x \in P , t \in Q\}$ is called a two fold hyper-under ideal (HU-ideal) of Δ_s.
If P=Q, then it is called a regular HU-ideal or (RHU-ideal).
Theorem:
Let P, Q, R, S be four ideals of $\mathbb{Z}$, then:
$\Delta^{(P)}_{(Q)} \cap \Delta^{(R)}_{(S)} = \Delta^{(P\cap R)}_{(Q\cap S)}$.
Proof:
Let $x_{\mu(t)} \in \Delta^{(P\cap R)}_{(Q\cap S)}$, then $x \in P \cap R$, $t \in Q \cap S$, thus:
$x \in \Delta^{(P)}_{(Q)} \cap \Delta^{(R)}_{(S)}$.
Conversely, if $x_{\mu(t)} \in \Delta^{(P)}_{(Q)} \cap \Delta^{(R)}_{(S)}$, then $x \in P$, $x \in R$, $t \in Q$, $t \in S$, so that $x_{\mu(t)} \in \Delta^{(P\cap R)}_{(Q\cap S)}$.
Theorem:
Let P,Q be two ideals of $\mathbb{Z}$, then:
$\Delta^{(P)} \cap \Delta_{(Q)} = \Delta^{(P)}_{(Q)}$.
Proof:
Let $x_{\mu(t)} \in \Delta^{(P)} \cap \Delta_{(Q)}$, then $\begin{cases} x_{\mu(t)} \in \Delta^{(P)} \\ x_{\mu(t)} \in \Delta_{(Q)} \end{cases}$
So that, $x \in P$, $t \in Q$ and $x_{\mu(t)} \in \Delta^{(P)}_{(Q)}$.
Conversely, if $x_{\mu(t)} \in \Delta^{(P)}_{(Q)}$, then $x \in P$, $t \in Q$ and
$\begin{cases} x_{\mu(t)} \in \Delta^{(P)} \\ x_{\mu(t)} \in \Delta_{(Q)} \end{cases}$, hence $x_{\mu(t)} \in \Delta^{(P)} \cap \Delta_{(Q)}$.
Definition:
Let w_s be a non empty subset Δ_S, then $(w_s,*)$ is called a two-fold subalgebra if and if w_s is closed under $*$.
Definition:
Let P,Q be two ideals of $\mathbb{Z}$, we define:
1] $\Delta^{(P)} \times \Delta^{(Q)} = \{(x_{\mu(t)}, y_{\mu(s)});\ x \in P, y \in Q, (t,s) \in \mathbb{Z}\}$.
2] $\Delta_{(P)} \times \Delta_{(Q)} = \{(x_{\mu(t)}, y_{\mu(s)});\ (x,y) \in \mathbb{Z}, t \in P,\ s \in Q\}$
3] $\Delta^{(P)} \times \Delta_{(Q)} = \{(x_{\mu(t)}, y_{\mu(s)});\ x \in P, t \in \mathbb{Z}, s \in Q, y \in \mathbb{Z}\}$
Definition:
Let $f: \Delta_s \rightarrow \Delta_s$ be a mapping, we say that (f) is a two-fold algebra homomorphism if:
1] $f(x_{\mu(t)} * y_{\mu(s)}) = f(x_{\mu(t)}) * f(y_{\mu(s)})$.
1] If (f) is a bijection, then it is called a two-fold algebra isomorphism.
Definition:
Let $g , h: \mathbb{Z} \rightarrow \mathbb{Z}$ such that:
$\begin{cases} g(x+y) = g(x) + g(y) \\ h(x.y) = h(x)h(y) \end{cases}$
We define the regular two-fold algebra homomorphism $f_{(g,h)}$ as follows:
$f_{(g,h)}: : \Delta_s \rightarrow \Delta_s$ with: $f_{(g,h)}(x_{\mu(t)}) = g(x_{\mu(h(t))})$.

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It is clear that $f_{(g,h)}\left(x_{\mu(t)} * y_{\mu(s)}\right) = f_{(g,h)}\left[(x+y)_{\frac{1}{|ts|}}\right] = g(x+y)_{\frac{1}{h(|ts|)}} = \left(g(x)+g(y)\right)_{\frac{1}{h(|t|)h(|s|)}} =$
$\left(g(x)\right)_{\frac{1}{h(|t|)}} * \left(g(y)\right)_{\frac{1}{h(|s|)}} = f_{(g,h)}\left(x_{\mu(t)}\right) * f_{(g,h)}\left(y_{\mu(s)}\right)$

Theorem:

Let $f_{(g,g)}$ be a regular two-fold algebra homomorphism, then:

1] If $\Delta^{(P)}$ is a hyper ideal, then $f_{(g,g)}\left(\Delta^{(P)}\right) \subseteq \Delta^{(g(P))}$

2] If $\Delta_{(P)}$ is an under ideal of Δ_S, then $f_{(g,g)}\left(\Delta_{(P)}\right) \subseteq \Delta_{(g(P))}$

3] If $\Delta_{(Q)}^{(P)}$ is a hyper-under ideal of Δ_S, then:

$f_{(g,g)}\left(\Delta_{(Q)}^{(P)}\right) \subseteq \Delta_{(g(Q))}^{(g(P))}$ is a hyper-under ideal of Δ_S.

Proof:

1] First, we must prove that $f_{(g,g)}\left(\Delta^{(P)}\right) \subseteq \Delta^{(g(P))}$

Let $y_{\mu(s)} \in f_{(g,g)}\left(\Delta^{(P)}\right)$, then there exists $x_{\mu(t)} \in \Delta^{(P)}$ such that $f_{(g,g)}\left(x_{\mu(t)}\right) = g(x)_{g(\frac{1}{|t|})} \in \Delta^{(g(P))}$, thus $f_{(g,g)}\left(\Delta^{(P)}\right) \subseteq \Delta^{g(P)}$

2] Let $x_{\mu(t)} \in f_{(g,g)}\left(\Delta_{(P)}\right)$, then there exists $y_{\mu(s)} \in \Delta_{(P)}$ such that: $x_{\mu(t)} = f_{(g,g)}\left(y_{\mu(s)}\right) = [g(y)]_{g(\frac{1}{|s|})}$, thus:

$x = g(y), t = g(|s|)$, hence $x_{\mu(t)} \in \Delta_{g(P)}$ and $f_{(g,g)}\left(\Delta_{(P)}\right) \subseteq \Delta_{g(P)}$

3] It holds directly from (1) and (2).

Example:

Consider $=< 2 >$, $Q =< 3 >$, then:

$\Delta^{(P)} = \{(2x)_{\mu(t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$,

$\Delta^{(Q)} = \{(3x)_{\mu(t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$,

$\Delta^{(P)} \cap \Delta^{(Q)} = \Delta^{(P\cap Q)} = \{(6x)_{\mu(t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$.

$\Delta_{(P)} = \{(x)_{\mu(2t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$,

$\Delta_{(Q)} = \{(x)_{\mu(3t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$,

$\Delta_{(P)} \cap \Delta_{(Q)} = \Delta_{(P\cap Q)} = \{(x)_{\mu(6t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$.

$\Delta_{(Q)}^{(P)} = \{(2x)_{\mu(3t)},\ x \in \mathbb{Z},\ t \in \mathbb{Z}\}$,

$\Delta_{(P)} \times \Delta_{(Q)} = \left((x)_{\mu(2t)}, (y)_{\mu(3s)}\right); x, y, t, s \in \mathbb{Z}$,

$\Delta^{(P)} \times \Delta^{(Q)} = \left((2x)_{\mu(t)}, (3y)_{\mu(s)}\right); x, y, t, s \in \mathbb{Z}$,

$\Delta^{(P)} \times \Delta_{(Q)} = \left((2x)_{\mu(t)}, (y)_{\mu(3s)}\right); x, y, t, s \in \mathbb{Z}$,

Example:

Consider:$g, h: \mathbb{Z} \to \mathbb{Z}$ such that:

$g(x) = 5x$, $h(y) = y^2$, we have:

$$\begin{cases} g(x+y) = g(x) + g(y) \\ \quad h(xy) = h(x)\, h(y) \end{cases}$$

Define $f_{(g,h)}: \Delta_s \to \Delta_s$ such that:

$f_{(g,h)}\left(x_{\mu(t)}\right) = \left(g(x)_{\mu(h(t))}\right) = (5x)_{\frac{1}{|t^2|}} = (5x)_{\frac{1}{t^2}}$.

The mapping $f_{(g,h)}$ is a two fold homomorphism.

Also, $ker\ (f_{(g,h)}) = \{x_{\mu(t)} \in \Delta_s\ such\ that\ f_{(g,h)}(x_{\mu(t)}) = O_{\mu(0)}\}$, so that $\begin{cases} 5x = 0 \ \ ,\ hence: \\ t^2 = 1 \Rightarrow t = 1 \end{cases}$

$ker\ (f_{(g,h)}) = \{O_{\mu(0)}, O_{\mu(1)}, O_{\mu(-1)}\}$.

Definition:

Let $f_{(g_1,h_1)}, f_{(g_2,h_2)}: \Delta_s \to \Delta_s$ be two-fold homomorphisms, then

We define. $f_{(g_1,h_1)} \circ f_{(g_2,h_2)}: \Delta_s \to \Delta_s$ such that:

$$f_{(g_1,h_1)} \circ f_{(g_2,h_2)}\left(x_{\mu(t)}\right) = f_{(g_1\circ g_2, h_1\circ h_2)}\left(x_{\mu(t)}\right) = \left(g_1 \circ g_2(x)_{\mu(h_1\circ h_2(t))}\right)$$

We denote to the set of all two fold algebra homomorphisms by F_S.

Remark:

Since (o) is associative, and non-commutative, then:

$O: F_S \times F_S \to F_S$ is associative and non-commutative operation, with $F_{(I,I)}$ as an identity, where $I: \mathbb{Z} \to \mathbb{Z}$ such that $T(x) = x$.

Example:

Consider $g_1, g_2, h_1, h_2: \mathbb{Z} \to \mathbb{Z}$ such that:

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$g_1(x) = 2x$, $g_2(x) = 3x$, $h_1(x) = x^2$, $h_2(x) = x^4$, we have:
$f_{(g_1,h_1)}: \Delta_s \to \Delta_s: f_{(g_1,h_1)}(x_{\mu(t)}) = (2x)_{\frac{1}{t^2}}$,
$f_{(g_2,h_2)}: \Delta_s \to \Delta_s: f_{(g_2,h_2)}(x_{\mu(t)}) = (3x)_{\frac{1}{t^4}}$,
$f_{(g_1,h_1)} \circ f_{(g_2,h_2)}: \Delta_s \to \Delta_s: f_{(g_1,h_1)} \circ f_{(g_2,h_2)}(x_{\mu(t)}) = (6x)_{\frac{1}{t^8}}$.

Theorem:

Let $\Delta^{(P)}, \Delta_{(Q)}$ be two hyper/under two fold ideals of Δ_s, then:
1] $\Delta^{(P)}$ is two fold subalgebra of Δ_s.
2] $\Delta_{(Q)}$ is two fold subalgebra of Δ_s.
3] $\Delta^{(P)}_{(Q)}$ is two fold subalgebra of Δ_s

Proof:

1] Let $x_{\mu(t)}, y_{\mu(s)} \in \Delta^{(P)}$, then $x, y \in P$, $t, s \in \mathbb{Z}$, thus:
$x_{\mu(t)} * y_{\mu(s)} = (x+y)_{\mu(ts)} \in \Delta^{(P)}$, that is because $x, y \in P$, hence $(\Delta^{(P)}.*)$ is closed under (*), and $\Delta^{(P)}$ is two fold subalgebra of Δ_s.
2] Let $x_{\mu(t)}, y_{\mu(s)} \in \Delta^{(P)}$, then $x, y \in \mathbb{Z}$, $t, s \in Q$, thus:
$x_{\mu(t)} * y_{\mu(s)} = (x+y)_{\mu(ts)} \in \Delta_{(Q)}$, that is because $x + y \in \mathbb{Z}$, $ts \in Q$ and $(\Delta_{(Q)},*)$ is closed and then it is a two fold subalgebra.
3] If holds directly from (1) and (2).

Remark:

If $x_{\mu(t)} \in \Delta_s$, then $[x_{\mu(t)}]^m = (mx)_{\frac{1}{t^m}}$.

Definition:

Let $x_{\mu(t)}, y_{\mu(s)} \in \Delta_s$, we say that $x_{\mu(t)} \sim y_{\mu(s)}$ if there exists $z_{\mu(c)} \in \Delta_s$ such that: $x_{\mu(t)} * z_{\mu(c)} = y_{\mu(s)}$.

Remark:

If $x_{\mu(t)} \sim y_{\mu(s)}$, then: there exists $z, c \in \mathbb{Z}$ such that,

$$\begin{cases} x + z = y \\ \dfrac{1}{|tc|} = \dfrac{1}{|s|} \end{cases} \Rightarrow \begin{cases} z = y - x \\ |c| = \dfrac{|s|}{|t|} \end{cases} \Rightarrow |t|\big||s|$$

Example:

$4_{\mu(5)} \sim 2_{\mu(10)}$, that is because: there exists $(-2)_{\mu(2)} \in \Delta_s$ such that: $4_{\mu(5)} \sim (-2)_{\mu(2)} = 2_{\mu(10)}$

Remark:

If $t|s, then\ x_{\mu(t)} \sim y_{\mu(s)}$ for all $x, y \in \mathbb{Z}$

Theorem:

Let $x_{\mu(t)}, y_{\mu(s)}, z_{\mu(c)} \in \Delta_s$, then:
1] $x_{\mu(t)} \sim x_{\mu(t)}$.
2] If $x_{\mu(t)} \sim y_{\mu(s)}$ and $y_{\mu(s)} \sim x_{\mu(t)}$, then $|s| = |t|$.
3] If $x_{\mu(t)} \sim y_{\mu(s)}$ and $y_{\mu(s)} \sim z_{\mu(c)}$, then $x_{\mu(t)} \sim z_{\mu(c)}$.

Proof:

1] Since $t|t$, we get that $x_{\mu(t)} \sim x_{\mu(t)}$.
2] If $x_{\mu(t)} \sim y_{\mu(s)}$ and $y_{\mu(s)} \sim x_{\mu(t)}$, then $t|s$ and $s|t$, thus $|s| = |t|$.
3] If $x_{\mu(t)} \sim y_{\mu(s)}$ and $y_{\mu(s)} \sim z_{\mu(c)}$, hence $\begin{cases} t|s \\ s|c \end{cases}$, thus $t|c$ and $x_{\mu(t)} \sim z_{\mu(c)}$.

3. Conclusion

In this paper, we have defined an algebraic structure generated from the fusion of two-fold algebras with the standard fuzzy number theoretical system with a well-defined binary operation. Also, many of its substructures were studied concerning the corresponding operation such as hyper/under ideals, and two-fold fuzzy homomorphisms. On the other hand, we have presented many to clarify the validity of our work.

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