On Some Symbolic 2-Plithogenic and 3-Plithogenic Real Series

Oliver Von Shtawzen[1], Bahadir Tantay[2,*], Foziljonov Ibrohimjon[3]

[1]University Of Nizwa, Department of Mathematics, Nizwa, Oman
[2]College of Business Administration, American University of the Middle East, Egaila, 54200, Kuwait
[3]Tashkent State University of Economics, Uzbekistan

Emails: Vonshtawzen1970abc@gmail.com; bahadir.tantay@aum.edu.kw; ifoziljonov@tsue.uz

Abstract

The objective of this paper is to study the expansion of symbolic 2-plithogenic and symbolic 3-plithogenic real functions in one variable with real series, where many famous expansions will be presented according to the Taylor series applied for symbolic plithogenic functions defined over symbolic plithogenic rings. Also, we provide many related examples to clarify and to explain the expansion method and properties.

Keywords: Taylor series; real function; symbolic 2-plithogenic function; symbolic 3-plithogenic function.

1. Introduction and preliminaries:

The concept of symbolic n-plithogenic set was proposed in [3], as a novel generalization of fuzzy sets with many interesting properties that are very similar to neutrosophic sets [4, 11-12]. These sets were used widely to study the algebraic generalized structures generated from them such as symbolic n-plithogenic rings [1,5,8,9], matrices [16-19], modules, and spaces [7].

In [21], symbolic 2-plithogenic and 3-plithogenic real functions were defined and formulated by using a general kind of AH-isometry, with many interesting applications through many different scientific fields [3, 14-15, 22-26]. Also, many generalizations of symbolic n-plithogenic numbers can be found in [27].

In this work, we use the formulas of some symbolic 2-plithogenic and 3-plithogenic real functions in one variable [21], and we present a method to expand those functions by symbolic w-plithogenic and 3-plithogenic version of Taylor's series. Also, we explain the efficiency of the novel expansion by many other examples applied to symbolic 3-plithogenic functions in one variable, and symbolic 2-plithogenic functions in one variable.
First, we recall some basic concepts and definitions:

Definition [1]

The symbolic 2-plithogenic ring of real numbers is defined as follows:

$$2-SP_R=\{t_0+t_1P_1+t_2P_2;t_i\in R,P_1\times P_2=P_2\times P_1=P_2,{P_1}^2={P_2}^2=P_2\}$$

The addition operation on $2-SP_R$ is defined as follows:

$(t_0+t_1P_1+t_2P_2)+(t_0'+t_1'P_1+t_2'P_2)=(t_0+t_0')+(t_1+t_1')P_1+(t_2+t_2')P_2$

The multiplication on $2-SP_R$ is defined as follows:

$(t_0+t_1P_1+t_2P_2)(t_0'+t_1'P_1+t_2'P_2)=t_0t_0'+(t_0t_1'+t_1t_0'+t_1t_1')P_1+(t_0t_2'+t_1t_2'+t_2t_2'+t_2t_0'+t_2t_1')P_2$

Remark.

If $T=t_0+t_1P_1+t_2P_2\in 2-SP_R$, then:

$T^{-1}=\frac{1}{T}=\frac{1}{t_0}+\left[\frac{1}{t_0+t_1}-\frac{1}{t_0}\right]P_1+\left[\frac{1}{t_0+t_1+t_2}-\frac{1}{t_0+t_1}\right]P_2$, with $t_0\neq 0,t_0+t_1\neq 0,t_0+t_1+t_2\neq 0$.

Definition. [21]

Let $2-SP_R=\{a+bP_1+cP_2;a,b,c\in R\}$ be the 2-plithogenic field of real numbers, a function $f=f(X):2-SP_R\rightarrow 2-SP_R$ is called one variable symbolic 2-plithogenic real function, with

Doi: https://doi.org/10.54216/IJNS.230422
Received: June 11, 2023 Revised: January 15, 2024 Accepted: March 13, 2024

$X = x_0 + x_1P_1 + x_2P_2 \in 2 - SP_R$.

Definition [21]

Let $2 - SP_R$ be the symbolic 2-plithogenic field of reals, we define its AH-isometry as follows:

$I: 2 - SP_R \to R \times R \times R$ such that:

$I(x + yP_1 + zP_2) = (x, x + y, x + y + z)$.

It is easy to see that I is a ring isomorphism with the inverse:

$I^{-1}: R \times R \times R \to 2 - SP_R$ such that:

$I^{-1}(x, y, z) = x + (y - x)P_1 + (z - y)P_2$

Definition [21]

Let $f: 2 - SP_R \to 2 - SP_R$ be a symbolic 2-plithogenic real function with one variable, we define the canonical formula as follows:

$$I^{-1} \circ I(f): 2 - SP_R \to 2 - SP_R$$

Definition [21]

Let $3 - SP_R = \{a + bP_1 + cP_2 + dP_2; a, b, c, d \in R\}$ be the 3-plithogenic field of real numbers, a function $f = f(X): 3 - SP_R \to 3 - SP_R$ is called one variable symbolic 3-plithogenic real function, with

$X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 \in 3 - SP_R$.

Definition. [21]

Let $3 - SP_R$ be the symbolic 3-plithogenic field of reals, we define its AH-isometry as follows:

$I: 3 - SP_R \to R \times R \times R \times R$ such that:

$I(x + yP_1 + zP_2 + tP_3) = (x, x + y, x + y + z, x + y + z + t)$.

It is easy to see that I is a ring isomorphism with the inverse:

$I^{-1}: R \times R \times R \times R \to 3 - SP_R$ such that:

$I^{-1}(x, y, z, t) = x + (y - x)P_1 + (z - y)P_2 + (t - z)P_3$

Definition [21]

Let $f: 3 - SP_R \to 3 - SP_R$ be a symbolic 3-plithogenic real function with one variable, we define the canonical formula as follows:

$$I^{-1} \circ I(f): 3 - SP_R \to 3 - SP_R$$

Example.

Consider$f(X) = X^2 - P_1 + P_3$, its canonical formula is:

$$I(f(X)) = [I(X)]^2 + I(-P_1 + P_3) = (x_0, x_0 + x_1, x_0 + x_1 + x_2, x_0 + x_1 + x_2 + x_3)^2 + (0, -1, -1, 0)$$
$$= ({x_0}^2, (x_0 + x_1)^2 - 1, (x_0 + x_1 + x_2)^2 - 1, (x_0 + x_1 + x_2 + x_3)^2)$$

$I^{-1} \circ I(f(X)) =$

$${x_0}^2 + P_1[(x_0 + x_1)^2 - {x_0}^2 - 1] + P_2[(x_0 + x_1 + x_2)^2 - (x_0 + x_1)^2]$$
$$+ P_3[(x_0 + x_1 + x_2 + x_3)^2 - (x_0 + x_1 + x_2)^2 + 1]$$

For example:

$f(1 + P_3) = (1 + P_3)^2 - P_1 + P_3 = 1 - P_1 + 4P_3$.

If we put values $x_0 = 1, x_1 = 0, x_2 = 0, x_2 = 1$

in the canonical formula, then we get:

$I^{-1} \circ I(f(X)) = (1)^2 + P_1[(1)^2 - (1)^2 - 1] + P_2[(1)^2 - (1)^2] + P_3[(2)^2 - (1)^2 + 1] = 1 - P_1 + 4P_3$.

Main discussion

Definition:

Let $2 - SP_{\mathbb{R}} = \{x + yp_1 + zp_2;\ x, y, z \in \mathbb{R}\}$ be the ring of symbolic 2-plithogenic real numbers, Consider the following sequence:

$U_n = x_n + y_nP_1 + z_nP_2$; x_n, y_n, z_n are three real sequences, we define:

$$\sum_{n=1}^{j} U_n = \sum_{n=1}^{j} x_n + \sum_{n=1}^{j} y_nP_1 + \sum_{n=1}^{j} z_nP_2. \quad (1)$$

It is called a finite series.

If $j = \infty$, then it is called an infinite series.

Example:

Consider: $U_n = \frac{1}{n} + \frac{1}{n^2}P_1 + \frac{3}{n}P_2$, then:

$$\sum_{n=1}^{\infty} U_n = \sum_{n=1}^{\infty} \frac{1}{n} + \sum_{n=1}^{\infty} \frac{1}{n^2}P_1 + \sum_{n=1}^{\infty} \frac{3}{n}P_2$$

Definition:

We say that $\sum_{n=1}^{\infty} U_n$ is convergent if and only if:

$\sum x_n, \sum y_n, \sum z_n$ are convergent.

Otherwise, it is called divergent.

Doi: https://doi.org/10.54216/IJNS.230422
Received: June 11, 2023 Revised: January 15, 2024 Accepted: March 13, 2024

Remark:

1] if $\sum x_n = a, \sum y_n = b, \sum z_n = c$, then $\sum U_n = a + bP_1 + cP_2$.

2] if $\sum U_n$ is divergent, we say that $\sum_{n=1}^{\infty} U_n = \infty$

Example:

Consider the function: $f: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$ such that:

$f(x_0 + x_1P_1 + x_2P_2) = e^{x_0} + e^{x_1}P_1 + e^{x_2}P_2$, then:

$$\begin{cases} e^{x_0} = \sum_{n=0}^{\infty} \frac{(x_0)^n}{n!} \\ e^{x_1} = \sum_{n=0}^{\infty} \frac{(x_1)^n}{n!} \\ e^{x_2} = \sum_{n=0}^{\infty} \frac{(x_2)^n}{n!} \end{cases}$$

So that:

$f(X) = \sum_{n=0}^{\infty} \frac{x_0{}^n}{n!} + \sum_{n=0}^{\infty} \frac{x_1{}^n}{n!} P_1 + \sum_{n=0}^{\infty} \frac{x_2{}^n}{n!} P_2 = \sum_{n=0}^{\infty} \frac{x_0{}^n + x_1{}^n P_1 + x_2{}^n P_2}{n!}$

Example:

Consider the functions: $f: 2 - SP_{\mathbb{R}} \rightarrow 2SP_{\mathbb{R}}$, $g: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$, $h: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$ such that:

$f(x_0 + x_1P_1 + x_2P_2) = sin(x_0) + sin(x_1)P_1 + sin(x_2)P_2$,

$g(x_0 + x_1P_1 + x_2P_2) = cos(x_0) + cos(x_1)P_1 + cos(x_2)P_2$,

$h(x_0 + x_1P_1 + x_2P_2) = sin(x_0) + cos(x_1)P_1 + sin(x_2)P_2$,

we have:

$$\begin{cases} sin(x_0) = \sum_{n=0}^{\infty} \frac{(-1)^n . x_0{}^{2n+1}}{(2n+1)!} \\ sin(x_1) = \sum_{n=0}^{\infty} \frac{(-1)^n . x_1{}^{2n+1}}{(2n+1)!} \\ sin(x_2) = \sum_{n=0}^{\infty} \frac{(-1)^n . x_2{}^{2n+1}}{(2n+1)!} \end{cases}$$

$$\begin{cases} cos(x_0) = \sum_{n=0}^{\infty} (-1)^n . \frac{x_0{}^{2n}}{(2n)!} \\ cos(x_1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} . x_1{}^{2n} \\ cos(x_2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} . x_2{}^{2n} \end{cases}$$

Hence, $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} [x_0{}^{2n+1} + x_1{}^{2n+1}P_1 + x_2{}^{2n+1}P_2]$

$$g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} [x_0{}^{2n} + x_1{}^{2n}P_1 + x_2{}^{2n}P_2]$$

$$h(x) = \sum_{n=0}^{\infty} [\frac{(-1)^n}{(2n+1)!} (x_0{}^{2n+1} + x_2{}^{2n+1}P_2) + \frac{(-1)^n}{(2n)!} x_1{}^{2n}P_1)]$$

Example:

Consider $f: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$ such that:

$f(x_0 + x_1P_1 + x_2P_2) = \ln(x_0) + \ln(x_1)P_1 + \ln(x_2)P_2$,

$g: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$ such that:

$g(x_0 + x_1P_1 + x_2P_2) = e^{x_0} + \ln(x_2)P_1 + \ln(x_1)P_2$,

$h: 2 - SP_{\mathbb{R}} \rightarrow 2 - SP_{\mathbb{R}}$ such that:

$h(x_0 + x_1P_1 + x_2P_2) = \ln(x_0) + e^{x_0}P_1 + e^{x_2+x_1}P_2$,

we have:

Doi: https://doi.org/10.54216/IJNS.230422
Received: June 11, 2023 Revised: January 15, 2024 Accepted: March 13, 2024

$$\begin{cases} \ln(x_0) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(x_0-1)^n \\ \ln(x_1) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(x_1-1)^n \\ \ln(x_2) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(x_2-1)^n \end{cases}$$

And: $\begin{cases} e^{x_0} = \sum_{n=0}^{\infty} \frac{{x_0}^n}{n!} \\ e^{x_2+x_1} = \sum_{n=0}^{\infty} \frac{(x_1+x_2)^n}{n!} \end{cases}$

So that:

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}[(x_0-1)^n + (x_1-1)^n P_1 + (x_2-1)^n P_2]$$

$$g(x) = \sum_{n=0}^{\infty} \frac{{x_0}^n}{n!} + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}[(x_2-1)^n P_1 + (x_1-1)^n P_2]$$

$$h(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x_0-1)^n + \sum_{n=0}^{\infty} \frac{1}{n!}({x_0}^n P_1 + (x_1+x_2)^n P_2)$$

Example:

Consider the functions:

$f: 2-SP_{\mathbb{R}} \to 2-SP_{\mathbb{R}}$

$g: 2-SP_{\mathbb{R}} \to 2-SP_{\mathbb{R}}$

$h: 2-SP_{\mathbb{R}} \to 2-SP_{\mathbb{R}}$

such that:

$f(x_0+x_1P_1+x_2P_2) = e^{x_1-x_0} + \sin(x_0-2x_2)\,P_1 + \cos(x_2)\,P_2,$

$g(x_0+x_1P_1+x_2P_2) = \sin(3x_1) + \ln(x_0+x_1+x_2)\,P_1 + \cos(2x_0)\,P_2,$

$h(x_0+x_1P_1+x_2P_2) = \cos(x_1+x_0) + e^{x_1+x_2-x_0}P_1 + \ln(2x_0-x_1)\,P_2,$

we have:

$$\begin{cases} e^{x_1-x_0} = \sum_{n=0}^{\infty} \frac{1}{n!}(x_1-x_0)^n \\ \sin(x_0-2x_2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}(x_0-2x_2)^{2n+1} \\ \cos(x_2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(x_2)^{2n} \\ \sin(3x_1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}(3x_1)^{2n+1} \\ \ln(x_0+x_1+x_2) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x_0+x_1+x_2-1)^n \\ \cos(2x_0) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(2x_0)^{2n} \\ \cos(x_1+x_0) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(x_1+x_0)^{2n} \\ e^{x_1+x_2-x_0} = \sum_{n=0}^{\infty} \frac{1}{n!}(x_1+x_2-x_0)^n \\ \ln(2x_0-x_1) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(2x_0-x_1-1)^n \end{cases}$$

Doi: https://doi.org/10.54216/IJNS.230422
Received: June 11, 2023 Revised: January 15, 2024 Accepted: March 13, 2024

So that:

$$f(x)=\sum_{n=0}^{\infty}\frac{1}{n!}(x_1-x_0)^n+\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(x_0-2x_1)^{2n+1}P_1+\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(x_2)^{2n}P_2$$

$$g(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}.(3x_1)^{2n+1}+\sum_{n=1}^{\infty}\frac{(-1)^n}{n}.(x_0+x_1+x_2-1)^nP_1+\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(2x_0)^{2n}P_2$$

$$h(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}.(x_1+x_0)^{2n}+\sum_{n=0}^{\infty}\frac{1}{n!}.(x_1+x_2-x_0)^nP_1+\sum_{n=1}^{\infty}\frac{(-1)^n}{n}(2x_0-x_1-1)^nP_2$$

Definition:

Let $3-SP_{\mathbb{R}}=\{x+yP_2+zP_2+tP_3; x,y,z,t\in\mathbb{R}\}$ be the ring of symbolic 3-plithogenic real numbers, consider the following sequence: $U_n=x_n+y_nP_1+z_nP_2+t_nP_3; x_n, y_n, z_n, t_n$ are four real sequences, we define:

$\sum_{n=1}^{j}U_n=\sum_{n=1}^{j}x_n+\sum_{n=1}^{j}y_nP_1+\sum_{n=1}^{j}z_nP_2+\sum_{n=1}^{j}t_nP_3$ (2)

It is a symbolic 3-plithogenic finite series.

For $j=\infty$, we get the infinite case.

The convergent and divergent symbolic 3-plithogenic series is defined as the case of symbolic 2-plithogenic case.

Example:

Consider the following functions:

$$f:3-SP_{\mathbb{R}}\to 3-SP_{\mathbb{R}}$$
$$g:3-SP_{\mathbb{R}}\to 3-SP_{\mathbb{R}}$$
$$h:3-SP_{\mathbb{R}}\to 3-SP_{\mathbb{R}}$$
$$k:3-SP_{\mathbb{R}}\to 3-SP_{\mathbb{R}}$$
$$l:3-SP_{\mathbb{R}}\to 3-SP_{\mathbb{R}}$$
$$s:3-SP_{\mathbb{R}}\to 3-SP_{\mathbb{R}}$$

Such that:

$f(x_0+x_1P_1+x_2P_2+x_3P_3)=e^{x_0+x_1}+\ln(x_1)P_1+\sin(x_2)P_2+\cos(x_3)P_3,$

$g(x_0+x_1P_1+x_2P_2+x_3P_3)=\ln(x_0-x_2)+e^{x_2}P_1+\cos(x_0+x_3)P_2+\sin(x_3)P_3,$

$h(x_0+x_1P_1+x_2P_2+x_3P_3)=e^{x_0}+\cos(x_3-x_2)P_1+\sin(x_2+2x_3)P_2+\ln(x_0)P_3,$

$k(x_0+x_1P_1+x_2P_2+x_3P_3)=\ln(x_0)+\ln(x_1)P_1+e^{x_2}P_2+e^{x_3}P_3,$

$l(x_0+x_1P_1+x_2P_2+x_3P_3)=\sin(x_0)+\sin(x_1)P_1+\cos(x_2)P_2+\cos(x_3)P_3$,

$s(x_0+x_1P_1+x_2P_2+x_3P_3)=e^{x_3+x_1}+e^{x_0-x_1}P_1+\ln(2x_2-5x_3)P_2+\cos(x_0+4x_2)P_3$,

We have:

$$\begin{cases} e^{x_0+x_1}=\sum_{n=0}^{\infty}\frac{1}{n!}(x_0+x_1)^n \\ \ln x_1=\sum_{n=1}^{\infty}\frac{(-1)^n}{n}(x_1-1)^n \\ \sin x_2=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}{x_2}^{2n+1} \\ \cos x_3=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}{x_3}^{2n+1} \\ \ln(x_0-x_2)=\sum_{n=1}^{\infty}\frac{(-1)^n}{n}(x_0-x_2-1)^n \\ e^{x_2}=\sum_{n=0}^{\infty}\frac{1}{n!}(x_2)^n \\ \cos(x_0+x_2)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(x_0+x_2)^{2n} \\ \sin x_3=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(x_3)^{2n+1} \end{cases}$$

And

Doi: https://doi.org/10.54216/IJNS.230422
Received: June 11, 2023 Revised: January 15, 2024 Accepted: March 13, 2024

$$\begin{cases} e^{x_0} = \sum_{n=0}^{\infty} \frac{1}{n!}(x_0)^n \\ \cos(x_3 - x_2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(x_3 - x_2)^{2n} \\ \sin(x_2 + 2x_3) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}(x_2 + 2x_3)^{2n+1} \\ \ln x_0 = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x_0 - 1)^n \end{cases}$$

And:

$$\begin{cases} \ln x_0 = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x_0 - 1)^n \\ \ln x_1 = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x_1 - 1)^n \\ e^{x_2} = \sum_{n=0}^{\infty} \frac{1}{n!}(x_2)^n \\ e^{x_3} = \sum_{n=0}^{\infty} \frac{1}{n!}(x_3)^n \end{cases}$$

And:

$$\begin{cases} \sin x_0 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}(x_0)^{2n+1} \\ \sin x_1 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}(x_1)^{2n+1} \\ \cos x_2 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(x_2)^{2n} \\ \cos x_3 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(x_3)^{2n} \end{cases}$$

And:

$$\begin{cases} e^{x_3+x_1} = \sum_{n=0}^{\infty} \frac{1}{n!}(x_3 + x_1)^n \\ e^{x_0-x_1} = \sum_{n=0}^{\infty} \frac{1}{n!}(x_0 - x_1)^n \\ \ln(2x_2 - 5x_3) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(2x_2 - 5x_3 - 1)^n \\ \cos(x_0 + 4x_2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(x_0 + 4x_2)^{2n} \end{cases}$$

Thus:

$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!}(x_0 + x_1)^n + \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x_1 - 1)^n P_1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} {x_2}^{2n+1} P_2 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} {x_3}^{2n} P_3$

$g(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x_0 - x_2 - 1)^n + \sum_{n=0}^{\infty} \frac{1}{n!} {x_2}^n P_1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(x_0 + x_2)^{2n} P_2 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} {x_3}^{2n+1} P_3$

$h(x) = \sum_{n=0}^{\infty} \frac{1}{n!} {x_0}^n + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(x_3 - x_2)^n P_1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}(x_2 + 2x_3)^{2n+1} P_2 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x_0 - 1)^n P_3$

$k(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x_0 - 1)^n + \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x_1 - 1)^n P_1 + \sum_{n=0}^{\infty} \frac{1}{n!}[{x_2}^n P_2 + {x_3}^n P_3]$

Doi: https://doi.org/10.54216/IJNS.230422
Received: June 11, 2023 Revised: January 15, 2024 Accepted: March 13, 2024

$l(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} [x_0{}^{2n+1} + x_1{}^{2n+1} P_1] + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} [x_2{}^{2n} P_2 + x_3{}^{2n} P_3]$

$s(x) = \sum_{n=0}^{\infty} \frac{1}{n!} [(x_3 + x_1)^n + (x_0 - x_1)^n P_1] + \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (2x_2 - 5x_3 - 1)^n P_2 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x_0 + 4x_2)^{2n}$.

Definition:

Let $4 - SP_{\mathbb{R}} = \{x + yP_2 + zP_2 + tP_3; x, y, z, t, s \in \mathbb{R}\}$ be the ring of symbolic 4-plithogenic real numbers, consider the following sequence: $U_n = x_n + y_n P_1 + z_n P_2 + t_n P_3 + s_n P_4; x_n, y_n, z_n, t_n, s_n$ are five real sequences, we define:

$\sum_{n=1}^{j} U_n = \sum_{n=1}^{j} x_n + \sum_{n=1}^{j} y_n P_1 + \sum_{n=1}^{j} z_n P_2 + \sum_{n=1}^{j} t_n P_3 + \sum_{n=1}^{j} s_n P_4$ (3)

It is a symbolic 3-plithogenic finite series.

For $j = \infty$, we get the infinite case.

The convergent and divergent symbolic 4-plithogenic series is defined as the case of symbolic 2-plithogenic case.

Example:

Consider the following functions:

$$f: 4 - SP_{\mathbb{R}} \to 4 - SP_{\mathbb{R}}$$
$$g: 4 - SP_{\mathbb{R}} \to 4 - SP_{\mathbb{R}}$$
$$h: 4 - SP_{\mathbb{R}} \to 4 - SP_{\mathbb{R}}$$
$$k: 4 - SP_{\mathbb{R}} \to 4 - SP_{\mathbb{R}}$$
$$l: 4 - SP_{\mathbb{R}} \to 4 - SP_{\mathbb{R}}$$

Such that:

$f(x_0 + x_1 P_1 + x_2 P_2 + x_3 P_3 + x_4 P_4) = e^{x_0 - x_4} + e^{x_1 + x_4} P_1 + e^{x_2 - x_3} P_2 + e^{x_3} P_3 + e^{x_4} P_4$,

We have:

$$\begin{cases} e^{x_0 - x_4} = \sum_{n=0}^{\infty} \frac{1}{n!} (x_0 - x_4)^n \\ e^{x_1 + x_4} = \sum_{n=0}^{\infty} \frac{1}{n!} (x_1 + x_4)^n \\ e^{x_2 - x_3} = \sum_{n=0}^{\infty} \frac{1}{n!} (x_2 - x_3)^n \\ e^{x_3} = \sum_{n=0}^{\infty} \frac{1}{n!} (x_3)^n \\ e^{x_4} = \sum_{n=0}^{\infty} \frac{1}{n!} (x_4)^n \end{cases}$$

Thus, $g(x) = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{(2n)!} (2x_1 + x_2)^{2n} + \sum_{n=0}^{\infty} \frac{(-1)^{2n+1}}{(2n+1)!} (x_3)^{2n+1} P_1 + \cdots$

$h(x) = \cos(x_4 - 5x_0) + \cos(x_1 + x_2) P_1 + \sin(x_2 + x_4) P_2 + \ln(x_0) P_3 + \ln(x_0 + 3x_1) P_4$,

$$\begin{cases} \cos(x_4 - 5x_0) = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{(2n)!} (x_4 - 5x_0)^{2n} \\ \cos(x_1 + x_2) = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{(2n)!} (x_1 + x_2)^{2n} \\ \sin(x_2 + x_3) = \sum_{n=0}^{\infty} \frac{(-1)^{2n+1}}{(2n+1)!} (x_2 + x_3)^{2n+1} \\ \ln x_0 = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 - 1)^n \\ \ln(x_0 + 3x_1) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x_0 + 3x_1 - 1)^n \end{cases}$$

$k(x) = e^{x_0 + x_1 - x_4} P_2 + e^{x_4 - 3x_1} P_4$, we have:

Doi: https://doi.org/10.54216/IJNS.230422
Received: June 11, 2023 Revised: January 15, 2024 Accepted: March 13, 2024

$$\begin{cases} e^{x_0+x_1-x_4} = \sum_{n=0}^{\infty} \frac{1}{n!}(x_0+x_1-x_4)^n \\ e^{x_4-3x_1} = \sum_{n=0}^{\infty} \frac{1}{n!}(e^{x_4-3x_1})^n \end{cases}$$

Thus, $k(x) = \sum_{n=0}^{\infty} \frac{1}{n!}[(x_0+x_1-x_4)^n P_2 + (x_4-3x_1)^n P_4]$

$l(x) = \ln(2x_1+x_2)P_1 + \ln(x_3-5x_4)P_3 + e^{x_3}P_4]$, we have:

$$\begin{cases} \ln(2x_1+x_2) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(2x_1+x_2-1)^n \\ \ln(x_3-5x_4) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x_3-5x_4-1)^n \\ e^{x_3} = \sum_{n=0}^{\infty} \frac{1}{n!}{x_3}^n \end{cases}$$

Thus, $l(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}[(2x_1+x_2-1)^n P_1 + (x_3-5x_4-1)^n P_3] + \sum_{n=0}^{\infty} \frac{1}{n!}(x_3)^n$

The expansion of some famous plithogenic real functions.

The following formulas are proved in [21].

1] $e^{x_0+x_1P_1+x_2P_2} = e^{x_0} + P_1[e^{x_0+x_1} - e^{x_0}] + P_2[e^{x_0+x_1+x_2} - e^{x_0+x_1}]$.

2] $e^{x_0+x_1P_1+x_2P_2+x_3P_3} = e^{x_0} + P_1[e^{x_0+x_1} - e^{x_0}] + P_2[e^{x_0+x_1+x_2} - e^{x_0+x_1}] + P_3[e^{x_0+x_1+x_2+x_3} - e^{x_0+x_1+x_2}]$

3] $\ln(x_0+x_1P_1+x_2P_2) = \ln(x_0) + P_1[\ln(x_0+x_1) - \ln x_0] + P_2[\ln(x_0+x_1+x_2) - \ln(x_0+x_1)]$

4] $\ln(x_0+x_1P_1+x_2P_2+x_3P_3) = \ln x_0 + P_1[\ln(x_0+x_1) - \ln x_0] + P_2[\ln(x_0+x_1+x_2) - \ln(x_0+x_1)] + P_3[\ln(x_0+x_1+x_2+x_3) - \ln(x_0+x_1+x_2)]$

5] $\sin(x_0+x_1P_1+x_2P_2) = \sin(x_0) + P_1[\sin(x_0+x_1) - \sin x_0] + P_2[\sin(x_0+x_1+x_2) - \sin(x_0+x_1)]$

6] $\sin(x_0+x_1P_1+x_2P_2+x_3P_3) = \sin x_0 + P_1[\sin(x_0+x_1) - \sin x_0] + P_2[\sin(x_0+x_1+x_2) - \sin(x_0+x_1)] + P_3[\sin(x_0+x_1+x_2+x_3) - \sin(x_0+x_1+x_2)]$

7] $\cos(x_0+x_1P_1+x_2P_2) = \cos(x_0) + P_1[\cos(x_0+x_1) - \cos x_0] + P_2[\cos(x_0+x_1+x_2) - \cos(x_0+x_1)]$

8] $\cos(x_0+x_1P_1+x_2P_2+x_3P_3) = \cos x_0 + P_1[\cos(x_0+x_1) - \cos x_0] + P_2[\cos(x_0+x_1+x_2) - \cos(x_0+x_1)] + P_3[\cos(x_0+x_1+x_2+x_3) - \cos(x_0+x_1+x_2)]$

The series formulas are:

1] $e^{x_0+x_1P_1+x_2P_2} = \sum_{n=0}^{\infty} \frac{1}{n!}[{x_0}^n + [(x_0+x_1)^n - {x_0}^n]P_1 + P_2[(x_0+x_1+x_2)^n - (x_0+x_1)^n$.

2] $e^{x_0+x_1P_1+x_2P_2+x_3P_3} = \sum_{n=0}^{\infty} \frac{1}{n!}[{x_0}^n + [(x_0+x_1)^n - {x_0}^n]P_1 + [(x_0+x_1+x_2)^n - (x_0+x_1)^n]P_2 + [(x_0+x_1+x_2+x_3)^n - (x_0+x_1+x_2)^n P_3]$

3] $\ln(x_0+x_1P_1+x_2P_2) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}[(x_0-1)^n + [(x_0+x_1-1)^n - (x_0-1)^n]P_1 + [(x_0+x_1+x_2-1)^n - (x_0+x_1-1)^n]P_2$

4] $\ln(x_0+x_1P_1+x_2P_2+x_3P_3) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}[(x_0-1)^n + [(x_0+x_1-1)^n - (x_0-1)^n]P_1 + [(x_0+x_1+x_2-1)^n - (x_0+x_1-1)^n]P_2 + [(x_0+x_1+x_2+x_3-1)^n - (x_0+x_1+x_2-1)^n]P_3]$

5] $\sin(x_0+x_1P_1+x_2P_2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}[{x_0}^{2n+1} + [(x_0+x_1)^{2n+1} - {x_0}^{2n+1}]P_1 + [(x_0+x_1+x_2)^{2n+1} - (x_0+x_1)^{2n+1}]P_2$

6] $\sin(x_0+x_1P_1+x_2P_2+x_3P_3) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}[{x_0}^{2n+1} + [(x_0+x_1)^{2n+1} - {x_0}^{2n+1}]P_1 + [(x_0+x_1+x_2)^{2n+1} - (x_0+x_1)^{2n+1}]P_2 + [(x_0+x_1+x_2+x_3)^{2n+1} - (x_0+x_1)^{2n+1}]P_3$

7] $\cos(x_0+x_1P_1+x_2P_2) = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{(2n)!}[{x_0}^{2n} + [(x_0+x_1)^{2n} - {x_0}^{2n}]P_1 + [(x_0+x_1+x_2)^{2n} - (x_0+x_1)^{2n}]P_2$

8] $\cos(x_0+x_1P_1+x_2P_2+x_3P_3) = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{(2n)!}[{x_0}^{2n} + [(x_0+x_1)^{2n} - {x_0}^{2n}]P_1 + [(x_0+x_1+x_2)^{2n} - (x_0+x_1)^{2n}]P_2 + [(x_0+x_1+x_2+x_3)^{2n} - (x_0+x_1+x_2)^{2n}]P_3]$.

5. Conclusion

In this work, we have studied the expansion of symbolic 2-plithogenic and symbolic 3-plithogenic real functions in one variable with real series, where we provided many related examples to clarify and to explain the expansion

Doi: https://doi.org/10.54216/IJNS.230422
Received: June 11, 2023 Revised: January 15, 2024 Accepted: March 13, 2024

method and properties.
In the future, we aim to generalize our study to symbolic n-plithogenic functions and symbolic n-plithogenic series.

References

[1] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.
[2] Soueycatt, M., Charchekhandra, B., Abu Hakmeh, R., " On The Algebraic Properties Of Symbolic 6-Plithogenic Integers", Neutrosophic Sets and Systems, vol. 59, 2023.
[3] Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 53, 2023.
[4] Abobala, M., " A Study of Maximal and Minimal Ideals of n-Refined Neutrosophic Rings", Journal of Fuzzy Extension and Applications, 2021.
[5] Albasheer, O., Hajjari., A., and Dalla., R., " On The Symbolic 3-Plithogenic Rings and TheirAlgebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.
[6] Olgun, N., Hatip, A., Bal, M., and Abobala, M., " A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined Neutrosophic Matrices", International Journal of neutrosophic Science, Vol. 16, pp. 72-79, 2021.
[7] Taffach, N., and Ben Othman, K., " An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol 54, 2023.
[8] Merkepci, M., and Abobala, M., " On Some Novel Results About Split-Complex Numbers, The Diagonalization Problem And Applications To Public Key Asymmetric Cryptography", Journal of Mathematics, Hindawi, 2023.
[9] Ben Othman, K., "On Some Algorithms For Solving Symbolic 3-Plithogenic Equations", Neoma Journal Of Mathematics and Computer Science, 2023.
[10] Rawashdeh, A., "An Introduction To The Symbolic 3-plithogenic Number Theory", Neoma Journal Of Mathematics and Computer Science, 2023.
[11] Abobala, M., " n-Cyclic Refined Neutrosophic Algebraic Systems of Sub-Indeterminacies, An Application to Rings and Modules", International Journal of Neutrosophic Science, 2020.
[12] Soueycatt, M., Charchekhandra, B., Abu Hakmeh, R., " On The Foundations Of Symbolic 5-Plithogenic Number Theory", Neutrosophic Sets and Systems, vol. 59, 2023.
[13] Abobala, M., and Allouf, A., " On A Novel Security Scheme for The Encryption and Decryption Of 2×2 Fuzzy Matrices with Rational Entries Based on The Algebra of Neutrosophic Integers and El-Gamal Crypto-System", Neutrosophic Sets and Systems, vol.54, 2023.
[14] Merkepci, M., Abobala, M., and Allouf, A., " The Applications of Fusion Neutrosophic Number Theory in Public Key Cryptography and the Improvement of RSA Algorithm ", Fusion: Practice and Applications, 2023.
[15] Merkepci, M., and Abobala, M., " Security Model for Encrypting Uncertain Rational Data Units Based on Refined Neutrosophic Integers Fusion and El Gamal Algorithm ", Fusion: Practice and Applications, 2023.
[16] Alfahal, A.; Alhasan, Y.; Abdulfatah, R.; Mehmood, A.; Kadhim, M. On Symbolic 2-Plithogenic Real Matrices and Their Algebraic Properties. Int. J. Neutrosophic Sci. 2023, 21.
[17] Merkepci, H., "On Novel Results about the Algebraic Properties of Symbolic 3-Plithogenic and 4-Plithogenic Real Square Matrices", Symmetry, MDPI, 2023.
[18] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Concept Of Symbolic 7-Plithogenic Real Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.
[19] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Symbolic 8-Plithogenic Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.
[20] Salman, N, K., " On The Computing Of Symbolic 2-Plithogenic And 3-Plithogenic Complex Roots Of Unity", Neutrosophic Sets and Systems, 2023.
[21] Salman, N, K., " On The Symbolic 2-Plithogenic Real Analysis By Using Algebraic Functions", Neutrosophic Sets and Systems, 2023.
[22] Sankari, H., and Abobala, M., " On The Classification of The Group of Units of Rational and Real 2-Cyclic Refined Neutrosophic Rings", Neutrosophic Sets and Systems, 2023.
[23] Sankari, H., and Abobala, M., " On The Algebraic Homomorphisms Between Symbolic 2-plithogenic Rings And 2-cyclic Refined Rings", Neutrosophic Sets and Systems, 2023.
[24] Ben Othman, K., Sarkis, M., and Ozcek, M., " On The Dual Symbolic 2-Plithogenic Numbers", Neutrosophic Sets and Systems, 2023.
[25] Nadweh, R., Von Shtawzen, O., Khaldi, A., and Ali, R., " A Study of Symbolic 2-Plithogenic Split-Complex Linear Diophantine Equations in Two Variables", Neutrosophic Sets and Systems, 2023.

Doi: https://doi.org/10.54216/IJNS.230422
Received: June 11, 2023 Revised: January 15, 2024 Accepted: March 13, 2024

[26] Von Shtawzen, O., Rawashdeh, A., and Zayood, K., " On The Symbolic 2-Plithogenic Split-Complex Real Square Matrices and Their Algebraic Properties", Neutrosophic Sets and Systems, 2023.
[27] Nadweh, R., Von Shtawzen, O., Khaldi, A., "An Introduction To The Split-Complex Symbolic 2-Plithogenic Numbers", Neutrosophic Sets and Systems, 2023.

Doi: https://doi.org/10.54216/IJNS.230422
Received: June 11, 2023 Revised: January 15, 2024 Accepted: March 13, 2024