



Studying the isotherm of the complementary Schaefer Ignaczak thermodynamical process of the first plane state of elastic strains for the unbounded micropolar body-Fourier Schaefer-Ignaczak formulas

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Abstract

This paper concerns the Ignaczak stress-temperature distribution [2] of the homogenous isotropic 2D micropolar thermodynamical in the first plane state of elastic strain, which discussed by Eringen [9] and Nowacki [8]. In [1] we provide this problem with new analytical method called Schaefer-Ignaczak method. In the paper, we do the following; We prove that the complementary Schaefer-Ignaczak process is an isothermal process for infinite 2D (E-N:5) [6,8], with no stresses and temperature at infinity, and then we find the related Fourier Schaefer-Ignaczak formulas [1] for the classical and complementary behavior of a two-dimensional infinite body (E-N:5), which is a micropolar body.

Keywords: isotherm; Schaefer Ignaczak; thermodynamical process; elastic strains

1. Introduction:

- In [10], the initial-boundary value problem of the elastic dynamical micropolar body was described by tow tensorial stress equations with the appropriate field relations and appropriate stress initial-boundary conditions.
- In [1], the stress-temperature initial-boundary value problem of Ignaczak type was discussed for the first plane state of elastic strain related to 2D (E-N:5), body [2]. discussed by Eringen [9] and Nowacki [8] and shortly named 2D (E-N:5).
- Next in [1] we provided the above missioned problem by new method, called Schaefer-Ignaczak method, which simplifies solving this problem.

2. Discussing:

1- The aim and the importunacy of this paper:

- a) The aim of the paper is to prove the isotherm of the complementary Schaefer-Ignaczak process for the unbounded 2D (E-N:6) body free of stresses and temperature at infinity. Next using the integral transform theory, we drive the Fourier Schaefer-Ignaczak formulas for the Scheafer-Ignaczak classical and complementary fields.
- b) The importunacy of paper: the results of this paper have important applications in shells theory and material laboratories.

2- The methods of the paper:

For the methods of this paper, we combine the integral transform theory [11] with Schaefer-Ignaczak method [1].

For the requirement of the results of the paper, need to introduce some following important results from the paper [5].

The Schaefer-Ignaczak method for the 2D (E-N:6) elastic body [1]:

In [5] we write the micropolar thermodynamical process for the 2D (E-N:6) elastic body occupying the initial configuration Ω , in the following form in $\Omega \times T$ ($T = [0, \infty[$):

$$\begin{aligned} \mathbf{u} &= \mathbf{u}^0 + \mathbf{u}', & \boldsymbol{\varphi} &= \boldsymbol{\varphi}^0 + \boldsymbol{\varphi}', & \theta &= \theta^0 + \theta', \\ \boldsymbol{\sigma} &= \boldsymbol{\sigma}^0 + \boldsymbol{\sigma}', & \boldsymbol{\mu} &= \boldsymbol{\mu}^0 + \boldsymbol{\mu}', & \boldsymbol{\gamma} &= \boldsymbol{\varepsilon}^0 + \boldsymbol{\gamma}', \\ \boldsymbol{\kappa} &= \boldsymbol{\kappa}^0 + \boldsymbol{\kappa}', & \mathbf{Y} &= \mathbf{Y}^0 + \mathbf{Y}', \end{aligned} \quad (3.1)$$

Where $(\mathbf{u}^0, \boldsymbol{\varphi}^0, \theta^0, \boldsymbol{\sigma}^0, \boldsymbol{\mu}^0, \boldsymbol{\varepsilon}^0, \boldsymbol{\kappa}^0)$, \mathbf{Y}^0 relates to the classical thermodynamical linear thermo elasticity (Thermodynamical Hooke Model) in the frame of first plane state of elastic strains, and $(\mathbf{u}', \boldsymbol{\varphi}', \theta', \boldsymbol{\sigma}', \boldsymbol{\mu}', \boldsymbol{\gamma}', \boldsymbol{\kappa}')$, \mathbf{Y}' are the micropolar complementary tensor fields. The above mentioned classical and complementary tensor fields can be written in $\Omega \times T$, and in the inertial frame \mathbf{e}_i ($i = 1, 2, 3$), in the following form:

$$\mathbf{u}^0 \equiv (u_1^0, u_2^0, 0), \quad \boldsymbol{\varphi}^0 \equiv (0, 0, \varphi_3^0), \quad (3.2) \quad \text{Where}$$

$\varphi_3^0 = \frac{1}{2} \epsilon_{\alpha\beta} u_{\beta,\alpha}^0$ and $\epsilon_{\alpha\beta}$ is the Levi-Chevita semi-tensor [8] of second order, the coma index denotes the partial derivative with the position coordinates; $f_{,\alpha} = \frac{\partial f}{\partial x_\alpha}$;

$$\boldsymbol{\varepsilon}^0 \equiv \begin{bmatrix} \varepsilon_{11}^0 & \varepsilon_{12}^0 & 0 \\ \varepsilon_{21}^0 & \varepsilon_{22}^0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\kappa}^0 \equiv \begin{bmatrix} 0 & 0 & \kappa_{13}^0 \\ 0 & 0 & \kappa_{23}^0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (3.3)$$

$$\boldsymbol{\sigma}^0 \equiv \begin{bmatrix} \sigma_{11}^0 & \sigma_{12}^0 & 0 \\ \sigma_{21}^0 & \sigma_{22}^0 & 0 \\ 0 & 0 & \sigma_{33}^0 \end{bmatrix}, \quad \boldsymbol{\mu}^0 \equiv \begin{bmatrix} 0 & 0 & \mu_{13}^0 \\ 0 & 0 & \mu_{23}^0 \\ \mu_{31}^0 & \mu_{32}^0 & 0 \end{bmatrix}, \quad (3.4)$$

where: $\sigma_{33}^0 = \nu \sigma_{\alpha\alpha}^0 - \frac{\mu}{\mu + \lambda} \nu_T \theta^0$, $\mu_{3\alpha}^0 = \frac{\gamma - \varepsilon}{\gamma + \varepsilon} \mu_{\alpha 3}^0$,

And the matrixes representing $\boldsymbol{\varepsilon}^0, \boldsymbol{\sigma}^0$ are symmetric, $\nu = \frac{\lambda}{2(\mu + \lambda)}$ is the Poisson's ratio,

$\nu_T = (3\lambda + 2\mu) a_t$, where a_t is the linear thermal coefficient of the body, and $\mu, \lambda, \gamma, \varepsilon \in \mathbb{R}_+$ are the material coefficients of the body. Finally, the all components in relations (3.2) -(3.4) depend on the position $\mathbf{x} \equiv (x_1, x_2) \in \Omega$ and the time t ; and the complementary Schaefer-Ignaczak tensor fields take the forms:

$$\mathbf{u}' \equiv (u'_1, u'_2, 0), \quad \boldsymbol{\varphi}' \equiv (0, 0, \varphi'_3), \quad (3.5)$$

$$\boldsymbol{\gamma}' \equiv \begin{bmatrix} \gamma'_{11} & \gamma'_{12} & 0 \\ \gamma'_{21} & \gamma'_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\kappa}' \equiv \begin{bmatrix} 0 & 0 & \kappa'_{13} \\ 0 & 0 & \kappa'_{23} \\ 0 & 0 & 0 \end{bmatrix}, \quad (3.6)$$

$$\boldsymbol{\sigma}' \equiv \begin{bmatrix} \sigma'_{11} & \sigma'_{12} & 0 \\ \sigma'_{21} & \sigma'_{22} & 0 \\ 0 & 0 & \sigma'_{33} \end{bmatrix}, \quad \boldsymbol{\mu}' \equiv \begin{bmatrix} 0 & 0 & \mu'_{13} \\ 0 & 0 & \mu'_{23} \\ \mu'_{31} & \mu'_{32} & 0 \end{bmatrix}, \quad (3.7)$$

$$\text{where } \sigma'_{33} = \nu \sigma'_{\alpha\alpha} - \frac{\mu}{\mu + \lambda} \nu_T \theta', \quad \mu'_{3\alpha} = \frac{\gamma - \varepsilon}{\gamma + \varepsilon} \mu'_{\alpha 3},$$

The matrixes representing $\boldsymbol{\gamma}'$, $\boldsymbol{\kappa}'$, are asymmetric. $\boldsymbol{\sigma}'$, $\boldsymbol{\mu}'$

3.a. The classical Schaefer-Ignaczak initial-boundary value problem:

Consists of the following:

Classical Schaefer-Ignaczak equation of motion $\Omega \times \mathbb{T}^+$:

$$\hat{c}_2^2 (R_{\alpha, \beta}^0 + R_{\beta, \alpha}^0) - \ddot{\sigma}_{\alpha\beta}^0 + (\lambda \ddot{e}_1^0 - \nu_T \ddot{\theta}^0) \delta_{\alpha\beta} = 0, \quad (3.8)$$

$$(3.9) \quad \theta_{, \alpha\alpha}^0 - \frac{1}{\kappa} \dot{\theta}^0 - \eta_0 \dot{e}_1^0 = -\frac{Q}{\kappa},$$

$$\text{where: } R_{\alpha}^0 = \hat{R}_{\alpha}^0 + X_{\alpha}, \quad \hat{R}_{\alpha}^0 = \sigma_{\beta\alpha, \beta}^0,$$

$$\dot{e}_1^0 = \frac{1}{2(\mu + \lambda)} (\dot{\sigma}_{\varepsilon\varepsilon}^0 + 2\nu_T \dot{\theta}^0), \quad \ddot{e}_1^0 = \frac{1}{2(\mu + \lambda)} (\ddot{\sigma}_{\varepsilon\varepsilon}^0 + 2\nu_T \ddot{\theta}^0), \quad \hat{c}_2^2 = \frac{\mu}{\rho},$$

ρ , J are the density and micro polar inertia of the body, $\mathbf{X} \equiv (X_1, X_2, 0)$ is the volume force vector of the body, $\mathbf{Y} \equiv (0, 0, Y_3)$ is the volume moment vector of the body, Q is the heat sources, W is the heat quantity, in the volume unit and time unit, λ_0 is the heat conduction coefficient, $C_{\mathbf{e}}$ the specific heat for through constant body deformation;

The boundary condition on $\partial\Omega \times \mathbb{T}$:

$$\sigma_{\beta\alpha}^0 n_{\beta} = p_{\alpha}, \quad \theta^0 = \mathfrak{G}, \quad (3.10)$$

where n_{β} are the components of the outer unit normal vector: $\mathbf{n} \equiv (n_1, n_2, 0)$.

In the boundary condition (3.10) we refer that p_{α} and \mathfrak{G} are the given components of the total traction vector and the given total thermal load on the surface of the body,

The initial condition in $\Omega \times \{0\}$:

$$\boldsymbol{\sigma}^0 = \mathbf{sym} \boldsymbol{\sigma}^{(0)}, \quad \theta^0 = \ell, \quad \dot{\boldsymbol{\sigma}}^0 = \mathbf{sym} \dot{\boldsymbol{\sigma}}^{(0)}, \quad (3.11)$$

where the symbol: \mathbf{sym} is the symmetric part of tensor field and $\boldsymbol{\sigma}^{(0)}$, $\dot{\boldsymbol{\sigma}}^{(0)}$, ℓ are the given initial total force stresses and total force stresses velocity and the given total initial temperature [1,2];

The inverse constitutive relations in $\Omega \times \mathbb{T}$:

$$2\mu \varepsilon_{\alpha\beta}^0 = \sigma_{\alpha\beta}^0 - (\lambda e_1^0 - \nu_T \theta^0) \delta_{\alpha\beta}, \quad (3.12) \text{ where:}$$

$$e_1^0 = \frac{1}{2(\mu + \lambda)} (\sigma_{\varepsilon\varepsilon}^0 + 2\nu_T \theta^0);$$

The convolution relations in $\bar{\Omega} \times \mathbf{T}$:

$$u_\alpha^0 = g_\alpha t + f_\alpha + \rho^{-1}(t * R_\alpha^0), \quad (3.13)$$

$$\varphi_3^0 = \frac{1}{2} \in_{\alpha\beta} [g_\beta t + f_\beta + \rho^{-1}(t * R_\beta^0)], \alpha \quad (3.14)$$

where f_α, g_α are the given initial total values of the total displacements u_α and its velocities.

3.b. The complementary Schaefer-Ignaczak initial-boundary value problem:

The complementary Schaefer-Ignaczak initial-boundary value problem consists of the following:

The Schaefer-Ignaczak equations of motion in $\Omega \times \mathbf{T}^+$:

$$\begin{aligned} \square_2^* \rho^{-1} R'_{\alpha,\beta} + J^{-1} \in_{\alpha\beta} \mathcal{R}'_3 - \square_2^* \left\{ \frac{1}{2\mu} \ddot{\sigma}'_{(\beta\alpha)} + \frac{1}{2\alpha} \ddot{\sigma}'_{[\beta\alpha]} \right. \\ \left. - \frac{1}{2\mu} (\lambda \ddot{e}'_1 - \nu_T \ddot{\theta}') \delta_{\alpha\beta} \right\} = 0, \end{aligned} \quad (3.15)$$

$$c_4^2 \mathcal{R}'_{3,\alpha} - \square_2^* \ddot{\mu}'_{\alpha 3} = 0, \quad (61.3)$$

$$D\theta' - \eta_0 \dot{e}'_1 = -\frac{\hat{Q}}{\kappa}, \quad (71.3)$$

where: $R'_\alpha = \hat{R}'_\alpha + \hat{X}'_\alpha$, $\hat{R}'_\alpha = \sigma'_{\beta\alpha,\beta}$, $\hat{X}'_\alpha \equiv 0$,

$$\mathcal{R}'_3 = \hat{\mathcal{R}}'_3 + \hat{Y}'_3, \quad \hat{\mathcal{R}}'_3 = \square_2^* \hat{R}'_3, \quad \hat{R}'_3 = \in_{\alpha\beta} \sigma'_{\alpha\beta} + \mu'_{\beta 3,\beta},$$

$$\hat{Y}'_3 = \square_2^* Y_3 - \frac{1}{2} \square_4^* \in_{\alpha\beta} X_{\beta,\alpha}, \quad \hat{Q} = 0,$$

$$\dot{e}'_1 = \frac{1}{2(\mu + \lambda)} (\dot{\sigma}'_{\varepsilon\varepsilon} + 2\nu_T \dot{\theta}'), \quad \ddot{e}'_1 = \frac{1}{2(\mu + \lambda)} (\ddot{\sigma}'_{\varepsilon\varepsilon} + 2\nu_T \ddot{\theta}'),$$

and: $\square_2^* = \mu \Delta_1 - \rho \partial_t^2$, $\square_4^* = (\gamma + \varepsilon) \Delta_1 - J \partial_t^2$,

$$D = \Delta_1 - \frac{1}{\kappa} \partial_t, \quad \Delta_1 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}, \quad \partial := \frac{\partial}{\partial t},$$

The boundary conditions on $\partial\Omega \times \mathbf{T}$:

$$\sigma'_{\beta\alpha} n_\beta = 0, \quad \mu'_{\alpha 3} n_\alpha = m_3 - m_3^0, \quad \theta' = 0 \quad (3.18)$$

where: $m_3^0 = \mu_{\alpha 3}^0 n_\alpha$, in which $\mu_{\alpha 3}^0$ results from the classical initial conditions (3.14)

and the classical relations [1]:

$$\kappa_{\alpha 3}^0 = \frac{1}{2} \in_{\gamma \varepsilon} [g_{\varepsilon} t + f_{\varepsilon} + \rho^{-1}(t * R_{\varepsilon}^0)]_{,\alpha \gamma} \quad (3.19)$$

$$\kappa_{\alpha 3}^0 = \frac{1}{\gamma + \varepsilon} \mu_{\alpha 3}^0, \quad (3.20)$$

The initial conditions in $\Omega \times \{0\}$ for: (σ', μ', θ') :

$$\begin{aligned} \sigma' &= \text{skew } \sigma^{(0)}, \quad \mu' = \mu^{(0)} - \mu^{0(0)}, \quad \theta' = 0, \\ \dot{\sigma}' &= \text{skew } \dot{\sigma}^{(0)}, \quad \dot{\mu}' = \dot{\mu}^{(0)} - \dot{\mu}^{0(0)}, \end{aligned} \quad (3.21)$$

where the symbol **skew** denotes the antisymmetric part of the tensor field, and $\sigma^{(0)}, \dot{\sigma}^{(0)}, \mu^{0(0)}, \dot{\mu}^{0(0)}$ are the given initial total force stress fields and couple stress fields and its velocities [1,2];

The inverse constitutes relations in $\Omega \times T$:

$$\begin{aligned} \gamma'_{\alpha \beta} &= \frac{1}{2\mu} \sigma'_{(\alpha \beta)} + \frac{1}{2\alpha} \sigma'_{[\alpha \beta]} - \frac{1}{2\mu} (\lambda e'_1 - \nu_T \theta') \delta_{\alpha \beta}, \\ \kappa'_{\alpha 3} &= \frac{1}{\gamma + \varepsilon} \mu'_{\alpha 3}, \quad e'_1 = \frac{1}{2(\mu + \lambda)} (\sigma'_{\varepsilon \varepsilon} + 2\nu_T \theta'), \end{aligned} \quad (3.22)$$

The convolution relation in $\bar{\Omega} \times T$:

$$\begin{aligned} u'_\alpha &= \rho^{-1}(t * R'_\alpha), \\ \varphi'_3 &= (g_3 - \frac{1}{2} \in_{\alpha \beta} g_{\beta, \alpha}) + (f_3 - \frac{1}{2} \in_{\alpha \beta} f_{\beta, \alpha}) + \\ &+ J^{-1}[t * (\hat{R}'_3 + Y_3)] + J^{-1}(t * \hat{R}_3^0) - \frac{1}{2} \in_{\alpha \beta} \rho^{-1}(t * R_{\beta, \alpha}^0) \end{aligned} \quad (3.23)$$

For the requirements of the following section, we use to rewrite the following integro- differential equations relating to equations (3.8), (3.9), which are satisfied in $\Omega \times T$ [1,2]:

$$\begin{aligned} \hat{c}_2^2(t * R_{\alpha, \beta}^0 + t * R_{\beta, \alpha}^0) - \sigma_{\alpha \beta}^0 + (\lambda e_1^0 - \nu_T \theta^0) \delta_{\alpha \beta} &= \\ = -\mu [(g_{\alpha, \beta} + g_{\beta, \alpha}) t + (f_{\alpha, \beta} + f_{\beta, \alpha})] , \\ \int_0^t \theta'^0_{,\alpha \alpha}(\mathbf{x}, \tau) d\tau - \frac{1}{\kappa} \theta^0(\mathbf{x}, t) - \eta_0 e_1^0(\mathbf{x}, t) &= \\ = -\frac{1}{\kappa} \ell(\mathbf{x}) - \eta_0 f_{\varepsilon, \varepsilon}(\mathbf{x}) - \frac{1}{\kappa} \int_0^t Q(\mathbf{x}, \tau) d\tau \end{aligned} \quad (3.24)$$

where: $e_1^0 = \frac{1}{2(\mu + \lambda)} (\sigma_{\varepsilon \varepsilon}^0 + 2\nu_T \theta^0)$

In paper, For the requirements of this section, we need to derive the integro-differential equations relating to the complementary Schaefer-Ignaczak differential equations (3.22) together with the initial conditions (3.22) in paper [1], starting from the following complementary equations of motion in $\Omega \times T^+$ [1]:

$$\begin{aligned} \frac{1}{2} \rho^{-1} (R'_{\alpha, \beta} + R'_{\beta, \alpha}) - \frac{1}{2\mu} \ddot{\sigma}'_{(\alpha \beta)} + \frac{1}{2\mu} (\lambda \ddot{e}'_1 - \nu_T \ddot{\theta}'_1) \delta_{\alpha \beta} + \frac{1}{2\alpha} \ddot{\sigma}'_{[\alpha \beta]} + \\ + \frac{1}{2} \rho^{-1} (R^0_{\alpha, \beta} + R^0_{\beta, \alpha}) + J^{-1} \in_{\alpha \beta} (\hat{R}'_1 + Y_3) = 0, \\ D \theta' - \eta_0 \dot{e}'_1 = -\frac{\hat{Q}}{\kappa}, \end{aligned} \quad (3.25)$$

$$c_4^2 \left(\hat{R}'_3 + Y_3 \right)_{,\alpha} - \ddot{\mu}'_{\alpha 3} = -c_4^2 \hat{R}_{3,\alpha}^0 + \ddot{\mu}_{\alpha 3}^0 \quad (3.26)$$

where: $\hat{R}'_3 = \epsilon_{\alpha\beta} \sigma'_{\alpha\beta} + \mu'_{\beta 3,\beta}$ $R_3^0 = \mu_{\beta 3,\beta}^0$

Applying the convolution operation to the above equation, and using the complementary initial conditions (3.21), we arrive to the following complementary Schaefer-Ignaczak system of integro-differential equations in $\Omega \times T$:

$$\begin{aligned} & \frac{1}{2} t^* \rho^{-1} \left(R'_{\alpha,\beta} + R'_{\beta,\alpha} \right) - \frac{1}{2\mu} \sigma'_{(\alpha\beta)} + \frac{1}{2\alpha} \sigma'_{[\alpha\beta]} + \frac{1}{2\mu} (\lambda e'_1 - \nu_T \theta') \delta_{\alpha\beta} + \\ & + \frac{1}{2} \left[t (g_{\beta,\alpha} - g_{\alpha,\beta}) + (f_{\beta,\alpha} - f_{\alpha,\beta}) \right] + \frac{1}{2} t^* \rho^{-1} \left(R^0_{\alpha,\beta} - R^0_{\beta,\alpha} \right) + \\ & + \frac{1}{2} t^* \rho^{-1} \left(R'_{\alpha,\beta} - R'_{\beta,\alpha} \right) + J^{-1} t^* \epsilon_{\alpha,\beta} \left(\hat{R}'_3 + Y_3 \right) = 0, \quad (3.27) \\ & \int_0^t \theta'_{,\alpha\alpha}(\mathbf{x}, \tau) d\tau - \frac{1}{\kappa} \theta'(\mathbf{x}, t) - \eta_0 e'_1(\mathbf{x}, t) = -\frac{1}{\kappa} \int_0^t \hat{Q}(\mathbf{x}, \tau) d\tau, \\ & t^* C_4^2 \left(\hat{R}'_3 + Y_3 \right)_{,\alpha} - \mu'_{\alpha 3} = -t^* C_4^2 \hat{R}_{3,\alpha}^0 + \mu_{\alpha 3}^0 - (\gamma + \varepsilon) \cdot (g_3 t + t_3)_{,\alpha}, \end{aligned}$$

3.c. separating the classical Schaefer-Ignaczak equations of motion in $\Omega \times T^+$:

For this purpose, we need to derive the independent equation for the quantities: $e^0, \theta^0, \rho^{-1}(t^* R_\alpha^0) + g_\alpha t + f_\alpha$, proceeding as following. Taking the contract of the equations (3.24)₁, for $\alpha = \beta$ we found:

$$\rho^{-1} \left(t^* R_{\beta,\beta}^0 \right) + g_{\beta,\beta} t + f_{\beta,\beta} = e_1^0 \quad (3.28)$$

Taking the partial derivative of (3.24)₁ with respect to , we find that: x_β

$$\begin{aligned} & \hat{c}_2^2 (t^* R_{\alpha,\beta\beta}^0 + t^* R_{\beta,\alpha\beta}^0) - \sigma_{\alpha\beta,\beta}^0 + (\lambda e_1^0 - \nu_T \theta^0)_{,\beta} = \\ & = -\mu \left[(g_{\alpha,\beta\beta} + g_{\beta,\alpha\beta}) t + (f_{\alpha,\beta\beta} + f_{\beta,\alpha\beta}) \right], \end{aligned}$$

or

$$\begin{aligned} & \mu \left\{ \rho^{-1} \left(t^* R_{\alpha,\beta\beta}^0 \right) + g_{\alpha,\beta\beta} t + f_{\alpha,\beta\beta} + \rho^{-1} \left(t^* R_{\beta,\beta\alpha}^0 \right) + g_{\beta,\alpha\beta} t + f_{\beta,\beta\alpha} \right\} \\ & - R_\alpha^0 + X_\alpha = 0, \quad (3.29) \end{aligned}$$

where we use the relations:

$$-R_\alpha^0 = -\rho \frac{\partial^2}{\partial t^2} \left[t^* \left(\rho^{-1} R_\alpha^0 \right) + g_\alpha t + f_\alpha \right],$$

Equations (3.29) take the following operator form:

$$\square_2^* \left[\rho^{-1} t^* R_\alpha^0 + g_\alpha t + f_\alpha \right] + \left\{ \mu \left(\rho^{-1} t^* R_{\beta\beta}^0 + g_{\beta\beta} t + f_{\beta\beta} \right) + \lambda e_1^0 - \nu_T \theta^0 \right\}_{,\alpha} + X_\alpha = 0 \quad (3.30)$$

Where: $\square_2^* f = \mu \Delta_1 f - \rho \frac{\partial^2}{\partial t^2}$

Using (3.28), (3.29) equation (3.30) takes the form:

$$\square_2^* \left[\rho^{-1} t^* R_\alpha^0 + g_\alpha t + f_\alpha \right] + \left\{ (\mu + \lambda) e_1^0 - \nu_T \theta^0 \right\}_{,\alpha} + X_\alpha = 0 \quad (3.31)$$

Or the following form in $\Omega \times T$:

$$\square_2^* \left[\rho^{-1} t^* R_\alpha^0 + g_\alpha t + f_\alpha \right] + (\mu + \lambda) e_{1\alpha}^0 - v_T \theta_{,\alpha}^0 + X_\alpha = 0 \tag{3.32}$$

Now, taking the partial derivative of (3.32), with respect to x_α , we find:

$$\square_2^* \left[\rho^{-1} t^* R_{\alpha,\alpha}^0 + g_{\alpha,\alpha} t + f_{\alpha,\alpha} \right] + (\mu + \lambda) e_{1,\alpha\alpha}^0 - v_T \theta_{,\alpha\alpha}^0 + X_{\alpha,\alpha} = 0 \tag{3.33}$$

or:

$$\square_2^* e_1^0 = (\mu + \lambda) \Delta_1 e_1^0 - v_T \Delta_1 \theta^0 + X_{\alpha,\alpha} = 0$$

or:

$$\square_1 e_1^0 - v_T \Delta_1 \theta^0 + X_{\alpha,\alpha} = 0, \tag{3.34}$$

where: $\square_1 e_1^0 = (\lambda + 2\mu) \Delta_1 e_1^0 - \rho \frac{\partial^2}{\partial t^2} e_1^0$

Taking the partial derivative of (3.24)₂ with respect to the time " t " we find the following equation in $\Omega \times T^+$:

$$\Delta_1 \theta^0 - \frac{1}{\kappa} \frac{\partial}{\partial t} \theta^0 - \eta_0 \dot{e}_1^0 = -\frac{1}{\kappa} Q, \tag{3.35}$$

or:

$$D \theta^0 - \eta_0 \dot{e}_1^0 = -\frac{1}{\kappa} Q, \tag{3.36}$$

where: $D \theta^0 = \Delta_1 - \frac{1}{\kappa} \frac{\partial}{\partial t} \theta^0$

Now applying operator D to the equation (3.34) and using (3.36), we find the following equation in $\Omega \times T^+$:

$$D e_1^0 = -\left(D X_{\alpha,\alpha} + \frac{v_T}{\kappa} \Delta_1 Q \right), \tag{3.37}$$

Where: $D_2 = D \square_1 - v_T \eta_0 \Delta_1 \partial_t$

On other hand applying the operator \square_1 on the equation (3.34) and using (3.36), we arrive to the following equation in $\Omega \times T^+$:

$$D_2 \theta^0 = -\left(\eta_0 \partial_t X_{\alpha,\alpha} + \frac{1}{\kappa} \square_1 Q \right), \tag{3.38}$$

Finally, we find the last separating equations for the quantity: $\rho^{-1} t^* R_\alpha^0 + g_\alpha t + f_\alpha$, applying operator D_2 on equation (3.32), when we find the following separated equation in $\Omega \times T$:

$$D_2 \square_2^* \left(\rho^{-1} t^* R_\alpha^0 + g_\alpha t + f_\alpha \right) = -D_2 X_\alpha + [(\mu + \lambda) D - v_T \eta_0 \partial_t] X_{\beta,\beta\alpha} + \frac{v_T}{\kappa} [(\mu + \lambda) \Delta_1 - \square_1] Q_{,\alpha} \tag{3.39}$$

or:

$$D_2 \square_2^* \left(\rho^{-1} t^* R_\alpha^0 + g_\alpha t + f_\alpha \right) = -D_2 X_\alpha + D_1 X_{\gamma,\gamma\alpha} - \frac{v_T}{\kappa} \square_2^* Q_{,\alpha},$$

where: $D_1 = (\mu + \lambda)D - \nu_T \eta_0 \partial_t$

Finally let us rewrite the first of the equation (3.24) in the form:

$$\mu \left\{ \left[\rho^{-1} (t * R_\alpha^0) + g_\alpha t + f_\alpha \right]_{,\beta} + \left[\rho^{-1} (t * R_\beta^0) + g_\beta t + f_\beta \right]_{,\alpha} \right\} - \sigma_{\alpha\beta}^0 + (\lambda e_1^0 - \nu_T \theta^0) \delta_{\alpha\beta} = 0 \tag{3.40}$$

Now applying the double operator $D_2 \square_2^*$ on equation (3.40), and using equations (3.37)-(3.38), we arrive to the following separated equation in $\Omega \times T^+$:

$$D_2 \square_2^* \sigma_{\alpha\beta}^0 = -\mu D_2 (X_{\alpha,\beta} + X_{\beta,\alpha}) + 2\mu D_1 X_{\gamma,\gamma\alpha\beta} - 2\mu \frac{\nu_T}{\kappa} \square_2^* Q_{,\alpha\beta} + \square_2^* \left\{ (\mu D - D_1) X_{\gamma,\gamma} + \frac{\nu_T}{\kappa} (\square_1 - \lambda \Delta_1) Q \right\} \delta_{\alpha\beta} \tag{3.41}$$

3.d. separating the complementary Scheafer-Ignaczak equations of motion in $\Omega \times T^+$:

Using similar way as in the previous one, from complementary integro-differential equations (3.27), we arrive to the following independent equations for the quantities e'_1, θ' in $\Omega \times T^+$:

$$D_2 e'_1 = - \left(D \hat{X}_{\alpha,\alpha} + \frac{\nu_T}{\kappa} \Delta_1 \hat{Q} \right) \tag{3.42}$$

$$D \theta' = - \left(\frac{1}{\kappa} \square_1 \hat{Q} + \eta_0 \partial_1 X_{\alpha,\alpha} \right) \tag{3.43}$$

By the view of: $\hat{X}_\alpha = 0, \hat{Q} = 0$, the above equations take the form in $\Omega \times T^+$:

$$D_2 e'_1 = 0, D \theta' = 0 \tag{3.44}$$

Theorem: The complementary Scheafer-Ignaczak proses for the unbounded (E-N:5) occupying \square^2 , of free complementary stresses and free complementary temperature at ∞ , is isothermal proses.

Proof: Applying the double Furrier integral trans form of third order of the position (x_1, x_2) and time t [11], and using the properties of this trans form, we arrive to the following trans formed equations:

$$\begin{aligned} & - (\lambda + 2\mu) W_4(\xi, \tau) \bar{\theta}'(\xi, \tau) = 0, \\ & - (\lambda + 2\mu) W_4(\xi, \tau) \bar{e}'(\xi, \tau) = 0, \end{aligned} \tag{3.46}$$

where the symbol:

$$F_3 [f(\mathbf{x}, t)] = \bar{f}(\xi, \tau) := \frac{1}{2\pi\sqrt{2\pi}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(\mathbf{x}, t) e^{i(\mathbf{x} \cdot \xi + \tau t)} d\mathbf{x} dt$$

where: $\mathbf{x} = (x_1, x_2), \xi = (\xi_1, \xi_2), d\mathbf{x} = dx_1 dx_2, \mathbf{x} \cdot \xi = x_\alpha \xi_\alpha, i = \sqrt{-1}$

and: $W_4(\xi, \tau) = \xi^4 - [\sigma_1^2(\tau) + (1 + \epsilon)q(\tau)] \xi^2 + q(\tau) \sigma_1^2(\tau)$

The equation (3.44),(3.45) take the form:

$$\bar{\theta}'(\xi, \tau) = 0, \bar{e}'(\xi, \tau) = 0$$

Now applying the invers double Furrier integral trans form of third order for the parameters $(\xi_1, \xi_2) \tau$ [11], we arrive to the following tow identities:

$$\theta'(\mathbf{x}, t) = 0, \quad e'(\mathbf{x}, t) = 0 \quad (3.47)$$

From the above we conclude that the complementary temperature and dilatation vanish, so the complementary Scheafer-Ignaczak proses for the unbounded considerable 2D (E-N:5) body is isothermal.

The independent complementary Scheafer-Ignaczak equations for the unbounded considerable 2D (E-N:5) body:

Proceeding in similar way as in the previous subsection 3.a, from the complementary Scheafer-Ignaczak integro-differential equations and using (3.47), we arrive to the following complementary Scheafer-Ignaczak independent equations for the isothermal complementary fields $\sigma'_{\alpha\beta}, \mu'_{\alpha3}$ in $\Omega \times T(\Omega = \square^2)$:

$$\square_2^* L \sigma'_{\alpha\beta} = 2\alpha \left[(\mu + \alpha) \epsilon_{\alpha\beta} \hat{Y}_{3,\gamma\alpha} + (\mu - \alpha) \epsilon_{\alpha\gamma} \hat{Y}_{3,\gamma\alpha} \right] + 2\alpha \epsilon_{\alpha\beta} \square_2 \hat{Y}_3 \quad (3.48)$$

$$\square_2^* L \mu'_{\alpha3} = -(\gamma + \epsilon) \square_2 \hat{Y}_{3,\alpha} \quad (3.49)$$

3.e. Fourier classical and complementary formulas for the Scheafer-Ignaczak fields:

In order to obtain the classical and complementary Fourier formulas for the Scheafer-Ignaczak fields, we apply the double Fourier integral transform of third order to the suppurating classical and complementary equations for: $\sigma_{\alpha\beta}^0, \theta^0, \sigma'_{\alpha\beta}, \mu'_{\alpha3}$, taking in to account that the variables are x_1, x_2, t , and using the transform relations:

$$\mathbf{F}_3(f, \beta) = (-i \xi_\beta) \bar{f}, \quad \mathbf{F}_3(\partial_t f) = (-i \tau) \bar{f},$$

$$\mathbf{F}_3(\Delta_1 f) = -\xi^2 \bar{f}, \quad \mathbf{F}_3(\square_2^* f) = -\mu [\xi^2 - \hat{\sigma}_2^2(\tau)] \bar{f},$$

$$\mathbf{F}_3(\square_1 f) = -(\lambda + 2\mu) [\xi^2 - \sigma_1^2(\tau)] \bar{f},$$

$$\mathbf{F}_3(\mathbf{D}f) = -[\xi^2 - q(\tau)] \bar{f},$$

$$\mathbf{F}_3(\mathbf{D}_1 f) = -\{(\lambda + \mu) [\xi^2 - q(\tau)] - (\lambda + 2\mu) \epsilon q(\tau)\} \bar{f}$$

$$= -\{(\lambda + 2\mu) [\xi^2 - (1 + \epsilon) q(\tau)] - \mu [\xi^2 - q(\tau)]\} \bar{f},$$

$$\mathbf{F}_3(\mathbf{D}_2 f) = (\lambda + 2\mu) \{[\xi^2 - q(\tau)] [\xi^2 - \sigma_1^2(\tau)] - \epsilon q(\tau) \xi^2\} \bar{f}$$

$$= (\lambda + 2\mu) W_4(\xi, \tau) \bar{f},$$

$$\text{Where: } \xi = (\xi_\alpha \xi_\alpha)^{\frac{1}{2}}, \quad \sigma_1(\tau) = \frac{\tau}{c_1}, \quad \hat{\sigma}_2(\tau) = \frac{\tau}{\hat{c}_2}, \quad c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}},$$

$$\hat{c}_2 = \sqrt{\frac{\mu}{\rho}}, \quad q(\tau) = \frac{i\tau}{\kappa}, \quad m = \frac{v_T}{\lambda + 2\mu}, \quad \epsilon = m \kappa \eta_0,$$

$$W_4(\xi, \tau) = \xi^4 - [\sigma_1^2(\tau) + (1 + \epsilon) q(\tau)] \xi^2 + q(\tau) \sigma_1^2(\tau)$$

We arrive to:

- Classical transformed stresses and temperature:

$$\begin{aligned}
 \bar{\sigma}_{\alpha\beta}^0 &= \frac{(-i \xi_\beta) \bar{X}_\alpha + (-i \xi_\alpha) \bar{X}_\beta}{\xi^2 - \hat{\sigma}_2^2(\tau)} + \frac{2}{(\lambda + 2\mu) [\xi^2 - \hat{\sigma}_2^2(\tau)] W_4(\xi, \tau)} \times \\
 &\times \{ (\lambda + 2\mu) [\xi^2 - (1+\epsilon) q(\tau)] - \mu [\xi^2 - q(\tau)] \} \cdot (-i \xi_\gamma) \cdot (-i \xi_\alpha) \cdot (-i \xi_\beta) \cdot \bar{X}_\gamma \\
 &- 2\mu \frac{\nu_T}{\kappa} \cdot \frac{1}{(\lambda + 2\mu) [\xi^2 - \hat{\sigma}_2^2(\tau)] W_4(\xi, \tau)} \cdot [\xi^2 - \sigma_1^2(\tau)] \cdot (-i \xi_\alpha) \cdot (-i \xi_\beta) \cdot \bar{Q} + \\
 &+ \frac{1}{(\lambda + 2\mu) W_4(\xi, \tau)} \left\{ \left[-\mu (\xi^2 - q(\tau)) + (\lambda + 2\mu) [\xi^2 - (1+\epsilon) q(\tau)] \right. \right. \\
 &\left. \left. - \mu (\xi^2 - q(\tau)) \right] \cdot (-i \xi_\gamma) \bar{X}_\gamma + \frac{\nu_T}{\kappa} \left[-(\lambda + 2\mu) [\xi^2 - q(\tau)] + \lambda \xi^2 \right] \cdot \bar{Q} \right\} \delta_{\alpha\beta} \\
 \bar{\theta}^0 &= \frac{\kappa \eta_0}{(\lambda + 2\mu) W_4(\xi, \tau)} q(\tau) (-i \xi_\alpha) \bar{X}_\alpha + \\
 &+ \frac{1}{\kappa W_4(\xi, \tau)} [\xi^2 - \sigma_1^2(\tau)] \bar{Q}
 \end{aligned} \tag{3.50}$$

$$\tag{3.51}$$

On the other hand, applying the double Fourier transform of the third order to the complementary Schaefer-Ignaczak equations (3.48), (3.49), and using the transform relations:

$$\begin{aligned}
 \mathbf{F}_3(\square_2 f) &= -(\mu + \alpha) [\xi^2 - \sigma_2^2(\tau)] \bar{f}, \\
 \mathbf{F}_3(\square_4 f) &= - [(\gamma + \epsilon) \xi^2 + 4\alpha - J\tau^2] \bar{f} = \\
 &= -(\gamma + \epsilon) [\xi^2 + \nu_0^2 - \sigma_4^2(\tau)] \bar{f}, \\
 \mathbf{F}_3(Lf) &= \mathbf{F}_3[(\square_2 \square_4 + 4\alpha^2 \Delta_1) f] = \\
 &= (\mu + \alpha)(\gamma + \epsilon) [\xi^2 - \sigma_2^2(\tau)] [\xi^2 + \nu_0^2 - \sigma_4^2(\tau)] \bar{f} \\
 &\quad - 4\alpha^2 \xi^2 \bar{f} = \\
 (\mu + \alpha)(\gamma + \epsilon) \{ &[\xi^2 - \sigma_2^2(\tau)] [\xi^2 + \nu_0^2 - \sigma_4^2(\tau)] - p s \} \bar{f} \\
 &= (\mu + \alpha)(\gamma + \epsilon) \Delta_4(\xi; \tau),
 \end{aligned}$$

where:

$$\begin{aligned}
 \Delta_4(\xi; \tau) &= \xi^4 - [\sigma_2^2(\tau) + \sigma_4^2(\tau) + \eta_0^2 - \nu_0^2] \xi^2 + \\
 &\quad + \sigma_2^2(\tau) [\sigma_4^2(\tau) - \nu_0^2], \\
 c_4 &= \sqrt{\frac{\gamma + \epsilon}{J}}, \quad c_2 = \sqrt{\frac{\mu + \alpha}{\rho}}, \quad \sigma_4(\tau) = \frac{\tau}{c_4}, \quad \sigma_2(\tau) = \frac{\tau}{c_2} \\
 \nu_0^2 &= 2p = \frac{4\alpha}{\gamma + \epsilon}, \quad \eta_0^2 = p s, \quad s = \frac{2\alpha}{\mu + \alpha}
 \end{aligned}$$

We arrive to the following complementary Fourier transforms for the stresses: $\sigma'_{\alpha\beta}, \mu'_{\alpha\beta}$:

$$\begin{aligned}
 -\mu [\xi^2 - \hat{\sigma}_2^2(\tau)] (\mu + \alpha)(\gamma + \epsilon) \Delta_4(\xi; \tau) \bar{\sigma}'_{\alpha\beta} &= \\
 2\alpha \left[(\mu + \alpha) (-i \xi_\alpha) (-i \xi_\gamma) \in_{\beta\gamma} \bar{Y}_3 + (\mu - \alpha) (-i \xi_\beta) (-i \xi_\gamma) \in_{\alpha\gamma} \bar{Y}_3 \right] \\
 -2\alpha (\mu + \alpha) \in_{\alpha\beta} [\xi^2 - \sigma_2^2(\tau)] \bar{Y}_3,
 \end{aligned}$$

$$-\mu(\mu+\alpha)(\gamma+\varepsilon)[\xi^2 - \hat{\sigma}_2^2(\tau)]\Delta_4(\xi; \tau)\bar{\mu}'_{\alpha 3} =$$

$$(\gamma+\varepsilon)(\mu+\alpha)[\xi^2 - \sigma_2^2(\tau)](-i\xi_\alpha)\bar{Y}_3, \quad (3.52)$$

or:

$$\bar{\sigma}'_{\alpha\beta} = \frac{-2\alpha}{\mu(\mu+\alpha)(\gamma+\varepsilon)\Delta_4(\xi; \tau)} \cdot \left[(\mu+\alpha)(-i\xi_\alpha)(-i\xi_\gamma) \in_{\beta\gamma} \bar{Y}_3 + \right.$$

$$\left. + (\mu-\alpha)(-i\xi_\beta)(-i\xi_\gamma) \in_{\alpha\gamma} \bar{Y} \right] +$$

$$+ \frac{2\alpha}{\mu(\gamma+\varepsilon)[\xi^2 - \hat{\sigma}_2^2(\tau)]\Delta_4(\xi; \tau)} \cdot \in_{\alpha\beta} [\xi^2 - \sigma_2^2(\tau)] \bar{Y}_3, \quad (3.52)$$

$$\bar{\mu}'_{\alpha 3} = \frac{-1}{\mu[\xi^2 - \hat{\sigma}_2^2(\tau)]\Delta_4(\xi; \tau)} \cdot [\xi^2 - \sigma_2^2(\tau)](-i\xi_\alpha)\bar{Y} \quad (3.53)$$

Now, suppose that:

$$\hat{F}_2(\mathbf{x}, t) := \mathbf{F}_3^{-1} \left[\frac{1}{\xi^2 - \hat{\sigma}_2^2(\tau)} \right], \quad F_1(\mathbf{x}, t) := \mathbf{F}_3^{-1} \left[\frac{1}{\xi^2 - \sigma_2^2(\tau)} \right]$$

$$G_2(\mathbf{x}, t) := \mathbf{F}_3^{-1} \left[\frac{1}{W_4(\xi, \tau)} \right], \quad G_1(\mathbf{x}, t) := \mathbf{F}_3^{-1} \left[\frac{1}{\Delta_4(\xi, \tau)} \right]$$

applying the inverse double Fourier transform with respect to $(\xi; \tau)$, then using the Fourier convolution theorem [11], we arrive to the following:

- Classical Schaefer-Ignaczak Fourier formulas for $\sigma_{\alpha\beta}^0, \theta^0$:

$$\sigma_{\alpha\beta}^0 = \partial_\beta(\hat{F}_2 * X_\alpha) + \partial_\alpha(\hat{F}_2 * X_\beta) - \frac{2}{\lambda+2\mu} \partial_\gamma \partial_\beta \partial_\alpha D_1(\hat{F}_2 * G_2 * X_\gamma)$$

$$) 54.3(- \frac{2\mu\nu_T}{\kappa} \partial_\alpha \partial_\beta (G_2 * Q) +$$

$$+ \frac{1}{\lambda+2\mu} \left\{ [\mu D - D_1] \partial_\alpha (G_2 * X_\gamma) + \frac{\nu_T}{\kappa} [(\lambda+2\mu)D - \lambda\Delta_1] (G_2 * Q) \right\} \delta_{\alpha\beta}$$

$$\theta^0 = - \frac{\kappa\eta_0}{(\lambda+2\mu)\kappa} \partial_t \partial_\alpha (G_2 * X_\alpha) - \frac{1}{\kappa(\lambda+2\mu)} \square (G_2 * Q) \quad (3.55)$$

- Complementary Schaefer-Ignaczak Fourier formulas for $\sigma'_{\alpha\beta}, \mu'_{\alpha 3}$:

$$\sigma'_{\alpha\beta} = - \frac{2\alpha}{\mu(\mu+\alpha)(\gamma+\varepsilon)} \left[(\mu+\alpha) \in_{\beta\gamma} \partial_\alpha + (\mu-\alpha) \in_{\alpha\gamma} \partial_\beta \right] \partial_\gamma (\hat{F}_2 * G_1 * \hat{Y}_3) -$$

$$\frac{2\alpha}{\mu(\mu+\alpha)(\gamma+\varepsilon)} \in_{\alpha\beta} \square_2 (\hat{F}_2 * G_1 * \hat{Y}_3), \quad (3.56)$$

$$\mu'_{\alpha 3} = \frac{1}{\mu(\mu+\alpha)} \square_2 (\hat{F}_2 * G_1 * \hat{Y}_3) \quad (3.57)$$

Remark1: We can arrive to the corresponding classical Schaefer-Ignaczak fields: $(u_\alpha^0, \varphi_3^0, \varepsilon_{\alpha\beta}^0, \kappa_{\alpha 3}^0, \mu_{\alpha 3}^0)$ using the Schaefer-Ignaczak relation (3.2), (3.11)-(3.14), (3.20), (3.21), and we arrive to the following results:

$$u_\alpha^0 = \frac{1}{\mu} (\hat{F}_2 * X_\alpha) - \frac{1}{\mu(\lambda+2\mu)} D_1 \partial_\alpha \partial_\beta (\hat{F}_2 * G_2 * X_\beta)$$

$$- \frac{m}{\kappa} \partial_\alpha (G_2 * Q), \quad (3.58)$$

$$\varphi_3^0 = \frac{1}{2\mu} \in_{\alpha\beta} \partial_\alpha (\hat{F}_2 * X_\beta) \quad , \quad (3.59)$$

$$\varepsilon_{\alpha\beta}^0 = \frac{1}{2\mu} \left[(\hat{F}_2 * X_\alpha)_{,\beta} + (\hat{F}_2 * X_\beta)_{,\alpha} \right] - \frac{1}{\mu(\lambda + 2\mu)} D_1 \partial_{\alpha\beta\gamma}^3 (\hat{F}_2 * G_2 * X_\gamma) - \frac{m}{\kappa} \partial_{\alpha\beta}^2 (G_2 * Q) \quad , \quad (3.60)$$

$$\mu_{\alpha 3}^0 = \frac{(\gamma + \varepsilon)}{2\mu} \in_{\gamma\beta} \partial_{\alpha\gamma}^2 (\hat{F}_2 * X_\beta) \quad , \quad (3.61)$$

$$\kappa_{\alpha 3}^0 = \frac{1}{2\mu} \in_{\gamma\beta} \partial_{\alpha\gamma}^2 (\hat{F}_2 * X_\beta) \quad (3.62)$$

Remark2: we also can arrive to the complementary Scheafer-Ignaczak fields $(u'_\alpha, \varphi'_3, \gamma'_{\alpha\beta}, \kappa'_{\alpha 3}, \mu'_{\alpha 3})$, using Scheafer-Ignaczak relations (3.22) -(3.24), when we arrive to the following results:

$$u'_\alpha = -\frac{2\alpha}{\mu(\gamma + \varepsilon)(\mu + \alpha)} \in_{\alpha\gamma} \partial_\gamma (\hat{F}_2 * G_1 * \hat{Y}_3) \quad (3.63)$$

$$\varphi'_3 = \frac{1}{\mu(\mu + \alpha)(\gamma + \varepsilon)} \square_2 (\hat{F}_2 * G_1 * \hat{Y}_3) \quad (3.64)$$

$$) 65.3(\gamma'_{\alpha\beta} = \frac{1}{\mu(\mu + \alpha)(\gamma + \varepsilon)} \left[-2\alpha \in_{\beta\gamma} \partial_{\alpha\gamma}^2 + \in_{\beta\alpha} \square_2 \right] (\hat{F}_2 * G_1 * \hat{Y}_3)$$

$$\kappa'_{\alpha 3} = \frac{1}{\mu(\mu + \alpha)(\gamma + \varepsilon)} \partial_\alpha \square_2 (\hat{F}_2 * G_1 * \hat{Y}_3) \quad (3.66)$$

Remark3: Finally substituting the results for: $(\sigma_{\alpha\beta}^0, \theta^0, u_\alpha^0, \varphi_3^0, \varepsilon_{\alpha\beta}^0, \kappa_{\alpha 3}^0, \mu_{\alpha 3}^0)$ and for:

$(\sigma'_{\alpha\beta}, u'_\alpha, \varphi'_3, \gamma'_{\alpha\beta}, \kappa'_{\alpha 3}, \mu'_{\alpha 3})$ in the relation (3.1), we arrive the final solution

$(\sigma_{\alpha\beta}, \theta, u_\alpha, \varphi_3, \kappa_{\alpha 3}, \gamma_{\alpha\beta}, \mu_{\alpha 3})$, in which:

$$\begin{aligned} \sigma_{\alpha\beta} &= \partial_\beta (\hat{F}_2 * X_\alpha) + \partial_\alpha (\hat{F}_2 * X_\beta) - \frac{2}{\lambda + 2\mu} \partial_\gamma \partial_\beta \partial_\alpha D_1 (\hat{F}_2 * G_2 * X_\gamma) \\ &- \frac{2\mu\nu_T}{\kappa} \partial_\alpha \partial_\beta (G_2 * Q) + \\ &+ \frac{1}{\lambda + 2\mu} \left\{ [\mu D - D_1] \partial_\alpha (G_2 * X_\gamma) + \frac{\nu_T}{\kappa} [(\lambda + 2\mu)D - \lambda\Delta_1] (G_2 * Q) \right\} \delta_{\alpha\beta} \\ &- \frac{2\alpha}{\mu(\mu + \alpha)(\gamma + \varepsilon)} \left[(\mu + \alpha) \in_{\beta\gamma} \partial_\alpha + (\mu - \alpha) \in_{\alpha\gamma} \partial_\beta \right] \partial_\gamma (\hat{F}_2 * G_1 * \hat{Y}_3) \\ &- \frac{2\alpha}{\mu(\mu + \alpha)(\gamma + \varepsilon)} \in_{\alpha\beta} \square_2 (\hat{F}_2 * G_1 * \hat{Y}_3) \quad , \end{aligned} \quad (3.67)$$

$$\theta = -\frac{\eta_0}{(\lambda + 2\mu)} \partial_t \partial_\alpha (G_2 * X_\alpha) - \frac{1}{\kappa(\lambda + 2\mu)} \square (G_2 * Q) \quad , \quad (3.68)$$

$$) 69.3(\mu_{\alpha 3} = \frac{\gamma + \varepsilon}{2\mu} \in_{\gamma\beta} \partial_{\alpha\gamma}^2 (\hat{F}_2 * X_\beta) + \frac{1}{\mu(\mu + \alpha)} \square_2 \partial_\alpha (\hat{F}_2 * G_1 * \hat{Y}_3) ,$$

$$\gamma_{\alpha\beta} = \frac{1}{2\mu} \left[(\hat{F}_2 * X_\alpha)_{,\beta} + (\hat{F}_2 * X_\beta)_{,\alpha} \right] - \frac{1}{\mu(\lambda + 2\mu)} D_1 \partial_{\alpha\beta\gamma}^3 (\hat{F}_2 * G_2 * X_\gamma) - \frac{m}{\kappa} \partial_{\alpha\beta}^2 (G_2 * Q) + \frac{1}{\mu(\mu + \alpha)(\gamma + \varepsilon)} \left[-2\alpha \in_{\beta\gamma} \partial_{\alpha\gamma}^2 + \in_{\beta\alpha} \square_2 \right] (\hat{F}_2 * G_1 * \hat{Y}_3), \quad (3.70)$$

$$\kappa_{\alpha 3} = \frac{1}{2\mu} \in_{\gamma\beta} \partial_{\alpha\gamma}^2 (\hat{F}_2 * X_\beta) + \frac{1}{\mu(\mu + \alpha)(\gamma + \varepsilon)} \partial_\alpha \square_2 (\hat{F}_2 * G_1 * \hat{Y}_3), \quad (3.71)$$

$$u_\alpha = \frac{1}{\mu} (\hat{F}_2 * X_\alpha) - \frac{1}{\mu(\lambda + 2\mu)} D_1 \partial_\alpha \partial_\beta (\hat{F}_2 * G_2 * X_\beta) - \frac{m}{\kappa} \partial_\alpha (G_2 * Q) - \frac{2\alpha}{\mu(\gamma + \varepsilon)(\mu + \alpha)} \in_{\alpha\gamma} \partial_\gamma (\hat{F}_2 * G_1 * \hat{Y}_3), \quad (3.72)$$

$$\varphi_3 = \frac{1}{2\mu} \in_{\alpha\beta} \partial_\alpha (\hat{F}_2 * X_\beta) + \frac{1}{\mu(\mu + \alpha)(\gamma + \varepsilon)} \square_2 (\hat{F}_2 * G_1 * \hat{Y}_3) \quad (3.73)$$

3. Conclusions and suggestions:

In paper, first we prove the isotherm of the complementary Schaefer-Ignaczak of the 2D(E-N:5) plane micropolar and unbounded body. Next for the same unbounded 2D(E-N:5) plane thermodynamical body, we derive the Fourier classical and complementary tensorial fields, using the combination of the transform theory and the method of Schaefer-Ignaczak. This result may have impotency in material resistance and composites theory.

Suggestions: we can suggest the following problems for discussing:

1. discuss the green functions (Fundamental Solutions) for the above Schaefer-Ignaczak initial-boundary problems.
2. Reputing the above problem for the second state of plane elastic strains

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