



Enhanced Academic Stress-Coping Skills Assessment in College Students: A Comparative Study of Neutrosophic Distance Measure and Proposed Cubic Pythagorean Fuzzy Hypersoft TOPSIS Method

E. Prabu^{1*}, M. Gopala Krishnan², A. Bobin³

¹ Department of Mathematics, Erode Arts and Science College, Erode, Tamil Nadu, India.

² Department of Mathematics, Erode Arts and Science College, Erode, Tamil Nadu, India.

³ Department of Mathematics, IFET College of Engineering, Villupuram, Tamil Nadu, India.

Emails: prbumathsrep@gmail.com; gkkrishnanmsc@gmail.com; bobinalbert@gmail.com

Abstract

In multi-criteria decision-making scenarios involving real numbers, interval numbers, and a combination of membership and non-membership grades, accurate decision-making is crucial yet challenging. The integration of diverse grade values into a single value poses a significant challenge for decision-makers. To address this issue, this study introduces the concept of a cubic Pythagorean fuzzy hypersoft set, facilitating information aggregation without ambiguity. The characteristics of correlation coefficients and aggregation operators are emphasized, underscoring their importance in decision-making processes. An algorithm based on correlation coefficients (CC) is proposed for the TOPSIS method, which ranks preferences based on their similarity to the ideal solution, applied here to examine how college students cope with academic stress. Furthermore, the efficiency of the proposed method is demonstrated through a comparative study, wherein the correlation coefficient in the TOPSIS method is contrasted with existing distance measures (DMs). Results indicate the superiority of CC in the TOPSIS method over DMs. In addition to comparing the proposed method with existing distance measures, the efficacy of the proposed approach is further demonstrated through a comparative analysis with established neutrosophic distance measures. This comprehensive evaluation highlights the robustness and versatility of the proposed method in addressing the complexities of multi-criteria decision-making scenarios, particularly in assessing stress management strategies among college students, thus providing valuable contributions to decision-making contexts. This study contributes to enhancing decision-making processes, particularly in evaluating stress management strategies among college students, thereby offering valuable insights for academic contexts.

Keywords: hypersoft set; Pythagorean fuzzy set; cubic Pythagorean fuzzy set

1 introduction

With the seminal introduction of the fuzzy set (FS) by Zadeh,¹ the study of uncertainty witnessed a significant surge in scientific exploration. This foundational concept paved the way for numerous theoretical extensions and generalizations, propelling its application across various domains. These extensions include interval-valued FS (IVFS),² intuitionistic FS (IFS),³ cubic set,⁴ Pythagorean FS (PFS),⁵ interval-valued PFS (IVPFS),⁶ cubic PFS (CPFS),⁷ among others. Notably, the introduction of the soft set (SS) by Molodsov⁸ revolutionized the parameterization of subsets within any universal set. Furthermore, Smarandache⁹ elucidated the concept of hypersoft set (HSS), underscoring its significance over traditional SS.

Songsaeng and Iampan¹⁰ delved into the intricate relations among neutrosophic cubic UP-subalgebras, shedding light on their interplay and significance within mathematical frameworks. Khan et al.¹¹ introduced the

innovative concept of b-picture fuzzy SS (b-PFSS) and its generalization, which is based on bijective SS, contributing to the advancement of fuzzy set theory. In a related vein, Chinnadurai et al.¹² proposed a solution to multi-criteria decision-making (MCDM) quandaries in candidate selection processes using cubic soft matrices (CSM), underscoring its efficacy in real-world decision scenarios. Building upon this, Chinnadurai and Bobin¹³ innovated the reversal ranking method employing max-min operations within CSM, offering a novel approach to decision-making optimization. Abbas et al.¹⁴ introduced CPFS, delineating membership degrees in IVPFS and non-membership degrees in PFS, thus enriching the understanding of hybrid fuzzy structures. Expanding the horizons of fuzzy set theory, Alhazaymeh et al.¹⁵ elucidated the concepts of internal-cubic vague sets (CVS) and external-CVS through illustrative examples, providing insights into their practical applications. Khan et al.¹⁶ explored the realm of Pythagorean cubic FS (PCFS), where membership and non-membership degrees manifest in cubic structures, extending the versatility of fuzzy set representations. Tehreem et al.¹⁷ contributed to this domain by establishing operational laws and Einstein weighted geometric aggregation operators on PCFS, enhancing its applicability in decision-making processes. Additionally, Kaur and Garg^{18,19} defined and investigated the properties of cubic IFS (CIFS) and generalized CIFS (GCIFS), offering a comprehensive understanding of their mathematical underpinnings. Joshi²⁰ further expanded the theoretical framework by discussing generalized PFS (GPFS) and defining average aggregation operators on GPFS, contributing to the arsenal of tools for fuzzy set analysis. The works of Bobin et al.,²⁷ Chinnadurai et al.,²⁸ Chinnadurai and Bobin and²⁹ contribute to the ongoing discourse in decision science by presenting innovative methodologies that extend the applicability of the TOPSIS method to diverse decision-making scenarios characterized by uncertainty, ambiguity, and complexity. Lastly, Zulqarnain et al.²¹⁻²³ introduced and explored the concepts of intuitionistic fuzzy HSS (IFHSS) and Pythagorean fuzzy HSS (PFHSS), as well as presented practical applications using aggregation operators on Pythagorean fuzzy hypersoft weighted average (PFHSSWA) and geometric (PFHSSWG) operators, highlighting their utility in decision-making contexts.

The motivation driving this study is to underscore the paramount significance of CPFS. To illustrate, we turn to the values delineated in Table 3, all of which adhere to the prescribed restriction conditions: $(0 \leq (\Upsilon_{\Gamma(\tilde{\psi})}(u))^2 + (\bar{\Lambda}_{\Gamma(\tilde{\psi})}(u))^2 \leq 1$ and $0 \leq (\Upsilon_{\Gamma(\tilde{\psi})}(u))^2 + (\Lambda_{\Gamma(\tilde{\psi})}(u))^2 \leq 1$) when presented within a CPFS environment, while failing to meet these criteria in a cubic IFS (CIFS) setting. This disparity underscores the heightened significance of CPFS over CIFS, as it enables decision-makers to furnish data within a CPFS environment with minimal restrictions. Moreover, we employ aggregation operators and TOPSIS approach based on CC to evaluate alternatives using cubic Pythagorean fuzzy HSS (CPFHSS) data. Remarkably, to our knowledge, the theory, related developments, and applications of CPFHSS constitute an entire research field. Consequently, the novel approach proposed in this study warrants examination and stands to offer decision-makers a practical solution. Specifically, we provide a pertinent methodology for analyzing students' stress-coping mechanism skills using the CPFHSS TOPSIS method. Additionally, we conduct a comparative study, replacing the CC in existing decision-making models, to demonstrate the effectiveness of our suggested method. Thus, CPFHSS emerges as a reliable tool for predicting uncertainty, particularly when grades are expressed in terms of membership and non-membership grades for specified attributes in a combination of interval and real number form.

This manuscript is structured into several sections, each contributing to the exploration and application of cubic Pythagorean fuzzy hypersoft sets (CPFHSS). Section 2 provides concise definitions, laying the groundwork for the subsequent discussions. In Section 3, we introduce the concept of CPFHSS, elaborating on various characteristic configurations (CC) and weighted CC of CPFHSS features. Moving forward, Section 4 outlines the operators of cubic Pythagorean fuzzy hypersoft weighted average (CPFHSSWA) and cubic Pythagorean fuzzy hypersoft weighted geometric (CPFHSSWG), elucidating their functionalities. In Section 5, we delve into the integration of CC with TOPSIS technique, showcasing its application in decision-making contexts. Furthermore, Section 6 underscores the significance of our proposed strategy through comparative analyses, highlighting its effectiveness over existing methodologies. Finally, Section 7 encapsulates our findings and contributions, with a conclusion to this study.

2 Preliminaries

To facilitate a clearer understanding of this study, we introduce the following notations: Let \mathcal{U} denote the universe, where $u_i \in \mathcal{U}$, and $P(\mathcal{U})$ signify the power set of \mathcal{U} . The symbol \mathbb{N} represents the set of natural numbers, while $[0,1]$ denotes the closed interval of real numbers inclusive of both endpoints. $C[0, 1]$ refers to

the collection encompassing all closed subintervals within [0,1]. Moreover, $\tilde{\Upsilon}\Gamma(u)$ and $\tilde{\Lambda}\Gamma(u)$ represent closed subintervals of [0,1], depicting the membership and non-membership grades, respectively, of the element $u \in \mathcal{U}$. Similarly, $\Upsilon\Gamma(u)$ and $\Lambda\Gamma(u)$ denote the membership and non-membership grades of the element $u \in \mathcal{U}$. Furthermore, \mathcal{C}^U signifies the collection of CPFS over \mathcal{U} .

Definition 2.1. ⁷ A CPFS can be represented as $\Gamma = \{(\langle \tilde{\Upsilon}\Gamma(u), \Upsilon\Gamma(u) \rangle, \langle \tilde{\Lambda}\Gamma(u), \Lambda\Gamma(u) \rangle), u \in \mathcal{U}\}$, where $\tilde{\Upsilon}\Gamma(u) : \mathcal{U} \rightarrow C[0, 1]$, $\tilde{\Lambda}\Gamma(u) : \mathcal{U} \rightarrow C[0, 1]$, $\Upsilon\Gamma(u) : \mathcal{U} \rightarrow [0, 1]$ and $\Lambda\Gamma(u) : \mathcal{U} \rightarrow [0, 1]$. The lower and upper ends of $\tilde{\Upsilon}\Gamma(u)$ and $\tilde{\Lambda}\Gamma(u)$ are denoted, respectively by $\underline{\Upsilon}\Gamma(u)$, $\overline{\Upsilon}\Gamma(u)$ and $\underline{\Lambda}\Gamma(u)$, $\overline{\Lambda}\Gamma(u)$, where $0 \leq (\overline{\Upsilon}\Gamma(u))^2 + (\overline{\Lambda}\Gamma(u))^2 \leq 1$ and $0 \leq (\Upsilon\Gamma(u))^2 + (\Lambda\Gamma(u))^2 \leq 1$.

Definition 2.2. ⁹ Let $\Theta_1, \Theta_2, \dots, \Theta_k$ be attribute sets and the sub-attributes can be represented as $\Theta_1 = \{\psi_{11}, \psi_{12}, \dots, \psi_{1l}\}$, $\Theta_2 = \{\psi_{21}, \psi_{22}, \dots, \psi_{2m}\}, \dots, \Theta_k = \{\psi_{k1}, \psi_{k2}, \dots, \psi_{kn}\}$, where $1 \leq l \leq x$, $1 \leq m \leq y$, $1 \leq n \leq z$ and $x, y, z \in \mathbb{N}$, such that $\Theta_p \cap \Theta_q = \emptyset$, for each $p, q \in \{1, 2, \dots, k\}$ and $p \neq q$. Then the collection of multi-attributes is given as $\Theta_1 \times \Theta_2 \times \dots \times \Theta_k = \tilde{\Theta} = \{\psi_{1l} \times \psi_{2m} \times \dots \times \psi_{kn}\}$. A pair $(\Gamma, \tilde{\Theta})$ is called a HSS over \mathcal{V} , if there exist a mapping $\Gamma : \tilde{\Theta} \rightarrow P(\mathcal{U})$. HSS is given by $(\Gamma, \tilde{\Theta}) = \{(\tilde{\psi}, \Gamma(\tilde{\psi})) | \tilde{\psi} \in \tilde{\Theta}, \Gamma(\tilde{\psi}) \in P(\mathcal{U})\}$.

3 Cubic Pythagorean fuzzy hypersoft set

The notion of cubic Pythagorean fuzzy hypersoft set CPFHSS and some fundamental aspects of CC and WCC on CPFHSS are given below.

Definition 3.1. A pair $(\Gamma, \tilde{\Theta})$ is called a CPFHSS over \mathcal{U} , if there exists a mapping $\Gamma : \tilde{\Theta} \rightarrow \mathcal{C}^U$. CPFHSS is represented as $(\Gamma, \tilde{\Theta}) = \{(\tilde{\psi}, \Gamma(\tilde{\psi})) | \tilde{\psi} \in \tilde{\Theta}, \Gamma(\tilde{\psi}) \in \mathcal{C}^U\}$, where $\Gamma(\tilde{\psi}) = \{(\langle \tilde{\Upsilon}_{\Gamma(\tilde{\psi})}(u), \Upsilon_{\Gamma(\tilde{\psi})}(u) \rangle, \langle \tilde{\Lambda}_{\Gamma(\tilde{\psi})}(u), \Lambda_{\Gamma(\tilde{\psi})}(u) \rangle) | u \in \mathcal{U}\}$, where $\tilde{\Upsilon}_{\Gamma(\tilde{\psi})}(u) : \mathcal{U} \rightarrow C[0, 1]$, $\tilde{\Lambda}_{\Gamma(\tilde{\psi})}(u) : \mathcal{U} \rightarrow C[0, 1]$, $\Upsilon_{\Gamma(\tilde{\psi})}(u) : \mathcal{U} \rightarrow [0, 1]$ and $\Lambda_{\Gamma(\tilde{\psi})}(u) : \mathcal{U} \rightarrow [0, 1]$. The lower and upper ends of $\tilde{\Upsilon}_{\Gamma(\tilde{\psi})}(u)$ and $\tilde{\Lambda}_{\Gamma(\tilde{\psi})}(u)$ are denoted, respectively by $\underline{\Upsilon}_{\Gamma(\tilde{\psi})}(u)$, $\overline{\Upsilon}_{\Gamma(\tilde{\psi})}(u)$ and $\underline{\Lambda}_{\Gamma(\tilde{\psi})}(u)$, $\overline{\Lambda}_{\Gamma(\tilde{\psi})}(u)$, where $0 \leq (\overline{\Upsilon}_{\Gamma(\tilde{\psi})}(u))^2 + (\overline{\Lambda}_{\Gamma(\tilde{\psi})}(u))^2 \leq 1$ and $0 \leq (\Upsilon_{\Gamma(\tilde{\psi})}(u))^2 + (\Lambda_{\Gamma(\tilde{\psi})}(u))^2 \leq 1$.

Example 3.2. Considering the present COVID-19 pandemic, a team of psychiatrists evaluate the students in two sessions to understand their stress-coping levels. A psychiatrist may have the following hindrances. (i) to provide values in interval form for the first session. (ii) to provide values in real number fuzzy form for the second session. (iii) to validate the intuitionistic restriction conditions for each session. Considering these hindrances, let's assume that the psychiatrist provides the values in interval for the first session (Table 1), with $0 \leq (\overline{\Upsilon}_{\Gamma(\tilde{\psi})}(u))^2 + (\overline{\Lambda}_{\Gamma(\tilde{\psi})}(u))^2 \leq 1$ and in real values (Table 2) for the second session with $0 \leq (\Upsilon_{\Gamma(\tilde{\psi})}(u))^2 + (\Lambda_{\Gamma(\tilde{\psi})}(u))^2 \leq 1$. Combine these values to form cubic Pythagorean fuzzy values as in Table 3.

Let a set of psychiatrists be represented as $\mathcal{U} = \{p_1, p_2, p_3, p_4\}$. They are responsible to examine students based on academic stress-coping skills. Let $\Theta_1, \Theta_2, \Theta_3$ and Θ_4 be distinct attribute sets and sub-attributes are given by

- $\Theta_1 =$ initial phase = $\{\psi_{11} = \text{sensitivity to stress}\}$,
- $\Theta_2 =$ intermediate phase = $\{\psi_{21} = \text{capacity for relaxation}, \psi_{22} = \text{self-reliance}\}$,
- $\Theta_3 =$ advanced phase = $\{\psi_{31} = \text{proactive mindset}, \psi_{32} = \text{adaptability and versatility}\}$ and
- $\Theta_4 =$ final phase = $\{\psi_{41} = \text{ability to evaluate situations}\}$.

Then $\tilde{\Theta} = \Theta_1 \times \Theta_2 \times \Theta_3 \times \Theta_4$ is the distinct attribute set given by

$$\begin{aligned} \tilde{\Theta} &= \Theta_1 \times \Theta_2 \times \Theta_3 \times \Theta_4 = \{\psi_{11}\} \times \{\psi_{21}, \psi_{22}\} \times \{\psi_{31}, \psi_{32}\} \times \{\psi_{41}\}. \\ &= \left\{ (\psi_{11}, \psi_{21}, \psi_{31}, \psi_{41}), (\psi_{11}, \psi_{21}, \psi_{32}, \psi_{41}), (\psi_{11}, \psi_{22}, \psi_{31}, \psi_{41}), (\psi_{11}, \psi_{22}, \psi_{32}, \psi_{41}) \right\}. \\ &= \left\{ \tilde{\psi}_1, \tilde{\psi}_2, \tilde{\psi}_3, \tilde{\psi}_4 \right\}. \end{aligned}$$

Then the values given by the psychiatrists for each student is in the form of CPFHSS.

Table 1: Shows stress-coping values in interval for first session.

\mathcal{U}	$\tilde{\psi}_1$	$\tilde{\psi}_2$	$\tilde{\psi}_3$	$\tilde{\psi}_4$
p_1	$\langle [0.64, 0.66], [0.72, 0.73] \rangle$	$\langle [0.91, 0.92], [0.32, 0.35] \rangle$	$\langle [0.36, 0.42], [0.61, 0.63] \rangle$	$\langle [0.48, 0.59], [0.49, 0.52] \rangle$
p_2	$\langle [0.72, 0.75], [0.41, 0.45] \rangle$	$\langle [0.52, 0.56], [0.81, 0.82] \rangle$	$\langle [0.19, 0.22], [0.91, 0.94] \rangle$	$\langle [0.11, 0.15], [0.91, 0.96] \rangle$
p_3	$\langle [0.22, 0.25], [0.81, 0.84] \rangle$	$\langle [0.77, 0.79], [0.46, 0.47] \rangle$	$\langle [0.42, 0.46], [0.72, 0.77] \rangle$	$\langle [0.34, 0.37], [0.64, 0.68] \rangle$
p_4	$\langle [0.37, 0.47], [0.73, 0.75] \rangle$	$\langle [0.24, 0.46], [0.49, 0.69] \rangle$	$\langle [0.58, 0.62], [0.71, 0.73] \rangle$	$\langle [0.54, 0.57], [0.71, 0.75] \rangle$

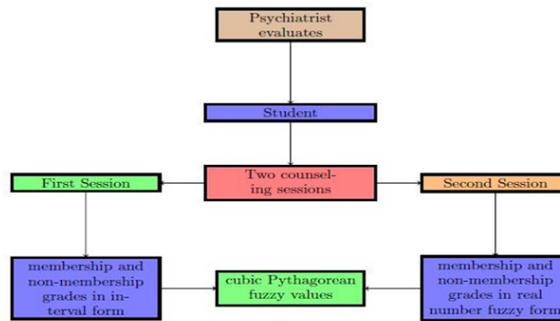
Table 2: Shows stress-coping values in fuzzy for second session.

\mathcal{U}	$\tilde{\psi}_1$	$\tilde{\psi}_2$	$\tilde{\psi}_3$	$\tilde{\psi}_4$
p_1	$\langle 0.49, 0.79 \rangle$	$\langle 0.81, 0.45 \rangle$	$\langle 0.57, 0.52 \rangle$	$\langle 0.69, 0.49 \rangle$
p_2	$\langle 0.55, 0.75 \rangle$	$\langle 0.25, 0.84 \rangle$	$\langle 0.34, 0.81 \rangle$	$\langle 0.23, 0.71 \rangle$
p_3	$\langle 0.72, 0.45 \rangle$	$\langle 0.34, 0.82 \rangle$	$\langle 0.56, 0.66 \rangle$	$\langle 0.41, 0.69 \rangle$
p_4	$\langle 0.56, 0.61 \rangle$	$\langle 0.55, 0.71 \rangle$	$\langle 0.49, 0.59 \rangle$	$\langle 0.64, 0.43 \rangle$

Table 3: Shows stress-coping skills of a student in CPFHSS $(\Gamma, \tilde{\Theta})$ form.

\mathcal{U}	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle \langle [0.64, 0.66], 0.49 \rangle, \langle [0.72, 0.73], 0.79 \rangle \rangle$	$\langle \langle [0.91, 0.92], 0.81 \rangle, \langle [0.32, 0.35], 0.45 \rangle \rangle$
p_2	$\langle \langle [0.72, 0.75], 0.55 \rangle, \langle [0.41, 0.45], 0.75 \rangle \rangle$	$\langle \langle [0.52, 0.56], 0.25 \rangle, \langle [0.81, 0.82], 0.84 \rangle \rangle$
p_3	$\langle \langle [0.22, 0.25], 0.72 \rangle, \langle [0.81, 0.84], 0.45 \rangle \rangle$	$\langle \langle [0.77, 0.79], 0.34 \rangle, \langle [0.46, 0.47], 0.82 \rangle \rangle$
p_4	$\langle \langle [0.37, 0.47], 0.56 \rangle, \langle [0.73, 0.75], 0.61 \rangle \rangle$	$\langle \langle [0.24, 0.46], 0.55 \rangle, \langle [0.49, 0.69], 0.71 \rangle \rangle$

\mathcal{U}	$\tilde{\psi}_3$	$\tilde{\psi}_4$
p_1	$\langle \langle [0.36, 0.42], 0.57 \rangle, \langle [0.61, 0.63], 0.52 \rangle \rangle$	$\langle \langle [0.48, 0.59], 0.69 \rangle, \langle [0.49, 0.52], 0.49 \rangle \rangle$
p_2	$\langle \langle [0.19, 0.22], 0.34 \rangle, \langle [0.91, 0.94], 0.81 \rangle \rangle$	$\langle \langle [0.11, 0.15], 0.23 \rangle, \langle [0.91, 0.96], 0.71 \rangle \rangle$
p_3	$\langle \langle [0.42, 0.46], 0.56 \rangle, \langle [0.72, 0.77], 0.66 \rangle \rangle$	$\langle \langle [0.34, 0.37], 0.41 \rangle, \langle [0.64, 0.68], 0.69 \rangle \rangle$
p_4	$\langle \langle [0.58, 0.62], 0.49 \rangle, \langle [0.71, 0.73], 0.59 \rangle \rangle$	$\langle \langle [0.54, 0.57], 0.64 \rangle, \langle [0.71, 0.75], 0.43 \rangle \rangle$



3.1 Correlation coefficient of CPFHSS

Let CPFHSSs over \mathcal{U} be represented as.

$$(\Gamma_1, \tilde{\Theta}_1) = \{ (u_i, \langle [\underline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_k)(u_i), \bar{\Upsilon}_{\Gamma_1}(\tilde{\psi}_k)(u_i)], \Upsilon_{\Gamma_1}(\tilde{\psi}_k)(u_i) \rangle, \langle [\underline{\Lambda}_{\Gamma_1}(\tilde{\psi}_k)(u_i), \bar{\Lambda}_{\Gamma_1}(\tilde{\psi}_k)(u_i)], \Lambda_{\Gamma_1}(\tilde{\psi}_k)(u_i) \rangle) \},$$

$$(\Gamma_2, \tilde{\Theta}_2) = \{ (u_i, \langle [\underline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_k)(u_i), \bar{\Upsilon}_{\Gamma_2}(\tilde{\psi}_k)(u_i)], \Upsilon_{\Gamma_2}(\tilde{\psi}_k)(u_i) \rangle, \langle [\underline{\Lambda}_{\Gamma_2}(\tilde{\psi}_k)(u_i), \bar{\Lambda}_{\Gamma_2}(\tilde{\psi}_k)(u_i)], \Lambda_{\Gamma_2}(\tilde{\psi}_k)(u_i) \rangle) \}.$$

Definition 3.3. The cubic Pythagorean fuzzy informational energies of $(\Gamma_1, \tilde{\Theta}_1)$ and $(\Gamma_2, \tilde{\Theta}_2)$ are represented as

$$\Phi(\Gamma_1, \tilde{\Theta}_1) = \sum_{k=1}^u \sum_{i=1}^n \left[(\underline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_k)(u_i))^2 + (\underline{\Lambda}_{\Gamma_1}(\tilde{\psi}_k)(u_i))^2 + (\bar{\Upsilon}_{\Gamma_1}(\tilde{\psi}_k)(u_i))^2 + (\bar{\Lambda}_{\Gamma_1}(\tilde{\psi}_k)(u_i))^2 + (\Upsilon_{\Gamma_1}(\tilde{\psi}_k)(u_i))^2 + (\Lambda_{\Gamma_1}(\tilde{\psi}_k)(u_i))^2 \right], \tag{1}$$

$$\Phi(\Gamma_2, \tilde{\Theta}_2) = \sum_{k=1}^u \sum_{i=1}^n \left[(\underline{\Upsilon}_{\Gamma_2(\tilde{\psi}_k)}(u_i))^2 + (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_k)}(u_i))^2 + (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_k)}(u_i))^2 + (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_k)}(u_i))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_k)}(u_i))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_k)}(u_i))^2 \right]. \tag{2}$$

Definition 3.4. The correlation measure between $(\Gamma_1, \tilde{\Theta}_1)$ and $(\Gamma_2, \tilde{\Theta}_2)$ is given as

$$\begin{aligned} \mathcal{C}_\Upsilon((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) &= \sum_{k=1}^u \sum_{i=1}^v \left[(\underline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_i) * (\underline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_i)) + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_i) * (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_i)) \right. \\ &\quad + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_i) * (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_i)) + (\overline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_i) * (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_i)) \\ &\quad \left. + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_i) * (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_i)) + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_i) * (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_i)) \right]. \tag{3} \end{aligned}$$

Proposition 3.5. Let $(\Gamma_1, \tilde{\Theta}_1)$ and $(\Gamma_2, \tilde{\Theta}_2)$ be two CPFHSS. Then,

- (i) $\mathcal{C}_\Upsilon((\Gamma_1, \tilde{\Theta}_1), (\Gamma_1, \tilde{\Theta}_1)) = \Phi(\Gamma_1, \tilde{\Theta}_1)$
- (ii) $\mathcal{C}_\Upsilon((\Gamma_2, \tilde{\Theta}_2), (\Gamma_2, \tilde{\Theta}_2)) = \Phi(\Gamma_2, \tilde{\Theta}_2)$.

Proof. Straight forward □

Definition 3.6. The CC between $(\Gamma_1, \tilde{\Theta}_1)$ and $(\Gamma_2, \tilde{\Theta}_2)$ is defined as $\mathcal{C}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = \frac{\mathcal{C}_\Upsilon((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))}{\sqrt{\Phi(\Gamma_1, \tilde{\Theta}_1)}\sqrt{\Phi(\Gamma_2, \tilde{\Theta}_2)}}$

Proposition 3.7. Let $(\Gamma_1, \tilde{\Theta}_1)$ and $(\Gamma_2, \tilde{\Theta}_2)$ represent CPFHSSs. Then,

- (i) $0 \leq \mathcal{C}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq 1$;
- (ii) $\mathcal{C}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = \mathcal{C}_C((\Gamma_2, \tilde{\Theta}_2), (\Gamma_1, \tilde{\Theta}_1))$;
- (iii) If $(\Gamma_1, \tilde{\Theta}_1) = (\Gamma_2, \tilde{\Theta}_2)$, then $\mathcal{C}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = 1$.

Proof. (i) Clearly, $\mathcal{C}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \geq 0$.
So, we show the proof of $\mathcal{C}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq 1$.

$$\begin{aligned} &\mathcal{C}_\Upsilon((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \\ &= \sum_{p=1}^u \sum_{q=1}^v \left[(\underline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q) * (\underline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q)) + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q) * (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q)) + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q) * (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q)) \right. \\ &\quad \left. + (\overline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q) * (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q)) + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q) * (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q)) + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q) * (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q)) \right]. \\ &= \sum_{p=1}^u \left[\left((\underline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_1) * (\underline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_1)) + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_1) * (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_1)) + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_1) * (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_1)) \right. \right. \\ &\quad \left. \left. + (\overline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_1) * (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_1)) + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_1) * (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_1)) + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_1) * (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_1)) \right) \right. \\ &\quad + \left((\underline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_2) * (\underline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_2)) + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_2) * (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_2)) + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_2) * (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_2)) \right. \\ &\quad \left. + (\overline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_2) * (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_2)) + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_2) * (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_2)) + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_2) * (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_2)) \right) + \dots \\ &\quad + \left((\underline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_v) * (\underline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_v)) + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_v) * (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_v)) + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_v) * (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_v)) \right. \\ &\quad \left. + (\overline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_v) * (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_v)) + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_v) * (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_v)) + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_v) * (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_v)) \right) \left. \right]. \end{aligned}$$

By using Cauchy-Schwarz inequality,
 $\mathcal{C}_M((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))^2$

$$\begin{aligned} &\leq \sum_{p=1}^u \left[\left\{ (\underline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_1))^2 + (\underline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_2))^2 + \dots + (\underline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_v))^2 \right\} \right. \\ &\quad + \left\{ (\underline{\Delta}_{\Gamma_1(\tilde{\psi}_p)}(u_1))^2 + (\underline{\Delta}_{\Gamma_1(\tilde{\psi}_p)}(u_2))^2 + \dots + (\underline{\Delta}_{\Gamma_1(\tilde{\psi}_p)}(u_v))^2 \right\} \\ &\quad + \left\{ (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_1))^2 + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_2))^2 + \dots + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_v))^2 \right\} \\ &\quad + \left\{ (\overline{\Delta}_{\Gamma_1(\tilde{\psi}_p)}(u_1))^2 + (\overline{\Delta}_{\Gamma_1(\tilde{\psi}_p)}(u_2))^2 + \dots + (\overline{\Delta}_{\Gamma_1(\tilde{\psi}_p)}(u_v))^2 \right\} \\ &\quad + \left\{ (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_1))^2 + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_2))^2 + \dots + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_v))^2 \right\} \\ &\quad + \left. \left\{ (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_1))^2 + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_2))^2 + \dots + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_v))^2 \right\} \right] \\ &\times \sum_{p=1}^u \left[\left\{ (\underline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_1))^2 + (\underline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_2))^2 + \dots + (\underline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_v))^2 \right\} \right. \\ &\quad + \left\{ (\underline{\Delta}_{\Gamma_2(\tilde{\psi}_p)}(u_1))^2 + (\underline{\Delta}_{\Gamma_2(\tilde{\psi}_p)}(u_2))^2 + \dots + (\underline{\Delta}_{\Gamma_2(\tilde{\psi}_p)}(u_v))^2 \right\} \\ &\quad + \left\{ (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_1))^2 + (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_2))^2 + \dots + (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_v))^2 \right\} \\ &\quad + \left\{ (\overline{\Delta}_{\Gamma_2(\tilde{\psi}_p)}(u_1))^2 + (\overline{\Delta}_{\Gamma_2(\tilde{\psi}_p)}(u_2))^2 + \dots + (\overline{\Delta}_{\Gamma_2(\tilde{\psi}_p)}(u_v))^2 \right\} \\ &\quad + \left\{ (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_1))^2 + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_2))^2 + \dots + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_v))^2 \right\} \\ &\quad + \left. \left\{ (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_1))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_2))^2 + \dots + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_v))^2 \right\} \right]. \end{aligned}$$

$$\mathcal{C}_M((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))^2$$

$$\begin{aligned} &\leq \sum_{p=1}^u \sum_{q=1}^v \left[(\underline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Delta}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Delta}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right. \\ &\quad + \left. (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right] \times \sum_{p=1}^u \sum_{q=1}^v \left[(\underline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Delta}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right. \\ &\quad + \left. (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Delta}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right]. \end{aligned}$$

$$\Rightarrow \mathcal{C}_M((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))^2 \leq \Phi(\Gamma_1, \tilde{\Theta}_1) \times \Phi(\Gamma_2, \tilde{\Theta}_2).$$

$$\Rightarrow \mathcal{C}_M((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq \sqrt{\Phi(\Gamma_1, \tilde{\Theta}_1)} \times \sqrt{\Phi(\Gamma_2, \tilde{\Theta}_2)}.$$

$$\Rightarrow \frac{\mathcal{C}_M((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))}{\sqrt{\Phi(\Gamma_1, \tilde{\Theta}_1)} \times \sqrt{\Phi(\Gamma_2, \tilde{\Theta}_2)}} \leq 1.$$

By Definition 3.6, $\mathcal{C}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq 1$.

Thus, $0 \leq \mathcal{C}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq 1$. □

Proof. (ii) Straight forward. □

$$\text{Proof. (iii) } \mathcal{C}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = \frac{\mathcal{C}_M((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))}{\sqrt{\Phi(\Gamma_1, \tilde{\Theta}_1)} \times \sqrt{\Phi(\Gamma_2, \tilde{\Theta}_2)}}.$$

Since, $(\Gamma_1, \tilde{\Theta}_1) = (\Gamma_2, \tilde{\Theta}_2)$.

$$\mathcal{C}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))$$

$$\begin{aligned} & \sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right. \\ & \quad \left. + (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right] \\ = & \frac{\sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right. \\ & \quad \left. + (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right]}{\sqrt{\left\{ \sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right. \right. \\ & \quad \left. \left. + (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right] \right\}}} \\ & \times \sqrt{\left\{ \sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right. \right. \\ & \quad \left. \left. + (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right] \right\}} \end{aligned}$$

$\Rightarrow \mathcal{C}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = 1.$ □

Definition 3.8. The CC between $(\Gamma_1, \tilde{\Theta}_1)$ and $(\Gamma_2, \tilde{\Theta}_2)$ is defined as

$$\tilde{\mathcal{C}}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = \frac{\mathcal{C}_R((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))}{\max \left\{ \Phi(\Gamma_1, \tilde{\Theta}_1), \Phi(\Gamma_2, \tilde{\Theta}_2) \right\}}. \tag{4}$$

$$\begin{aligned} \tilde{\mathcal{C}}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = & \frac{\sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q)) + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q)) \right. \\ & + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q)) + (\overline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q)) \\ & \left. + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q)) + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q)) \right]}{\max \left\{ \sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right. \right. \\ & \quad \left. \left. + (\overline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right], \right. \\ & \left. \sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right. \right. \\ & \quad \left. \left. + (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right] \right\}} \end{aligned}$$

Proposition 3.9. Let $(\Gamma_1, \tilde{\Theta}_1)$ and $(\Gamma_2, \tilde{\Theta}_2)$ be CPFHSSs. Then,

- (i) $0 \leq \tilde{\mathcal{C}}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq 1$;
- (ii) $\tilde{\mathcal{C}}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = \tilde{\mathcal{C}}_C((\Gamma_2, \tilde{\Theta}_2), (\Gamma_1, \tilde{\Theta}_1))$;
- (iii) If $(\Gamma_1, \tilde{\Theta}_1) = (\Gamma_2, \tilde{\Theta}_2)$, then $\tilde{\mathcal{C}}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = 1$.

Proof. (i) Clearly, $\tilde{\mathcal{C}}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \geq 0$.
 So, we show the proof of $\tilde{\mathcal{C}}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq 1$.
 $\mathcal{C}_R((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))$

$$\begin{aligned}
 &= \sum_{p=1}^u \sum_{q=1}^v \left[(\underline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_q)) * (\underline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_q)) + (\underline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_q)) * (\underline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_q)) + (\overline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_q)) * (\overline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_q)) \right. \\
 &\quad \left. + (\overline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_q)) * (\overline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_q)) + (\Upsilon_{\Gamma_1}(\tilde{\psi}_p)(u_q)) * (\Upsilon_{\Gamma_2}(\tilde{\psi}_p)(u_q)) + (\Lambda_{\Gamma_1}(\tilde{\psi}_p)(u_q)) * (\Lambda_{\Gamma_2}(\tilde{\psi}_p)(u_q)) \right]. \\
 &= \sum_{p=1}^u \left[\left((\underline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_1)) * (\underline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_1)) + (\underline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_1)) * (\underline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_1)) + (\overline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_1)) * (\overline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_1)) \right) \right. \\
 &\quad \left. + (\overline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_1)) * (\overline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_1)) + (\Upsilon_{\Gamma_1}(\tilde{\psi}_p)(u_1)) * (\Upsilon_{\Gamma_2}(\tilde{\psi}_p)(u_1)) + (\Lambda_{\Gamma_1}(\tilde{\psi}_p)(u_1)) * (\Lambda_{\Gamma_2}(\tilde{\psi}_p)(u_1)) \right) \\
 &\quad + \left((\underline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_2)) * (\underline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_2)) + (\underline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_2)) * (\underline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_2)) + (\overline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_2)) * (\overline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_2)) \right) \\
 &\quad \left. + (\overline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_2)) * (\overline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_2)) + (\Upsilon_{\Gamma_1}(\tilde{\psi}_p)(u_2)) * (\Upsilon_{\Gamma_2}(\tilde{\psi}_p)(u_2)) + (\Lambda_{\Gamma_1}(\tilde{\psi}_p)(u_2)) * (\Lambda_{\Gamma_2}(\tilde{\psi}_p)(u_2)) \right) + \dots \\
 &\quad + \left((\underline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_v)) * (\underline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_v)) + (\underline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_v)) * (\underline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_v)) + (\overline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_v)) * (\overline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_v)) \right) \\
 &\quad \left. + (\overline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_v)) * (\overline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_v)) + (\Upsilon_{\Gamma_1}(\tilde{\psi}_p)(u_v)) * (\Upsilon_{\Gamma_2}(\tilde{\psi}_p)(u_v)) + (\Lambda_{\Gamma_1}(\tilde{\psi}_p)(u_v)) * (\Lambda_{\Gamma_2}(\tilde{\psi}_p)(u_v)) \right)].
 \end{aligned}$$

By using Cauchy-Schwarz inequality,

$$\mathcal{C}_M((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))$$

$$\begin{aligned}
 &\leq \left\{ \sum_{p=1}^u \left[\left\{ (\underline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_1))^2 + (\underline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_2))^2 + \dots + (\underline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_v))^2 \right\} \right. \right. \\
 &\quad \left. \left. + \left\{ (\underline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_1))^2 + (\underline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_2))^2 + \dots + (\underline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_v))^2 \right\} \right. \right. \\
 &\quad \left. \left. + \left\{ (\overline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_1))^2 + (\overline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_2))^2 + \dots + (\overline{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_v))^2 \right\} \right. \right. \\
 &\quad \left. \left. + \left\{ (\overline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_1))^2 + (\overline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_2))^2 + \dots + (\overline{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_v))^2 \right\} \right. \right. \\
 &\quad \left. \left. + \left\{ (\Upsilon_{\Gamma_1}(\tilde{\psi}_p)(u_1))^2 + (\Upsilon_{\Gamma_1}(\tilde{\psi}_p)(u_2))^2 + \dots + (\Upsilon_{\Gamma_1}(\tilde{\psi}_p)(u_v))^2 \right\} \right. \right. \\
 &\quad \left. \left. + \left\{ (\Lambda_{\Gamma_1}(\tilde{\psi}_p)(u_1))^2 + (\Lambda_{\Gamma_1}(\tilde{\psi}_p)(u_2))^2 + \dots + (\Lambda_{\Gamma_1}(\tilde{\psi}_p)(u_v))^2 \right\} \right] \right\} \times \\
 &\sum_{p=1}^u \left[\left\{ (\underline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_1))^2 + (\underline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_2))^2 + \dots + (\underline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_v))^2 \right\} \right. \\
 &\quad \left. + \left\{ (\underline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_1))^2 + (\underline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_2))^2 + \dots + (\underline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_v))^2 \right\} \right. \\
 &\quad \left. + \left\{ (\overline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_1))^2 + (\overline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_2))^2 + \dots + (\overline{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_v))^2 \right\} \right. \\
 &\quad \left. + \left\{ (\overline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_1))^2 + (\overline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_2))^2 + \dots + (\overline{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_v))^2 \right\} \right. \\
 &\quad \left. + \left\{ (\Lambda_{\Gamma_2}(\tilde{\psi}_p)(u_1))^2 + (\Lambda_{\Gamma_2}(\tilde{\psi}_p)(u_2))^2 + \dots + (\Lambda_{\Gamma_2}(\tilde{\psi}_p)(u_v))^2 \right\} \right. \\
 &\quad \left. + \left\{ (\Upsilon_{\Gamma_2}(\tilde{\psi}_p)(u_1))^2 + (\Upsilon_{\Gamma_2}(\tilde{\psi}_p)(u_2))^2 + \dots + (\Upsilon_{\Gamma_2}(\tilde{\psi}_p)(u_v))^2 \right\} \right. \\
 &\quad \left. + \left\{ (\Lambda_{\Gamma_2}(\tilde{\psi}_p)(u_1))^2 + (\Lambda_{\Gamma_2}(\tilde{\psi}_p)(u_2))^2 + \dots + (\Lambda_{\Gamma_2}(\tilde{\psi}_p)(u_v))^2 \right\} \right] \right\}^{\frac{1}{2}}.
 \end{aligned}$$

$$\mathcal{C}_M((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))$$

$$\begin{aligned} &\leq \left\{ \sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right. \right. \\ &\quad \left. \left. + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right] \right. \\ &\quad \times \sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right. \\ &\quad \left. \left. + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right] \right\}^{\frac{1}{2}}. \\ &\leq \left\{ \left(\max \left\{ \sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right. \right. \right. \right. \\ &\quad \left. \left. + (\overline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right] \right. \right. \\ &\quad \times \sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right. \\ &\quad \left. \left. + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right] \right\}^{\frac{1}{2}}. \\ &= \max \left\{ \sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right. \right. \\ &\quad \left. \left. + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right] \right. \\ &\quad \times \sum_{p=1}^u \sum_{q=1}^v \left[(\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\overline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right. \\ &\quad \left. \left. + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right] \right\}. \\ &\Rightarrow \mathcal{C}_M((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq \max \left\{ \Phi(\Gamma_1, \tilde{\Theta}_1) \times \Phi(\Gamma_2, \tilde{\Theta}_2) \right\}. \\ &\Rightarrow \frac{\mathcal{C}_M((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))}{\max \left\{ \Phi(\Gamma_1, \tilde{\Theta}_1) \times \Phi(\Gamma_2, \tilde{\Theta}_2) \right\}} \leq 1. \end{aligned}$$

By using Definition 3.8, $\tilde{\mathcal{C}}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq 1$.

Hence, $0 \leq \tilde{\mathcal{C}}_C((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq 1$.

Proofs of (ii) and (iii) can be worked out with the proposed method given in Proposition 3.7. □

3.2 Weighted correlation coefficient for CPFHSS

Decision-makers (DMs) may assign different weights for each alternative, to facilitate the same, WCC plays a significant role which is presented in the following section. Consider $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_u\}$ and $\mathcal{W} = \{\mathcal{W}_1, \mathcal{W}_2, \dots, \mathcal{W}_v\}$ as weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_p, \mathcal{W}_q > 0$ and $\sum_{p=1}^u \mathcal{D}_p = 1, \sum_{q=1}^v \mathcal{W}_q = 1$.

Definition 3.10. The WCC between $(\Gamma_1, \tilde{\Theta}_1)$ and $(\Gamma_2, \tilde{\Theta}_2)$ can be represented as

$$\mathcal{C}_{C_{\mathcal{W}}}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = \frac{\mathcal{C}_{\Upsilon}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))}{\sqrt{\Phi(\Gamma_1, \tilde{\Theta}_1)}\sqrt{\Phi(\Gamma_2, \tilde{\Theta}_2)}} \tag{5}$$

$$\mathcal{C}_{C\mathcal{W}}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))$$

$$\begin{aligned} & \sum_{p=1}^u \mathcal{D}_p \left(\sum_{q=1}^v \mathcal{W}_q \left[\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q) * \Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q) + \underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q) * \underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q) \right. \right. \\ & \quad \left. \left. + \bar{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q) * \bar{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q) + \bar{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q) * \bar{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q) \right. \right. \\ & \quad \left. \left. + \Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q) * \Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q) + \Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q) * \Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q) \right] \right) \\ = & \frac{\sum_{p=1}^u \mathcal{D}_p \left(\sum_{q=1}^v \mathcal{W}_q \left[(\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\bar{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right. \right. \\ & \quad \left. \left. + (\bar{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right] \right)}{\sqrt{\left\{ \sum_{p=1}^u \mathcal{D}_p \left(\sum_{q=1}^v \mathcal{W}_q \left[(\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\bar{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right. \right. \right. \\ & \quad \left. \left. + (\bar{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right] \right\}}} \\ & \times \sqrt{\left\{ \sum_{p=1}^u \mathcal{D}_p \left(\sum_{q=1}^v \mathcal{W}_q \left[(\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\bar{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right. \right. \right. \\ & \quad \left. \left. + (\bar{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right] \right\}}. \end{aligned}$$

Proposition 3.11. Let $(\Gamma_1, \tilde{\Theta}_1)$ and $(\Gamma_2, \tilde{\Theta}_2)$ be CPFHSSs. Then,

(i) $0 \leq \mathcal{C}_{C\mathcal{W}}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq 1$;

(ii) $\mathcal{C}_{C\mathcal{W}}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = \mathcal{C}_{C\mathcal{W}}((\Gamma_2, \tilde{\Theta}_2), (\Gamma_1, \tilde{\Theta}_1))$;

(iii) If $(\Gamma_1, \tilde{\Theta}_1) = (\Gamma_2, \tilde{\Theta}_2)$, then $\mathcal{C}_{C\mathcal{W}}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = 1$.

Proof. Follow the same steps presented in Proposition 3.7. □

Definition 3.12. The WCC between $(\Gamma_1, \tilde{\Theta}_1)$ and $(\Gamma_2, \tilde{\Theta}_2)$ can be represented as

$$\mathcal{C}_{\tilde{C}\mathcal{W}}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = \frac{\mathcal{C}_{\Upsilon}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))}{\max \left\{ \Phi(\Gamma_1, \tilde{\Theta}_1), \Phi(\Gamma_2, \tilde{\Theta}_2) \right\}}. \tag{6}$$

$$\mathcal{C}_{\tilde{C}\mathcal{W}}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2))$$

$$\begin{aligned} & \sum_{p=1}^u \mathcal{D}_p \left(\sum_{q=1}^v \mathcal{W}_q \left[(\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q)) + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q)) \right. \right. \\ & \quad \left. \left. + (\bar{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\bar{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q)) + (\bar{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\bar{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q)) \right. \right. \\ & \quad \left. \left. + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q)) + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q)) * (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q)) \right] \right) \\ = & \frac{\sum_{p=1}^u \mathcal{D}_p \left(\sum_{q=1}^v \mathcal{W}_q \left[(\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\bar{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right. \right. \\ & \quad \left. \left. + (\bar{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right] \right)}{\max \left\{ \sum_{p=1}^u \mathcal{D}_p \left(\sum_{q=1}^v \mathcal{W}_q \left[(\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\bar{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right. \right. \right. \\ & \quad \left. \left. + (\bar{\Lambda}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 \right] \right\},} \\ & \sum_{p=1}^u \mathcal{D}_p \left(\sum_{q=1}^v \mathcal{W}_q \left[(\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\underline{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\bar{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right. \right. \\ & \quad \left. \left. + (\bar{\Lambda}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 + (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right] \right) \left. \right\}. \end{aligned}$$

Proposition 3.13. Let $(\Gamma_1, \tilde{\Theta}_1)$ and $(\Gamma_2, \tilde{\Theta}_2)$ be CPFHSSs. Then,

(i) $0 \leq \mathcal{C}_{\tilde{C}\mathcal{W}}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) \leq 1$;

(ii) $\mathcal{C}_{\tilde{C}\mathcal{W}}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = \mathcal{C}_{\tilde{C}\mathcal{W}}((\Gamma_2, \tilde{\Theta}_2), (\Gamma_1, \tilde{\Theta}_1))$;

(iii) If $(\Gamma_1, \tilde{\Theta}_1) = (\Gamma_2, \tilde{\Theta}_2)$, then $\mathcal{C}_{\tilde{C}\mathcal{W}}((\Gamma_1, \tilde{\Theta}_1), (\Gamma_2, \tilde{\Theta}_2)) = 1$.

Proof. Follow the same steps presented in Proposition 3.7. □

4 Aggregation operators for CPFHSS

By using operational laws, the concept of cubic Pythagorean fuzzy hypersoft weighted average operator(CPFHSWAO) and cubic Pythagorean fuzzy hypersoft weighted geometric operator (CPFHSWGGO) are presented in this section. Let κ denote the collection of cubic Pythagorean fuzzy hypersoft numbers (CPFHSNs).

4.1 Operational laws for CPFHSS

Definition 4.1. Let $\Gamma_{e_{11}} = (\langle [\underline{\Upsilon}_{11}, \overline{\Upsilon}_{11}], \Upsilon_{11} \rangle, \langle [\underline{\Lambda}_{11}, \overline{\Lambda}_{11}], \Lambda_{11} \rangle)$ and $\Gamma_{e_{12}} = (\langle [\underline{\Upsilon}_{12}, \overline{\Upsilon}_{12}], \Upsilon_{12} \rangle, \langle [\underline{\Lambda}_{12}, \overline{\Lambda}_{12}], \Lambda_{12} \rangle)$ be two CPFHSS and δ a positive integer. Then,

- (i) $\Gamma_{e_{11}} \oplus \Gamma_{e_{12}} = (\langle [\underline{\Upsilon}_{11} + \underline{\Upsilon}_{12} - \underline{\Upsilon}_{11}\underline{\Upsilon}_{12}, \overline{\Upsilon}_{11} + \overline{\Upsilon}_{12} - \overline{\Upsilon}_{11}\overline{\Upsilon}_{12}], (\Upsilon_{11} + \Upsilon_{12} - \Upsilon_{11}\Upsilon_{12}) \rangle, \langle [\underline{\Lambda}_{11}\underline{\Lambda}_{12}, \overline{\Lambda}_{11}\overline{\Lambda}_{12}], (\Lambda_{11}\Lambda_{12}) \rangle)$;
- (ii) $\Gamma_{e_{11}} \otimes \Gamma_{e_{12}} = (\langle [\underline{\Upsilon}_{11}\underline{\Upsilon}_{12}, \overline{\Upsilon}_{11}\overline{\Upsilon}_{12}], (\Upsilon_{11}\Upsilon_{12}) \rangle, \langle [\underline{\Lambda}_{11} + \underline{\Lambda}_{12} - \underline{\Lambda}_{11}\underline{\Lambda}_{12}, \overline{\Lambda}_{11} + \overline{\Lambda}_{12} - \overline{\Lambda}_{11}\overline{\Lambda}_{12}], (\Lambda_{11} + \Lambda_{12} - \Lambda_{11}\Lambda_{12}) \rangle)$;
- (iii) $\delta\Gamma_{e_{11}} = (\langle [(1 - (1 - \underline{\Upsilon}_{11})^\delta), (1 - (1 - \overline{\Upsilon}_{11})^\delta)], (1 - (1 - \Upsilon_{11})^\delta) \rangle, \langle [(\underline{\Lambda}_{11})^\delta, (\overline{\Lambda}_{11})^\delta], (\Lambda_{11})^\delta \rangle)$;
- (iv) $(\Gamma_{e_{11}})^\delta = (\langle [(\underline{\Upsilon}_{11})^\delta, (\overline{\Upsilon}_{11})^\delta], (\Upsilon_{11})^\delta \rangle, \langle [(1 - (1 - \underline{\Lambda}_{11})^\delta), (1 - (1 - \overline{\Lambda}_{11})^\delta)], (1 - (1 - \Lambda_{11})^\delta) \rangle)$.

4.2 Cubic Pythagorean fuzzy hypersoft weighted average operator

Definition 4.2. Let \mathcal{D}_p and \mathcal{W}_q represent the weight vectors for alternatives and experts, with conditions $\mathcal{D}_p, \mathcal{W}_q > 0$ and $\sum_{p=1}^u \mathcal{D}_p = 1, \sum_{q=1}^v \mathcal{W}_q = 1$. Let $\Gamma_{e_{ik}} = (\langle [\underline{\Upsilon}_{ik}, \overline{\Upsilon}_{ik}], \Upsilon_{ik} \rangle, \langle [\underline{\Lambda}_{ik}, \overline{\Lambda}_{ik}], \Lambda_{ik} \rangle)$ be a CPFHSN, where $q = \{1, 2, \dots, v\}, p = \{1, 2, \dots, u\}$. Then, $\mathcal{A} : \kappa^v \rightarrow \kappa$, CPFHSWAO is represented as $\mathcal{A}(\Gamma_{e_{11}}, \Gamma_{e_{12}}, \dots, \Gamma_{e_{nm}}) = \bigoplus_{p=1}^u \mathcal{D}_p \left(\bigoplus_{q=1}^v \mathcal{W}_q \Gamma_{e_{ik}} \right)$.

Theorem 4.3. Let $\Gamma_{e_{ik}} = (\langle [\underline{\Upsilon}_{ik}, \overline{\Upsilon}_{ik}], \Upsilon_{ik} \rangle, \langle [\underline{\Lambda}_{ik}, \overline{\Lambda}_{ik}], \Lambda_{ik} \rangle)$ be a CPFHSN, where $q = \{1, 2, \dots, v\}, p = \{1, 2, \dots, u\}$. Then, the aggregated value of CPFHSWAO is also a CPFHSN, which is given by $\mathcal{A}(\Gamma_{e_{11}}, \Gamma_{e_{12}}, \dots, \Gamma_{e_{nm}})$

$$= \left(\left\langle \left[1 - \prod_{p=1}^u \left(\prod_{q=1}^v (1 - \underline{\Upsilon}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p}, 1 - \prod_{p=1}^u \left(\prod_{q=1}^v (1 - \overline{\Upsilon}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right], 1 - \prod_{p=1}^u \left(\prod_{q=1}^v (1 - \Upsilon_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right\rangle, \left\langle \left[\prod_{p=1}^u \left(\prod_{q=1}^v (\underline{\Lambda}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p}, \prod_{p=1}^u \left(\prod_{q=1}^v (\overline{\Lambda}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right], \prod_{p=1}^u \left(\prod_{q=1}^v (\Lambda_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right\rangle \right)$$

Example 4.4. Consider the values given in Example 3.2. Assume the weights of the psychiatrists and attributes as $\mathcal{W}_q = \{0.16, 0.28, 0.34, 0.22\}$ and $\mathcal{D}_p = \{0.26, 0.32, 0.14, 0.28\}$, respectively. Then, $\mathcal{A}(\Gamma_{e_{11}}, \Gamma_{e_{12}}, \dots, \Gamma_{e_{44}})$

$$= \left(\left\langle \left[1 - \prod_{p=1}^4 \left(\prod_{q=1}^4 (1 - \underline{\Upsilon}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p}, 1 - \prod_{p=1}^4 \left(\prod_{q=1}^4 (1 - \overline{\Upsilon}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right], 1 - \prod_{p=1}^4 \left(\prod_{q=1}^4 (1 - \Upsilon_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right\rangle, \left[\prod_{p=1}^4 \left(\prod_{q=1}^4 (\underline{\Lambda}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p}, \prod_{p=1}^4 \left(\prod_{q=1}^4 (\overline{\Lambda}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right], \prod_{p=1}^4 \left(\prod_{q=1}^4 (\Lambda_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right\rangle \right)$$

$$= (\langle [0.53, 0.57], 0.52 \rangle, \langle [0.62, 0.67], 0.65 \rangle)$$

4.3 Cubic Pythagorean fuzzy hypersoft weighted geometric operator

Definition 4.5. Let \mathcal{D}_p and \mathcal{W}_q represent the weight vectors for alternatives and experts with conditions $\mathcal{D}_p, \mathcal{W}_q > 0$ and $\sum_{p=1}^u \mathcal{D}_p = 1, \sum_{q=1}^v \mathcal{W}_q = 1$. Let $\Gamma_{e_{ik}} = (\Upsilon_{ik}, \mathcal{E}_{ik}, \Lambda_{ik})$ be a CPFHSN, where $q = \{1, 2, \dots, v\}, p = \{1, 2, \dots, u\}$. Then, $\mathcal{G} : \kappa^v \rightarrow \kappa$, CPFHSWGGO is defined as $\mathcal{G}(\Gamma_{e_{11}}, \Gamma_{e_{12}}, \dots, \Gamma_{e_{nm}}) = \bigotimes_{p=1}^u \left(\bigotimes_{q=1}^v \left(\Gamma_{e_{ik}} \right)^{\mathcal{W}_q} \right)^{\mathcal{D}_p}$.

Theorem 4.6. Let $\Gamma_{e_{ik}} = (\langle [\underline{\Upsilon}_{ik}, \overline{\Upsilon}_{ik}], \Upsilon_{ik} \rangle, \langle [\underline{\Lambda}_{ik}, \overline{\Lambda}_{ik}], \Lambda_{ik} \rangle)$ be a CPFHSN, where $q = \{1, 2, \dots, v\}$, $p = \{1, 2, \dots, u\}$. Then, the aggregated value of CPFHSWGGO is also a CPFHSN, which is given by

$$\begin{aligned} &\mathcal{G}(\Gamma_{e_{11}}, \Gamma_{e_{12}}, \dots, \Gamma_{e_{nm}}) \\ &= \left(\left\langle \left[\prod_{p=1}^u \left(\prod_{q=1}^v (\underline{\Upsilon}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p}, \prod_{p=1}^u \left(\prod_{q=1}^v (\overline{\Upsilon}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right], \prod_{p=1}^u \left(\prod_{q=1}^v (\Upsilon_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right\rangle, \right. \\ &\quad \left. \left\langle \left[1 - \prod_{p=1}^u \left(\prod_{q=1}^v (1 - \underline{\Lambda}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p}, 1 - \prod_{p=1}^u \left(\prod_{q=1}^v (1 - \overline{\Lambda}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right], 1 - \prod_{p=1}^u \left(\prod_{q=1}^v (1 - \Lambda_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right\rangle \right). \end{aligned}$$

Example 4.7. Consider the values given in Example 3.2. Let the weight of psychiatrists and attributes be same as in Example 4.4. Then,

$$\begin{aligned} &\mathcal{G}(\Gamma_{e_{11}}, \Gamma_{e_{12}}, \dots, \Gamma_{e_{44}}) \\ &= \left(\left\langle \left[\prod_{p=1}^4 \left(\prod_{q=1}^4 (\underline{\Upsilon}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p}, \prod_{p=1}^4 \left(\prod_{q=1}^4 (\overline{\Upsilon}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right], \prod_{p=1}^4 \left(\prod_{q=1}^4 (\Upsilon_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right\rangle \right. \\ &\quad \left. \left\langle \left[1 - \prod_{p=1}^4 \left(\prod_{q=1}^4 (1 - \underline{\Lambda}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p}, 1 - \prod_{p=1}^4 \left(\prod_{q=1}^4 (1 - \overline{\Lambda}_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right], 1 - \prod_{p=1}^4 \left(\prod_{q=1}^4 (1 - \Lambda_{ik})^{\mathcal{W}_q} \right)^{\mathcal{D}_p} \right\rangle \right). \\ &= (\langle [0.40, 0.46], 0.46 \rangle, \langle [0.70, 0.75], 0.69 \rangle). \end{aligned}$$

5 MCDM problems based on TOPSIS and CC method

An algorithm and a case study are provided to demonstrate the dependability of CC when integrated with the CPFHSS TOPSIS method. This novel approach aids in enhancing the selection of the optimal alternative by considering the minimum and maximum distances from the cubic Pythagorean fuzzy positive ideal solution (CPFPIIS) and cubic Pythagorean fuzzy negative ideal solution (CPFNIIS).

5.1 Algorithm to solve MCDM problems with CPFHSS data based on TOPSIS and CC method

Let $\mathcal{A} = \{\mathcal{A}^1, \mathcal{A}^2, \dots, \mathcal{A}^x\}$ represent set of college students and $\mathcal{U} = \{p_1, p_2, \dots, p_v\}$ be a set of psychiatrists responsible to evaluate the academic stress-coping skills on college students with weights $\mathcal{W}_q = (\mathcal{W}_1, \mathcal{W}_2, \dots, \mathcal{W}_v)$, such that $\mathcal{W}_q > 0$ and $\sum_{q=1}^v \mathcal{W}_q = 1$. Let $\tilde{\Theta} = \{\tilde{\psi}_1, \tilde{\psi}_2, \dots, \tilde{\psi}_u\}$ be a set of multi-valued sub-attributes with weights $\mathcal{D}_p = (\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_u)$, such that $\mathcal{D}_p > 0$ and $\sum_{p=1}^u \mathcal{D}_p = 1$. The evaluation of students \mathcal{A}^t , ($t = 1, 2, \dots, x$) performed by the psychiatrists p_q , ($q = 1, 2, \dots, v$) based on the multi-valued sub-attributes $\tilde{\psi}_p$, ($p = 1, 2, \dots, u$) are given in CPFHSS form and represented as $\mu_{ik}^t = (\langle [\underline{\Upsilon}_{ik}^t, \overline{\Upsilon}_{ik}^t], \Upsilon_{ik}^t \rangle, \langle [\underline{\Lambda}_{ik}^t, \overline{\Lambda}_{ik}^t], \Lambda_{ik}^t \rangle)$, such that $0 \leq ((\overline{\Upsilon})_{ik}^t + ((\overline{\Lambda})_{ik}^t) \leq 1$ and $0 \leq (\Upsilon)_{ik}^t + (\Lambda)_{ik}^t \leq 1 \forall q, p$.

Step 1. Create the matrix in CPFHSS format as depicted below:

$$\begin{aligned} &[\mathcal{A}^t, \tilde{\Theta}]_{v \times u} = [\mathcal{A}^t]_{v \times u} = ccccc\tilde{\psi}_1\tilde{\psi}_2 \dots \tilde{\psi}_u \\ &c[cccc]p_1\mu_{11}\mu_{12} \dots \mu_{1m} \\ &p_2\mu_{21}\mu_{22} \dots \mu_{2m} \\ &\vdots \vdots \vdots \\ &p_v\mu_{v1}\mu_{v2} \dots \mu_{vm} \\ &, \end{aligned}$$

such that $[\mathcal{A}^t]_{v \times u} = \mu_{ik}^t = \langle [\underline{\Upsilon}_{ik}^t, \overline{\Upsilon}_{ik}^t], \Upsilon_{ik}^t \rangle, \langle [\underline{\Lambda}_{ik}^t, \overline{\Lambda}_{ik}^t], \Lambda_{ik}^t \rangle, q = 1, 2, \dots, v$ and $p = 1, 2, \dots, u$.

Step 2. Generate the weighted decision matrix for every multi-valued sub-attribute, $[\tilde{A}_{ik}^t]_{v \times u}$

$$\begin{aligned}
 &= \left\langle \left[1 - \prod_{p=1}^u \left(\prod_{q=1}^v (1 - \mathcal{P}_{ik})^{W_q} \right)^{D_p}, 1 - \prod_{p=1}^u \left(\prod_{q=1}^v (1 - \bar{\mathcal{P}}_{ik})^{W_q} \right)^{D_p} \right], 1 - \prod_{p=1}^u \left(\prod_{q=1}^v (1 - \mathcal{P}_{ik})^{W_q} \right)^{D_p} \right\rangle, \\
 &\quad \left\langle \left[\prod_{p=1}^u \left(\prod_{q=1}^v (\underline{\Lambda}_{ik})^{W_q} \right)^{D_p}, \prod_{p=1}^u \left(\prod_{q=1}^v (\bar{\Lambda}_{ik})^{W_q} \right)^{D_p} \right], \prod_{p=1}^u \left(\prod_{q=1}^v (\Lambda_{ik})^{W_q} \right)^{D_p} \right\rangle \\
 &= \left\langle \left[\tilde{\Upsilon}_{ik}, \tilde{\Upsilon}_{ik} \right], \tilde{\Upsilon}_{ik} \right\rangle, \left\langle \left[\tilde{\Lambda}_{ik}, \tilde{\Lambda}_{ik} \right], \tilde{\Lambda}_{ik} \right\rangle.
 \end{aligned}$$

Step 3. Determine the CPFPIIS and CPFNIS values for the weighted CPFHSS representation:

$$\begin{aligned}
 \tilde{A}^+ &= \left\langle \left[\tilde{\Upsilon}^+, \tilde{\Upsilon}^+ \right], \tilde{\Upsilon}^+ \right\rangle, \left\langle \left[\tilde{\Lambda}^+, \tilde{\Lambda}^+ \right], \tilde{\Lambda}^+ \right\rangle_{v \times u} \\
 &= \left\langle \left[\tilde{\Upsilon}^{(\tilde{V}_{ij})}, \tilde{\Upsilon}^{(\tilde{V}_{ij})} \right], \tilde{\Upsilon}^{(\tilde{V}_{ij})} \right\rangle, \left\langle \left[\tilde{\Lambda}^{(\tilde{\wedge}_{ij})}, \tilde{\Lambda}^{(\tilde{\wedge}_{ij})} \right], \tilde{\Lambda}^{(\tilde{\wedge}_{ij})} \right\rangle, \\
 \tilde{A}^- &= \left\langle \left[\tilde{\Upsilon}^-, \tilde{\Upsilon}^- \right], \tilde{\Upsilon}^- \right\rangle, \left\langle \left[\tilde{\Lambda}^-, \tilde{\Lambda}^- \right], \tilde{\Lambda}^- \right\rangle_{v \times u} \\
 &= \left\langle \left[\tilde{\Upsilon}^{(\tilde{\wedge}_{ij})}, \tilde{\Upsilon}^{(\tilde{\wedge}_{ij})} \right], \tilde{\Upsilon}^{(\tilde{\wedge}_{ij})} \right\rangle, \left\langle \left[\tilde{\Lambda}^{(\tilde{V}_{ij})}, \tilde{\Lambda}^{(\tilde{V}_{ij})} \right], \tilde{\Lambda}^{(\tilde{V}_{ij})} \right\rangle,
 \end{aligned}$$

where $\vee_{ij} = \arg \max_t \{ \varphi_{ij}^t \}$ and $\wedge_{ij} = \arg \min_t \{ \varphi_{ij}^t \}$.

Step 4. Assess the CC for the alternatives based on CPFPIIS and CPFNIS.

$$\begin{aligned}
 \chi^t &= C_C(\tilde{A}^t, \tilde{A}^+) = \frac{C_{\Upsilon}(\tilde{A}^t, \tilde{A}^+)}{\sqrt{\Phi(\tilde{A}^t)} * \sqrt{\Phi(\tilde{A}^+)}} \text{ and} \\
 \psi^t &= C_C(\tilde{A}^t, \tilde{A}^-) = \frac{C_{\Upsilon}(\tilde{A}^t, \tilde{A}^-)}{\sqrt{\Phi(\tilde{A}^t)} * \sqrt{\Phi(\tilde{A}^-)}}
 \end{aligned}$$

Step 5. Calculate the closeness coefficient for the cubic Pythagorean fuzzy ideal solution:

$$\epsilon^t = \frac{1 - \psi^t}{2 - \chi^t - \psi^t}$$

Step 6. Compare the ϵ^t scores with the norms provided in Table 4 to ascertain the stress-coping level for each alternative \tilde{A}^t , where t ranges from 1 to x . Those exhibiting lower stress-coping skills might necessitate the assistance of a psychiatrist to manage academic stress effectively.

Table 4: Stress-coping norms.

Scores	Level
0-0.30	Low
0.31-0.50	Average
0.51-0.70	Good
0.71-1.00	Excellent

5.2 Application based on TOPSIS and CC method

Let $\mathcal{A} = \{ \mathcal{A}^1, \mathcal{A}^2, \mathcal{A}^3, \mathcal{A}^4 \}$ represent a set of college students. Let $\mathcal{U} = \{ p_1, p_2, p_3, p_4 \}$ represent a set of psychiatrists who evaluate the students based on the stress-coping skills with weights $\mathcal{W}_q = (0.16, 0.28, 0.34, 0.22)$. Let $\Theta_1, \Theta_2, \Theta_3$ and Θ_4 be distinct attribute sets. The corresponding sub-attributes are given by $\Theta_1 =$ initial phase = $\{ \psi_{11} =$ sensitivity to stress $\}$, $\Theta_2 =$ intermediate phase = $\{ \psi_{21} =$ capacity for relaxation, $\psi_{22} =$ self-reliance $\}$, $\Theta_3 =$ advanced phase = $\{ \psi_{31} =$ proactive mindset, $\psi_{32} =$ adaptability and versatility $\}$ and

$\Theta_4 =$ final phase = $\{\psi_{41} =$ ability to evaluate situations $\}$. Then $\tilde{\Theta} = \Theta_1 \times \Theta_2 \times \Theta_3 \times \Theta_4$ is the distinct attribute set given by

$$\begin{aligned} \tilde{\Theta} &= \Theta_1 \times \Theta_2 \times \Theta_3 \times \Theta_4 = \{\psi_{11}\} \times \{\psi_{21}, \psi_{22}\} \times \{\psi_{31}, \psi_{32}\} \times \{\psi_{41}\}. \\ &= \left\{ (\psi_{11}, \psi_{21}, \psi_{31}, \psi_{41}), (\psi_{11}, \psi_{21}, \psi_{32}, \psi_{41}), (\psi_{11}, \psi_{22}, \psi_{31}, \psi_{41}), (\psi_{11}, \psi_{22}, \psi_{32}, \psi_{41}) \right\}. \\ &= \left\{ \tilde{\psi}_1, \tilde{\psi}_2, \tilde{\psi}_3, \tilde{\psi}_4 \right\} \text{ with weights } \mathcal{D}_p = (0.26, 0.32, 0.14, 0.28). \end{aligned}$$

This framework aids in identifying students who may benefit from psychiatric support to effectively manage stress amidst academic pressures.

Step 1. Construct $\mathcal{A}^1, \mathcal{A}^2, \mathcal{A}^3$ and \mathcal{A}^4 matrices in CPFHSS form.

Table 5: Shows values in CPFHSS form for \mathcal{A}^1 .

\mathcal{A}^1	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.64, 0.66], 0.49 \rangle, \langle [0.72, 0.73], 0.79 \rangle$	$\langle [0.91, 0.92], 0.81 \rangle, \langle [0.32, 0.35], 0.45 \rangle$
p_2	$\langle [0.72, 0.75], 0.55 \rangle, \langle [0.41, 0.45], 0.75 \rangle$	$\langle [0.52, 0.56], 0.25 \rangle, \langle [0.81, 0.82], 0.84 \rangle$
p_3	$\langle [0.41, 0.45], 0.72 \rangle, \langle [0.81, 0.84], 0.45 \rangle$	$\langle [0.77, 0.79], 0.81 \rangle, \langle [0.46, 0.47], 0.55 \rangle$
p_4	$\langle [0.92, 0.93], 0.56 \rangle, \langle [0.31, 0.36], 0.61 \rangle$	$\langle [0.77, 0.79], 0.71 \rangle, \langle [0.49, 0.59], 0.61 \rangle$

\mathcal{A}^1	$\tilde{\psi}_3$	$\tilde{\psi}_4$
p_1	$\langle [0.36, 0.42], 0.57 \rangle, \langle [0.61, 0.63], 0.52 \rangle$	$\langle [0.48, 0.59], 0.69 \rangle, \langle [0.49, 0.52], 0.49 \rangle$
p_2	$\langle [0.73, 0.75], 0.55 \rangle, \langle [0.61, 0.62], 0.81 \rangle$	$\langle [0.81, 0.83], 0.23 \rangle, \langle [0.52, 0.55], 0.71 \rangle$
p_3	$\langle [0.42, 0.46], 0.56 \rangle, \langle [0.72, 0.77], 0.66 \rangle$	$\langle [0.34, 0.37], 0.41 \rangle, \langle [0.64, 0.68], 0.69 \rangle$
p_4	$\langle [0.58, 0.62], 0.49 \rangle, \langle [0.71, 0.73], 0.59 \rangle$	$\langle [0.54, 0.57], 0.64 \rangle, \langle [0.71, 0.75], 0.43 \rangle$

Table 6: Shows values in CPFHSS form for \mathcal{A}^2 .

\mathcal{A}^2	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.35, 0.41], 0.55 \rangle, \langle [0.59, 0.64], 0.55 \rangle$	$\langle [0.56, 0.61], 0.71 \rangle, \langle [0.47, 0.54], 0.51 \rangle$
p_2	$\langle [0.21, 0.25], 0.32 \rangle, \langle [0.88, 0.95], 0.88 \rangle$	$\langle [0.51, 0.53], 0.24 \rangle, \langle [0.79, 0.81], 0.72 \rangle$
p_3	$\langle [0.43, 0.48], 0.45 \rangle, \langle [0.71, 0.73], 0.67 \rangle$	$\langle [0.35, 0.39], 0.43 \rangle, \langle [0.62, 0.67], 0.68 \rangle$
p_4	$\langle [0.57, 0.64], 0.65 \rangle, \langle [0.69, 0.75], 0.62 \rangle$	$\langle [0.55, 0.59], 0.66 \rangle, \langle [0.63, 0.72], 0.42 \rangle$

\mathcal{A}^2	$\tilde{\psi}_3$	$\tilde{\psi}_4$
p_1	$\langle [0.84, 0.91], 0.83 \rangle, \langle [0.31, 0.36], 0.46 \rangle$	$\langle [0.91, 0.94], 0.52 \rangle, \langle [0.31, 0.32], 0.81 \rangle$
p_2	$\langle [0.48, 0.55], 0.24 \rangle, \langle [0.72, 0.74], 0.88 \rangle$	$\langle [0.71, 0.76], 0.58 \rangle, \langle [0.39, 0.46], 0.77 \rangle$
p_3	$\langle [0.72, 0.78], 0.36 \rangle, \langle [0.48, 0.58], 0.89 \rangle$	$\langle [0.21, 0.24], 0.74 \rangle, \langle [0.79, 0.86], 0.44 \rangle$
p_4	$\langle [0.71, 0.75], 0.58 \rangle, \langle [0.59, 0.66], 0.77 \rangle$	$\langle [0.74, 0.75], 0.58 \rangle, \langle [0.61, 0.64], 0.64 \rangle$

Table 7: Shows values in CPFHSS form for \mathcal{A}^3 .

\mathcal{A}^3	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.46, 0.61], 0.71 \rangle, \langle [0.46, 0.53], 0.69 \rangle$	$\langle [0.61, 0.62], 0.47 \rangle, \langle [0.76, 0.77], 0.81 \rangle$
p_2	$\langle [0.14, 0.17], 0.22 \rangle, \langle [0.72, 0.77], 0.92 \rangle$	$\langle [0.16, 0.17], 0.57 \rangle, \langle [0.46, 0.48], 0.69 \rangle$
p_3	$\langle [0.15, 0.21], 0.43 \rangle, \langle [0.66, 0.71], 0.71 \rangle$	$\langle [0.21, 0.27], 0.77 \rangle, \langle [0.84, 0.88], 0.49 \rangle$
p_4	$\langle [0.52, 0.59], 0.66 \rangle, \langle [0.68, 0.69], 0.49 \rangle$	$\langle [0.34, 0.48], 0.57 \rangle, \langle [0.74, 0.78], 0.69 \rangle$

\mathcal{A}^3	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.39, 0.44], 0.54 \rangle, \langle [0.56, 0.64], 0.59 \rangle$	$\langle [0.51, 0.61], 0.68 \rangle, \langle [0.47, 0.55], 0.45 \rangle$
p_2	$\langle [0.24, 0.29], 0.34 \rangle, \langle [0.86, 0.95], 0.81 \rangle$	$\langle [0.16, 0.19], 0.29 \rangle, \langle [0.89, 0.96], 0.77 \rangle$
p_3	$\langle [0.46, 0.49], 0.44 \rangle, \langle [0.64, 0.78], 0.64 \rangle$	$\langle [0.38, 0.39], 0.44 \rangle, \langle [0.62, 0.69], 0.72 \rangle$
p_4	$\langle [0.62, 0.63], 0.64 \rangle, \langle [0.65, 0.77], 0.54 \rangle$	$\langle [0.57, 0.59], 0.67 \rangle, \langle [0.63, 0.79], 0.45 \rangle$

Table 8: Shows values in CPFHSS form for \mathcal{A}^4 .

\mathcal{A}^4	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.89, 0.92], 0.79 \rangle, \langle [0.32, 0.35], 0.52 \rangle$	$\langle [0.84, 0.92], 0.88 \rangle, \langle [0.31, 0.34], 0.46 \rangle$
p_2	$\langle [0.51, 0.56], 0.24 \rangle, \langle [0.81, 0.82], 0.77 \rangle$	$\langle [0.48, 0.56], 0.26 \rangle, \langle [0.72, 0.82], 0.88 \rangle$
p_3	$\langle [0.75, 0.79], 0.33 \rangle, \langle [0.46, 0.47], 0.84 \rangle$	$\langle [0.72, 0.77], 0.44 \rangle, \langle [0.48, 0.62], 0.89 \rangle$
p_4	$\langle [0.45, 0.46], 0.56 \rangle, \langle [0.49, 0.69], 0.81 \rangle$	$\langle [0.81, 0.82], 0.61 \rangle, \langle [0.54, 0.55], 0.77 \rangle$

\mathcal{A}^4	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.56, 0.66], 0.77 \rangle, \langle [0.47, 0.57], 0.55 \rangle$	$\langle [0.31, 0.44], 0.58 \rangle, \langle [0.59, 0.64], 0.59 \rangle$
p_2	$\langle [0.89, 0.94], 0.25 \rangle, \langle [0.31, 0.32], 0.77 \rangle$	$\langle [0.35, 0.39], 0.38 \rangle, \langle [0.88, 0.91], 0.81 \rangle$
p_3	$\langle [0.35, 0.49], 0.44 \rangle, \langle [0.62, 0.69], 0.69 \rangle$	$\langle [0.41, 0.47], 0.48 \rangle, \langle [0.71, 0.73], 0.61 \rangle$
p_4	$\langle [0.55, 0.69], 0.67 \rangle, \langle [0.63, 0.71], 0.45 \rangle$	$\langle [0.51, 0.65], 0.68 \rangle, \langle [0.69, 0.75], 0.69 \rangle$

Step 2. Evaluate $\tilde{A}^1, \tilde{A}^2, \tilde{A}^3$ and \tilde{A}^4 , the weighted matrices for multi-valued sub-attributes.

Table 9: Shows weighted values in CPFHSS form for \tilde{A}^1 .

\tilde{A}^1	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.0416, 0.0439], 0.0276 \rangle, \langle [0.9864, 0.9870], 0.9902 \rangle$	$\langle [0.1160, 0.1213], 0.0815 \rangle, \langle [0.9433, 0.9477], 0.9599 \rangle$
p_2	$\langle [0.0885, 0.0960], 0.0565 \rangle, \langle [0.9372, 0.9435], 0.9793 \rangle$	$\langle [0.0636, 0.0709], 0.0254 \rangle, \langle [0.9813, 0.9824], 0.9845 \rangle$
p_3	$\langle [0.0456, 0.0515], 0.1064 \rangle, \langle [0.9815, 0.9847], 0.9318 \rangle$	$\langle [0.1478, 0.1562], 0.1653 \rangle, \langle [0.9190, 0.9211], 0.9370 \rangle$
p_4	$\langle [0.1345, 0.1411], 0.0459 \rangle, \langle [0.9352, 0.9432], 0.9721 \rangle$	$\langle [0.0983, 0.1040], 0.0835 \rangle, \langle [0.9510, 0.9635], 0.9658 \rangle$

\tilde{A}^1	$\tilde{\psi}_3$	$\tilde{\psi}_4$
p_1	$\langle [0.0099, 0.0121], 0.0187 \rangle, \langle [0.9890, 0.9897], 0.9855 \rangle$	$\langle [0.0289, 0.0392], 0.0511 \rangle, \langle [0.9685, 0.9711], 0.9685 \rangle$
p_2	$\langle [0.0500, 0.0529], 0.0308 \rangle, \langle [0.9808, 0.9814], 0.9918 \rangle$	$\langle [0.1221, 0.1297], 0.0203 \rangle, \langle [0.9500, 0.9542], 0.9735 \rangle$
p_3	$\langle [0.0256, 0.0289], 0.0383 \rangle, \langle [0.9845, 0.9876], 0.9804 \rangle$	$\langle [0.0388, 0.0430], 0.0490 \rangle, \langle [0.9584, 0.9640], 0.9653 \rangle$
p_4	$\langle [0.0264, 0.0294], 0.0205 \rangle, \langle [0.9895, 0.9904], 0.9839 \rangle$	$\langle [0.0467, 0.0507], 0.0610 \rangle, \langle [0.9791, 0.9824], 0.9493 \rangle$

Table 10: Shows weighted values in CPFHSS form for \tilde{A}^2 .

\tilde{A}^2	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.01780, 0.0217], 0.0327 \rangle, \langle [0.97830, 0.9816], 0.9754 \rangle$	$\langle [0.0412, 0.0471], 0.0614 \rangle, \langle [0.9621, 0.9689], 0.9661 \rangle$
p_2	$\langle [0.01700, 0.0207], 0.0277 \rangle, \langle [0.99070, 0.9963], 0.9907 \rangle$	$\langle [0.0619, 0.0654], 0.0243 \rangle, \langle [0.9791, 0.9813], 0.9710 \rangle$
p_3	$\langle [0.04850, 0.0562], 0.0515 \rangle, \langle [0.97020, 0.9726], 0.9652 \rangle$	$\langle [0.0458, 0.0524], 0.0593 \rangle, \langle [0.9493, 0.9574], 0.9589 \rangle$
p_4	$\langle [0.04710, 0.0568], 0.0583 \rangle, \langle [0.97900, 0.9837], 0.9730 \rangle$	$\langle [0.0547, 0.0608], 0.0731 \rangle, \langle [0.9680, 0.9771], 0.9408 \rangle$

\tilde{A}^2	$\tilde{\psi}_3$	$\tilde{\psi}_4$
p_1	$\langle [0.0402, 0.0525], 0.0389 \rangle, \langle [0.9741, 0.9774], 0.9828 \rangle$	$\langle [0.1023, 0.1184], 0.0323 \rangle, \langle [0.9489, 0.9502], 0.9906 \rangle$
p_2	$\langle [0.0253, 0.0308], 0.0107 \rangle, \langle [0.9872, 0.9883], 0.9950 \rangle$	$\langle [0.0925, 0.1059], 0.0658 \rangle, \langle [0.9288, 0.9409], 0.9797 \rangle$
p_3	$\langle [0.0588, 0.0695], 0.0210 \rangle, \langle [0.9657, 0.9744], 0.9945 \rangle$	$\langle [0.0222, 0.0258], 0.1204 \rangle, \langle [0.9778, 0.9857], 0.9248 \rangle$
p_4	$\langle [0.0374, 0.0418], 0.0264 \rangle, \langle [0.9839, 0.9873], 0.9920 \rangle$	$\langle [0.0796, 0.0819], 0.0520 \rangle, \langle [0.9700, 0.9729], 0.9729 \rangle$

Table 11: Shows weighted values in CPFHSS form for \tilde{A}^3 .

\tilde{A}^3	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.0253, 0.0384], 0.0502 \rangle, \langle [0.9682, 0.9739], 0.9847 \rangle$	$\langle [0.0471, 0.0483], 0.0320 \rangle, \langle [0.9860, 0.9867], 0.9893 \rangle$
p_2	$\langle [0.0109, 0.0135], 0.0179 \rangle, \langle [0.9764, 0.9812], 0.9939 \rangle$	$\langle [0.0155, 0.0166], 0.0728 \rangle, \langle [0.9328, 0.9364], 0.9673 \rangle$
p_3	$\langle [0.0143, 0.0206], 0.0485 \rangle, \langle [0.9639, 0.9702], 0.9702 \rangle$	$\langle [0.0253, 0.0337], 0.1478 \rangle, \langle [0.9812, 0.9862], 0.9253 \rangle$
p_4	$\langle [0.0411, 0.0497], 0.0598 \rangle, \langle [0.9782, 0.9790], 0.9600 \rangle$	$\langle [0.0288, 0.0450], 0.0577 \rangle, \langle [0.9790, 0.9827], 0.9742 \rangle$

\tilde{A}^3	$\tilde{\psi}_3$	$\tilde{\psi}_4$
p_1	$\langle [0.0110, 0.0129], 0.0172 \rangle, \langle [0.9871, 0.9901], 0.9883 \rangle$	$\langle [0.0315, 0.0413], 0.0498 \rangle, \langle [0.9667, 0.9736], 0.9649 \rangle$
p_2	$\langle [0.0107, 0.0133], 0.0162 \rangle, \langle [0.9941, 0.9980], 0.9918 \rangle$	$\langle [0.0136, 0.0164], 0.0265 \rangle, \langle [0.9909, 0.9968], 0.9797 \rangle$
p_3	$\langle [0.0289, 0.0315], 0.0272 \rangle, \langle [0.9790, 0.9882], 0.9790 \rangle$	$\langle [0.0445, 0.0460], 0.0537 \rangle, \langle [0.9555, 0.9653], 0.9692 \rangle$
p_4	$\langle [0.0294, 0.0302], 0.0310 \rangle, \langle [0.9868, 0.9920], 0.9812 \rangle$	$\langle [0.0507, 0.0534], 0.0660 \rangle, \langle [0.9719, 0.9856], 0.9520 \rangle$

Table 12: Shows weighted values in CPFHSS form for \tilde{A}^4 .

\tilde{A}^4	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.0877, 0.0997], 0.0629 \rangle, \langle [0.9537, 0.9573], 0.9732 \rangle$	$\langle [0.0896, 0.1213], 0.1029 \rangle, \langle [0.9418, 0.9463], 0.9610 \rangle$
p_2	$\langle [0.0506, 0.0580], 0.0198 \rangle, \langle [0.9848, 0.9857], 0.9812 \rangle$	$\langle [0.0569, 0.0709], 0.0266 \rangle, \langle [0.9710, 0.9824], 0.9886 \rangle$
p_3	$\langle [0.1153, 0.1289], 0.0348 \rangle, \langle [0.9337, 0.9354], 0.9847 \rangle$	$\langle [0.1293, 0.1478], 0.0611 \rangle, \langle [0.9232, 0.9493], 0.9874 \rangle$
p_4	$\langle [0.0336, 0.0346], 0.0459 \rangle, \langle [0.9600, 0.9790], 0.9880 \rangle$	$\langle [0.1103, 0.1137], 0.0641 \rangle, \langle [0.9575, 0.9588], 0.9818 \rangle$

\tilde{A}^4	$\tilde{\psi}_3$	$\tilde{\psi}_4$
p_1	$\langle [0.0182, 0.0239], 0.0324 \rangle, \langle [0.9832, 0.9875], 0.9867 \rangle$	$\langle [0.0165, 0.0256], 0.0381 \rangle, \langle [0.9766, 0.9802], 0.9766 \rangle$
p_2	$\langle [0.0829, 0.1044], 0.0112 \rangle, \langle [0.9551, 0.9563], 0.9898 \rangle$	$\langle [0.0332, 0.0380], 0.0368 \rangle, \langle [0.9900, 0.9926], 0.9836 \rangle$
p_3	$\langle [0.0203, 0.0315], 0.0272 \rangle, \langle [0.9775, 0.9825], 0.9825 \rangle$	$\langle [0.0490, 0.0587], 0.0604 \rangle, \langle [0.9679, 0.9705], 0.9540 \rangle$
p_4	$\langle [0.0243, 0.0354], 0.0336 \rangle, \langle [0.9859, 0.9895], 0.9757 \rangle$	$\langle [0.0430, 0.0626], 0.0678 \rangle, \langle [0.9774, 0.9824], 0.9774 \rangle$

Step 3. Evaluate the CPFPIs and CPFNIS from the matrices, $\tilde{A}^1, \tilde{A}^2, \tilde{A}^3$ and \tilde{A}^4 .

Table 13: Shows the values of CPFPIs (\tilde{A}^+).

\tilde{A}^+	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.0877, 0.0997], 0.0629 \rangle, \langle [0.9537, 0.9573], 0.9732 \rangle$	$\langle [0.1160, 0.1213], 0.1029 \rangle, \langle [0.9418, 0.9463], 0.9599 \rangle$
p_2	$\langle [0.0885, 0.0960], 0.0565 \rangle, \langle [0.9372, 0.9435], 0.9793 \rangle$	$\langle [0.0636, 0.0709], 0.0728 \rangle, \langle [0.9328, 0.9364], 0.9673 \rangle$
p_3	$\langle [0.1153, 0.1289], 0.1064 \rangle, \langle [0.9337, 0.9354], 0.9318 \rangle$	$\langle [0.1478, 0.1562], 0.1653 \rangle, \langle [0.9190, 0.9211], 0.9253 \rangle$
p_4	$\langle [0.1345, 0.1411], 0.0598 \rangle, \langle [0.9352, 0.9432], 0.9600 \rangle$	$\langle [0.1103, 0.1137], 0.0835 \rangle, \langle [0.9510, 0.9588], 0.9408 \rangle$

$\tilde{\mathcal{A}}^+$	$\tilde{\psi}_3$	$\tilde{\psi}_4$
p_1	$\langle [0.0402, 0.0525], 0.0389 \rangle, \langle [0.9741, 0.9774], 0.9828 \rangle$	$\langle [0.1023, 0.1184], 0.0511 \rangle, \langle [0.9489, 0.9502], 0.9649 \rangle$
p_2	$\langle [0.0829, 0.1044], 0.0308 \rangle, \langle [0.9551, 0.9563], 0.9898 \rangle$	$\langle [0.1221, 0.1297], 0.0658 \rangle, \langle [0.9288, 0.9409], 0.9735 \rangle$
p_3	$\langle [0.0588, 0.0695], 0.0383 \rangle, \langle [0.9657, 0.9744], 0.9790 \rangle$	$\langle [0.0490, 0.0587], 0.1204 \rangle, \langle [0.9555, 0.9640], 0.9248 \rangle$
p_4	$\langle [0.0374, 0.0418], 0.0336 \rangle, \langle [0.9839, 0.9873], 0.9757 \rangle$	$\langle [0.0796, 0.0819], 0.0678 \rangle, \langle [0.9700, 0.9729], 0.9493 \rangle$

Table 14: Shows the values of CPFPIIS ($\tilde{\mathcal{A}}^-$).

$\tilde{\mathcal{A}}^-$	$\tilde{\psi}_1$	$\tilde{\psi}_2$
p_1	$\langle [0.0178, 0.0217], 0.0629 \rangle, \langle [0.9864, 0.9870], 0.9902 \rangle$	$\langle [0.0412, 0.0471], 0.1029 \rangle, \langle [0.9860, 0.9867], 0.9893 \rangle$
p_2	$\langle [0.0109, 0.0135], 0.0327 \rangle, \langle [0.9907, 0.9963], 0.9939 \rangle$	$\langle [0.0155, 0.0166], 0.0320 \rangle, \langle [0.9813, 0.9824], 0.9886 \rangle$
p_3	$\langle [0.0143, 0.0206], 0.0179 \rangle, \langle [0.9815, 0.9847], 0.9847 \rangle$	$\langle [0.0253, 0.0337], 0.0243 \rangle, \langle [0.9812, 0.9862], 0.9874 \rangle$
p_4	$\langle [0.0336, 0.0346], 0.0348 \rangle, \langle [0.9790, 0.9837], 0.9880 \rangle$	$\langle [0.0288, 0.0450], 0.0593 \rangle, \langle [0.9790, 0.9827], 0.9818 \rangle$

$\tilde{\mathcal{A}}^-$	$\tilde{\psi}_3$	$\tilde{\psi}_4$
p_1	$\langle [0.0099, 0.0121], 0.0389 \rangle, \langle [0.9890, 0.9901], 0.9883 \rangle$	$\langle [0.0165, 0.0256], 0.0511 \rangle, \langle [0.9766, 0.9802], 0.9906 \rangle$
p_2	$\langle [0.0107, 0.0133], 0.0172 \rangle, \langle [0.9941, 0.9980], 0.9950 \rangle$	$\langle [0.0136, 0.0164], 0.0323 \rangle, \langle [0.9909, 0.9968], 0.9836 \rangle$
p_3	$\langle [0.0203, 0.0289], 0.0107 \rangle, \langle [0.9845, 0.9882], 0.9945 \rangle$	$\langle [0.0222, 0.0258], 0.0265 \rangle, \langle [0.9778, 0.9857], 0.9692 \rangle$
p_4	$\langle [0.0243, 0.0294], 0.0210 \rangle, \langle [0.9895, 0.9920], 0.9920 \rangle$	$\langle [0.0430, 0.0507], 0.0537 \rangle, \langle [0.9791, 0.9856], 0.9774 \rangle$

Step 4. Evaluate the CC by using the values of CPFPIIS and CPFNIS.

$$\chi^1 = 0.9992, \chi^2 = 0.9984, \chi^3 = 0.9976 \text{ and } \chi^4 = 0.99878.$$

$$\lambda^1 = 0.9980, \lambda^2 = 0.9991, \lambda^3 = 0.9995 \text{ and } \lambda^4 = 0.99870.$$

Step 5. Determine the closeness coefficient of cubic Pythagorean fuzzy ideal solution.

$$\epsilon^1 = 0.7294, \epsilon^2 = 0.3538, \epsilon^3 = 0.1449 \text{ and } \epsilon^4 = 0.5157.$$

Step 6. Compare the scores with the norms given in Table 4 and determine the stress-coping levels.

$$\epsilon^1 - \text{Excellent}, \epsilon^2 - \text{Average}, \epsilon^3 - \text{Low} \text{ and } \epsilon^4 - \text{Good}.$$

$$\Rightarrow \mathcal{A}^1 - \text{Excellent}, \mathcal{A}^2 - \text{Average}, \mathcal{A}^3 - \text{Low} \text{ and } \mathcal{A}^4 - \text{Good}$$

Hence, \mathcal{A}^3 may require the help of a psychiatrist to manage the stress effectively during academic.

6 Comparative Analysis

The comparative analysis between the proposed cubic Pythagorean fuzzy TOPSIS method and established DMs serves to highlight the efficacy of CC.

Example 6.1. Considering the values and weights outlined in Section 5.2, we integrate the proposed TOPSIS method with existing DMs⁷ to rank the alternatives.

$$\begin{aligned} \mathcal{S}_1(\Theta_1, \Theta_2) = \frac{1}{6n} \sum_{q=1}^v \left\{ \left| (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right| + \left| (\Delta_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\Delta_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right| \right. \\ \left. + \left| (\bar{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\bar{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right| + \left| (\bar{\Delta}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\bar{\Delta}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right| \right. \\ \left. + \left| (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right| + \left| (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right| \right\}. \end{aligned}$$

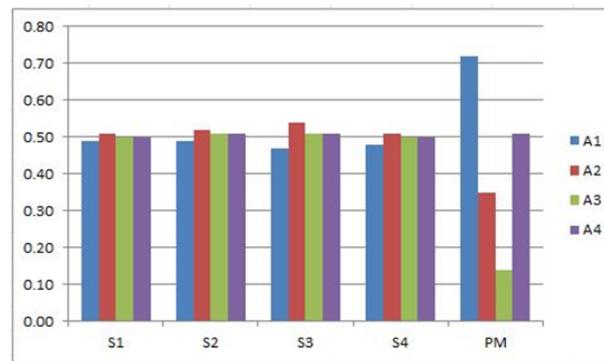
$$\begin{aligned} \mathcal{S}_2(\Theta_1, \Theta_2) = \frac{1}{6} \sum_{q=1}^v \max \left\{ \left| (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right|, \left| (\Delta_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\Delta_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right|, \right. \\ \left| (\bar{\Upsilon}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\bar{\Upsilon}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right|, \left| (\bar{\Delta}_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\bar{\Delta}_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right|, \\ \left. \left| (\Upsilon_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\Upsilon_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right|, \left| (\Lambda_{\Gamma_1(\tilde{\psi}_p)}(u_q))^2 - (\Lambda_{\Gamma_2(\tilde{\psi}_p)}(u_q))^2 \right| \right\}. \end{aligned}$$

$$S_3(\Theta_1, \Theta_2) = \frac{1}{6} \sum_{q=1}^v \left\{ \left((\Upsilon_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\Upsilon_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 + \left((\Delta_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\Delta_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 \right. \\ \left. + \left((\bar{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\bar{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 + \left((\bar{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\bar{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 \right. \\ \left. + \left((\Upsilon_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\Upsilon_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 + \left((\Delta_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\Delta_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 \right\}.$$

$$S_4(\Theta_1, \Theta_2) = \frac{1}{6n} \sum_{q=1}^v \left\{ \left((\Upsilon_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\Upsilon_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 + \left((\Delta_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\Delta_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 \right. \\ \left. + \left((\bar{\Upsilon}_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\bar{\Upsilon}_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 + \left((\bar{\Delta}_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\bar{\Delta}_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 \right. \\ \left. + \left((\Upsilon_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\Upsilon_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 + \left((\Delta_{\Gamma_1}(\tilde{\psi}_p)(u_q))^2 - (\Delta_{\Gamma_2}(\tilde{\psi}_p)(u_q))^2 \right)^2 \right\}.$$

Table 15: Comparison study between proposed study and existing DMs.

Tabulation of values computed using existing DMs	
$S_1(\Theta_1, \Theta_2) \Rightarrow \mathcal{A}^1 = 0.49, \mathcal{A}^3 = 0.51$	and $\mathcal{A}^2 = \mathcal{A}^4 = 0.50$
$S_2(\Theta_1, \Theta_2) \Rightarrow \mathcal{A}^1 = 0.49, \mathcal{A}^3 = 0.52$	and $\mathcal{A}^2 = \mathcal{A}^4 = 0.51$
$S_3(\Theta_1, \Theta_2) \Rightarrow \mathcal{A}^1 = 0.47, \mathcal{A}^3 = 0.54$	and $\mathcal{A}^2 = \mathcal{A}^4 = 0.51$
$S_4(\Theta_1, \Theta_2) \Rightarrow \mathcal{A}^1 = 0.48, \mathcal{A}^3 = 0.51$	and $\mathcal{A}^2 = \mathcal{A}^4 = 0.50$



Analysis : As discerned from Table 15, it is evident that conventional DMs struggle to pinpoint the optimal alternative, unlike CC which effectively identifies the best alternative. Consequently, CC outperforms DMs by providing precise rankings for each alternative.

7 Comparative Analysis with existing neutrosophic DMs

Table 16: Representation of values in neutrosophic form for \mathcal{A}^i .

\mathcal{A}^i	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$	$\tilde{\lambda}_5$	$\tilde{\lambda}_6$
\mathcal{A}^1	$\langle 0.45, 0.46, 0.54 \rangle$	$\langle 0.45, 0.48, 0.45 \rangle$	$\langle 0.34, 0.38, 0.45 \rangle$	$\langle 0.34, 0.45, 0.55 \rangle$	$\langle 0.24, 0.26, 0.56 \rangle$	$\langle 0.22, 0.24, 0.28 \rangle$
\mathcal{A}^1	$\langle 0.66, 0.70, 0.34 \rangle$	$\langle 0.15, 0.17, 0.13 \rangle$	$\langle 0.23, 0.34, 0.64 \rangle$	$\langle 0.20, 0.30, 0.25 \rangle$	$\langle 0.36, 0.45, 0.23 \rangle$	$\langle 0.65, 0.67, 0.68 \rangle$
\mathcal{A}^1	$\langle 0.66, 0.67, 0.72 \rangle$	$\langle 0.43, 0.45, 0.41 \rangle$	$\langle 0.22, 0.25, 0.55 \rangle$	$\langle 0.11, 0.18, 0.14 \rangle$	$\langle 0.33, 0.43, 0.53 \rangle$	$\langle 0.21, 0.23, 0.35 \rangle$
\mathcal{A}^1	$\langle 0.43, 0.45, 0.25 \rangle$	$\langle 0.35, 0.45, 0.21 \rangle$	$\langle 0.33, 0.43, 0.36 \rangle$	$\langle 0.23, 0.33, 0.45 \rangle$	$\langle 0.44, 0.54, 0.64 \rangle$	$\langle 0.43, 0.44, 0.45 \rangle$

The ranking of the proposed method is as follows:

$$\mathcal{A}^2 = 0.65, > \mathcal{A}^3 = 0.54, > \mathcal{A}^4 = 0.32, > \mathcal{A}^1 = 0.28.$$

Analysis: By using Table 17, let's compare the proposed study with each existing study ($\mathcal{S}_1(\psi_1, \psi_2)$ to $\mathcal{S}_7(\psi_1, \psi_2)$).

$$\mathcal{S}_1(\psi_1, \psi_2): \mathcal{A}^1 = \mathcal{A}^2 = 0.50 \text{ and } \mathcal{A}^3 = \mathcal{A}^4 = 0.49.$$

Table 17: Comparison of existing neutrosophic similarity measures with proposed method.

Unable to rank using existing similarity measures	
$\mathcal{S}_1(\psi_1, \psi_2)^{24} \Rightarrow \mathcal{A}^1 = 0.50, \mathcal{A}^2 = 0.49, \mathcal{A}^3 = 0.50, \mathcal{A}^4 = 0.49.$	
$\mathcal{S}_2(\psi_1, \psi_2)^{24} \Rightarrow \mathcal{A}^1 = 0.50, \mathcal{A}^2 = 0.50, \mathcal{A}^3 = 0.49, \mathcal{A}^4 = 0.49.$	
$\mathcal{S}_3(\psi_1, \psi_2)^{25} \Rightarrow \mathcal{A}^1 = 0.50, \mathcal{A}^2 = 0.49, \mathcal{A}^3 = 0.50, \mathcal{A}^4 = 0.49.$	
$\mathcal{S}_4(\psi_1, \psi_2)^{25} \Rightarrow \mathcal{A}^1 = 0.50, \mathcal{A}^2 = 0.49, \mathcal{A}^3 = 0.50, \mathcal{A}^4 = 0.50.$	
$\mathcal{S}_5(\psi_1, \psi_2)^{26} \Rightarrow \mathcal{A}^1 = 0.51, \mathcal{A}^2 = 0.48, \mathcal{A}^3 = 0.50, \mathcal{A}^4 = 0.50.$	
$\mathcal{S}_6(\psi_1, \psi_2)^{26} \Rightarrow \mathcal{A}^1 = 0.50, \mathcal{A}^2 = 0.49, \mathcal{A}^3 = 0.50, \mathcal{A}^4 = 0.50.$	
$\mathcal{S}_7(\psi_1, \psi_2)^{30} \Rightarrow \mathcal{A}^1 = 0.46, \mathcal{A}^2 = 0.50, \mathcal{A}^3 = 0.50, \mathcal{A}^4 = 0.50.$	

$$\mathcal{S}_2(\psi_1, \psi_2): \mathcal{A}^1 = \mathcal{A}^3 = 0.50 \text{ and } \mathcal{A}^2 = \mathcal{A}^4 = 0.49.$$

$$\mathcal{S}_3(\psi_1, \psi_2): \mathcal{A}^1 = \mathcal{A}^3 = 0.50 \text{ and } \mathcal{A}^2 = \mathcal{A}^4 = 0.49.$$

$$\mathcal{S}_4(\psi_1, \psi_2): \mathcal{A}^1 = \mathcal{A}^3 = \mathcal{A}^4 = 0.50 \text{ and } \mathcal{A}^2 = 0.49.$$

$$\mathcal{S}_5(\psi_1, \psi_2): \mathcal{A}^3 = \mathcal{A}^4 = 0.50, \mathcal{A}^1 = 0.51 \text{ and } \mathcal{A}^2 = 0.48.$$

$$\mathcal{S}_6(\psi_1, \psi_2): \mathcal{A}^1 = \mathcal{A}^3 = \mathcal{A}^4 = 0.50 \text{ and } \mathcal{A}^2 = 0.49.$$

$$\mathcal{S}_7(\psi_1, \psi_2): \mathcal{A}^2 = \mathcal{A}^3 = \mathcal{A}^4 = 0.50, \text{ and } \mathcal{A}^1 = 0.46.$$

From the analysis, it is evident that the existing studies ($\mathcal{S}_1(\psi_1, \psi_2)$ to $\mathcal{S}_7(\psi_1, \psi_2)$) struggle to differentiate ranks for the data, while the proposed study provides distinct ranks for the same dataset.

8 Conclusions

Based on the comprehensive exploration and analysis conducted in this study, we have successfully defined CPFHSS and elucidated its key properties. Through the application of aggregation operators and the implementation of the TOPSIS method, we have effectively assessed the academic stress-coping skills of college students. In our examination of closeness coefficients, we have utilized CC as a fundamental metric, diverging from conventional DMs typically employed in similar studies. The culmination of our research is highlighted in the comparative study between our proposed method and established DMs, showcasing the reliability and efficacy of our model. In addition, it's worth noting that our proposed method was also compared with existing neutrosophic study methodologies. This comparison further solidifies the efficacy and reliability of our model in evaluating academic stress-coping mechanisms among college students. By demonstrating superior performance compared to both established DMs and existing neutrosophic approaches, our study offers a comprehensive and advanced framework for assessing coping strategies in higher education settings. Through meticulous analysis and rigorous experimentation, we have demonstrated the superiority of our approach in evaluating academic stress-coping mechanisms among college students. In conclusion, our study not only contributes to the academic discourse on stress management but also provides a robust framework for assessing coping strategies in higher education settings. The insights gleaned from this research hold significant implications for educators, counselors, and policymakers seeking to enhance student well-being and academic performance.

References

- [1] L.A.Zadeh, Fuzzy sets, Information and Control. 8 (1965) 338–353.
- [2] L.A.Zadeh, The concept of a linguistic variable and its application to approximate reasoning I, Information Sciences. 8 (1975) 199–249.

- [3] K.T.Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*. 20 (1986) 87–96.
- [4] Y.B.Jun, C.S.Kim, K.O.Yang, Cubic sets, *Annals of Fuzzy Mathematics and Informatics*. 4 (2012) 83–98.
- [5] R. R. Yager, Pythagorean fuzzy subsets, *Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*. (2013) 57–61.
- [6] X.Peng, Y.Yang, Fundamental Properties of interval-valued Pythagorean fuzzy aggregation operators, *International Journal of Intelligent Systems*. 31 (2016) 444–487.
- [7] P.Talukdar, P. Dutta, Distance measures for cubic Pythagorean fuzzy sets and its applications to multicriteria decision making. *Granul. Comput.* 6 (2021) 267–284.
- [8] D. Molodsov, Soft set theory—first results, *Computers and Mathematics with Applications*. 37 (1999) 19–31.
- [9] F.Smarandache, Extension of soft set to hypersoft set, and then to plithogenic hypersoft set, *Neutrosophic Sets and Systems*. 22 (2018) 168–170.
- [10] M.Songsaeng, A.Iampan, Neutrosophic Cubic Set Theory Applied to UP-Algebras, *Thai Journal of Mathematics*. 18 (2020) 1447–1474.
- [11] M.J.Khan, S.Phiangsungnoen, H.Rehman, W.Kumam, Applications of generalized picture fuzzy soft set in concept selection, *Thai Journal of Mathematics*. 18 (2020) 296–314.
- [12] V. Chinnadurai, A. Swaminathan, A. Bobin, N.Thillaigovindan, Multi-criteria decision making process using cubic soft matrices, *Poincare Journal of Analysis and Applications* Vol. 7(1), 2020, 119-147.
- [13] V.Chinnadurai, A.Bobin, A novel selection process using combinations of max-min operations in cubic soft matrices, *AIP Conference Proceedings* 2277, 140007 (2020).
- [14] S.Z.Abbas, M.S.A.Khan, S.Abdullah, H.Sun, F.Hussain, Cubic Pythagorean fuzzy sets and their application to multi-attribute decision making with unknown weight information, *Journal of Intelligent and Fuzzy Systems*. 37 (2019) 1529–1544.
- [15] K.Alhazaymeh, Y.Al-Qudah, N.Hassan, A. M. Nasruddin, Cubic vague set and its application in decision making, *Entropy* 22, 963 (2020) 1–17.
- [16] F.Khan, M.S.A.Khan, M.Shahzad, S.Abdullah, Pythagorean cubic fuzzy aggregation operators and their application to multi-criteria decision making problems, *Journal of Intelligent and Fuzzy Systems* 36 (2019) 595–607.
- [17] A.Hussain, Tehreem, J.R.Lee, M.S.A.Khan, D.Y.Shin, Analysis of social networks by using Pythagorean cubic fuzzy Einstein weighted geometric aggregation operators, *Journal of Mathematics*. 2021 Article ID 5516869 1–18.
- [18] G.Kaur, H.Garg, Multi-attribute decision-making based on Bonferroni mean operators under cubic intuitionistic fuzzy set environment, *Entropy*. 20 (2018) 65.
- [19] G.Kaur, H.Garg, Generalized cubic intuitionistic fuzzy aggregation operators using t-Norm operations and their applications to group decision-making process. *Arab J Sci Eng* 44 (2019) 2775–2794.
- [20] B.P.Joshi, Pythagorean fuzzy average aggregation operators based on generalized and group-generalized parameter with application in MCDM problems, *Int J Intell Syst*. 34 (2019) 895–919.
- [21] R.M.Zulqarnain, X.L.Xin, M.Saeed, Extension of TOPSIS method under intuitionistic fuzzy hypersoft environment based on correlation coefficient and aggregation operators to solve decision making problem, *AIMS Mathematics*. 6 (2021) 2732–2755.
- [22] R.M.Zulqarnain, X.L.Xin, M.Saeed, A development of Pythagorean fuzzy hypersoft set with basic operations and decision-making approach based on the correlation coefficient, *Neutrosophic Sets and Systems*. 40 (2021) 149–168.
- [23] R.M.Zulqarnain, I.Siddique, F.Jarad, R.Ali, T.Abdeljawad, Development of TOPSIS Technique under Pythagorean Fuzzy Hypersoft Environment Based on Correlation Coefficient and Its Application towards the Selection of Antivirus Mask in COVID-19 Pandemic, *Complexity*. (2021) Article ID 6634991 1-27.

- [24] Ye, J. (2015). Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses, *Artificial Intelligence in Medicine*, 63, 171-179.
- [25] Ye, J and Fu, J. (2016). Multi-period medical diagnosis method using a single valued neutrosophic similarity measure based on Tangent function, *Computer Methods and Programs in Biomedicine*, 123, 142-149.
- [26] Ye, J. (2015). Single-valued neutrosophic similarity measures based on Cotangent function and their application in the fault diagnosis of steam turbine, *Soft Computing*, 21, 817-825.
- [27] Bobin A., Thangaraja P., Prathab H., and Thayalan S. (2023). Decision Making Using Cubic Hypersoft Topsis Method, *Journal of Applied Mathematics and Informatics*, Vol. 41, No. 5, pp. 973 - 988.
- [28] Chinnadurai V., Bobin A., and Cokilavany D. (2022). Simplified intuitionistic neutrosophic hypersoft TOPSIS method based on correlation coefficient, Vol 51, pp 570-591.
- [29] Chinnadurai V., and Bobin A., (2022). Picture Fuzzy Hypersoft TOPSIS Method Based on Correlation Coefficient. *Journal of Hyperstructures*, 10(2).
- [30] Elhassouny, A., and Smarandache, F. (2016). Neutrosophic-simplified-TOPSIS multi-criteria decision-making using combined simplified-TOPSIS method and neutrosophics, *IEEE International Conference on Fuzzy Systems*, 2468-2474.