Quaternion Framework of Neutrosophic Information with its Distance Measures and Decision-Making Model

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Abstract

Neutrosophic sets can be used to model uncertain data in real-world applications. To increase the use of complex neutrosophic sets, the space of quaternion numbers is investigated in this work. Analysts in complex contexts can benefit from the knowledge and direction that quaternion neutrosophic sets can offer by modeling complicated systems and capturing the interactions between various factors. Division algebras are used in some applications, such as particular formulations of class field theory, but they are generally far less important than quaternion numbers. Three-dimensional information with imaginary membership, imaginary indeterminacy, and imaginary non-membership functions is represented using quaternion neutrosophic sets. Intriguing quaternion numbers give us useful results when we analyze complicated data. Some basic characteristics of the derived concepts are examined. Novel quaternion-based operations and the analysis of order relations and logic operations are also explored based on neutrosophic set theory. For modeling uncertainty in quaternion-based systems, quaternion neutrosophic sets are helpful. Other fuzzy sets are unable to adequately capture the sophisticated fuzzy information that they can represent, such as uncertainty in both size and direction. The capacity to define fuzzy distance and similarity metrics is one of its intriguing qualities. We also present two quaternion distance measures and evaluate their properties. We use quaternion representations and measurements in a neutrosophic framework for decision-making models, and the results are excellent. Additionally, it shows readers how to construct the connections between traits and alternatives that are used in decision-making issues. An example is provided at the end to help illustrate the suggested strategy and provide additional context. Finally, we employ a different distance metric that is illustrated in the reliability section to validate the developed methodologies. It is possible to address the findings of studies on the application of quaternion neutrosophic sets for addressing various types of uncertainty in optimization problems related to the design and management of complex systems.

Keywords: Neutrosophics information; Quaternion numbers; Decision making; Distance measures

1 Introduction

Making a decision is like working through a puzzle to get an answer that seems right, or at least good enough. Therefore, it's a procedure that can be more or less rational or irrational, depending on whether the facts and

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ideas upon which it is founded are made known or kept hidden. When making a difficult choice, people typically rely on their tacit knowledge to help fill in the blanks. In most cases, when making a choice, both tacit and explicit knowledge are considered. A substantial component of decision-making involves analyzing a small set of possibilities that are described in terms of criteria for assessment. The next step could be to rank these choices according to how desirable they appear to the decision-maker(s) whenever all the relevant considerations are taken into account. Finding the best option or assessing the relative or overall significance of every choice (for instance, if choices represent projects vying for money) would be a challenge when all the factors have been taken into consideration at once.[1] Solving such problems is the aim of multiple-criteria decision analysis (MCDA). Because several MCDA methods can produce drastically different results when applied to the same set of data, this topic of decision-making has attracted the interest of many scholars and practitioners despite being quite ancient. As a result, a dilemma regarding decision-making is produced. All jobs based on science demand rational decision-making, in which professionals use their knowledge of a given topic to make informed decisions. For instance, selecting the appropriate course of action and making a diagnosis are usually required while making medical decisions. Naturalistic decision-making research, however, reveals that in situations with greater time restraints, stakes, or uncertainties, professionals may choose intuitive rather than organized approaches. Rather than weighing their options, they can choose a course to pursue based on a recognition-driven decision that fits their experience.[2]

The decision-makers circumstances may have an impact on their choices. For instance, the environment's complexity is a factor that influences cognitive function. An intricate environment has a wide range of potential states that may evolve throughout time. A simple space contained fewer of these items; in one experiment, the number of small pieces of equipment and items in the room functioned as a substitute for the area's complexity. A setting with a higher degree of complexity affects cognitive function significantly, making it easier to assess the situation and make better decisions.

The majority of systems in use today are categorized as complex systems with many variables, elements, and large dimensions. It is commonly acknowledged that traditional approaches to modeling and regulating contemporary systems have made significant contributions to research and the resolution of numerous control issues.[3,4] They have only been able to have a small impact on the problems that complex dynamic systems are causing, though. For complex systems, new approaches have been put forth that draw on existing knowledge and human experience, are capable of learning, and include cutting-edge features like the ability to recognize and identify failures. For modeling and managing complex systems, fuzzy quaternion maps (FQM) are suggested in this work. The use of FQM may help in the pursuit of more intelligent control strategies and the creation of autonomous systems. To depict the model and the behavior of the system, an FQM creates a causal diagram. Imprecise rules govern how the concepts in an FQM interact and how complicated system operations are replicated.[5]

Fuzzy quaternion maps are a type of symbolic representation that is used to describe and model complicated systems. They are made up of ideas that depict various facets of the system's behavior, and these ideas interact with one another to demonstrate the dynamics of the system. Because of the way by which it is developed, i.e., utilizing human specialists who are familiar with the operation of the system and its behavior in various conditions, the human experience and knowledge of the operation of the system are used to produce FQMs. An FQM uses the knowledge that has been acquired about the complex system to simply and symbolically characterize the system's behavior using a graph that depicts cause and effect along concepts.[6]

2 Literature Review

One of the most beneficial resources of what has become known as contemporary mathematics, Cantor's theory regarding sets is founded on the idea that substances belong to sets and has allowed us to examine modeling and develop in other domains.[7] In addition, unit membership within a set is a bivalent switch concept that can only take on values between 0 (no membership) and 1 (membership), which eliminates other set possibilities that have been investigated in the fields of logic models. Zadeh advocates the use of fuzzy sets (FSs)[8] by assuming that an element's membership in a set may be represented by a number between 0 and 1, with 0 representing non-membership, 1 denoting membership, and the digits between 0 and 1 denoting varying degrees of membership. Both explaining phenomena governed by ambiguous parameters and creating non-bivalent logic models have proven to be extremely effective with these sets. The concept of degree of membership, $\mu : \Delta \to [0, 1]$, is critical to this theory. In addition, Zadeh introduces the idea of interval-valued fuzzy sets, in which an element's level of membership in a set is established by a closed subinterval of [0,

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1] rather than by an element of [0, 1]. If Δ is a non-empty set, also referred to as a referential set, then an expression is given that is an interval-valued fuzzy set P on Δ.

$$P = \{(\mathbb{U}, M_P(\mathbb{U}))|\mathbb{U} \in \Delta\}$$

where $M_P : \mathbb{U} \to D[0, 1]$, such that
$x \to M_P(\mathbb{U}) = [M_{PL}(\mathbb{U}), M_{PU}(\mathbb{U})]$, D[0,1] being the set of all closed subintervals of [0,1], $M_{PL}(\mathbb{U})$ and $M_{PU}(\mathbb{U})$ are the lower extreme and upper extreme respectively of the interval $M_P(\mathbb{U})$. The set of all interval valued fuzzy sets on Δ shall be referred to as IVFSs (Δ). These sets have received a lot of attention and are commonly utilized. Gorzalczany's work on approximate reasoning,[10] Sanchez's,[11] Sambuc's,[12] Roy and Biswas' work on medical diagnostics,[13] Turksen's work on multivalent logic,[14] Ponsard's, and the Chinese school of Julong's are all worth mentioning. It's worth noting that Julong refers to these sets as grey sets. Atanassov proposed the intuitionistic fuzzy set (IFS), which is a broadening of Zadeh's fuzzy sets. Each IFS element is represented as an ordered pair (μ, ν) that meets the criterion $\mu + \nu \leq 1$. IFS is most commonly used in practical MCDA issues, and academic research on the subject has progressed significantly.

The neutrosophic set (NS), an idea in philosophy and mathematical instrument for understanding the genesis, nature, and scope of neutralities, was initially put forth by Smarandache.[15] It examines neutralities' development, characteristics, and uses as well as how they interact with other ideational spectrum. However, in some real-world situations, an option satisfying a decision maker feature may have membership and nonmembership degrees that add up to more than 1, but their square sum is less than or equal to 1. To support this theory, Pythagorean fuzzy sets have been characterized.[16] It will be difficult to apply NS in actual scientific and engineering contexts, despite the fact that it philosophically generalised the ideas of FS, IFS, and all existing structures. This concept is essential in a variety of circumstances, such as details fusion, which combines data from many sensors. Neutosophic sets have been heavily utilized in engineering and other sectors in recent years for decision-making. Wang et al.[17] suggested a single-valued neutrosophic set (SV-NS) that can handle challenges with erroneous, unclear, and conflicting data. Its extensions have been specified by numerous other academics; for instance, see.[18] Contrarily, an SV-NS is a kind of NS that allows us to depict uncertainty, imprecision, incompleteness, and unpredictable behavior in real life. It would be more suitable to employ contradictory data as well as an information matrix while making decisions. On the other hand, SV-NSs can be employed in scientific and technological applications since the SV-NS theory can be helpful in modeling confusing, imprecise, and inconsistent data. Due to its simplicity in capturing the ambiguous nature of subjective assessments, the SV-NS is well suited for gathering vague, ambiguous, and inconsistent data in MCDA.[20–22]

A DM gives his endorsement for membership of a substitute is $\frac{\sqrt{3}}{2}$ and his sympathy opposed membership is $\frac{1}{2}$. Yager[23] provided an example to illustrate this circumstances: a DM gives his endorsement for membership of a substitute is $\frac{\sqrt{3}}{2}$ and his support against membership is $\frac{1}{2}$. They are not accessible for IFS since the total of two values is more than 1, but they are accessible for PFS because $(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 \leq 1$. Clearly, PFSs is better in modeling the ambiguity in real-world problems. After carefully examining all of the possibilities, a method called "MCDA" is used to resolve real-world issues. The MCDA techniques have gained acceptance and are now widely employed in a variety of fields, including health, architecture, economics, and a number of other scientific and professional ones. Due to the complexity and ambiguity of the gathered data, which made it challenging for decision-makers to arrive at the best conclusions, the MCDA technique has recently advanced. The standard MCDA procedures produced results that were so confused and surprising that they were unusable.[28]

A more recent field of fuzzy logic study is complex fuzzy sets (CFSs). According to Ramot et al.,[24,25] CFSs are subsets of some universal set with a membership function whose codomain is the unit disc of the complex plane. A CFS is correspondingly a set of ordered pairs $(x, \mu(x))$ in which $x \in X$ is an element of some universal set and $\mu(x)$ is membership in the CFSs, $\mu(x) \in \{c \in C||c| \leq 1$, for C the complex plane. CFSs and isomorphic complex fuzzy logic have been demonstrated to produce accurate and parsimonious models for time series forecasting,[26] data mining,[27] and image processing. By asserting that, at least in some circumstances, a second dimension is necessary to express membership, complex fuzzy set theory extends the original idea of fuzzy membership. The core idea of fuzzyness, however, is unaffected by this additional dimension. The membership of a complicated fuzzy set is "just as fuzzy" as that of a regular fuzzy set. Unfortunately, it is difficult to understand the idea of a complex-valued membership, which presents a significant obstacle to achieving its full potential. As a result, a sizeable chunk of this book is devoted to discussing the intuitive understanding of membership grades with complex values. To help people grasp the benefits and potential of complex-valued membership functions, a number of examples are also given. It should be mentioned that combining complex numbers with fuzzy sets has been the subject of extensive research. But the characteristics of using complex numbers with fuzzy sets that are presented in the literature frequently diverge greatly from

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those that are mentioned. We use Quaternion numbers with neutrosophic information in this study. Quaternion numbers are the generalization of complex numbers and non-commutative four-dimensional algebra.
Quaternion numbers and their applications to rotations were first brought to light in print by Olinde Rodrigues et al in 1840, but an Irish mathematician Sir William Rowan Hamilton[29] discovered it independently in 1843 and applied it to mechanics in 3D space. A quaternion can be used to inscribe any rotation in a 3D coordinate system. Technically, a quaternion is the combination of three complex elements and one real element. Also, it can be used for much more than rotations. For rotations, quaternion are superior to using Euler angles, and the gamble lock problem is handled. There is extensive use of quaternion in computer animation; in which quaternion are used to represent transformations of orientations of graphical objects. They provide an inventive solution to problems that calamity early animated programs, convenient interpolation, gimbal lock, and instability.

2.1 Motivation

Real and imaginary numbers are added together to form complex numbers. charting a complex number on the plane of complex numbers is analogous to charting a real number on a number line: the x and y coordinates stand in for the real and imaginary halves of the number, respectively. As a result, it is simple to use complex numbers for expressing two-dimensional quantities (such as a location on a map). There is a strong connection between points on a sphere's surface and complex numbers. Although complex numbers have a long history in mathematics, they also have numerous uses in engineering and physics that are too numerous to go into here. Instead, let's focus on one specific example: describing rotational in two dimensions. As far as I can tell from my historical readings, when Hamilton began attempting to generalised complex numbers to higher dimensions, he wasn't particularly considering applications to science. Instead, he was simply interested in seeing if he could find a system of numbers with two fictitious units, i,j and k, that might represent points in three dimensions. But he was unable to locate one that was reliable. On that walk on October 16, 1843, he came to the realization that he could accomplish it using three separate fictitious numbers: i, j, and k. Using this concept, we apply the neutrosophic information to construct the relationship between neutrosophic numbers with quaternion numbers to represent the three dimension complex information. Use of quaternion neutrosophic sets (QNSs) in decision-making has a number of justifications, including:

- The capacity of QNSs to deal with data ambiguity is one of their main advantages. Fuzzy quaternion sets can capture more complicated and subtle interactions between many aspects and produce more accurate answers by expressing data in a three-dimensional space.
- For decision-making processes including several criteria or considerations, QNSs can be especially helpful. These sets can produce more thorough findings that take into consideration all pertinent elements by merging many criteria in a single model.
- In complicated network decision-making tasks like those in industrial applications or financial modeling, QNSs are also well suited. Such type of information can produce predictions and insights that are more precise because they are able to capture the intricate interactions between various variables.
- QNSs can be applied to long-term strategic planning, including organizational development and market analysis. These can assist firms in making wise judgments regarding the direction of their future by producing precise projections based on a variety of parameters.
- QNSs can be utilized for traffic management, including pattern prediction, flow optimization, and congestion reduction. Such information can shorten travel times and increase safety by taking into consideration a variety of factors, including weather, accidents, and construction.

The following are some of the study endowments that are consequential:

- Quaternion neutrosophic set is a useful novel approach of fuzzy set when identifying the imaginary membership of an element to a set is challenging due to disagreement between a few alternative values. order relations, set-theoretic operations, and some additional operations that take advantage of QNSs are given.

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- Algebraic quaternion distance measures are proposed.
- There are numerous illustrated examples available to explain various novel concepts connected to the newly constructed AOs for information fusion. In comparison to previous techniques, the proposed operators provide more generalised, reliable, and correct information then the complex sets.
- With the assistance of specified relations and distance measures, a new MCDA technique for modeling uncertainty in real-life settings was devised. Quaternion neutrosophic numbers (QNNs) are being utilized to solve a range of MCDA problems. To demonstrate the applicability of the suggested approach, an illustrative example for medical diagnosis based on the proposed representations based is offered to investigate.

Why this method is powerful then the previous?
We cannot sketch Hamilton's quaternion on a piece of paper since it appears that it requires four coordinates (one actual portion and three fictitious parts) to plot. But we can draw that if we have imaginary sections of the quaternion (leaving out the real bit) and the three spatial coordinates (x, y, and z). In fact, unit vectors mathematical constructions that represent the most basic perpendicular directions in three dimensional space have been designated with the (i, j, k) notation, as those of you who have taken beginning physics or specific math subjects may be aware. Quaternion numbers are far more effective because they allow you to indicate rotations in both directions (using only the imaginary components) or the entire quaternion. The study of the rotation and the algebraic structure of the complex numbers is much more difficult. In the context of complex neutrosophic sets, the study is very complicated. Quaternion numbers allow us to easily solve complex problems using neutrosophic information. As a result, the model we introduced to solve complex neutrosophic problems is extremely useful.
The following sections make up the remaining portion of the article. In Section 3, background ideas are defined. In Section 4, a new neutrosophic set representation based on quaternion numbers is presented. Section 5 investigates neutrosophic-based quaternion distance measurements. Section 6 presents tests using benchmark medical diagnosis data-sets and introduces a novel decision-making model based on the proposed quaternion distance metrics also demonstrates the reliability of our suggested approach in this section. Advantages and limitations of the proposed model is described in section 7. Section 8 concludes with recommendations for additional research.

3 Preliminary

Basic definitions and associated concepts utilized in the work are presented in this part.

Definition 3.1. (See[8]) Assume that, D is a fuzzy set (FS) over Z is mathematically described as;

$$D = \{(\mathbb{U}, \mu_D(\mathbb{U})) : \mathbb{U} \in Z\}, \tag{1}$$

Here $\mu_D(\mathbb{U}) \in [0,1]$ is membership degree of $\mathbb{U}$ in Z.

Definition 3.2. (See[8]) Assume D is an Intuitionistic fuzzy set (IFS) over Z is mathematically described as;

$$D = \{(\mathbb{U}, \mu_D(\mathbb{U}), \nu_D(\mathbb{U})) : \mathbb{U} \in Z\}, \tag{2}$$

Here $(\mu_D(\mathbb{U}), \nu_D(\mathbb{U})) \to [0,1]$ are membership degree (MD) and non membership degree (NMD) of $\mathbb{U}$ in Z. Satisfy that $0 \leq \mu_D(\mathbb{U}) + \nu_D(\mathbb{U}) \leq 1$

Definition 3.3. (See[24,25]) Assume that, D is a Complex fuzzy set (CFS) over Z is mathematically described as;

$$D = \{(\mathbb{U}, \mu_D(\mathbb{U})) : \mathbb{U} \in Z\}, \tag{3}$$

Here $\mu_D(\mathbb{U})$ is complex valued grade of MD of $\mathbb{U}$ in Z. By definition, the value $\mu_D(\mathbb{U})$ may receive all lie with in the unit circle in the complex plane, and are thus of the form $r_D(\mathbb{U}).e^{j\omega_D(\mathbb{U})}$ where $j = \sqrt{-1}$, $r_D(\mathbb{U})$ and $P_D(\mathbb{U})$ are both real valued and $r_D(\mathbb{U}) \in [0,1]$

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Definition 3.4. (See[30]) Assume that, D is a Complex intuitionistic fuzzy set (CIFS) over Z is distinguished by a MD $\zeta_D(\Bbb U)$ and NMD $\eta_D(\Bbb U)$, respectively, that assign an element $\Bbb U \in Z$ a complex-valued grade to both MD and NMD in Z. The values of $\zeta_D(\Bbb U)$ and $\eta_D(\Bbb U)$ all lie with in the unit circle in the complex plane and are of the form $\zeta_D(\Bbb U) = P_D(\Bbb U)e^{j\omega_D(\Bbb U)}$ and $\eta_D(\Bbb U) = r_D(\Bbb U)e^{j\mu_D(\Bbb U)}$, where $\zeta_D(\Bbb U), \eta_D(\Bbb U), P_D(\Bbb U),$ and $r_D(\Bbb U)$ all are real values and $P_D(\Bbb U)$ and $r_D(\Bbb U) \in [0, 1]$ with $j = \sqrt{-1}$ is denoted as

$$D = \{(\Bbb U, \zeta_D(\Bbb U), \eta_D(\Bbb U) : \Bbb U \in Z\}, \tag{4}$$

Similarly, the pure complex NMD was added to the concept of complex fuzzy class to create the concept of complex intuitionistic fuzzy class.[31]

Definition 3.5. (See[33]) Assume that D is a Pythagorean fuzzy set (PFS) over Z is mathematically described as;

$$D = \{(\Bbb U, \mu_D(\Bbb U), \nu_D(\Bbb U)) : \Bbb U \in Z\}, \tag{5}$$

Here $\mu_D(\Bbb U) \to [0, 1]$ and $\nu_D(\Bbb U) \to [0, 1]$, is MD and NMD of $\Bbb U$ in Z. Satisfy that $0 \leq (\mu_D(\Bbb U))^2 + (\nu_D(\Bbb U))^2 \leq 1$, Moreover the hesitancy degree is defined as $\eta_D(\Bbb U) = \sqrt{1 - ((\mu_D(\Bbb U))^2 + (\nu_D(\Bbb U))^2)}$.

Definition 3.6. (See[30]) A Complex Pythagorean fuzzy set (CPFS) D over Z is distinguished by a MD $\zeta_D(\Bbb U)$ and NMD $\eta_D(\Bbb U)$, respectively, that assign an element $\Bbb U \in Z$ a complex-valued grade to both MD and NMD in Z. The values of $\zeta_D(\Bbb U)$ and $\eta_D(\Bbb U)$ all lie with in the unit circle in the complex plane and are of the form $\zeta_D(\Bbb U) = P_D(x)e^{j\omega_D(\Bbb U)}$ and $\eta_D(\Bbb U) = r_D(\Bbb U)e^{j\mu_D(\Bbb U)}$, where $\zeta_D(\Bbb U), \eta_D(\Bbb U), P_D(\Bbb U),$ and $r_D(\Bbb U)$ all are real values and $P_D(\Bbb U)$ and $r_D(\Bbb U) \in [0, 1]$ with $j = \sqrt{-1}$ is denoted as

$$D = \{(\Bbb U, \zeta_D(\Bbb U), \eta_D(\Bbb U) : \Bbb U \in Z\}, \tag{6}$$

Here $\zeta_D(\Bbb U) \to [0, 1]$ and $\eta_D(\Bbb U) \to [0, 1]$, is MD and NMD of $\Bbb U$ in Z. Satisfy that $0 \leq (\zeta_D(\Bbb U))^2 + (\eta_D(\Bbb U))^2 \leq 1$, Moreover the hesitancy degree is defined as $\eta_D(\Bbb U) = \sqrt{1 - (\zeta_D(\Bbb U))^2 + (\eta_D(\Bbb U))^2)}$.

Definition 3.7. A quaternion q is a four dimensional complex number also called a hyper complex number, introduced by Hamilton[34] in 1843. Let a, b, c, d are real numbers and i, j, k are imaginary units then a quaternion is expressed as $q = a + bi + cj + dk$. The imaginary units i, j, k are mutually orthogonal unit vectors. These imaginary units have the following properties:

- $i^2 = j^2 = k^2 = ijk = -1$
- $ij = -ji = k, jk = -kj = i, ki = -ik = j.$

Definition 3.8. Let Z be space and $\Bbb U \in Z$. A neutrosophic set[15] D in Z is categorized as MD T_D, indeterminacy membership (IM) I_D and NMD F_D. T_D, I_D and F_D are there any genuine standard or non-standard subsets of $]0^{-1}, 1^+[$. That is,

$$T_D : Z \to]0^{-1}, 1^+[, I_D : Z \to]0^{-1}, 1^+[, F_D : Z \to]0^{-1}, 1^+[.$$

The overall value is unrestricted, T_D, I_D and F_D, so $0^- \leq$ sup T_D + sup I_D + sup $F_D \leq 3^+$.

Definition 3.9. Assume that Z represents finite universe of discourse (*UOD*). The D of an SV-NFSs in Z is specified in.[3]

$$D = \{T_D, I_D, F_D | \in Z\}$$
$$T_D : Z \to [0, 1], I_D : Z \to [0, 1], F_D : Z \to [0, 1].$$

such that,

$$0 \leq T_D + I_D + F_D \leq 3.$$

Definition 3.10. Assume that M represents finite universe of discourse (*UOD*). The N of an SVNFSs in M is specified in.[35]

$$N = \{\varrho, \alpha_A(\varrho), \beta_A(\varrho), \gamma_A(\varrho) | \varrho \in M\}$$
$$\alpha_A : M \to [0, 1], \beta_A : M \to [0, 1], \gamma_A : M \to [0, 1].$$

such that,

$$0 \leq \alpha_A(\varrho) + \beta_A(\varrho) + \gamma_A(\varrho) \leq 3.$$

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4 Development of Neutrosophic Information in term of Quaternion Numbers

In this section, we generalize the neutrosophic sets by the help of Quaternion numbers. This model improves on the previous model[31] by representing NS with pure quaternion numbers (hyper complex) rather than complex numbers. The suggested model is more powerful than the two or three parameter description of IFSs based on complex numbers provided in.[30] In all below mentioned state we discussed about neutrosophic sets with the help of quaternion numbers. We denoted them as quaternion neutrosophic sets (QNSs). We consider $\Re$ as the set of real numbers and $\mathbb{Q}$ as the set of quaternion numbers.

Definition 4.1. A fuzzy quaternion number[30] is given by $\acute{Q} : \mathbb{B} \to [0, 1]$ such that

$$\acute{Q}(\omega + \alpha i + \beta j + \gamma k) = min\{\overline{W}(\omega), \overline{X}(\alpha), \overline{Y}(\beta), \overline{Z}(\gamma)\}$$

for some $(\omega, \alpha, \beta, \gamma) \in \mathbb{B}_{\mathbb{Q}}$ and $\overline{W}, \overline{X}, \overline{Y}, \overline{Z} \in \Re_F$, here $\Re_F$ fuzzy quaternion numbers.

Definition 4.2. A fuzzy pure quaternion number[31] is given by $\acute{Q} : \mathbb{B} \to [0, 1]$ such that

$$\acute{Q}(\alpha i + \beta j + \gamma k) = min\{\overline{X}(\alpha), \overline{Y}(\beta), \overline{Z}(\gamma)\}$$

for some $(\omega, \alpha, \beta, \gamma) \in \mathbb{B}_{\mathbb{Q}}$ and $\overline{X}, \overline{Y}, \overline{Z} \in \Re_F$, here $\Re_F$ fuzzy pure quaternion numbers.

Definition 4.3. A quaternion neutrosophic set (QNS) is given by $\acute{N}_Q : \mathbb{N} \to [0, 1]$ such that

$$\acute{N}_Q(\alpha i + \beta j + \gamma k) = min\{\overline{X}(\alpha), \overline{Y}(\beta), \overline{Z}(\gamma)\}$$

for some $(\alpha, \beta, \gamma) \in \mathbb{N}_{\mathbb{Q}}$ and $\overline{X}, \overline{Y}, \overline{Z} \in \Re_{N_Q}$, here $\Re_{N_Q}$ is quaternion neutrosophic set. For the better understanding $\overline{X}$ is called membership function, $\overline{Y}$ is called indeterminacy function, and $\overline{Z}$ is called non membership function. Here the function satisfied the following condition for Quaternion Neutrosophic set (QNS) as

$$0 \leq \alpha + \beta + \gamma \leq 3$$

and

$$(\alpha, \beta, \gamma) \to [0, 1]$$

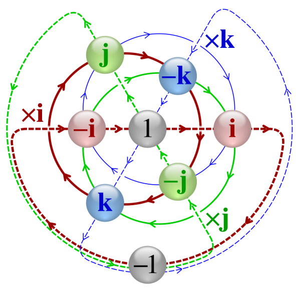

Figure 1: Graphical representation of the complex degrees for Quaternion 3-D

Remarks: When studying scenarios involving rotations in N^3, quaternions are particularly effective. A triplet, or pure quaternion, is a more succinct illustration than a rotation matrix. The fact that the rotation axis and angle can be easily determined makes its geometric significance more clear. We will be able to compose rotations with ease using the quaternion algebra that will be introduced. This is so because just sixteen multiplications and twelve additions are required for quaternion composition. As described a quaternion operation is $\acute{N}_Q(\alpha i + \beta j + \gamma k)$ here i,j & k satisfying condition $i^2 = j^2 = k^2 = ijk = -1 \& ij = -ji = k, jk = -kj = i, ki = -ik = j$. Figure 1 provides an accessible illustration of the unit quaterions' byproducts.

Theorem 4.4. *A quaternion neutrosophic number is normal if and only if* $\mathbb{Z}$ *exists for every* $\zeta \in \Xi_Q$ *such that* $\zeta(\mathbb{Z}) = 1$.

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Proof. Suppose $\zeta \in \Xi_Q$, then

$$\zeta = (\alpha, \beta, \gamma), \alpha, \beta, \gamma \in N_Q.$$

Given that any ambiguous real number is normal, there must exist $\flat, \ell, \partial \in N$ such that;

$$\alpha(\flat) = \beta(\ell) = \gamma(\partial) = 1.$$

Let

$$\mathbb{Z} = \flat i + \ell j + \partial k$$

Hence, we deduce that $\zeta(\mathbb{Z}) = 1$. Suppose $\delta \in (0, 1]$ and $\alpha \in N_Q$. Then the set

$$\alpha[\delta] = \{\kappa \in N : \alpha(\kappa) \geq \delta\}$$

is the $\delta - cut$ of α. The set of the $\delta - cut$ will apply to fuzzy quaternion numbers as

$$\zeta[\delta] = \{\mathbb{Z} \in \Xi : \zeta(\mathbb{Z}) \geq \delta\}.$$

□

Theorem 4.5. *For each $\zeta \in \Xi_Q$ and $\delta \in (0, 1]$, $\zeta[\delta] = \alpha[\delta] \times \beta[\delta] \times \gamma[\delta]$, such that $\zeta[\delta]$ is a hiper-cube in N^3.*

Proof. suppose that

$$\zeta = (\alpha, \beta, \gamma)$$

and

$$\mathbb{Z} = \flat i + \ell j + \partial k \in \zeta[\delta],$$

then

$$\zeta(\mathbb{Z}) = min\{\alpha(\flat), \beta(\ell), \gamma(\partial)\} \geq \delta$$

and

$$\{\alpha(\flat), \beta(\ell), \gamma(\partial)\} \geq \delta,$$

such that

$$\flat \in \alpha[\delta],\ \ell \in \beta[\delta], \partial \in \gamma[\delta].$$

Hence,

$$\mathbb{Z} \in \{\alpha[\delta] \times \beta[\delta] \times \gamma[\delta]\}.$$

Instead, if

$$\mathbb{Z} \in \{\alpha[\delta] \times \beta[\delta] \times \gamma[\delta]\},$$

Then

$$\mathbb{Z} = \flat i + \ell j + \partial k,\ \flat \in \alpha[\delta], \ell \in \beta[\delta], \partial \in \gamma[\delta]$$

It is simple to draw the conclusion that $\mathbb{Z} \in \zeta[\delta]$, if we proceed along the opposite method of the first portion.

□

4.1 Algebraic Operations for Quaternion Neutrosophic Sets

In this section, we discuss the algebraic operations for Quaternion Neutrosophic sets (QNSs). We devolved some algebraic operations on the newly generalized quaternion neutrosophic information such as complement, Conjugate, Magnitude, Inverse, Union, Intersection, and product.

Definition 4.6. Let $D = \{(\Bbbk, T_D(\Bbbk)i, I_D(\Bbbk)j, F_D(\Bbbk))k : \Bbbk \in \Delta\}$ is a set of Quaternion Neutrosophic numbers (QNNs). The complement of D is as

$$D^C = \{(\Bbbk, F_D(\Bbbk)k, (1 - I_D(\Bbbk))j, T_D(\Bbbk))i : \Bbbk \in \Delta\}$$

Proposition 4.7. *Assume that D be a QNS on P. Then, $(D^C)^C = D$.*

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Proof. Using definition 4.6, We will easily demonstrate it. □

Example 4.8. Let $D_q = (0.5i, 0.3j, 0.4k)$ is a QNs then the Complement of QNNs D_q is as

$$D^C = (0.4k, 0.7j, 0.5i)$$

Remarks: By definition 4.6, we have $D = \{(\Bbbk, T_D(\Bbbk)i, I_D(\Bbbk)j, F_D(\Bbbk))k : \Bbbk \in \Delta\}$, where $T_D(\Bbbk)i, I_D(\Bbbk)j, F_D(\Bbbk)$ are imaginary MD, imaginary IM and imaginary NMD of the quaternion neutrosophic function respectively.

Definition 4.9. Let $D_Q = \alpha i, \beta j, \gamma k$ be a set of QNNs, then its conjugate, denoted by $\overline{D_Q}$, is defined as $\overline{D} = -\alpha i, -\beta j, -\gamma k$. Geometrically, it represents the reflection of quaternion number around the imaginary directions see in Figure 1.

Example 4.10. Let $D_q = 0.5i, 0.3j, 0.4k$ is a QNs then the Conjugate of QNs D_q is as

$$\overline{D_q} = 0.5i, -0.7j, -0.4k$$

Definition 4.11. Let $D = \alpha i, \beta j, \gamma k$ be a set of QNNs, where $\omega = 0$ for pure Quaternion function, then its magnitude is described as

$$|| D ||= \sqrt{\alpha^2 + \beta^2 + \gamma^2}$$

It is also called length or norm of a Quaternion neutrosophic numbers. Alternatively this is described as

$$|| D ||= \sqrt{D\overline{D}}$$

Example 4.12. Let $D_q = 0.5i, 0.3j, 0.4k$ is a QNs then the magnitude of QNNs D_Q is as

$$|| D_Q ||= \sqrt{0.5^2 + 0.3^2 + 0.4^2}$$

Definition 4.13. Let D_Q and $\overline{D_Q}$ are set of QNNs and its conjugate respectively, then the inverse of a conjugate is defined as

$$D_Q{}^{-1} = \frac{\overline{D_Q}}{| D_Q |}$$

Example 4.14. Let $D_Q = 0.5i, 0.3j, 0.4k$ is a set of QNNs then the Inverse of QNNs D_Q is as

$$D_Q{}^{-1} = \frac{0.5i, 0.7j, 0.4k}{\sqrt{0.5^2 + 0.3^2 + 0.4^2}}$$

Definition 4.15. A QNN is said to be unit QNN if its magnitude is one. i.e.,

$$|| D_Q ||= 1$$

, here $D_Q = \{\Bbbk, \alpha i, \beta j, \gamma k\}$

Definition 4.16. Union of Quaternion neutrosophic sets. Let D_{q1} and D_{q2} be two Quaternion neutrosophic sets. The union is defined as,

$$T_{D_{q1}(\Bbbk) \cup D_{q2}(\Bbbk)} = [p_{q1}(\Bbbk) \vee p_{q1}(\Bbbk)]i$$
$$I_{D_{q1}(\Bbbk) \cup D_{q2}(\Bbbk)} = [q_{q1}(\Bbbk) \wedge q_{q1}(\Bbbk)]j$$
$$F_{D_{q1}(\Bbbk) \cup D_{q2}(\Bbbk)} = [r_{q1}(\Bbbk) \wedge r_{q1}(\Bbbk)]k$$

Here $\vee$ and $\wedge$ denote the max and min operators respectively.

Proposition 4.17. *Assume that D and E be the two QNSs on P. Then, $(D \cup E)^C = D^C \cup E^C$.*

Proof. Using definition 4.6 and definition 4.16, We will easily demonstrate it. □

Definition 4.18. The intersection of Quaternion neutrosophic sets. Let D_{q1} and D_{q2} be two Quaternion neutrosophic sets. The intersection is defined as,

$$T_{D_{q1}(\Bbbk) \cap D_{q2}(\Bbbk)} = [p_{q1}(\Bbbk) \wedge p_{q1}(\Bbbk)]i$$
$$I_{D_{q1}(\Bbbk) \cap D_{q2}(\Bbbk)} = [q_{q1}(\Bbbk) \vee q_{q1}(\Bbbk)]j$$
$$F_{D_{q1}(\Bbbk) \cap D_{q2}(\Bbbk)} = [r_{q1}(\Bbbk) \vee r_{q1}(\Bbbk)]k$$

Here $\vee$ and $\wedge$ denote the max and min operators respectively.

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Proposition 4.19. *Let N_{Q_1} and N_{Q_2} are two QNS in Δ then,*

$$(N_{Q_1} \cup N_{Q_2}) \cap N_{Q_1} = N_{Q_1}$$

$$(N_{Q_1} \cap N_{Q_2}) \cup N_{Q_1} = N_{Q_1}$$

Proof: *Suppose N_{Q_1} and N_{Q_2} are two QNS in Δ then $\alpha(\Bbbk), \beta(\Bbbk)$, and $\gamma(\Bbbk)$ are quaternion MD, IM and NMD respectively, then*

$$\alpha((N_{Q_1} \cup N_{Q_2}) \cap N_{Q_1} = N_{Q_1})(\Bbbk) = min(max(N_{Q_1}, N_{Q_2}), N_{Q_1}) = \alpha(\Bbbk)$$

Similarly we can find $\beta(\Bbbk)$ and $\gamma(\Bbbk)$ respectively.

Definition 4.20. Suppose N_{Q_1} and N_{Q_2} are two QNS in Δ and $T_{N_{Q_1}}(\Bbbk) = p_{N_{Q_1}}(\Bbbk)i$, $I_{N_{Q_1}}(\Bbbk) = q_{N_{Q_1}}(\Bbbk)j$, $F_{N_{Q_1}}(\Bbbk) = r_{N_{Q_1}}(\Bbbk)k$ and $T_{N_{Q_2}}(\Bbbk) = p_{N_{Q_2}}(\Bbbk)i$, $I_{N_{Q_2}}(\Bbbk) = q_{N_{Q_2}}(\Bbbk)j$, $F_{N_{Q_2}}(\Bbbk) = r_{N_{Q_2}}(\Bbbk)k$ be their quaternion MD, IM and NMD respectively, then the quaternion neutrosophic product is denoted as $N_{Q_1} \circ N_{Q_2}$ and specified by the function as

$$T_{N_{Q_1} \circ N_{Q_2}}(\Bbbk) = p_{N_{Q_1} \circ N_{Q_2}}(\Bbbk) = p(\Bbbk)i$$

$$I_{N_{Q_1} \circ N_{Q_2}}(\Bbbk) = q_{N_{Q_1} \circ N_{Q_2}}(\Bbbk) = q(\Bbbk)j$$

$$F_{N_{Q_1} \circ N_{Q_2}}(\Bbbk) = p_{N_{Q_1} \circ N_{Q_2}}(\Bbbk) = r(\Bbbk)k$$

Example 4.21. Suppose $\Delta = \{\Bbbk_1, \Bbbk_2\}$ and $N_{Q_1} = \{\frac{0.3i, 0.6j, 0.9k}{\Bbbk_1}, \frac{0.2i, 0.7j, 0.3k}{\Bbbk_2}\}$ and $N_{Q_2} = \{\frac{0.7i, 0.3j, 0.9k}{\Bbbk_1}, \frac{0.8i, 0.7j, 0.6k}{\Bbbk_2}\}$ then $N_{Q_1} \circ N_{Q_2}(\Bbbk) = (\frac{0.21i, 0.18j, 0.81k}{\Bbbk_1}, \frac{0.16i, 0.49j, 0.18k}{\Bbbk_2})$

Definition 4.22. Suppose N_n be the n quaternion neutrosophic sets on X and $\{n = 1, 2, 3, ...N\}$ $T_{N_n}(\Bbbk) = p_{N_n}(\Bbbk)i$, $I_{N_n}(\Bbbk) = q_{N_n}(\Bbbk)j$, $F_{N_n}(\Bbbk) = p_{N_n}(\Bbbk)k$ be their quaternion MD, IM and NMD respectively, the cartesian product of N_n is mathematically written as $N_1 \times N_2 \times N_3 \times, ..., N_n$ specified as

$$T_{N_1 \times N_2 \times N_3 \times, ..., N_n}(\Bbbk)i = p_{N_1 \times N_2 \times N_3 \times, ..., N_n}(\Bbbk)i$$

$$= min(p_{N_1}(\Bbbk), p_{N_2}(\Bbbk), p_{N_3}(\Bbbk), ..., p_{N_n}(\Bbbk))i$$

$$I_{N_1 \times N_2 \times N_3 \times, ..., N_n}(\Bbbk)j = q_{N_1 \times N_2 \times N_3 \times, ..., N_n}(\Bbbk)j$$

$$= max(q_{N_1}(\Bbbk), q_{N_2}(\Bbbk), q_{N_3}(\Bbbk), ..., q_{N_n}(\Bbbk))j$$

$$F_{N_1 \times N_2 \times N_3 \times, ..., N_n}(\Bbbk)k = q_{N_1 \times N_2 \times N_3 \times, ..., N_n}(\Bbbk)k$$

$$= max(r_{N_1}(\Bbbk), r_{N_2}(\Bbbk), r_{N_3}(\Bbbk), ..., r_{N_n}(\Bbbk))k$$

5 Distance Measure of Quaternion Neutrosophic Sets

We introduce numerous quaternion neutrosophic set distance metrics in this section. Initially, we provide the following definition of distance between quaternion neutrosophic sets:

Definition 5.1. A distance of quaternion neutrosophic sets is a function $d : QNS \times QNS \longrightarrow Q_R \cup \{0\}$ which satisfied the following conditions for any N_{Q_1}, N_{Q_2} and N_{Q_3} in QNS.

$d(N_{Q_1}, N_{Q_2} \geq 0)$ and $d(N_{Q_1}, N_{Q_2} = 0)$ iff $N_{Q_1} = N_{Q_2}$.

$d((N_{Q_1}, N_{Q_2}) = d((N_{Q_2}, N_{Q_1})$.

$d((N_{Q_1}, N_{Q_2}) + d((N_{Q_1}, N_{Q_3}) \geq d((N_{Q_2}, N_{Q_3})$.

In this part, we suggest a novel distance metric for QNS based on the Jensen-Shannon divergence. Additionally, we deduce and establish the merits and traits of suggested distance metrics.

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Definition 5.2. N_{Q_1} and N_{Q_2} be two QNSs are given in Δ then,

$$\begin{aligned} N_{Q_1} &= \Big\{(\mathbb{U}, \alpha_{N_{Q_1}}(\mathbb{U})i, \beta_{N_{Q_1}}(\mathbb{U})j, \gamma_{N_{Q_1}}(\mathbb{U})k)|\mathbb{U} \in \Delta\Big\}, and \\ N_{Q_2} &= \Big\{(\mathbb{U}, \alpha_{N_{Q_2}}(\mathbb{U})i, \beta_{N_{Q_2}}(\mathbb{U})j, \gamma_{N_{Q_2}}(\mathbb{U}))k|\mathbb{U} \in \Delta\Big\}. \end{aligned}$$

Here $\gamma_{N_{Q_1}}(\mathbb{U})$ and $\gamma_{N_{Q_2}}(\mathbb{U})$ are two QNSs, the divergence measure between the QNSs N_{Q_1} and N_{Q_2} denotes as $JS_{QNS}(N_{Q_1}, N_{Q_2})$, and defined as:

$$JS_{QNS}(N_{Q_1}, N_{Q_2}) = \frac{1}{2}\Big[KL(N_{Q_1}, \frac{N_{Q_1}+N_{Q_2}}{2}) + KL(N_{Q_2}, \frac{N_{Q_1}+N_{Q_2}}{2})\Big],$$

with

$$KL({}_{N_{Q_1}}(\mathbb{U}), N_{Q_1}(\mathbb{U})) = \alpha_{N_{Q_1}}(\mathbb{U})log\frac{\alpha_{N_{Q_1}}(\mathbb{U})}{\alpha_{N_{Q_2}}(\mathbb{U})}i + \beta_{N_{Q_1}}(\mathbb{U})log\frac{\beta_{N_{Q_1}}(\mathbb{U})}{\beta_{N_{Q_2}}(\mathbb{U})}j + \gamma_{N_{Q_1}}(\mathbb{U})log\frac{\gamma_{N_{Q_1}}(\mathbb{U})}{\gamma_{N_{Q_2}}(\mathbb{U})}k.$$

where $KL(N_{Q_1}, N_{Q_2})$ is the divergence measure between N_{Q_1}, N_{Q_2}. $JS_{QNS}(N_{Q_1}, N_{Q_2})$ can alternatively be represented using the formula below:

$$\begin{aligned} JS_{QNS}(N_{Q_1}, N_{Q_2}) &= A(\frac{N_{Q_1}+N_{Q_2}}{2}) - \frac{1}{2}A(N_{Q_1}) - \frac{1}{2}A(N_{Q_2}) \\ &= \frac{1}{2}\Big[\alpha_{N_{Q_1}}(\mathbb{U})log\frac{2\alpha_{N_{Q_2}}(\mathbb{U})}{\alpha_{N_{Q_1}}(\mathbb{U})+\alpha_{N_{Q_2}}(\mathbb{U})}i + \alpha_{N_{Q_2}}(\mathbb{U})log\frac{2\alpha_{N_{Q_2}}(\mathbb{U})}{\alpha_{N_{Q_2}}(\mathbb{U})+\alpha_{N_{Q_2}}(\mathbb{U})}i \\ &\quad +\beta_{N_{Q_1}}(\mathbb{U})log\frac{2\beta_{N_{Q_1}}(\mathbb{U})}{\beta_{N_{Q_1}}(\mathbb{U})+\beta_{N_{Q_2}}(\mathbb{U})}j + \beta_{N_{Q_2}}(\mathbb{U})log\frac{2\beta_{N_{Q_2}}(\mathbb{U})}{\beta_{N_{Q_1}}(\mathbb{U})+\beta_{N_{Q_2}}(\mathbb{U})}j \\ &\quad +\gamma_{N_{Q_1}}(\mathbb{U})log\frac{2\gamma_{N_{Q_1}}(\mathbb{U})}{\gamma_{N_{Q_1}}(\mathbb{U})+\gamma_{N_{Q_2}}(\mathbb{U})}k + \gamma_{N_{Q_2}}(\mathbb{U})log\frac{2\gamma_{N_{Q_2}}(\mathbb{U})}{\gamma_{N_{Q_1}}(\mathbb{U})+\gamma_{N_{Q_2}}(\mathbb{U})}k\Big]. \end{aligned}$$

such that

$$\begin{aligned} A(N_{Q_1}) &= -(\alpha_{N_{Q_1}}(\mathbb{U})log\alpha_{N_{Q_1}}(\mathbb{U})i + \beta_{N_{Q_1}}(\mathbb{U})log\beta_{N_{Q_1}}(\mathbb{U})j + \gamma_{N_{Q_1}}(\mathbb{U})log\gamma_{N_{Q_1}}(\mathbb{U}))k. \\ A(N_{Q_2}) &= -((\alpha_{N_{Q_2}}(\mathbb{U})log\alpha_{N_{Q_2}}(\mathbb{U})i + \beta_{N_{Q_2}}(\mathbb{U})log\beta_{N_{Q_2}}(\mathbb{U})j + \gamma_{N_{Q_2}}(\mathbb{U})log\gamma_{N_{Q_2}}(\mathbb{U}))k. \end{aligned}$$

where $A(N_{Q_1})$ and $A(N_{Q_2})$ are the Entropies for N_{Q_1} and N_{Q_2}. Then, we defined a new distance measure for the QNSs in accordance with neutrosophic fuzzy divergence.

Definition 5.3. Let N_{Q_1} and N_{Q_2} be two QNSs in Δ, $\Delta = \{\mathbb{U}_1, \mathbb{U}_2, \mathbb{U}_3, ..., \mathbb{U}_n\}$ where

$$N_{Q_1} = \{\langle\mathbb{U}_\ell, \alpha_{N_{Q_1}}(\mathbb{U})i, \beta_{N_{Q_1}}(\mathbb{U})j, \gamma_{N_{Q_1}}(\mathbb{U})k\rangle|\mathbb{U}_\ell \in \Delta\} \text{ and}$$
$$N_{Q_2} = \{\langle\mathbb{U}_\ell, \alpha_{N_{Q_2}}(\mathbb{U})i, \beta_{N_{Q_2}}(\mathbb{U})j, \gamma_{N_{Q_2}}(\mathbb{U})k\rangle|\mathbb{U}_\ell \in \Delta\}.$$

Normalized quaternion neutrosophic distance d_η measure between N_{Q_1} and N_{Q_2} is defined by

$$d_\eta(N_{Q_1}, N_{Q_2}) = \frac{1}{n}\Sigma_{\ell=1}^{n} d_\eta(N_{Q_1}, N_{Q_2})$$

$$\begin{aligned} = \frac{1}{n}[\frac{1}{2}(\alpha_{N_{Q_1}}(\mathbb{U}_\ell)log\frac{2\alpha_{N_{Q_1}}(\mathbb{U}_\ell)}{\alpha_{N_{Q_1}}(\mathbb{U}_\ell)+\alpha_{N_{Q_2}}(\mathbb{U}_\ell)}i + \alpha_{N_{Q_2}}(\mathbb{U}_\ell)log\frac{2\alpha_{N_{Q_2}}(\mathbb{U}_\ell)}{\alpha_{N_{Q_1}}(\mathbb{U}_\ell)+\alpha_{N_{Q_2}}(\mathbb{U}_\ell)}i \\ +\beta_{N_{Q_1}}(\mathbb{U}_\ell)log\frac{2\beta_{N_{Q_1}}(\mathbb{U}_\ell)}{\beta_{N_{Q_1}}(\mathbb{U}_\ell)+\beta_{N_{Q_2}}(\mathbb{U}_\ell)}j + \beta_{N_{Q_2}}(\mathbb{U}_\ell)log\frac{2\beta_{N_{Q_2}}(\mathbb{U}_\ell)}{\beta_{N_{Q_1}}(\mathbb{U}_\ell)+\beta_{N_{Q_2}}(\mathbb{U}_\ell)}j \\ +\gamma_{N_{Q_1}}(\mathbb{U}_\ell)log\frac{2\gamma_{N_{Q_1}}(\mathbb{U}_\ell)}{\gamma_{N_{Q_1}}(\mathbb{U}_\ell)+\gamma_{N_{Q_2}}(\mathbb{U}_\ell)}k + \gamma_{N_{Q_2}}(\mathbb{U}_\ell)log\frac{2\gamma_{N_{Q_2}}(\mathbb{U}_\ell)}{\gamma_{N_{Q_1}}(\mathbb{U}_\ell)+\gamma_{N_{Q_2}}(\mathbb{U}_\ell)}k)]^{\frac{1}{2}}. \end{aligned}$$

Definition 5.4. Let N_{Q_1} and N_{Q_2} be two QNSs in Δ, $\Delta = \{\mathbb{U}_1, \mathbb{U}_2, \mathbb{U}_3, ..., \mathbb{U}_n\}$ where

$$N_{Q_1} = \{\langle\mathbb{U}_\ell, \alpha_{N_{Q_1}}(\mathbb{U})i, \beta_{N_{Q_1}}(\mathbb{U})j, \gamma_{N_{Q_1}}(\mathbb{U})k\rangle|\mathbb{U}_\ell \in \Delta\} \text{ and}$$
$$N_{Q_2} = \{\langle\mathbb{U}_\ell, \alpha_{N_{Q_2}}(\mathbb{U})i, \beta_{N_{Q_2}}(\mathbb{U})j, \gamma_{N_{Q_2}}(\mathbb{U})k\rangle|\mathbb{U}_\ell \in \Delta\}.$$

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Normalized Quaternion Neutrosophic distance measure d_ζ from N_{Q_1} and N_{Q_2} is defined as

$$d_\zeta(N_{Q_1}, N_{Q_2}) = \frac{1}{4n}\Sigma_{\ell=1}^{n} d_\zeta(N_{Q_1}, N_{Q_2}),$$

$$= \frac{1}{4n}\Sigma_{\ell=1}^{n}(|\alpha_{N_{Q_1}}(\mathbb{U}_\ell)i - \alpha_{N_{Q_2}}(\mathbb{U}_\ell)i| + |\beta_{N_{Q_1}}(\mathbb{U}_\ell)j - \beta_{N_{Q_2}}(\mathbb{U}_\ell)j| + |\gamma_{N_{Q_1}}(\mathbb{U}_\ell)k - \gamma_{N_{Q_2}}(\mathbb{U}_\ell)k|$$
$$+2max\{|\alpha_{N_{Q_1}}(\mathbb{U}_\ell)i - \alpha_{N_{Q_2}}(\mathbb{U}_\ell)i|, |\beta_{N_{Q_1}}(\mathbb{U}_\ell)j - \beta_{N_{Q_2}}(\mathbb{U}_\ell)j|, |\gamma_{N_{Q_1}}(\mathbb{U}_\ell) - \gamma_{N_{Q_2}}(\mathbb{U}_\ell)|\}).$$

Definition 5.5. Let N_{Q_1} and N_{Q_2} be two QNSs in Δ, $\Delta = \{\mathbb{U}_1, \mathbb{U}_2, \mathbb{U}_3, ..., \mathbb{U}_n\}$ where

$$N_{Q_1} = \{\langle \mathbb{U}_\ell, \alpha_{N_{Q_1}}(\mathbb{U})i, \beta_{N_{Q_1}}(\mathbb{U})j, \gamma_{N_{Q_1}}(\mathbb{U})k\rangle | \mathbb{U}_\ell \in \Delta\} \text{ and}$$
$$N_{Q_2} = \{\langle \mathbb{U}_\ell, \alpha_{N_{Q_2}}(\mathbb{U})i, \beta_{N_{Q_2}}(\mathbb{U})j, \gamma_{N_{Q_2}}(\mathbb{U})k\rangle | \mathbb{U}_\ell \in \Delta\}.$$

Euclidean quaternion neutrosophic distance measure d_δ from N_{Q_1} and N_{Q_2} is defined as $d_\delta(N_{Q_1}, N_{Q_2})$

$$= |N_{Q_1} - N_{Q_2}| = \frac{1}{4n}\Sigma_{\ell=1}^{n}\sqrt{[\alpha_{N_{Q_1}}(\mathbb{U}_\ell) - \alpha_{N_{Q_2}}(\mathbb{U}_\ell)]^2 + [\beta_{N_{Q_1}}(\mathbb{U}_\ell) - \beta_{N_{Q_2}}(\mathbb{U}_\ell)]^2 + [\gamma_{N_{Q_1}}(\mathbb{U}_\ell) - \gamma_{N_{Q_2}}(\mathbb{U}_\ell)]^2}$$

Definition 5.6. Let N_{Q_1} and N_{Q_2} be two QNSs in Δ, $\Delta = \{\mathbb{U}_1, \mathbb{U}_2, \mathbb{U}_3, ..., \mathbb{U}_n\}$ where

$$N_{Q_1} = \{\langle \mathbb{U}_\ell, \alpha_{N_{Q_1}}(\mathbb{U})i, \beta_{N_{Q_1}}(\mathbb{U})j, \gamma_{N_{Q_1}}(\mathbb{U})k\rangle | \mathbb{U}_\ell \in \Delta\} \text{ and}$$
$$N_{Q_2} = \{\langle \mathbb{U}_\ell, \alpha_{N_{Q_2}}(\mathbb{U})i, \beta_{N_{Q_2}}(\mathbb{U})j, \gamma_{N_{Q_2}}(\mathbb{U})k\rangle | \mathbb{U}_\ell \in \Delta\}.$$

The divergence quaternion neutrosophic distance measure $d_\Im$ from N_{Q_1} and N_{Q_2} is defined as

$$d_{\Im(N_{Q_1}, N_{Q_2})} = -ln_2(\frac{1}{2} + \frac{1}{2n}\Sigma_{\ell=1}^{n}(\sqrt{\alpha_{N_{Q_1}}(\mathbb{U})\alpha_{N_{Q_2}}(\mathbb{U})} + \sqrt{\beta_{N_{Q_1}}(\mathbb{U})\beta_{N_{Q_2}}(\mathbb{U})} + \sqrt{\gamma_{N_{Q_1}}(\mathbb{U})\gamma_{N_{Q_2}}(\mathbb{U})}))$$

Definition 5.7. Let N_{Q_1} and N_{Q_2} be two QNSs in Δ, $\Delta = \{\mathbb{U}_1, \mathbb{U}_2, \mathbb{U}_3, ..., \mathbb{U}_n\}$ where

$$N_{Q_1} = \{\langle \mathbb{U}_\ell, \alpha_{N_{Q_1}}(\mathbb{U})i, \beta_{N_{Q_1}}(\mathbb{U})j, \gamma_{N_{Q_1}}(\mathbb{U})k\rangle | \mathbb{U}_\ell \in \Delta\} \text{ and}$$
$$N_{Q_2} = \{\langle \mathbb{U}_\ell, \alpha_{N_{Q_2}}(\mathbb{U})i, \beta_{N_{Q_2}}(\mathbb{U})j, \gamma_{N_{Q_2}}(\mathbb{U})k\rangle | \mathbb{U}_\ell \in \Delta\}.$$

The $\gamma - max$ quaternion neutrosophic distance measure $d_\gamma - max$ from N_{Q_1} and N_{Q_2} is defined as

$$d_\gamma - max(N_{Q_1}, N_{Q_2}) = \frac{1}{3n}\Sigma_{\ell=1}^{n}(|\alpha_{N_{Q_1}}(\mathbb{U}) - \alpha_{N_{Q_2}}(\mathbb{U})| + |\beta_{N_{Q_1}}(\mathbb{U}) - \beta_{N_{Q_2}}(\mathbb{U})| + |\gamma_{N_{Q_1}}(\mathbb{U}) - \gamma_{N_{Q_2}}(\mathbb{U})|$$

$$+|max\{\alpha_{N_{Q_1}}(\mathbb{U}), \beta_{N_{Q_2}}(\mathbb{U})\gamma_{N_{Q_2}}(\mathbb{U}),\} - max\{\alpha_{N_{Q_2}}(\mathbb{U}), \beta_{N_{Q_1}}(\mathbb{U})\gamma_{N_{Q_1}}(\mathbb{U}),\}|)$$

Definition 5.8. Let N_{Q_1} and N_{Q_2} be two QNSs in Δ, $\Delta = \{\mathbb{U}_1, \mathbb{U}_2, \mathbb{U}_3, ..., \mathbb{U}_n\}$ where

$$N_{Q_1} = \{\langle \mathbb{U}_\ell, \alpha_{N_{Q_1}}(\mathbb{U})i, \beta_{N_{Q_1}}(\mathbb{U})j, \gamma_{N_{Q_1}}(\mathbb{U})k\rangle | \mathbb{U}_\ell \in \Delta\} \text{ and}$$
$$N_{Q_2} = \{\langle \mathbb{U}_\ell, \alpha_{N_{Q_2}}(\mathbb{U})i, \beta_{N_{Q_2}}(\mathbb{U})j, \gamma_{N_{Q_2}}(\mathbb{U})k\rangle | \mathbb{U}_\ell \in \Delta\}.$$

The Q-max (QM) quaternion neutrosophic distance measure $d_\wp$ from N_{Q_1} and N_{Q_2} is defined as

$$d_\wp(N_{Q_1}, N_{Q_2}) = \frac{1}{n}\Sigma_{\ell=1}^{n}(\frac{|\alpha_{N_{Q_1}}(\mathbb{U}) - \alpha_{N_{Q_2}}(\mathbb{U})| + |\beta_{N_{Q_1}}(\mathbb{U}) - \beta_{N_{Q_2}}(\mathbb{U})| + |\gamma_{N_{Q_1}}(\mathbb{U}) - \gamma_{N_{Q_2}}(\mathbb{U})|}{8}$$

$$+|max\{\alpha_{N_{Q_1}}(\mathbb{U}), \beta_{N_{Q_2}}(\mathbb{U})\gamma_{N_{Q_2}}(\mathbb{U}),\} - max\{\alpha_{N_{Q_2}}(\mathbb{U}), \beta_{N_{Q_1}}(\mathbb{U})\gamma_{N_{Q_1}}(\mathbb{U}),\}|)$$

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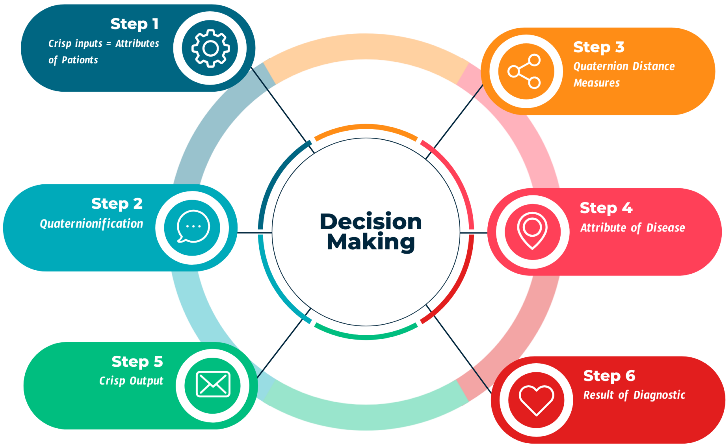

Figure 2: Graphical representation of decision making strategy

6 Decision Making Strategy based on Quaternion Neutrosophic Distance Measure

This section presents a novel decision-making paradigm based on the quaternion neutrosophic distance measure. All the steps are shown in Figure 2.

Example 6.1. Considerations are given five records which are referenced in Table 1 data. Label 1 indicates that the patient has a loss of taste in Table 1 titled "Class of Loss of Taste." Label 2 in Table 1 column for "Class of Cough" indicates that the patient has a cough. Labe3 indicates that the patient has a fever or a chill in Table 1 Class of Fever or Chill column. Label 4 in the Class of Body Aches column of Table 1 indicates that the patient has body aches. Label 5 in Table 1 Vomiting column indicates that the patient has vomitus. Considerations are given five records which are referenced in Table 2 data. Label 1 indicates that the patient

S	Loss of Taste	Cough	Fever or Chill	Body Aches	Vomiting
Wade	(0.8,0.3,0.6)	(0.4,0.6,0.9)	(0.9,0.8,0.6)	(0.5,0.5,0.8)	(0.6,0.4,0.6)
Ivan	(0.3,0.4,0.8)	(0.5,0.3,0.6)	(0.7,0.9,0.9)	(0.8,0.9,0.3)	(0.4,0.5,0.5)
Jorge	(0.3,0.2,0.2)	(0.1,0.6,0.8)	(0.9,0.3,0.4)	(0.3,0.6,0.9)	(0.6,0.9,0.8)
Dave	(0.6,0.4,0.8)	(0.3,0.4,0.1)	(0.2,0.6,0.1)	(0.8,0.3,0.9)	(0.4,0.3,0.8)
Gilbert	(0.6,0.4,0.3)	(0.4,0.9,0.9)	(0.8,0.8,0.9)	(0.8,0.3,0.7)	(0.6,0.7,0.6)

Table 1: Quaternion Neutrosophic fuzzy relation S(Patients→Symptoms)

has a viral infection in Table 2 column under "Class of Viral Infection," Label 2 in Table 2 Class of Malaria column indicates that the patient has malaria. Labe3 indicates that the patient has Covid-19 in the Table 2 column labelled "Class of Covid-19." Label 4 in Table 2 column for "Class of Dengue" indicates that the patient has dengue. Label 5 indicates that the patient has chest issues in Table 2 column "Chest Issues." Table 3 displays how Table 1 and Table 2 are related.

We can find the outcomes of the diagnosis in Table 4. Using the following formulae on Quaternion neutrosophic numbers, the results of Table 3 become apparent.

$$\alpha_T(P_i, F_k) = \bigvee_{s\in S} [\alpha_T(P_i, T)\Lambda\alpha_R(T, F_k)],$$

$$\beta_T(P_i, F_k) = \bigwedge_{s\in S} [\beta_T(P_i, T)\bigvee \beta_R(T, F_k)],$$

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T	Viral Infection	Malaria	Covid-19	Dengue	Chest Issues
Loss of Taste	(0.3,0.5,0.9)	(0.7,0.8,0.9)	(0.6,0.7,0.8)	(0.9,0.3,0.5)	(0.5,0.3,0.7)
Cough	(0.6,0.7,0.7)	(0.8,0.3,0.4)	(0.7,0.3,0.4)	(0.5,0.6,0.7)	(0.9,0.8,0.2)
Fever or Chill	(0.9,0.6,0.3)	(0.7,0.8,0.3)	(0.0.8,0.3,0.1)	(0.8,0.6,0.2)	(0.2,0.3,0.9)
Body Aches	(0.8,0.3,0.7)	(0.9,0.3,0.4)	(0.7,0.3,0.2)	(0.6,0.4,0.5)	(0.3,0.4,0.5)
Vomiting	(0.6,0.7,0.3)	(0.5,0.5,0.6)	(0.6,0.2,0.5)	(0.4,0.3,0.5)	(0.5,0.3,0.4)

Table 2: Quaternion Neutrosophic fuzzy relation T(Symptoms→Diagnosis)

R	Viral Infection	Malaria	Covid-19	Dengue	Chest Issues
Wade	(0.9,0.5,0.6)	(0.7,0.5,0.6)	(0.8,0.4,0.6)	(0.8,0.3,0.6)	(0.5,0.3,0.6)
Ivan	(0.8,0.5,0.5)	(0.7,0.3,0.4)	(0.7,0.3,0.5)	(0.7,0.4,0.5)	(0.5,0.4,0.5)
Jorge	(0.9,0.5,0.4)	(0.7,0.6,0.4)	(0.8,0.3,0.4)	(0.8,0.3,0.3)	(0.5,0.3,0.7)
Dave	(0.8,0.6,0.3)	(0.8,0.6,0.3)	(0.7,0.3,0.3)	(0.6,0.3,0.4)	(0.5,0.3,0.4)
Gilbert	(0.8,0.3,0.6)	(0.8,0.3,0.6)	(0.8,0.3,0.6)	(0.8,0.3,0.6)	(0.5,0.3,0.6)

Table 3: Quaternion Neutrosophic fuzzy relation R(Patients→Diagnosis)

$$\gamma_T(P_i, F_k) = \bigwedge_{s \in S} [\gamma_T(P_i, T) \bigvee \gamma_R(T, F_k)].$$

After getting the information relationship between Patients→Diagnosis then To produce Table 4, we apply the following formula to the neutrosophic quaternion numbers.

The procedure that follows is applied to the neutrosophic quaternion numbers after obtaining the data on the relationship between Patients→Diagnosis to produce Table 4. $S_R = \alpha_R - \beta_R.\gamma_R$ by observing the quaternion numbers' rule, $ij = k, jk = i, ki = j$.
In this Table 4 we can find that, IVAN and Dave are suffering in Malaria, Wada is infected with Dengue,Jorge

	Viral Infection	Malaria	Covid-19	Dengue	Chest Issues
Wade	0.6	0.4	0.56	0.62	0.32
Ivan	0.55	0.58	0.55	0.5	0.3
Jorge	0.7	0.46	0.68	0.71	0.29
Dave	0.58	0.62	0.61	0.48	0.38
Gilbert	0.58	0.62	0.62	0.62	0.32

Table 4: Results of Diagnosis

diagnosis Viral Infection and Gilbert have Symptoms of Malaria, Covid-19 and Dengue. The graphical behavior of the results are shown in Figure 3.

6.1 Reliability

In this section, we test our results validly using the proposed distance measures (Euclidean Quaternion distance measure) and compared our results with table 4. We use the data set of Table 1 and Table 2. Using proposed Euclidean Quaternion distance measure the result is as In this Table 6 we can find that, IVAN is suffering in Malaria, Wada and Jorge is infected with Dengue,Dave diagnosis Viral Infection and Gilbert have Symptoms of Malaria, and Dengue. Using proposed divergence Quaternion distance measure the result is as In this Table 7 we can find that, IVAN and Dave are suffering in Malaria, Wada is infected with Dengue,Jorge diagnosis Viral Infection and Gilbert have Symptoms of Covid-19 and Dengue. The graphical behavior of the results are shown in Figure 4.

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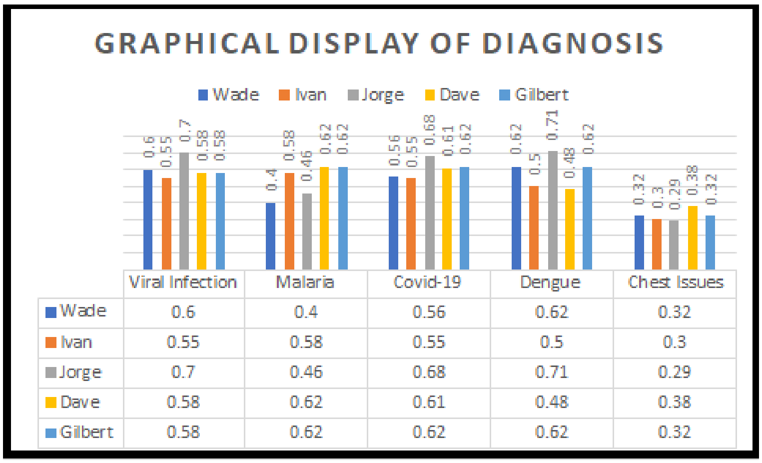

Figure 3: Graphical representation of the results of diagnosis

R	Viral Infection	Malaria	Covid-19	Dengue	Chest Issues
Wade	(0.9,0.5,0.6)	(0.7,0.5,0.6)	(0.8,0.4,0.6)	(0.8,0.3,0.6)	(0.5,0.3,0.6)
Ivan	(0.8,0.5,0.5)	(0.7,0.3,0.4)	(0.7,0.3,0.5)	(0.7,0.4,0.5)	(0.5,0.4,0.5)
Jorge	(0.9,0.5,0.4)	(0.7,0.6,0.4)	(0.8,0.3,0.4)	(0.8,0.3,0.3)	(0.5,0.3,0.7)
Dave	(0.8,0.6,0.3)	(0.8,0.6,0.3)	(0.7,0.3,0.3)	(0.6,0.3,0.4)	(0.5,0.3,0.4)
Gilbert	(0.8,0.3,0.6)	(0.8,0.3,0.6)	(0.8,0.3,0.6)	(0.8,0.3,0.6)	(0.5,0.3,0.6)
Maximize solution	(0.9,0.6,0.6)	(0.8,0.6,0.6)	(0.8,0.4,0.6)	(0.8,0.4,0.6)	(0.5,0.4,0.7)
Minimize Solution	(0.8,0.3,0.3)	(0.7,0.3,0.3)	(0.7,0.3,0.3)	(0.6,0.3,0.3)	(0.5,0.3,0.4)

Table 5: Quaternion Neutrosophic fuzzy relation R(Patients→Diagnosis)

R	Viral Infection	Malaria	Covid-19	Dengue	Chest Issues
Wade	0.025	0.035	0	0.04	0.03
Ivan	0.035	0.09	0.04	0.035	0.05
Jorge	0.05	0.06	0.06	0.08	0.025
Dave	0.079	0.083	0.075	0.075	0.079
Gilbert	0.079	0.08	0.025	0.08	0.035

Table 6: Results of Diagnosis

R	Viral Infection	Malaria	Covid-19	Dengue	Chest Issues
Wade	0.36	0.45	0.28	0.48	0.38
Ivan	0.76	0.92	0.81	0.64	0.68
Jorge	0.85	0.79	0.79	0.83	0.42
Dave	0.82	0.85	0.79	0.75	0.73
Gilbert	0.63	0.64	0.68	0.68	0.42

Table 7: Results of Diagnosis

7 Advantages of proposed method

Compared to other methods of decision-making, quaternion neutrosophic sets have a number of important benefits.

1. For characterizing unpredictability in quaternion-based structures, QNSs are helpful. They can capture intricate fuzzy information that other fuzzy sets are unable to, such as uncertainty in magnitude

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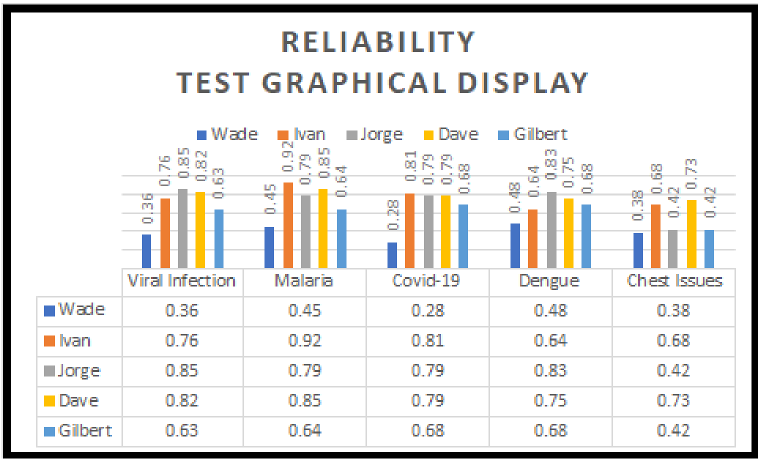

Figure 4: Graphical representation of the reliability test of diagnosis

and direction. Intriguing characteristics of these data include the capacity to define fuzzy distance and similarity measurements.

2. QNS can manage both data and decision-making process ambiguity. As a result, they are able to make decisions when there is a lack of information, insufficient data, or uncertainty about the outcome.

3. These types of data permit the mixing of several components in a single model and can handle multiple criteria and weighting. This offers a more thorough decision-making process that takes into account all pertinent considerations.

4. In order to help decision-makers in complicated situations, QNSs may model complex systems and capture the interactions between many factors.

5. Particularly when dealing with complicated or unclear data, QNSs can produce more accurate answers than other decision-making strategies. They are thus particularly beneficial for applications involving financial modeling, traffic control, and strategic planning.

6. However, the application-specific use and the kind of uncertainty being modeled determine whether QNSs are better than other complex fuzzy sets. The benefits and drawbacks of various kinds of fuzzy sets must therefore be carefully weighed in each individual situation.

7. Increasing the effectiveness of decision-making processes by lowering the time and resources needed for complicated data analysis and interpretation is another key goal of the QNS. This is especially important in today's fast-paced, data-driven environment, where decision-makers must quickly and precisely analyze a lot of information.

7.1 Limitations of the Model

- QNSs required substantial processing resources, which limits their applicability in real-time decision-making contexts.
- Fuzzy quaternion sets are not widely used, which might make it challenging to compare and assess the outcomes of various decision-making procedures.
- Usually, only specific kinds of decision-making procedures, such those involving complex systems or multi-criteria decision-making, employ this kind of data.
- Three-dimensional illustrations of data are used in the QNSs, which may complicate decision-making and make it more difficult to understand the findings.

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8 Conclusion

We come with complex and intricate data every day. We have developed methods and tools designed to manage such complex data to increase our effectiveness and process comprehensive information. To reduce voluminous data to a single value, the technique of aggregation entails costs. A powerful merger of a complicated neutrosophic set gave rise to the idea of the quaternion neutrosophic set. This is especially helpful in situations when each element has a range of possible values that can be affected by variables like an imagined degree of membership, an imagined degree of indeterminacy, or an imagined degree of non-membership. The first step is to build a quaternion neutrosophic set based on imagined membership degree, imagined indeterminacy, and imagined non-membership degree. Also covered with examples are several fundamental operators, such as complement, conjugate, magnitude, union, intersection, and the cartesian product. A novel order relation and the concept of quaternion neutrosophic-based distance measurements were presented. These theoretical methods provide a thorough method for deciphering unclear data. We suggested a quaternionifying procedure to increase decision-making precision. To show the applicability of the suggested strategy, the decision-making model is created using a novel quaternion distance measure. When employing neutrosophic data, it has been recognized that fuzziness is a crucial step in the selection process. In addition, compared to present approaches, which are unwilling to take into account the interrelationships of qualities in practical applications, the decision-making procedures proposed in this study exhibit higher precision and a wider threshold. This suggests that many more connections between traits could be found when the decision-making methods presented in this work are applied. Future research on tailored individual homogeneity control agreement challenges, reaching agreements with difficult behavior control problems, and two-sided corresponding decision-making with multi-granular and incomplete criterion weight information may benefit from the strategies presented here. When analyzing the limitations imposed by suggested strategies, the levels of involvement, abstention, and non-membership are irrelevant. On the foundation of the current research, future studies can explore more quaternionification algorithms and improve the quaternionification parameters that are applied to decision-making. One might also consider using quaternion numerical representations for expressing the neutrosophic theory of sets and logic. In addition, linear programming strategies and neurosophic linguistic sets will be examined together.
This study serves as an introduction to quaternion neutrosophic sets, and it is true that additional research is still necessary to fully understand quaternion neutrosophic sets. The quaternion neutrosophic set discussed in this work is a comprehensive generic idea that is not restricted to a single application.
Appendix: An addition to neutrosophic set theory known as the quaternion neutrosophic model (QNM) integrates uncertainty, indeterminacy, and inconsistent decision-making. It is a mathematical strategy that seeks to effectively integrate and handle complex situations involving several criteria. The imagined membership, imaginary indeterminacy, and imaginary non-membership degrees make up the QNM. A three-dimensional quaternion numeric value that reflects the level of membership of each set member is used to represent these components. The QNM has been effectively used to address decision-making issues in a variety of domains, including image processing, pattern recognition, robotics, and finance. In conclusion, the quaternion neutrosophic model offers a practical mathematical framework that can assist in handling challenging decision-making issues by allowing for ambiguity, inconsistent behavior, and indeterminacy.

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Data availability: The manuscript includes all the data that were used to support this study.
Conflicts of interest: The authors declare no conflicts of interest.

References

[1] Naseem, A., Akram, M., Ullah, K., & Ali, Z. (2023). Aczel-Alsina Aggregation Operators Based on Complex Single-valued Neutrosophic Information and Their Application in Decision-Making Problems. Decision Making Advances, 1(1), 86-114.

[2] Khalifa, H.A.E.W., Kumar, P. and Mirjalili, S., 2021. A KKM approach for inverse capacitated transportation problem in neutrosophic environment. Sadhana, 46(3), p.166.

Received: June 26, 2023 Revised: September 19, 2023 Accepted: December 28, 2023

[3] Kamran, M., Ismail, R., Al-Sabri, E.H.A., Salamat, N., Farman, M. and Ashraf, S., 2023. An Optimization Strategy for MADM Framework with Confidence Level Aggregation Operators under Probabilistic Neutrosophic Hesitant Fuzzy Rough Environment. Symmetry, 15(3), p.578.

[4] Khalifa, H. A., Kumar, P., and Smarandache, F. (2020). On optimizing neutrosophic complex programming using lexicographic order. Neutrosophic Sets and Systems, 32: 330-343.

[5] Lan, L.T.H., Thong, N.T., Smarandache, F. and Giang, N.L., 2023. An ANP-TOPSIS model for tourist destination choice problems under Temporal Neutrosophic environment. Applied Soft Computing, 136, p.110146.

[6] Khalifa, H.A.E.W. and Kumar, P., 2022. A fuzzy programming approach to neutrosophic complex nonlinear programming problem of real functions in complex variables via lexicographic order. OPSEARCH, 59(4), pp.1396-1412.

[7] Hallett, M. (1984). Cantorian set theory and limitation of size (Vol. 28). Oxford: Clarendon Press.

[8] Zadeh, L. A., Klir, G. J., & Yuan, B. (1996). Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers (Vol. 6). World Scientific.

[9] Dat, L. Q., Thong, N. T., Ali, M., Smarandache, F., Abdel-Basset, M., & Long, H. V. (2019). Linguistic approaches to interval complex neutrosophic sets in decision making. IEEE access, 7, 38902-38917.

[10] Gorzalczany, M. B. (1989). An interval-valued fuzzy inference method some basic properties. Fuzzy Sets and Systems, 31(2), 243-251.

[11] Sanchez, E. (1979). Inverses of fuzzy relations. Application to possibility distributions and medical diagnosis. Fuzzy sets and systems, 2(1), 75-86.

[12] Sambuc, R. (1975). Fonctions and floues: application a l'aide au diagnostic en pathologie thyroidienne (Doctoral dissertation, Faculte de Medecine de Marseille).

[13] Roy, M. K., & Biswas, R. (1992). Iv fuzzy relations and Sanchez's approach for medical diagnosis. Fuzzy Sets and Systems, 47(1), 35-38.

[14] Yazdani, M., Torkayesh, A. E., Stevic, Ž., Chatterjee, P., Ahari, S. A., & Hernandez, V. D. (2021). An interval valued neutrosophic decision-making structure for sustainable supplier selection. Expert Systems with Applications, 183, 115354.

[15] Smarandache, F., Quiroz-Martinez, M. A., Ricardo, J. E., Hernandez, N. B., & Vazquez, M. Y. L. (2020). Application of neutrosophic offsets for digital image processing. Infinite Study.

[16] Zulqarnain, R. M., Siddique, I., Jarad, F., & Iampan, A. (2022). Aggregation Operators for Interval-Valued Intuitionistic Fuzzy Hypersoft Set with Their Application in Material Selection. Mathematical Problems in Engineering, 2022.

[17] Wang, H., Smarandache, F., Zhang, Y. and Sunderraman, R., 2010. Single valued neutrosophic sets. Infinite study.

[18] Zulqarnain, R. M., Siddique, I., Ahmad, S., Ayaz, S., & Broumi, S. (2021). An Algorithm Based on Correlation Coefficient Under Neutrosophic hypersoft set environment with its Application for Decision-Making. Neutrosophic Sets and Systems, 46, 128-140.

[19] Maqbool, H., Yousaf, A., Asif, R. M., Rehman, A. U., Eldin, E. T., Shafiq, M., & Hamam, H. (2022). An optimized fuzzy based control solution for frequency oscillation reduction in electric grids. Energies, 15(19), 6981.

[20] Akram, M., Shahzadi, S., & Smarandache, F. (2018). Multi-attribute decision-making method based on neutrosophic soft rough information. Axioms, 7(1), 19.

[21] Saqlain, M., Saeed, M., Zulqarnain, R. M., & Moin, S. (2021). Neutrosophic Hypersoft Matrix Theory: Its Definition, Operators, and Application in Decision-Making of Personnel Selection Problem. In Neutrosophic Operational Research (pp. 449-470). Springer, Cham.

Received: June 26, 2023 Revised: September 19, 2023 Accepted: December 28, 2023

[22] Sahin, R., & Liu, P. (2017). Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making. Neural Computing and Applications, 28, 1387-1395.

[23] Smarandache, F. (2019). Neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), pythagorean fuzzy set, spherical fuzzy set, and q-rung orthopair fuzzy set, while neutrosophication is a generalization of regret theory, grey system theory, and three-ways decision (revisited). Journal of New Theory, (29), 1-31.

[24] Dai, S. (2019). On partial orders in complex fuzzy logic. IEEE Transactions on Fuzzy Systems, 29(3), 698-701.

[25] Dai, S. (2020). A generalization of rotational invariance for complex fuzzy operations. IEEE Transactions on Fuzzy Systems, 29(5), 1152-1159.

[26] Yazdanbakhsh, O., & Dick, S. (2019). FANCFIS: Fast adaptive neuro-complex fuzzy inference system. International Journal of Approximate Reasoning, 105, 417-430.

[27] Kumar, A., Singh, A. K., Ahmad, I., Kumar Singh, P., Anushree, Verma, P. K., ... & Tag-Eldin, E. (2022). A novel decentralized blockchain architecture for the preservation of privacy and data security against cyberattacks in healthcare. Sensors, 22(15), 5921.

[28] Lashin, M. M., Khan, M. I., Khedher, N. B., & Eldin, S. M. (2022). Optimization of Display Window Design for Females' Clothes for Fashion Stores through Artificial Intelligence and Fuzzy System. Applied Sciences, 12(22), 11594.

[29] Hamilton, W. R. (1866). Elements of quaternions. London: Longmans, Green, & Company.

[30] Liu, J. B., Salamat, N., Kamran, M., Ashraf, S., & Khan, R. H. (2023). Single-Valued Neutrosophic Set With Quaternion Information: A Promising Approach To Assess Image Quality. FRACTALS (fractals), 31(06), 1-10.

[31] Liu, J. B., Ismail, R., Kamran, M., Al-Sabri, E. H. A., Ashraf, S., & Cangul, I. N. (2023). An optimization strategy with SV-neutrosophic quaternion information and probabilistic hesitant fuzzy rough Einstein aggregation operator. AIMS Mathematics, 8(9), 20612-20653.

[32] Hata, R., Islam, M. M., & Murase, K. (2016). Quaternion neuro-fuzzy learning algorithm for generation of fuzzy rules. Neurocomputing, 216, 638-648.

[33] Rasheed, M., Tag-Eldin, E., Ghamry, N. A., Hashmi, M. A., Kamran, M., & Rana, U. (2023). Decision-making algorithm based on Pythagorean fuzzy environment with probabilistic hesitant fuzzy set and Choquet integral. AIMS Mathematics, 8(5), 12422-12455.

[34] Hardy, A. S. (1881). Elements of quaternions.

[35] Ye, J. (2018). Multiple-attribute decision-making method using similarity measures of single-valued neutrosophic hesitant fuzzy sets based on least common multiple cardinality. Journal of Intelligent & Fuzzy Systems, 34(6), 4203-4211.

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