Neutrosophic Lindley distribution with application for Alloying Metal Melting Point

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Abstract

In the field of survival analysis, the Lindley distribution is used to mimic methods used with human lifespan data. A variety of survival statistics with indeterminacies are intended to be characterized by the neutrosophic Lindley distribution (NLD). In example, modeling unknown data that is roughly positively skewed makes use of the established distribution. The neutrosophic survival function, neutrosophic hazard rate, and neutrosophic moments are three of the developed NLD's major statistical features that are discussed in this article. Additionally, the well-known maximum likelihood estimation method is used to estimate the neutrosophic parameters. A simulation study is conducted to see whether the projected neutrosophic parameters were attained. Not to mention that discussions of prospective NLD real-world applications have made use of actual data. To demonstrate how well the suggested model performed in comparison to the existing distributions, actual data were used.

Keywords: Neutrosophic statistics; Lindley distribution; survival analysis; hazard function; metal melting point.

1. Introduction

A subfield of statistics known as "neutosophic statistics" focuses on using neutrosophic reasoning to handle uncertainty and incompleteness in data. The use of fuzzy logic was expanded by [1] to create neutrosophy, which enables the depiction of uncertainty, ambiguity, and contradiction. When using traditional statistics, it is frequently assumed that the data is clear-cut, in which case each observation is given a specific value. Real-world information, however, occasionally contains ambiguous or insufficient facts. In order to overcome these restrictions, neurosophic statistics provides a paradigm for handling ambiguous, insufficient, and inconsistent data [2-4].

Truth membership, indeterminacy membership, and falsity membership are the three variables taken into account by non-empirical statistics. Each element represents the level of veracity, ambiguity, or falsity connected to an observation or a hypothesis. Similar to fuzzy sets, membership functions are employed to represent these degrees [2, 3]. Numerous industries, such as decision-making, pattern identification, data mining, and image processing, use neutrosophic statistics [4-7]. It offers a versatile mathematical tool for modeling and analyzing complicated systems with a high degree of uncertainty and imprecision.

The examination of survival statistics is one of the crucial uses of neutrosophic information. A statistical method called survival analysis looks at how long it will be until an event occurs [8]. The foundation for the entire survival analysis is provided by the probability distributions of the temporal data. A neutrosophic survival

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probability distribution combines survival analysis and neutrosophic reasoning. The survival probability distribution in the context of neutrosophic represents the possibility of an event occurring at various dates. The survival statistics' ambiguity and uncertainty are taken into consideration using neutosophic logic. It enables the depiction of just having a limited or hazy comprehension of events. Both neutrosophic factors and the already available survival statistics must take into account the degree of truth, falsity, and uncertainty related to the survival probability at various time points. Neutosophic logic-specific mathematical models and methods can be used for this. The neutrosophic probability distribution is covered in many papers [8-20].

The Lindley distribution has applications in various fields, such as survival analysis. In this paper, we expanded the uses of the Lindley distribution when the data is in interval form and has some degree of indeterminacy in the form of neutrosophy. With the aid of simulated and real data application based on metal melting point, a number of properties are examined under the newly proposed distribution and their applications are discussed.

2. Neutrosophic Lindley distribution

The Lindley distribution specified by the probability density function (p.d.f.)

$$f(x)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x},\ x>0,\theta>0 \qquad (1)$$

was introduced by Lindley [21] .The corresponding cumulative distribution function (c.d.f.) is:

$$F(x)=1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x},\ x>0,\theta>0. \qquad (2)$$

The mode of the Lindley distribution is:

$$\text{mode}(X)=\begin{cases}\frac{1-\theta}{\theta}, & \text{if} 0<\theta<1\\ 0, & \text{otherwise.}\end{cases} \qquad (3)$$

The mean of the Lindley distribution is:

$$\mu=E(X)=((\theta+2)/\theta(\theta+1)) \qquad (4)$$

The concept of neutrosophic probability as a function $NP:\rightarrow [0,1]^3$ was originally presented by [2], where U is a neutrosophic sample space and defined the probability mapping to take the form $NP(S) = (ch(S), ch(neut\ S), ch(anti\ S)) = (\eta, \beta, \tau)$ $where\ 0 \le \eta, \beta, \tau \le 1\ and\ 0 \le \eta+\beta+\tau \le 3$. The term Ψ represents the set of sample space, R represents the set of real numbers, and Υ denotes a sample space event, X_N and Y_N denote neutrosophic r.v. Furthermore, we demonstrate certain renowned definitions and characteristics of neutrosophic probability and logic that will be important in creating this neutrosophic probability model.

Definition 1:
Consider the real-valued crisp r.v. X , which has the following definition: $X:\Psi\rightarrow R$
where Ψ is the event space and X_N neutrosophic r.v. as follows:

$X_N:\Psi\rightarrow R(I)$

and

$X_N = X + I$

The term I represents indeterminacy.

Theorem 1:
Let the neutrosophic r.v. $X_N = X + I$ and the CDF of X is $F_X(x) = P(X \le x)$ [13]. The following assertions are correct:

$FX_N(x) = F_X(x-I)$,

$f_{X_N}(x) = f_X(x-I)$,

where F_{X_N} and f_{X_N} are the CDF and PDF of a neutrosophic r.v. X_N , respectively.

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Theorem 2 :

Let $X_N = X + I$, is the neutrosophic r.v., then the expected value and variance can be derived as follows: $E(X_N) = E(X) + I$ and $V(X_N) = V(X)$ [13].

Suppose the neutrosophic variable could be expressed as: $x_N = x_L + x_U I_N$ where $I_N \in \{I_L, I_U\}$ and x_L and $x_U I_N$ denote the determined and indeterminate parts, respectively. Assume that the neutrosophic random variable $x_N \in \{x_L, x_U\}$ follows the LD having neutrosophic scale parameter $\theta_N \in \{\theta_L, \theta_U\}$ where the letters L and U are the lower values and the upper values, respectively. Then, the neutrosophic probability density function (NPDF) of neutrosophic LD (NLD) is given by the Lindley distribution specified by the probability density function (p.d.f.)

$$f(x_N) = \frac{\theta_N^{\;2}}{\theta_N + 1}(1 + x_N)e^{-\theta_N x_N}, \; x_N > 0, \theta_N > 0 \qquad (5)$$

3. Statistical Properties of NLD

In this section, statistical properties of the NLD are covered.

Moments: The r^{th} moment about origin of the Lindley distribution is:

$$\mu_r^{'} = E\left(X_N^{\;r}\right) = \frac{r!(\theta_N + r + 1)}{\theta_N^r(\theta_N + 1)}, \; r = 1, 2, \ldots \qquad (6)$$

In particular, we have

$$\mu_1^{'} = \frac{\theta_N + 2}{\theta_N(\theta_N + 1)} = \mu, \qquad (7)$$

$$\mu_2^{'} = \frac{2(\theta_N + 3)}{\theta_N^2(\theta_N + 1)}, \qquad (8)$$

$$\mu_3^{'} = \frac{6(\theta_N + 4)}{\theta_N^3(\theta_N + 1)}, \qquad (9)$$

$$\mu_4^{'} = \frac{24(\theta_N + 5)}{\theta_N^4(\theta_N + 1)}. \qquad (10)$$

The central moments of the Lindley distribution are:

$$\mu_k = E\left\{(X_N - \mu)^k\right\} = \sum_{r=0}^{k}\binom{k}{r}\mu_r^{'}(-\mu)^{k-r}. \qquad (11)$$

In particular, we have

$$\mu_2 = \frac{\theta_N^{\;2} + 4\theta_N + 2}{\theta_N^2(\theta_N + 1)^2} = \sigma^2, \qquad (12)$$

$$\mu_3 = \frac{2\left(\theta_N^3 + 6\theta_N^2 + 6\theta_N + 2\right)}{\theta_N^3(\theta_N + 1)^3}, \qquad (13)$$

$$\mu_4 = \frac{3\left(3\theta_N^4 + 24\theta_N^3 + 44\theta_N^2 + 32\theta_N + 8\right)}{\theta_N^4(\theta_N + 1)^4}. \qquad (14)$$

The coefficient of variation (γ), skewness $\left(\sqrt{\beta_1}\right)$ and the kurtosis (β_2) are:

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$$\gamma = \frac{\sqrt{\theta_N^2 + 4\theta_N + 2}}{\theta_N + 2}, \tag{15}$$

$$\sqrt{\beta_1} = \frac{2\left(\theta_N^3 + 6\theta_N^2 + 6\theta_N + 2\right)}{\left(\theta_N^2 + 4\theta_N + 2\right)^{3/2}} \tag{16}$$

$$\beta_2 = \frac{3\left(3\theta_N^4 + 24\theta_N^3 + 44\theta_N^2 + 32\theta_N + 8\right)}{\left(\theta_N^2 + 4\theta_N + 2\right)^2} \tag{17}$$

For the Lindley distribution, the hazard rate function is

$$h(x_N) = \frac{\theta_N^2 (1 + x_N)}{\theta_N + 1 + \theta_N x_N}. \tag{18}$$

$h(x_N)$ is an increasing function in x_N and θ_N and $\theta_N^2/(\theta_N + 1) < h(x_N) < \theta_N$

4. Parameter Estimation of NLD

Suppose that n identical units are placed on a life test with the corresponding lifetimes $x_{N1}, x_{N2}, ..., x_{Nn}$ It is assumed that these variables are independent and identically distributed as Lindley (θ_N). Let $\mathbf{X}_N = (X_{N1}, \ldots, X_{Nn})$ denotes the vector of lifetimes. If a realization $\mathbf{x}$ of $\mathbf{X}_N$ was known exactly, we could obtain the complete-data likelihood function as

$$\ell(\theta_N) = \left(\frac{\theta_N^2}{1+\theta_N}\right)^n \exp\left(-\theta_N \sum_{i=1}^{n} x_i\right) \prod_{i=1}^{n} (1 + x_i) \tag{19}$$

Consider the situation where the available information about $\mathbf{x}$ can not be exactly perceived, but that rather it may be assimilated with fuzzy numbers $\tilde{x}_1, \ldots, \tilde{x}_n$ with the corresponding membership functions $\mu_{\tilde{x}_1}(\cdot), \ldots, \mu_{\tilde{x}_m}(\cdot)$ Then, we can obtain the likelihood function of θ_N as

$$L(\theta_N) = \prod_{i=1}^{n} \int \frac{\theta_N^2}{1+\theta_N} (1 + x_N) \exp(-\theta_N x_N) \mu_{\tilde{x}_{Ni}}(x_N) dx_N \tag{20}$$

and the corresponding log-likelihood function $L^*(\theta_N) = \log L(\theta_N)$ becomes

$$L^*(\theta_N) = 2n\log\theta_N - n\log(1+\theta_N) + \sum_{i=1}^{n} \log \int (1 + x_N) \exp(-\theta_N x_N) \mu_{\tilde{x}_{Ni}}(x_N) dx_N. \tag{21}$$

The maximum likelihood estimate of the parameter θ_N can be computed as any value maximizing the observed-data log-likelihood (21). Equating the derivative of the log-likelihood L^* with respect to θ_N to zero, we have

$$\frac{\partial}{\partial\theta_N} L^*(\theta_N) = \frac{2n}{\theta_N} - \frac{n}{1+\theta_N} - \sum_{i=1}^{n} \frac{\int x_N (1 + x_N) \exp(-\theta_N x_N) \mu_{\tilde{x}_{Ni}}(x_N) dx_N}{\int (1 + x_N) \exp(-\theta_N x_N) \mu_{\tilde{x}_{Ni}}(x) dx_N} = 0. \tag{22}$$

5. Simulation results

A Monte Carlo simulation is run in R software with several sample sizes, $n = 30, 50, 150, 250$ and neutrosophic parameters in two cases: (1) $\theta_N \in [1, 2.5]$ and (2) $\theta_N \in [1.5, 2]$. The simulation is replicated for 1000 times. Performance measures, such as the neutrosophic average of the estimators, the neutrosophic average bias (NAB) and neutrosophic Mean Square Error (NMSE) are attained for all values of n . The results are given in Tables 1

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and 2. From Tables 1 and 2, It is seen that, as expected, the NAB and NMSE fall for both neutrosophic parameters as sample sizes rise. Furthermore, according to the study's findings, the neutrosophic MLE for the LD offers accurate estimation with a higher sample size..

Table 1: Average NAB and NMSE for case 1

n	NAB	NMSE
	θ_N	θ_N
30	[0.0183, 0.0193]	[0.0371, 0.0381]
50	[0.0121, 0.0132]	[0.0331, 0.0341]
150	[0.0109, 0.0118]	[0.0321, 0.0328]
250	[0.0041, 0.0056]	[0.0252, 0.0268]

Table 2: Average NAB and NMSE for case 2

n	NAB	NMSE
	θ_N	θ_N
30	[0.0242, 0.0251]	[0.0452, 0.0461]
50	[0.0203, 0.0210]	[0.0410, 0.0421]
150	[0.0183, 0.0197]	[0.0403, 0.0411]
250	[0.0132, 0.0148]	[0.0331, 0.0347]

6. Applications

The carefully crafted data set relates to information on alloy melting points that was obtained from [22] and used for the first time by [23]. An alloy is a mixture of material components, containing at least one metal. These alloys may possess properties that let them stand out from pure metals, which helps them increase strength or hardness while also bringing down the price of the material. Red gold, made of a copper and gold alloy, white gold, made of a silver and gold alloy, etc. are a few examples of alloys. Manufacturing engineers involved in the production of bimetals frequently take the information on alloy melting points from a distribution with a set of aggregate melting values. Because it might be difficult to determine melting points in general, observations are indeterministic and can be reported in intervals. For quick reference, the following is a list of the 18 questionable data observations of alloy melting points: [563.3, 545.5], [529.4, 511.6], [523.1, 503.5], [470.1, 449.2], [506.7, 489.0], [495.6, 479.1], [495.3, 467.9], [520.9, 495.6], [496.9, 472.8], [542.9, 519.1], [505.4, 484.0], [550.7, 525.9], [517.7, 500.9], [499.2, 483.0], [500.6, 480.0], [516.8, 499.6], [535.0, 515.1], [489.3, 464.4].

The model adequacy of the proposed NLD is compared with the neutrosophic exponential distribution (NED) applications for complicated data analysis investigated by [12] and neutrosophic Log-Logistic distribution (NLLD) by [17]. The log-likelihood value (LogL), Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), and Kolmogorov-Smirnov (KS) test are the criteria selection methods used to determine which model fits the data the best. The criteria for the best fitting model are the highest LogL values and the lowest AIC, BIC, and KS statistic values. Additionally, a higher p-value suggests that the model that best fits the neutrosophic data. Table 3 lists the neutrosophic maximum likelihood estimators and model sufficiency metrics. The findings show that the NLD is more effective than the NED and NLLD for data on alloy melting points. The table's bold values demonstrate the effectiveness of the suggested model.

Table 3: The criteria selection neutrosophic distributions for alloy melting points data

NED	NLLD	NLD

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Parameter	$\theta_N = [461.958, 491.642]$	$\sigma_N = [492.696, 512.952]$ $\beta_N = [36.418, 38.934]$	$\theta_N = [490.547, 503.721]$
LogL	[129.6745, 130.392]	[82.1435, 82.581]	**[79.3826, 81.053]**
AIC	[261.349, 262.784]	[168.287, 169.162]	**[154.894, 157.881]**
BIC	[262.239, 263.674]	[170.067, 170.943]	**[153.227, 155.793]**
KS-value	[0.6156, 0.62181]	[0.101, 0.117]	**[0.131, 0.135]**
KS- p-value	$[3.032\times10^{-7}, 4.289\times10^{-7}]$	[0.942, 0.984]	**[0.957, 0.986]**

7. Conclusions

This paper proposes a neutrosophic Lindley distribution (NLD). This well-known distribution can be used to account for survival and reliability issues in a variety of application data. The neutrosophic survival function, neutrosophic hazard rate, and neutrosophic moments have all been studied as the main statistical characteristics of the evolved NLD. Neutosophic average bias and MSEs have been shown for a range of sample sizes using the neutrosophic MLEs, which have been developed. A simulation study was done to see if the computed neutrosophic parameters were met. According to simulation results, the sample size and neutrosophic parametric value are important elements in accurately estimating an unknown parameter. The assortment of melting points employed is additional evidence in favor of the NLD's application in neutrosophic conditions..

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