MBJ-Neutrosophic Positive Implicative LI-ideals, Associative LI-ideals, and Fantastic LI-ideals

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Abstract

The MBJ-Neutrosophic positive implicative LI-ideals, the MBJ-Neutrosophic associative LI-ideals, and the MBJ-Neutrosophic fantastic LI-ideals are all introduced in this study. Of all these ideals, several their qualities and corresponding conditions were explored. We have demonstrated how every positive implicative MBJ-Neutrosophic LI-ideal evolved into an MBJ-Neutrosophic LI-ideal, MBJ-Neutrosophic implicative LI-ideal, and MBJ-Neutrosophic fantastic LI-ideal. Additionally, it was demonstrated that any MBJ-Neutrosophic fantastic LI-ideal is a MBJ-Neutrosophic associative LI-ideal.

Keywords: Lattice implication algebra (LIA); MBJ-Neutrosophic LI-ideal (MBJ-NLII), MBJ-Neutrosophic implicative LI-ideal (MBJ-NILII), MBJ-Neutrosophic positive implicative LI-ideal (MBJ-NPILII), MBJ-Neutrosophic associative LI-ideal (MBJ-NALII), and MBJ-Neutrosophic fantastic LI-ideal (MBJ-NFLII) .

1. Introduction:

The concept of Neutrosophic set (Nset), which Smarandache proposed in 1998 [15], includes the interval-valued fuzzy (intuitionistic) set and the standard set (fuzzy). According to [11], "Jun, his colleagues examined, properties and relations of BCK/BCI-algebras using the idea of Neutrosophic set theory" (Nset) was defined after the Neutrosophic elements T, I, and F were innovated. It indicates the truth-membership, indeterminancy, and false-membership values individually. Then, according to [15], "the interval Neutrosophic set was inaugurated. Additionally, "Interval Neutrosophic ideals were interpreted, and several features examined" in [11]. They showed a variety of interval Neutrosophic ideals in [10], examined some of their traits, and found a connection between them.

In 1993, Xu [16] put up the idea of a logical algebra-lattice implication algebra. In lattice implication algebra, the lattice is defined to explain uncertainty, notably beyond comparison, and is intended to capture human reasoning. The use of Neutrosophic sets in lattice implication algebra offers a fresh method for identifying uncertainty while avoiding many of the drawbacks of earlier theoretical methods. Applications of neutrosophic set theory include information theory, probability theory, control theory, decision-making, measurement theory, and others. Since then, a considerable body of literature on the theory of Neutrosophic sets has been published as a result of the work of several researchers in this area. Smarandache introduces the idea of the MBJ-Neutrosophic set as another generalisation of the Neutrosophic set.

In [9], Jun also suggested the notion of LI-ideals and examined a number of their characteristics as well as their interrelationships. The Neutrosophic set theory idea was also used to LIA.The MBJ Neutrosophic set abstraction is a novel generalisation of the N set.

We examined the properties and corresponding circumstances of the MBJ-Neutrosophic LI-Ideals in Lattice Implication Algebras in [1] and introduced them.

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The MBJ-Neutrosophic Implicative LI-Ideals in Lattice Implication Algebras are explored in [2], where their features are provided and related relations are analysed.

The MBJ-Neutrosophic positive implicative LI ideal was introduced in section 3.1 of this work, and it was demonstrated that any MBJ-Neutrosophic positive implicative LI ideal is both an MBJ-Neutrosophic LI ideal and an MBJ-Neutrosophic implicative LI-ideal. We introduced the MBJ-Neutrosophic associative LI-ideal in section 3.2. We study the MBJ-Neutrosophic fantastic LI-ideal of a LIA in section 3.3 and look at a number of its features. Additionally, it demonstrated how these ideals linked to one another.

2. Related Work:

Definition-2.1. ([18]) : LIA is described as a Lattice (L, V, Λ, 0, 1) with bounds (0 , 1), order-reversing involution " ′ " comforting propositions given below under a binary operation " → " are

(I1) $\varsigma \rightarrow (\varrho \rightarrow \varphi) = \varrho \rightarrow (\varsigma \rightarrow \varphi)$,

(I2) $\varsigma \rightarrow \varsigma = 1$,

(I3) $\varsigma \rightarrow \varrho = \varrho' \rightarrow \varsigma'$,

(I4) $\varsigma \rightarrow \varrho = \varrho \rightarrow \varsigma = 1 \Rightarrow \varsigma = \varrho$,

(I5) $(\varsigma \rightarrow \varrho) \rightarrow \varrho = (\varrho \rightarrow \varsigma) \rightarrow \varsigma$,

(L1) $(\varsigma \vee \varrho) \rightarrow \varphi = (\varsigma \rightarrow \varphi) \wedge (\varrho \rightarrow \varphi)$,

(L2) $(\varsigma \wedge \varrho) \rightarrow \varphi = (\varsigma \rightarrow \varphi) \vee (\varrho \rightarrow \varphi)$,

A partial ordering ≤ on 'L' defined by the condition $\varsigma \leq \varrho$ iff $\varsigma \rightarrow \varrho = 1$.

A LIA 'L' satisfies the propositions cited below (see [12]):

(a1) $0 \rightarrow \varsigma = 1, 1 \rightarrow \varsigma = \varsigma$ & $\varsigma \rightarrow 1 = 1$.

(a2) $\varsigma \rightarrow \varrho \leq (\varrho \rightarrow \varphi) \rightarrow (\varsigma \rightarrow \varrho)$.

(a3) $\varsigma \leq \varrho$ implies $\varrho \rightarrow \varphi \leq \varsigma \rightarrow \varphi$ and $\varphi \rightarrow \varsigma \leq \varphi \rightarrow \varrho$.

(a4) $\varsigma' = \varsigma \rightarrow 0$.

(a5) $\varsigma \vee \varrho = (\varsigma \rightarrow \varrho) \rightarrow \varrho$.

(a6) $((\varrho \rightarrow \varsigma) \rightarrow \varsigma')' = \varsigma \wedge \varrho = ((\varsigma \rightarrow \varrho) \rightarrow \varsigma')'$.

(a7) $\varsigma \leq (\varsigma \rightarrow \varrho) \rightarrow \varrho$.

(a8) $((\varsigma \rightarrow \varrho) \rightarrow \varrho) \rightarrow \varrho = \varsigma \rightarrow \varrho$

Definition 2.2. ([9]) : A LI-ideal of 'L'is a nonempty subset χ of 'L' holds the propositions given by

(L3) $0 \in \chi$.

(L4) $(\forall \varsigma \in$ 'L'$) (\forall \varrho \in \chi) ((\varsigma \rightarrow \varrho)c\in \chi \Rightarrow \varsigma \in \chi)$.

MBJ-Neutrosophic LI-ideals:

We suppose 'L' as LIA except as otherwise indicated.

Definition 2.3. ([1]) : A MBJ-NLII of 'L' is an MBJ-Nset

A = $(M_A, \widetilde{\mathcal{B}_A}, J_A)$ in 'L' if the following attributes are true.

$(\forall \varsigma \in L)\ (M_A(0) \geq M_A(\varsigma), \tilde{B}_A(0) \geq \tilde{B}_A(\varsigma), J_A(0) \leq J_A(\varsigma))$

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and $(\forall\, \varsigma, \varrho \in L) \begin{pmatrix} M_A(\varsigma) \geq \min\{M_A((\varsigma \to \varrho)'), M_A(\varrho)\} \\ \tilde{B}_A(\varsigma) \geq r\min\{\tilde{B}_A((\varsigma \to \varrho)'), \tilde{B}_A(\varrho)\} \\ J_A(\varsigma) \leq \max\{J_A((\varsigma \to \varrho)'), J_A(\varrho)\} \end{pmatrix}$

We use the notation MBJ-NLI 'L' to represent the collection of all MBJ-NLI-ideals of 'L'.

Definition 2.4. ([2]) : An implicative LI-Ideal of 'L' is a nonempty subset A of 'L' which satisfies 0∈L &∀ς, ϱ, φ ∈ L, $(((\varsigma \to \varrho)' \to \varrho)' \to \varphi)' \in A$, $\varphi \in A \Rightarrow (\varsigma \to \varrho)' \in A$.

Definition 2.5. ([2]) **: MBJ-Neutrosophic Implicative LI-ideal :** An MBJ-N set 'A' of 'L' is known as an MBJ-NILII of 'L' if it holds the inequalities given below for all ς ∈ L, we have

$(M_A(0) \geq M_A(\varsigma), \widetilde{\mathcal{B}_A}(0) \geq \widetilde{\mathcal{B}_A}(\varsigma), J_A(0) \leq J_A(\varsigma))$, ∀ς, ϱ, φ ∈L and

$$\begin{cases} M_A((\varsigma \to \varrho)') \geq \min\{M_A((((\varsigma \to \varrho)' \to \varrho)' \to \varphi)'), M_A(\varphi)\}, \\ \widetilde{\mathcal{B}_A}((\varsigma \to \varrho)') \geq r\min\{\widetilde{\mathcal{B}_A}((((\varsigma \to \varrho)' \to \varrho)' \to \varphi)'), \widetilde{\mathcal{B}_A}(\varphi)\}, \\ J_A((\varsigma \to \varrho)') \leq \max\{J_A((((\varsigma \to \varrho)' \to \varrho)' \to \varphi)'), J_A(\varphi)\}. \end{cases}$$

3. MBJ-Neutrosophic

3.1. MBJ-Neutrosophic positive implicative LI-ideals :

3.1.1. Definition :

Let X be the universe. A M$\mathcal{B}$J-Neutrosophic set $A = \{\langle \varsigma; M_A(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma), J_A(\varsigma) / \varsigma \in X \rangle\}$ of a LIA 'L' is called a M$\mathcal{B}$J-Neutrosophic positive implicative LI-ideal of 'L' if it satisfies

$M_A(0) \geq M_A(\varsigma)$, $\widetilde{\mathcal{B}_A}(0) \geq \widetilde{\mathcal{B}_A}(\varsigma)$, $J_A(0) \leq J_A(\varsigma)$

$$M_A(\varrho) \geq \min\{M_A((\varrho \longrightarrow (\varphi \longrightarrow \varrho)')' \longrightarrow \varsigma)'), M_A(\varsigma)\}$$

$$\widetilde{\mathcal{B}_A}(\varrho) \geq r\min\{\widetilde{\mathcal{B}_A}((\varrho \longrightarrow (\varphi \longrightarrow \varrho)')' \longrightarrow \varsigma)'), \widetilde{\mathcal{B}_A}(\varsigma)\}$$

$$J_A(\varrho) \leq \max\{J_A((\varrho \longrightarrow (\varphi \longrightarrow \varrho)')' \longrightarrow \varsigma)'), J_A(\varsigma)\}$$

3.1.2. Example : Let L= {0, a, b, c, 1}. Define partial order on L as 0 < a < b < c < 1 and define $\varsigma \wedge \varrho = \min\{\varsigma, \varrho\}$ and $\varsigma \vee \varrho = \max\{\varsigma, \varrho\}$ for all ς,ϱ ∈ L and " ' "and "⟶" as follows

Table 1: Illustration of Compliment of element

ς	ς′
0	1
a	c
b	b
c	a
1	0

Table 2: Illustration of Implication

⟶	**0**	a	b	c	**1**

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0	1	1	1	1	1
a	*c*	1	1	1	1
b	*b*	*c*	1	1	1
c	*a*	*b*	*c*	1	1
1	0	*a*	*b*	*c*	1

Then $(L, \vee, \wedge, ', \longrightarrow)$ is LIA.

Define $M\mathcal{B}J$-Neutrosophic set A in L by Tabular representation of A.

Table 3: Illustration of Neutrosophic set $A = (M_A, \widetilde{\mathcal{B}}_A, J_A)$

L	**0**	***a***	***b***	***c***	**1**
$\mathbf{M_A}$	0.7	0.7	0.7	0.5	0.5
$\widetilde{\mathcal{B}}_A$	[0.3, 0.5]	[0.3, 0.5]	[0.3, 0.5]	[0.2, 0.3]	[0.2, 0.3]
$\mathbf{J_A}$	0.5	0.5	0.7	0.7	0.7

Then $A = (M_A, \widetilde{\mathcal{B}}_A, J_A)$ is a $M\mathcal{B}J$-NPILII on L,

As it satisfies all the attributes of the $M\mathcal{B}J$-NPILII.

3.1.3. Theorem: Every $M\mathcal{B}J$-NPILII of a lattice implication algebra is a MBJ-NLII.

Proof : Let A be a $M\mathcal{B}J$-NPILII of a LIA 'L'.

Then $M_A(0) \geq M_A(\varsigma)$, $\widetilde{\mathcal{B}}_A(0) \geq \widetilde{\mathcal{B}}_A(\varsigma)$, $J_A(0) \leq J_A(\varsigma)$

And let $\varphi = \varrho$ then $M_A(\varrho) \geq \min\{M_A((\varrho \longrightarrow (\varrho \longrightarrow \varrho)')' \longrightarrow \varsigma)'), M_A(\varsigma)\}$

$= \min\{M_A((\varrho \longrightarrow 0)' \longrightarrow \varsigma)'), M_A(\varsigma)\}$

$= \min\{M_A((\varrho')' \longrightarrow \varsigma)'), M_A(\varsigma)\}$

$= \min\{M_A(\varrho \longrightarrow \varsigma)', M_A(\varsigma)\}$

$\widetilde{\mathcal{B}}_A(\varrho) \geq r\min\{\widetilde{\mathcal{B}}_A((\varrho \longrightarrow (\varrho \longrightarrow \varrho)')' \longrightarrow \varsigma)'), \widetilde{\mathcal{B}}_A(\varsigma)\}$

$= r\min\{\widetilde{\mathcal{B}}_A((\varrho \longrightarrow 0)' \longrightarrow \varsigma)'), \widetilde{\mathcal{B}}_A(\varsigma)\}$

$= r\min\{\widetilde{\mathcal{B}}_A((\varrho')' \longrightarrow \varsigma)'), \widetilde{\mathcal{B}}_A(\varsigma)\}$

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$= r \min\{\widetilde{\mathcal{B}}_A(\varrho \longrightarrow \varsigma)', \widetilde{\mathcal{B}}_A(\varsigma)\}$

$J_A(\varrho) \leq \max\{J_A((\varrho \longrightarrow (\varrho \longrightarrow \varrho)')' \longrightarrow \varsigma)'), J_A(\varsigma)\}$

$= \max\{J_A((\varrho \longrightarrow 0)' \longrightarrow \varsigma)'), J_A(\varsigma)\}$

$= \max\{J_A((\varrho')' \longrightarrow \varsigma)'), J_A(\varsigma)\}$

$= \max\{J_A(\varrho \longrightarrow \varsigma)', J_A(\varsigma)\}$

Therefore, A is a MBJ-NLII.

Note: Converse of the above theorem is need not be true. It will be followed by the below example.

3.1.4. Example: The Hasse diagram and Cayley tables of a poset, L = {0, a, b, c, d, 1} are given below as follows:

Table 4: Illustration of compliment of element

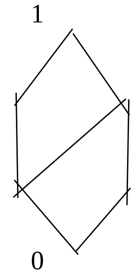

Figure 1: Hasse diagram of L

a b

d

ς	ς'
0	1
a	c
b	d
c	a
d	b
1	0

Table 5: Illustration of Implication

→	**0**			$\boldsymbol{a}$		
0	1	1	1	1	1	1
a	c	1	b	c	b	1
b	d	a	1	b	a	1
c	a	a	1	1	a	1
d	b	1	1	b	1	1
1	0	a	b	c	d	1

The operations V and Λ on L are defined as follows

$\varsigma \vee \varrho = (\varsigma \to \varrho) \to \varrho$, $\varsigma \wedge \varrho = ((\varsigma' \to \varrho') \to \varrho')'$, for all $\varsigma, \varrho \in$ L.

Clearly we can observe that L is a LIA.

Then (L,∨,∧,′ ,⟶) is LIA.

Define M$\mathcal{B}$J-Neutrosophic set A in L by

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Table 6 : Illustration of Neutrosophic set A = $(M_A, \widetilde{\mathcal{B}}_A, J_A)$

L	0	a	b	c	d	1
M_A	0.7	0.5	0.5	0.7	0.5	0.5
$\widetilde{\mathcal{B}}_A$	[0.5, 0.6]	[0.3, 0.4]	[0.3, 0.4]	[0.3, 0.4]	[0.5, 0.6]	[0.3, 0.4]
J_A	0.3	0.5	0.5	0.5	0.3	0.5

Then A is a MBJ-NLII but it is not a M$\mathcal{B}$J-NPILII on L, because

For $0, d, a \in$ L, we can observe that

$$M_A(d) \ngeq \min\{M_A(((d \rightarrow (a \rightarrow (d)')' \rightarrow 0)'), M_A(0)\}$$

$$0.5 \ngeq 0.7.$$

3.1.5. Theorem: Let A be a M$\mathcal{B}$J-NLII of a LIA 'L'. Then A is a M$\mathcal{B}$J-NPILII of L if and only if it satisfies

$$M_A(\varsigma) \geq M_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')')$$

$$\widetilde{\mathcal{B}}_A(\varsigma) \geq \widetilde{\mathcal{B}}_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')')$$

$$J_A(\varsigma) \leq J_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')')$$

Proof : Assume that A is a M$\mathcal{B}$J-NPILII of L and let $\varsigma, \varrho, \varphi \in$ L. Let $\varphi = 0$, then

$$M_A(\varsigma) \geq \min\{M_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')' \rightarrow 0)'), M_A(0)$$

$$= \min\{M_A(((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')'), M_A(0)\}$$

$$= M_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')')$$

$$\widetilde{\mathcal{B}}_A(\varsigma) \geq r\min\{\widetilde{\mathcal{B}}_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')' \rightarrow 0)'), \widetilde{\mathcal{B}}_A(0)$$

$$= r\min\{\widetilde{\mathcal{B}}_A(((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')'), \widetilde{\mathcal{B}}_A(0)\}$$

$$= \widetilde{\mathcal{B}}_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')')$$

$$J_A(\varsigma) \leq \max\{J_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')' \rightarrow 0)'), J_A(0)$$

$$= \max\{J_A(((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')'), J_A(0)\}$$

$$= J_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')')$$

Conversely suppose that

$$M_A(\varsigma) \geq M_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')')$$

$$\widetilde{\mathcal{B}}_A(\varsigma) \geq \widetilde{\mathcal{B}}_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')')$$

$$J_A(\varsigma) \leq J_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')')$$

Since $M_A(\varsigma) \geq M_A((\varsigma \rightarrow (\varrho \rightarrow \varsigma)')')$

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$= \min\{M_A(((\varsigma \longrightarrow (\varrho \longrightarrow \varsigma)')'), M_A(0)\}$

$= \min\{M_A\ (((\varsigma \longrightarrow (\varrho \longrightarrow \varsigma)')' \longrightarrow 0)'), M_A(0)\}$

$$\widetilde{\mathcal{B}_A}(\varsigma) \geq \widetilde{\mathcal{B}_A}((\varsigma \longrightarrow (\varrho \longrightarrow \varsigma)')')$$

$= r \min\{\widetilde{\mathcal{B}_A}(((\varsigma \longrightarrow (\varrho \longrightarrow \varsigma)')'), \widetilde{\mathcal{B}_A}(0)\}$

$= r \min\{\widetilde{\mathcal{B}_A}((\varsigma \longrightarrow (\varrho \longrightarrow \varsigma)')' \longrightarrow 0)'), \widetilde{\mathcal{B}_A}(0)\}$

$J_A(\varsigma) \leq J_A(((\varsigma \longrightarrow (\varrho \longrightarrow \varsigma)')')$

$= \max\{J_A(((\varsigma \longrightarrow (\varrho \longrightarrow \varsigma)')'), J_A(0)\}$

$= \max\{J_A((\varsigma \longrightarrow (\varrho \longrightarrow \varsigma)')' \longrightarrow 0)'), J_A(0)\}$

Hence A is a M$\mathcal{B}$J-NPILII of L.

3.1.6. Theorem : Every M$\mathcal{B}$J-NPILII of a lattice implication algebra is a MBJ-NILII.

Proof : Let A be a M$\mathcal{B}$J-NPILII of a lattice implication algebra L.

$$M_A(\varsigma) \geq \min\{M_A(((\varsigma \to (\varrho \to \varsigma)')' \to \varphi)'), M_A(\varphi)\}$$

Put $\varphi = 0$ in the above equation, we get

$$M_A(\varsigma) \geq \min\{M_A(((\varsigma \to (\varrho \to \varsigma)')' \to 0)'), M_A(0)\}$$

$= M_A(((\varsigma \to (\varrho \to \varsigma)')' \to 0)') = M_A((\varsigma \to (\varrho \to \varsigma)')'$

On the other hand $((\varsigma \to \varrho)' \to ((\varsigma \to (\varsigma \to \varrho)')')' = ((\varsigma \to \varrho)' \to \varrho)'$

we obtain $M_A(\varsigma \longrightarrow \varrho)' \geq M_A(((\varsigma \longrightarrow \varrho)' \to (\varsigma \to (\varsigma \longrightarrow \varrho)')')')$

$= M_A((\varsigma \to \varrho)' \to \varrho)'$

$\geq \min\{M_A(((\varsigma \longrightarrow \varrho)' \longrightarrow \varrho)' \longrightarrow \varphi)'), M_A(\varphi)\}$

$$\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varphi)' \geq \widetilde{\mathcal{B}_A}(((\varsigma \longrightarrow \varphi)' \to \varphi)' \to (\varsigma \longrightarrow \varphi)')$$

$= \widetilde{\mathcal{B}_A}((\varsigma \to \varrho)' \to \varrho)'$

$= r \min\{M_A(((\varsigma \longrightarrow \varrho)' \longrightarrow \varrho)' \longrightarrow \varphi)'), M_A(\varphi)\}$

$$J_A(\varsigma \longrightarrow \varphi)' \leq J_A(((\varsigma \longrightarrow \varphi)' \to \varphi)' \to (\varsigma \longrightarrow \varphi)')$$

$= J_A((\varsigma \to \varrho)' \to \varrho)'$

$= \max\{M_A(((\varsigma \longrightarrow \varrho)' \longrightarrow \varrho)' \longrightarrow \varphi)'), M_A(\varphi)\}$

Hence A is a MBJ-NILII.

3.2. MBJ-Neutrosophic associative LI-ideal :

3.2.1. Definition : Let ς be a fixed element of a LIA 'L'. Let X be the universe. A M$\mathcal{B}$J-Nutrosophic set $A = \{\langle \varsigma; M_A(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma), J_A(\varsigma)/\varsigma \in X \rangle\}$ of a LIA L is called a M$\mathcal{B}$J-Neutrosophic associative LI-ideal of L with respect to ς if it satisfies

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$M_A(0) \geq M_A(\varsigma)$, $\widetilde{\mathcal{B}_A}(0) \geq \widetilde{\mathcal{B}_A}(\varsigma)$, $J_A(0) \leq J_A(\varsigma)$

$$M_A(\varphi) \geq \min\{M_A((\varphi \to \varrho)' \to \varsigma)'), M_A(\varrho \to \varsigma)'\}$$

$$\widetilde{\mathcal{B}_A}(\varphi) \geq r\min\{\widetilde{\mathcal{B}_A}((\varphi \to \varrho)' \to \varsigma)'), \widetilde{\mathcal{B}_A}(\varrho \to \varsigma)'\}$$

$$J_A(\varphi) \leq \max\{J_A((\varphi \to \varrho)' \to \varsigma)'), J_A(\varrho \to \varsigma)'\}$$

3.2.2. Proposition : Every MBJ-NALII of a LIA ‘L’ with respect to 0 is a M$\mathcal{B}$J-NLII.

Proof : Let A be a MBJ-NALII of a LIA ‘L’ with respect to 0. Then

$$M_A(\varphi) \geq \min\{M_A((\varphi \to \varrho)' \to 0)'), M_A(\varrho \to 0)'\} = \min\{M_A(\varphi \to \varrho)', M_A(\varrho)\}$$

$$\widetilde{\mathcal{B}_A}(\varphi) \geq r\min\{\widetilde{\mathcal{B}_A}((\varphi \to \varrho)' \to 0)'), \widetilde{\mathcal{B}_A}(\varrho \to 0)'\} = r\min\{\widetilde{\mathcal{B}_A}(\varphi \to \varrho)', \widetilde{\mathcal{B}_A}(\varrho)\}$$

$$J_A(\varphi) \leq \max\{J_A((\varphi \to \varrho)' \to 0)'), J_A(\varrho \to 0)'\} = \max\{J_A(\varphi \to \varrho)', J_A(\varrho)\}$$

Hence A is MBJ-LII.

3.2.3. Theorem : Let A be a MBJ-NLII of a LIA ‘L’. Then A is a MBJ-NALII of a LIA ‘L’ if and only if it satisfies

$$M_A(\varphi \to (\varrho \to \varsigma)')' \geq M_A((\varphi \to \varrho)' \to \varsigma)')$$

$$\widetilde{\mathcal{B}_A}(\varphi \to (\varrho \to \varsigma)')' \geq \widetilde{\mathcal{B}_A}((\varphi \to \varrho)' \to \varsigma)')$$

$$J_A(\varphi \to (\varrho \to \varsigma)')' \leq J_A((\varphi \to \varrho)' \to \varsigma)')$$

Proof : Consider $M_A(\varphi \to (\varrho \to \varsigma)')' \geq M_A((\varphi \to \varrho)' \to \varsigma)')$

$$\widetilde{\mathcal{B}_A}(\varphi \to (\varrho \to \varsigma)')' \geq \widetilde{\mathcal{B}_A}((\varphi \to \varrho)' \to \varsigma)')$$

$$J_A(\varphi \to (\varrho \to \varsigma)')' \leq J_A((\varphi \to \varrho)' \to \varsigma)')$$

Now we have to prove that A is a MBJ-NALII of a LIA ‘L’.

Since A is a MBJ-NLII of a LIA ‘L’ then

$$M_A(\varphi) \geq \min\{M_A(\varphi \to (\varrho \to \varsigma)')', M_A(\varrho \to \varsigma)'\}$$

$$\widetilde{\mathcal{B}_A}(\varphi) \geq r\min\{\widetilde{\mathcal{B}_A}(\varphi \to (\varrho \to \varsigma)')', \widetilde{\mathcal{B}_A}(\varrho \to \varsigma)'\}$$

$$J_A(\varphi) \leq \max\{J_A(\varphi \to (\varrho \to \varsigma)')', J_A(\varrho \to \varsigma)'\}$$

That implies $M_A(\varphi) \geq \min\{M_A((\varphi \to \varrho)' \to \varsigma)'), M_A(\varrho \to \varsigma)'\}$

$$\widetilde{\mathcal{B}_A}(\varphi) \geq r\min\{\widetilde{\mathcal{B}_A}((\varphi \to \varrho)' \to \varsigma)'), \widetilde{\mathcal{B}_A}(\varrho \to \varsigma)'\}$$

$$J_A(\varphi) \leq \max\{J_A((\varphi \to \varrho)' \to \varsigma)'), J_A(\varrho \to \varsigma)'\}$$

Hence A is a MBJ-NALII of a LIA ‘L’.

Conversely consider A is a MBJ-NALII of a LIA ‘L’.

Now we have to prove $M_A(\varphi \to (\varrho \to \varsigma)')' \geq M_A((\varphi \to \varrho)' \to \varsigma)')$

$$\widetilde{\mathcal{B}_A}(\varphi \to (\varrho \to \varsigma)')' \geq \widetilde{\mathcal{B}_A}((\varphi \to \varrho)' \to \varsigma)')$$

$$J_A(\varphi \to (\varrho \to \varsigma)')' \leq J_A((\varphi \to \varrho)' \to \varsigma)')$$

Consider $((\varphi \to (\varrho \to \varsigma)')' \to (\varphi \to \varrho)')' \to \varsigma)))$

$$= (\varsigma' \to ((\varphi \to (\varrho \to \varsigma)')' \to (\varphi \to \varrho)')$$

$$= \varsigma' \to ((\varphi \to \varrho) \to (\varphi \to (\varrho \to \varsigma)'))$$

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$$= (\varphi \rightarrow \varrho) \rightarrow \left(\varsigma' \rightarrow ((\varrho \rightarrow \varsigma) \rightarrow \varphi')\right)$$

$$= (\varphi \rightarrow \varrho) \rightarrow \left((\varrho \rightarrow \varsigma) \rightarrow (\varsigma' \rightarrow \varphi')\right)$$

$$= (\varphi \rightarrow \varrho) \rightarrow (\varphi \rightarrow \varrho) = 1$$

Then $M_A(\varphi \rightarrow (\varrho \rightarrow \varsigma)')'$

$$\geq \min\{M_A(((\varphi \rightarrow (\varrho \rightarrow \varsigma)')' \rightarrow (\varphi \rightarrow \varrho)')' \rightarrow \varsigma))), M_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)'))\}$$

$$= \min\{M_A(0), M_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')\}$$

$$= M_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')$$

Hence $M_A(\varphi \rightarrow (\varrho \rightarrow \varsigma)')' \geq M_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')$

$$\widetilde{\mathcal{B}}_A(\varphi \rightarrow (\varrho \rightarrow \varsigma)')'$$

$$\geq r\min\{\widetilde{\mathcal{B}}_A(((\varphi \rightarrow (\varrho \rightarrow \varsigma)')' \rightarrow (\varphi \rightarrow \varrho)') \rightarrow \varsigma))), \widetilde{\mathcal{B}}_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)'))\}$$

$$= r\min\{\widetilde{\mathcal{B}}_A(0), \widetilde{\mathcal{B}}_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')\}$$

$$= \widetilde{\mathcal{B}}_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')$$

Hence $\widetilde{\mathcal{B}}_A(\varphi \rightarrow (\varrho \rightarrow \varsigma)')' \geq \widetilde{\mathcal{B}}_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')$

$$J_A(\varphi \rightarrow (\varrho \rightarrow \varsigma)')'$$

$$\leq \max\{J_A(((\varphi \rightarrow (\varrho \rightarrow \varsigma)')' \rightarrow (\varphi \rightarrow \varrho)') \rightarrow \varsigma))), J_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)'))\}$$

$$= \max\{J_A(0), J_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')\}$$

$$= J_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')$$

Hence $J_A(\varphi \rightarrow (\varrho \rightarrow \varsigma)')' \leq J_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')$

3.2.4. Theorem : Let A be a MBJ-NLII of a LIA 'L'. Then A is a MBJ-NALII of a LIA 'L' if and only if it satisfies

$$M_A(\varrho) \geq M_A((\varrho \rightarrow \varsigma)' \rightarrow \varsigma)')$$

$$\widetilde{\mathcal{B}}_A(\varrho) \geq \widetilde{\mathcal{B}}_A((\varrho \rightarrow \varsigma)' \rightarrow \varsigma)')$$

$$J_A(\varrho) \leq J_A((\varrho \rightarrow \varsigma)' \rightarrow \varsigma)')$$

Proof : Let A be a MBJ-NALII of a LIA 'L'. Then by above theorem we have

$$M_A(\varphi \rightarrow (\varrho \rightarrow \varsigma)')' \geq M_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')$$

$$\widetilde{\mathcal{B}}_A(\varphi \rightarrow (\varrho \rightarrow \varsigma)')' \geq \widetilde{\mathcal{B}}_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')$$

$$J_A(\varphi \rightarrow (\varrho \rightarrow \varsigma)')' \leq J_A((\varphi \rightarrow \varrho)' \rightarrow \varsigma)')$$

Now take $\varsigma = \varrho$ then

$$M_A(\varphi \rightarrow (\varrho \rightarrow \varrho)')' \geq M_A((\varphi \rightarrow \varrho)' \rightarrow \varrho)')$$

$$\widetilde{\mathcal{B}}_A(\varphi \rightarrow (\varrho \rightarrow \varrho)')' \geq \widetilde{\mathcal{B}}_A((\varphi \rightarrow \varrho)' \rightarrow \varrho)')$$

$$J_A(\varphi \rightarrow (\varrho \rightarrow \varrho)')' \leq J_A((\varphi \rightarrow \varrho)' \rightarrow \varrho)')$$

Since $\varrho \rightarrow \varrho = 1$ then

$M_A(\varphi \rightarrow (1)')' \geq M_A((\varphi \rightarrow \varrho)' \rightarrow \varrho)')$

$\widetilde{\mathcal{B}}_A(\varphi \rightarrow (1)')' \geq \widetilde{\mathcal{B}}_A((\varphi \rightarrow \varrho)' \rightarrow \varrho)')$

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$J_A(\varphi \longrightarrow (1)')' \leq J_A((\varphi \longrightarrow \varrho)' \longrightarrow \varrho)')$

$M_A(\varphi \longrightarrow 0)' \geq M_A((\varphi \longrightarrow \varrho)' \longrightarrow \varrho)')$

$\widetilde{\mathcal{B}_A}(\varphi \longrightarrow 0)' \geq \widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho)' \longrightarrow \varrho)')$

Since $\varphi \longrightarrow 0 = \varphi'$ then

$J_A(\varphi \longrightarrow 0)' \leq J_A((\varphi \longrightarrow \varrho)' \longrightarrow \varrho)')$

$$M_A(\varphi) \geq M_A((\varphi \longrightarrow \varrho)' \longrightarrow \varrho)')$$

$$\widetilde{\mathcal{B}_A}(\varphi) \geq \widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho)' \longrightarrow \varrho)')$$

$$J_A(\varphi) \leq J_A((\varphi \longrightarrow \varrho)' \longrightarrow \varrho)')$$

Conversely suppose that

$$M_A(\varphi) \geq M_A((\varphi \longrightarrow \varrho)' \longrightarrow \varrho)')$$

$$\widetilde{\mathcal{B}_A}(\varphi) \geq \widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho)' \longrightarrow \varrho)')$$

$$J_A(\varphi) \leq J_A((\varphi \longrightarrow \varrho)' \longrightarrow \varrho)')$$

Now we have to show that A is a MBJ-NALII of a LIA 'L'.

Consider $(((\varphi \longrightarrow \varsigma)' \longrightarrow (\varrho \longrightarrow \varsigma)')' \longrightarrow (\varphi \longrightarrow \varrho)') = 0$

And $((\varphi \longrightarrow \varrho)' \longrightarrow (\varphi \longrightarrow \varsigma)')' \leq (\varsigma \longrightarrow \varrho)'$

It follows that

$((((\varphi \longrightarrow (\varrho \longrightarrow \varsigma)')' \longrightarrow \varsigma)' \longrightarrow \varsigma)' \longrightarrow ((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)')' = 0$

$M_A((\varphi \longrightarrow (\varrho \longrightarrow \varphi)')') \geq \{M_A(((\varphi \longrightarrow (\varrho \longrightarrow \varsigma)')' \longrightarrow \varsigma)' \longrightarrow \varsigma)' \longrightarrow ((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)')', M_A((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)'\}$

$\geq \min\{M_A(0), M_A((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)')\} = M_A((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)'$

$\widetilde{\mathcal{B}_A}((\varphi \longrightarrow (\varrho \longrightarrow \varphi)')') \geq \{\widetilde{\mathcal{B}_A}(((\varphi \longrightarrow (\varrho \longrightarrow \varsigma)')' \longrightarrow \varsigma)' \longrightarrow \varsigma)' \longrightarrow (((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)')', \widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)'\}$

$\geq r\min\{\widetilde{\mathcal{B}_A}(0), \widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)')\} = \widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)'$

$J_A((\varphi \longrightarrow (\varrho \longrightarrow \varphi)')') \leq \{J_A(((\varphi \longrightarrow (\varrho \longrightarrow \varsigma)')' \longrightarrow \varsigma)' \longrightarrow \varsigma))' \longrightarrow ((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)')', J_A((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)')\}$

$\leq \max\{J_A(0), J_A((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)')\} = J_A((\varphi \longrightarrow \varrho)' \longrightarrow \varsigma)'$

Therefore A is a MBJ-NALII of a LIA 'L'.

3.3. MBJ-Neutrosophic Fantastic LI-ideal :

3.3.1. Definition : Let X be the universe. A M$\mathcal{B}$J-Neutrosophic set

$A = \{\langle \varsigma; M_A(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma), J_A(\varsigma) / \varsigma \in X \rangle\}$of a LIA 'L' is called a M$\mathcal{B}$J-Neutrosophic fantastic LI-ideal of 'L' if it satisfies

$M_A(0) \geq M_A(\varsigma)$, $\widetilde{\mathcal{B}_A}(0) \geq \widetilde{\mathcal{B}_A}(\varsigma)$, $J_A(0) \leq J_A(\varsigma)$ and

$$M_A(\varsigma \longrightarrow (\varrho \longrightarrow (\varrho \longrightarrow \varsigma)')')' \geq \min\{M_A((\varsigma \longrightarrow \varrho)' \longrightarrow \varphi)', M_A(\varphi)\}$$

$$\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow (\varrho \longrightarrow (\varrho \longrightarrow \varsigma)')')' \geq r\min\{\widetilde{\mathcal{B}_A}((\varsigma \longrightarrow \varrho)' \longrightarrow \varphi)', \widetilde{\mathcal{B}_A}(\varphi)\}$$

$$J_A(\varsigma \longrightarrow (\varrho \longrightarrow (\varrho \longrightarrow \varsigma)')')' \leq \max\{J_A((\varsigma \longrightarrow \varrho)' \longrightarrow \varphi)', J_A(\varphi)\}$$

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3.3.2. Example : The Hasse diagram and Cayley tables of a poset, L = {0, a, b, c, d, 1} are given below as follows:

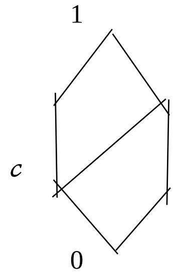

Figure 2: Hasse diagram of L

Table 7: Illustration of complement of element

a b

d

ς	ς′
0	1
a	c
b	d
c	a
d	b
1	0

Table 8: Illustration of Implication

→	**0**			***a***		
0	1	1	1	1	1	1
a	c	1	b	c	b	1
b	d	a	1	b	a	1
c	a	a	1	1	a	1
d	b	1	1	b	1	1
1	0	a	b	c	d	1

The operations V and Λ on L are defined as follows

$ς \vee ϱ = (ς \rightarrow ϱ) \rightarrow ϱ$, $ς \wedge ϱ = ((ς' \rightarrow ϱ') \rightarrow ϱ')'$, for all $ς, ϱ \in$ L.

Clearly we can observe that L is a LIA.

Assume A = (M_A,$\widetilde{\mathcal{B}_A}$, JA) is a MBJ-Neutrosophic set on L defined by Table given below

Table 9 : Illustration of Neutrosophic set A = (M_A, $\widetilde{\mathcal{B}_A}$, J_A)

L	0	a	b	c	d	1
$M_A(ς)$	0.8	0.4	0.4	0.6	0.4	0.4
$\widetilde{\mathcal{B}_A}(ς)$	[0.8, 0.9]	[0.3, 0.4]	[0.3, 0.4]	[0.3, 0.4]	[0.3, 0.4]	[0.2, 0.4]
$J_A(ς)$	0.1	0.3	0.5	0.5	0.3	0.5

In general we verify that L is a MBJ-NFLII

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3.3.3 Theorem : Every MBJ-NFLIIis a M$\mathcal{B}$J-NLII.

Proof : Let A be a MBJ-NFLII of a LIA 'L'.

Taking $\varrho = 0$ we get

$M_A(\varsigma) = M_A\big(((\varsigma \to (0 \to (0 \to \varsigma)')')' \geq \min\{M_A((\varsigma \to 0)' \to \varphi)'), M_A(\varphi)\}$

$= \min\{M_A(\varsigma \to \varphi)', M_A(\varphi)\}$

$\widetilde{\mathcal{B}_A}(\varsigma) = \widetilde{\mathcal{B}_A}\big(((\varsigma \to (0 \to (0 \to \varsigma)')')' \geq r\min\{\widetilde{\mathcal{B}_A}((\varsigma \to 0)' \to \varphi)'), \widetilde{\mathcal{B}_A}(\varphi)\}$

$= r\min\{\widetilde{\mathcal{B}_A}(\varsigma \to \varphi)', \widetilde{\mathcal{B}_A}(\varphi)\}$

$J_A(\varsigma) = J_A\big(((\varsigma \to (0 \to (0 \to \varsigma)')')' \leq \max\{J_A((\varsigma \to 0)' \to \varphi)'), J_A(\varphi)\}$

$= \max\{J_A(\varsigma \to \varphi)', J_A(\varphi)\}$

Hence A is a M$\mathcal{B}$J-NLII.

Note : Converse of the above theorem is also true. It will be followed by the below example.

3.3.4. Example : Let L = $\{0, a, b, c, 1\}$. Define partial order on L as $0 < a < b < c < 1$ and define $\varsigma \wedge \varrho = \min\{\varsigma, \varrho\}$ and $\varsigma \vee \varrho = \max\{\varsigma, \varrho\}$ for all $\varsigma, \varrho \in L$ and " ' "and "⟶" as follows

Define M$\mathcal{B}$J-Neutrosophic set A in L by

Table 10 : Illustration of Neutrosophic set A = $(M_A, \widetilde{\mathcal{B}_A}, J_A)$

L	0	a	b	c	1
M_A	0.5	0.4	0.4	0.4	0.2
$\widetilde{\mathcal{B}_A}$	[0.4, 0.5]	[0.3, 0.4]	[0.3, 0.4]	[0.3, 0.4]	[0.2, 0.4]
J_A	0.2	0.4	0.4	0.4	0.5

Then A is a MBJ-NFLII on L, if A is a M$\mathcal{B}$J-NLII.

3.3.5. Theorem : A M$\mathcal{B}$J-NLII A of L is a MBJ-NFLII if and only if it satisfies

$$M_A\big(((\varsigma \to (\varrho \to (\varrho \to \varsigma)')')' \geq M_A(\varsigma \to \varrho)'$$

$$\widetilde{\mathcal{B}_A}\big(((\varsigma \to (\varrho \to (\varrho \to \varsigma)')')' \geq \widetilde{\mathcal{B}_A}(\varsigma \to \varrho)'$$

$$J_A\big(((\varsigma \to (\varrho \to (\varrho \to \varsigma)')')' \geq J_A(\varsigma \to \varrho)'$$

Proof : Let A be a MBJ-NFLII of a LIA 'L'.

Taking $\varphi = 0$ in above definition

$$M_A\big(((\varsigma \to (\varrho \to (\varrho \to \varsigma)')')' \geq \min\{M_A((\varsigma \to \varrho)' \to 0)'), M_A(0)\} \geq M_A(\varsigma \to \varrho)'$$

$$\widetilde{\mathcal{B}_A}\big(((\varsigma \to (\varrho \to (\varrho \to \varsigma)')')\big)' \geq r\min\{\widetilde{\mathcal{B}_A}((\varsigma \to \varrho)' \to 0)', \widetilde{\mathcal{B}_A}(\varphi)\} \geq \widetilde{\mathcal{B}_A}(\varsigma \to \varrho)'$$

$$J_A\big(((\varsigma \to (\varrho \to (\varrho \to \varsigma)')')\big)' \leq \max\{J_A((\varsigma \to \varrho)' \to 0)', J_A(\varphi)\} \geq J_A(\varsigma \to \varrho)'$$

Conversely suppose that A is a M$\mathcal{B}$J-NLII of 'L' and

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$$M_A(((\varsigma \to (\varrho \to (\varrho \to \varsigma)')')' \geq M_A(\varsigma \to \varrho)' \geq \min\{M_A((\varsigma \to \varrho)' \to \varphi)', M_A(\varphi)\}$$

$$\widetilde{\mathcal{B}}_A(((\varsigma \to (\varrho \to (\varrho \to \varsigma)')')' \geq \widetilde{\mathcal{B}}_A(\varsigma \to \varrho)' \geq \operatorname{rmin}\{\widetilde{\mathcal{B}}_A((\varsigma \to \varrho)' \to \varphi)', \widetilde{\mathcal{B}}_A(\varphi)\}$$

$$J_A(((\varsigma \to (\varrho \to (\varrho \to \varsigma)')')' \geq J_A(\varsigma \to \varrho)' \geq \max\{J_A((\varsigma \to \varrho)' \to \varphi)', J_A(\varphi)\}$$

Hence A is a MBJ-NFLII of L.

3.3.6. Theorem : Every MBJ-NPILII of 'L' is a MBJ-NFLII.

Proof : Let A be a MBJ-NPILII of L.

Since every MBJ-NPILII of L is a MBJ-NLII

Since $\varsigma \leq (\varsigma \to (\varrho \to (\varrho \to \varsigma)')')'$

we get $(\varrho \to (\varsigma \to (\varrho \to (\varrho \to \varsigma)')')')' \leq (\varrho \to \varsigma)'$

Hence $(\varrho \to (\varsigma \to (\varrho \to (\varrho \to \varsigma)')')')' \to (\varsigma \to (\varrho \to (\varrho \to \varsigma)')')'$

$$\geq (\varsigma \to (\varrho \to (\varrho \to \varsigma)')')' \to (\varrho \to \varsigma)'$$

$$= (\varsigma \to (\varrho \to \varsigma)')' \to (\varrho \to (\varrho \to \varsigma)')'$$

$\geq (\varsigma \to \varrho)'$ and thus

$$M_A(\varrho \to \varsigma)' \geq M_A(\varrho \to (\varsigma \to (\varrho \to (\varrho \to \varsigma)')')')' \to (\varsigma \to (\varrho \to (\varrho \to \varsigma)')')'$$

$$\geq M_A(\varsigma \to (\varrho \to (\varrho \to \varsigma)')')'$$

$$\widetilde{\mathcal{B}}_A(\varrho \to \varsigma)' \geq \widetilde{\mathcal{B}}_A(\varrho \to (\varsigma \to (\varrho \to (\varrho \to \varsigma)')')')' \to (\varsigma \to (\varrho \to (\varrho \to \varsigma)')')'$$

$$\geq \widetilde{\mathcal{B}}_A(\varsigma \to (\varrho \to (\varrho \to \varsigma)')')'$$

$$J_A(\varrho \to \varsigma)' \leq J_A(\varrho \to (\varsigma \to (\varrho \to (\varrho \to \varsigma)')')')' \to (\varsigma \to (\varrho \to (\varrho \to \varsigma)')')'$$

$$\leq J_A(\varsigma \to (\varrho \to (\varrho \to \varsigma)')')'$$

Therefore, by above theorem A is a MBJ-NFLII.

4. Conclusion:

In this paper we proved the relation between the ideals that "Every MBJ-NPILII is a MBJ-NLII", "Every MBJ-NPILII is a MBJ-NILII","Every MBJ-NPILII is a MBJ-NFII", "Every MBJ-NALII is a MBJ-NFII" and "Every MBJ-NFII is a MBJ-NLII".

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