An Introduction to The Symbolic 3-Plithogenic Vector Spaces

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Abstract

The objective of this paper is to define and study for the first time the concept of symbolic 3-plithogenic vector spaces based on symbolic 3-plithogenic sets and classical vector spaces.Also, many related substructures will be defined and handled such as AH-functions, AH-spaces, and symbolic 3-plithogenic basis.

Keywords: 3-plithogenic symbolic set; 3-plithogenic vector space; 3-plithogenic dimension

1. Introduction

The concept of symbolic plithogenic sets was defined by Smarandache in [13-17 ,30], and he suggested an algebraic approach of these sets. Laterally, the concept of symbolic 2-plithogenic rings [31], where the concepts such as symbolic AH-ideals, and AH-homomorphisms were presented and discussed. In [35-41] many algebraic structures about symbolic 2-plithogenic structures were studied such as number theory, algebraic equations, and symbolic 3-plithogenic rings.

In general, we can say that symbolic plithogenic structures are very close to neutrosophic algebraic structures with many differences in the definition of multiplication operation [1-10].

Let R be a ring, the symbolic 3-plithogenic ring is defined as follows:
$3-SP_R = \{a_0 + a_1P_1 + a_2P_2 + a_3P_3;\ a_i \in R, {P_j}^2 = P_j, P_i \times P_j = P_{max(i,j)}\}$.
Smarandache has defined algebraic operations on $3-SP_R$ as follows:
Addition:
$[a_0 + a_1P_1 + a_2P_2 + a_3P_3] + [b_0 + b_1P_1 + b_2P_2 + b_3P_3] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2 + (a_3 + b_3)P_3$.
Multiplication:
$[a_0 + a_1P_1 + a_2P_2 + a_3P_3].[b_0 + b_1P_1 + b_2P_2 + b_3P_3] = a_0b_0 + P_1[a_0b_1 + a_1b_0 + a_1b_1] + P_2[a_0b_2 + a_1b_2 + a_2b_2 + a_2b_0 + a_2b_1] + P_3[a_0b_3 + a_1b_3 + a_2b_3 + a_3b_3 + a_3b_0 + a_3b_1 + a_3b_2]$.
It is clear that $(3-SP_R)$ is a ring.
If R is a field, then $3-SP_R$ is called a symbolic 3-plithogenic field.

Main Discussion

Definition.
Let V be a vector space over the field F, let $3-SP_F$ be the corresponding symbolic 3-plithogenic field.
$3-SP_F = \{x + yP_1 + zP_2 + tP_3;\ x, y, z, t \in F, {P_i}^2 = P_i, P_1P_2 = P_2P_1 = P_2\}$.
We define the symbolic 3-plithogenic vector space as follows:
$3-SP_V = V + VP_1 + VP_2 + VP_3 = \{a + bP_1 + cP_2 + dP_3;\ a, b, c, d \in V\}$.
Operations on $3-SP_V$ can be defined as follows:
Addition: $(+): 3-SP_V \to 3-SP_V$, such that:

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$[x_0 + x_1P_1 + x_2P_2 + x_3P_3] + [y_0 + y_1P_1 + y_2P_2 + y_3P_3] = (x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2 + (x_3 + y_3)P_3$.

Multiplication: $(.): 3 - SP_F \times 3 - SP_V \to 3 - SP_V$, such that:

$[a + bP_1 + cP_2 + dP_3].[x_0 + x_1P_1 + x_2P_2 + x_3P_3] = ax_0 + (ax_1 + bx_0 + bx_1)P_1 + (ax_2 + bx_2 + cx_0 + cx_1 + cx_2)P_2 + (ax_3 + bx_3 + cx_3 + dx_0 + dx_1 + dx_2 + dx_3)P_3$.

where $x_i, y_i \in V, a, b, c, d \in F$

Theorem.

Let $(3 - SP_V, +, .)$ Is a module over the ring $3 - SP_F$.

Proof.

Let$X = x_0 + x_1P_1 + x_2P_2 + x_3P_3, Y = y_0 + y_1P_1 + y_2P_2 + y_3P_3 \in 3 - SP_V$, $A = a_0 + a_1P_1 + a_2P_2 + a_3P_3, B = b_0 + b_1P_1 + b_2P_2 + b_3P_3 \in 3 - SP_F$ we have:

$1. X = X, (X + Y) + Z = X + (Y + Z), X + (-X) = -X + X = 0, X + 0 = 0 + X = X$

Also

$A(X + Y) = (a_0 + a_1P_1 + a_2P_2 + a_3P_3)[(x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2 + (x_3 + y_3)P_3] = A.X + A.Y$

$(A + B)X = A.X + B.X$

$(A.B).X = A(B.X)$

Example.

Let $V = R^3$ be the Euclidean space over the field $F = R$.

The corresponding symbolic 3-plithogenic vector space over $3 - SP_F$ is:

$3 - SP_{R^3} = \{(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2 + (x_3, y_3, z_3)P_3; x_i, y_i, z_i \in R\}$

Definition.

Let $3 - SP_V$ be a symbolic 3-plithogenic vector space over $3 - SP_F$, let V_0, V_1, V_2, V_3 be the three subspaces of V, we define the AH-subspace as follows:

$$W = V_0 + V_1P_1 + V_2P_2 + V_3P_3 = \{x + yP_1 + zP_2 + tP_3;\ x \in V_0, y \in V_1, z \in V_2, t \in V_3\}$$

If $V_0 = V_1 = V_2 = V_3$, then W is called an AHS-subspace.

Example.

Consider $3 - SP_{R^3}$, we have $V_0 = \{(a, 0,0);\ a \in R\}, V_1 = \{(0, b, 0);\ b \in R\}, V_2 = \{(0,0, c);\ c \in R\}$ are three subspaces of $V = R^3$.

$W = V_0 + V_1P_1 + V_2P_2 + V_2P_3 = \{(a, 0,0) + (0, b, 0)P_1 + (0,0, c)P_2 + (0,0, d)P_2;\ a, b, c, d \in R\}$ is an AH-subspace of $3 - SP_{R^3}$.

$T = V_1 + V_1P_1 + V_1P_2 + V_1P_3 = \{(0, a, 0) + (0, b, 0)P_1 + (0, c, 0)P_2 + (0, d, 0)P_2;\ a, b, c, d \in R\}$ is an AHS-subspace.

Theorem.

Let $3 - SP_V$ be a symbolic 3-plithogenic vector space over $3 - SP_F$, let W be an AHS-subspace of $3 - SP_V$, then W is a submodule of $3 - SP_V$.

Proof.

The proof is similar to the case of 2-plithogenic spaces.

Definition.

Let V, W be two vector spaces over the field F. Let $3 - SP_V$, $3 - SP_W$ be the corresponding symbolic 3-plithogenic vector spaces over $3 - SP_F$.

Let $L_0, L_1, L_2, L_3: V \to W$ be three linear transformations, we define the AH-linear transformation as follows:

$L: 3 - SP_V \to 3 - SP_W, L = L_0 + L_1P_1 + L_2P_2 + L_3P_3; L(x + yP_1 + zP_2 + dP_3) = L_0(x) + L_1(y)P_1 + L_2(z)P_2 + L_3(d)P_3$.

If $L_0 = L_1 = L_2 = L_3$, then L is called AHS-linear transformation.

Definition.

Let $L = L_0 + L_1P_1 + L_2P_2 + L_3P_3: 3 - SP_V \to 3 - SP_W$ be an AH-linear transformation, we define:

1. $AH - ker(L) = ker(L_0) + ker(L_1)P_1 + ker(L_2)P_2 + ker(L_3)P_3 = \{x + yP_1 + zP_2 + dP_3\}; x \in ker(L_0), y \in ker(L_1), z \in ker(L_2), d \in ker(L_3)$.
2. $AH - Im(L) = Im(L_0) + Im(L_1)P_1 + Im(L_2)P_2 + Im(L_3)P_3 = \{a + bP_1 + cP_2 + dP_3\}; a \in Im(L_0), b \in Im(L_1), c \in Im(L_2), d \in Im(L_3)$

If L is AHS-linear transformation, then we get $AHS - kernel, AHS - Image$.

Theorem.

Let $L = L_0 + L_1P_1 + L_2P_2 + L_3P_3: 3 - SP_V \to 3 - SP_W$ be an AH-linear transformation, then:

1. $AH - ker(L)$ is AH-subspace of $3 - SP_V$.
2. $AH - Im(L)$ is AH-subspace of $3 - SP_W$.

Example.

Take $V = R^3, W = R^3$, $L_0, L_1, L_2: V \to W$ such that:

$L_0(x, y, z) = (x, y), L_1(x, y, z) = (2x, z), L_2(x, y, z) = (x - y, y - z)$

The corresponding AH-linear transformation is:

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$L = L_0 + L_1P_1 + L_2P_2 + L_2P_3: 3 - SP_{R^3} \to 3 - SP_{R^2}:$

$$L[(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2 + +(x_3, y_3, z_3)P_3]$$
$$= L_0(x_0, y_0, z_0) + L_1(x_1, y_1, z_1)P_1 + L_2(x_2, y_2, z_2)P_2 + L_2(x_3, y_3, z_3)P_3$$
$$= (x_0, y_0) + (2x_1, z_1)P_1 + (x_2 - y_2, y_2 - z_2)P_2 + (x_3 - y_3, y_3 - z_3)P_3$$

$$\begin{cases} ker(L_0) = \{(0,0,z_0);\ z_0 \in R\} \\ ker(L_1) = \{(0, y_1, 0);\ y_1 \in R\} \\ ker(L_2) = \{(x_2, x_2, x_2);\ x_2 \in R\} \\ AH - ker(L) = \{(0,0,z_0) + (0, y_1, 0)P_1 + (x_2, x_2, x_2)P_2 + (x_3, x_3, x_3)P_3; z_0, y_1, x_2, x_3 \in R\} \end{cases}$$

Also,

$$\begin{cases} Im(L_0) = R^2 \\ Im(L_1) = R^2 \\ Im(L_2) = R^2 \\ AH - Im(L) = R^2 + R^2P_1 + R^2P_2 + R^2P_3 = 3 - SP_W \end{cases}$$

3. Conclusion

In this paper we have defined the concept of symbolic 3-plithogenic vector spaces over a symbolic 3-plithogenic field, where we have presented some of their elementary properties such as basis, linear transformations, and AH-subspaces. On the other hand, we have suggested many examples to clarify the validity of our work.

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