Implementation of Neutrosophic Bipolar Pentagonal Fuzzy Set on Multi-Criteria Decision-Making Scenario

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Abstract

Several others have already discussed various types of new techniques in multi-criteria decision-making problems. De-fuzzification, De-eutrophication, and De-Bipolarisation all of these are new approaches that have been established. But this work depicts the concept and features of the Neutrosophic Bipolar Pentagonal Fuzzy number. The relationships and products of the picture fuzzy set, as well as their associated results, are investigated. The different kinds of neutrosophic single-typed bipolar pentagonal numbers (nbpnum) were also discussed. A multi-criteria choice is also done in a triangular Bipolar Neutrosophic Fuzzy Set to identify the ideal output depending on several attributes. A multi-criteria choice is also done in a triangular Bipolar Neutrosophic Fuzzy Set to identify the ideal output depending on several attributes.

Keywords: Neutrosohic set; Bipolar Fuzzy Set; Pentagonal Fuzzy Set; Decision Making

1. Introduction

To manage data and information with non-statistical uncertainty, Zadeh [36] investigated fuzzy sets in 1965. In 1986, Atanassov [5] symbolized an enlarged idea, which is an intuitionistic fuzzy set [InF]. Zhang[37] contributed to the development of bipolar fuzzy sets, which discusses the resolutions of bipolar level sets and their relationships. To rank trapezoidal fuzzy numbers, Abbasbandy [2] presented an innovative approach. Maissam et .al [22] described some significant neutrosophic rules for decision-making in the case of uncertain data. In 2015, Lee [21] extended the concept of BCK/BCI fuzzy algebras and fuzzy ideals in a bipolar fuzzy set. Wang et al. [31] pioneered the came up with the concept of single-valued neutrosophic sets. Somen Debnath [26] is devoted to introducing a novel concept as a restricted neutrosophic set. In 2013, Aslam et al. [4] suggested Bipolar fuzzy soft sets and their applications in decision-making problems. Abdullah et al. [1] discussed the properties of Bipolar fuzzy soft sets and their applications in decision-making problems. Ye [34] discussed the use of prioritized trapezoidal InF aggregation operators in multi-criteria decision-making (M_C_D_M). The software's utility and effectiveness in choosing different prioritized relationships between decision-making factors are also shown through an example. New operations and ranking have been found on the Pentagonal fuzzy number by Helen [20]. The refined fuzzy concept also emerged in later days it is referred

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to as a neutrosophic set. In 2015 [16,17], Deli et al. presented Bipolar neutrosophic sets and their implementation based on M_C_D_M situations. Triangular, Reverse Order, Pentagonal and Trapezoidal Fuzzy Numbers are a significant development of Pathinathan P and K. Ponnivalavan [25]. Wang et al. [33] proposed the TODIM approach with multi-valued neutrosophic sets. The Trapezoidal neutrosophic set and associated methods for multiple-character decision-making were proposed by Yun Ye [35]. Vigin Raj et al. [30] described using a neural network to choose the ideal machine for a job by incorporating an extension of pentagonal fuzzy numbers. Extended bipolar single-valued in neutrosophic graph theory is described in [6] by Broumi et al. The relation between the Trapezoidal Fuzzy Numbers in the Massive Level intelligent similarity model was generalized by Tourad et al. [28]. Ulucay et al. [29] utilized the applications of similarity measurements of bipolar neutrosophic sets in the multiple criteria decision-making process. In M_C_D_M applications, Wang et al. [32] applied the Frank choguet Bonferroni mean operators of bipolar neutrosophic sets. Chakraborty et. al invented Pentagonal [7,11,12], Triangular [8,10], and Trapezoidal [9] fuzzy numbers have been researched, as well as their numerous uses in fuzzy and neutrosophic fuzzy fields. Further, the authors have discussed many types of new techniques such as De-fuzzification, De-eutrophication, and De-Bipolarisation [13] in M_C_D_M. Additionally, the discussion of Pentagonal Neutrosophic Number Arithmetic and Geometric Operators and their Application in the Mobile Communication Based MCGDM Problem [14] has been made. Furthermore, the use of cylindrical neutrosophic single-valued numbers in networking problems is explored [15]. The idea of n-valued Refined Neutrosophic Logic and its uses in Physics was introduced by Florentin Smarandache [18]. Haque has started a new method for solving multi-criteria group decision-making problems using exponential operational law in a generalized spherical fuzzy environment [19]. Ali et al. described an M_C_D_M strategy that uses single-valued neutrosophic Schweizer-Sklar Muirhead mean aggregation operators. [3]. Deli et al. [16] studied the use of M_C_D_M problems on bipolar neutrosophic sets, and their M_C_D_M problems methods were established based on this aggregation operator in it. The ranking of pentagonal neutrosophic numbers and their applications to solve assignment problems by employing their magnitude were pioneered by Radhika et al. [27]. Necmiya Merve Sahin et al [24] explained M_C_D_M applications based on generalized hamming measures for Law. Mani Parimala et al. [23] interpreted the M_C_D_M algorithm via complex neutrosophic nano topological spaces. After considering all of these concepts and techniques, this study presents M_C_D_M based on a pentagonal bipolar neutrosophic fuzzy set.

1.1 Motivation:

In mathematical modeling, resolving technical issues, and resolving medical diagnosis issues as well as multi-criteria medical diagnosis problems, the interpretation of uncertainty theory is crucial. When considering a neutrosophic bipolar pentagonal number, an essential question arises: what are the linear and non-linear forms, as well as the geometrical figure? How can we logically and scientifically turn a nbpnum comparable to a crisp number? When the membership functions are connected, how should the type-1,2,3 nbpnum be classified? We wrote the research paper from this point of view. We came up with some more intriguing discoveries on De-formulation techniques and other uses of neutrosophic bipolar pentagonal numbers.

1.2 Novelties:

On the pentagonal neutrosophic number arena, several studies have already been discussed. This approach was also implemented in many areas by researchers from various domains. The theory of a nbpnum is new in the world of M_C_D_M.

As a final decision, the following model for M_C_D_M is as follows,

(i) Formulation and the definition of linear nbpnum in various instances.
(ii) De- Formulation of linear neutrosophic bipolar pentagonal number.
(iii) Application-oriented in M_C_D_M problem.

1.3 Verbal phrase on Neutrosophic Arena:

The fascinating question arises regularly in everyday situations: how the concept of vagueness and neutrosophic theory can be applied in the real world, and what are the suitable verbal phrases?

Example:

Consider the challenge of choosing a move for a national award. Assume that one film must be chosen from a finite number of options during the selection process. Every member of the selecting committee will have a different point of view, feelings, emotions, ethics, dreams, and so on. According to them, the result might be

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any fuzzy number, including an interval number, a triangular fuzzy number, an intuitionistic number [InN], or a neutrosophic fuzzy number. Let's now look at the terminology used to describe the Time problem in each of the cases.

1.4 Verbal Phrases:

Table 1: Verbal phrase representation

Distinct Parameter	Verbal Phrases	Information
Interval number	[Low, High]	Depending on their initial priority, Zuari members will select from a variety of moves, such as [2nd,3rd].
Pentagonal bipolar fuzzy number	[Low, Median, High]	Zuari members will make decisions depending on their highest priorities, with an intermediate move [1st, 2nd, 3rd].
InFN-Intuitionistic fuzzy number	[Standard, Median, High; Very low, Poor, Low]	According to their viewpoints, Zuari members will automatically select some cinemas and reject others.
Neutrosophic fuzzy number [pentagonal bipolar]	[Very low, Low, Median, High, Very high; Very low, Low, Median, High; Very low, Low, Median, High, Very high]	Depending on their own interests, some Zuari members will choose films without hesitation, while others would hesitate and reject certain films.

2. Preliminaries

This section addresses some fundamental terms and findings that are important for comprehending the sections that follow.

Definition 2.1A set $\widetilde{\mathfrak{W}}$, defined as $\widetilde{\mathfrak{W}} = \left\{\left(\mathscr{k}, \psi_{\widetilde{\mathfrak{W}}}(\mathscr{k})\right): \mathscr{k} \in \widetilde{\mathfrak{W}}, \psi_{\widetilde{\mathfrak{W}}}(\mathscr{k}) \in [0,1]\right\}$ and it is usually represented by the pair$\left(\mathscr{k}, \psi_{\widetilde{\mathfrak{W}}}(\mathscr{k})\right)$, $\mathscr{k}$ corresponds to the crisp set $\mathfrak{W}$ and $\psi_{\widetilde{\mathfrak{W}}}(\mathscr{k}) \in [0,1]$, then $\widetilde{\mathfrak{W}}$ is referred as a fuzzy set.

Definition 2.2

The set $\mathbb{N}\widetilde{eut\,\mathcal{N}}$ is to be referred to as a neutrosophic set in the universal discourse X, commonly denoted by Ω if $\mathbb{N}\widetilde{eut\,\mathcal{N}} = \{\langle\Omega, [\varphi_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega), \eta_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega), \psi_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega)]\rangle: \Omega \in \mathfrak{M}\}$ where $\varphi_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega): \mathfrak{M} \to [0,1]$ is defined to be the degree of confidence, $\eta_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega): \mathfrak{M} \to [0,1]$ denoted by the degree of hesitation and $\psi_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega): \mathfrak{M} \to [0,1]$ is referred as the degree of the falseness of decision. Where $\varphi_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega), \eta_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega), \psi_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega)$ supports the relation $0 \le \varphi_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega) + \eta_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega) + \psi_{\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega) \le 3+$.

Definition 2.3

A neutrosophic set $N\widetilde{eut}\,N$ in definition 2.1 is referred to be a single-valued neutrosophic set ($\mathbb{N}\widetilde{eut\,\mathcal{N}}$) if Ω is a single-valued independent variable. $\mathfrak{W}\mathbb{N}\widetilde{eut\,\mathcal{N}} = \{\langle\Omega, [\varphi_{\mathfrak{W}\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega), \eta_{\mathfrak{W}\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega), \psi_{\mathfrak{W}\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega)]\rangle: \Omega \in \mathfrak{M}\}$, Where $\varphi_{\mathfrak{W}\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega), \eta_{\mathfrak{W}\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega), \psi_{\mathfrak{W}\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\Omega)$ referred to the idea ofconfidence of confidence, hesitation, falseness membership function respectively.

If there exist three points $\varphi_{\mathfrak{W}\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\alpha_0) = 1$, $\eta_{\mathfrak{W}\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\beta_0) = 1$, $\psi_{\mathfrak{W}\mathbb{N}\widetilde{eut\,\mathcal{N}}}(\gamma_0) = 1$, then the $\mathfrak{W}\mathbb{N}\widetilde{eut\,\mathcal{N}}$ is called neut-normal.

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Definition 2.4

The single valued pentagonal neturosophic number $\tilde{\zeta}$ is exact as

$\tilde{\zeta} = \{\langle[(p^1, q^1, r^1, s^1, t^1); \tau], [(p^2, q^2, r^2, s^3, t^2); \epsilon], [(p^3, q^3, r^3, s^3, t^3); \rho]\ \rangle\}$ where , $\epsilon, \rho \in [0,1]$.

The confidence membership function $(l_{\tilde{\zeta}}): \mathbb{R} \to [0, \tau]$,the hesitation membership function $(m_{\tilde{\zeta}}): \mathbb{R} \to [\epsilon, 1]$ and the false membership function $(n_{\tilde{\zeta}}): \mathbb{R} \to [\rho, 1]$ are given as

$$(l_{\tilde{\zeta}})(\Omega) = \begin{cases} l_{\widetilde{\zeta a_1}}(\Omega) & p^1 \le \Omega \le q^1 \\ l_{\widetilde{\zeta a_2}}(\Omega) & q^1 \le \Omega < r^1 \\ \tau & \Omega = r^1 \\ l_{\widetilde{\zeta b_2}}(\Omega) & r^1 \le \Omega < s^1 \\ l_{\widetilde{\zeta b_1}}(\Omega) & s^1 \le \Omega < t^1 \\ 0 & other\ wise \end{cases}, \quad (m_{\tilde{\zeta}})(\Omega) = \begin{cases} m_{\widetilde{\zeta a_1}}(\Omega) & p^2 \le \Omega \le q^2 \\ m_{\widetilde{\zeta a_2}}(\Omega) & q^2 \le \Omega < r^2 \\ \epsilon & x = r^2 \\ m_{\widetilde{\zeta b_2}}(\Omega) & r^2 \le \Omega < s^2 \\ m_{\widetilde{\zeta b_1}}(\Omega) & s^2 \le \Omega < t^2 \\ 1 & other\ wise \end{cases}$$

$$(n_{\tilde{\zeta}})(\Omega) = \begin{cases} n_{\widetilde{\zeta a_1}}(\Omega) & p^3 \le \Omega \le q^3 \\ n_{\widetilde{\zeta a_2}}(\Omega) & q^3 \le \Omega < r^3 \\ \rho & \Omega = r^3 \\ n_{\widetilde{\zeta b_2}}(\Omega) & r^3 \le \Omega < s^3 \\ n_{\widetilde{\zeta b_1}}(\Omega) & s^3 \le \Omega < t^3 \\ 1 & other\ wise \end{cases}$$

Definition 2.5

A bipolar neutrosophic set is characterized as,

$$Bp\widetilde{Neut}\,N = \{\langle \Omega, [\varphi^+_{Bp\widetilde{Neut}\,N}(\Omega), \eta^+_{Bp\widetilde{Neut}\,N}(\Omega), \psi^+_{Bp\widetilde{Neut}\,N}(\Omega), \varphi^-_{Bp\widetilde{Neut}\,N}(\Omega), \eta^-_{Bp\widetilde{Neut}\,N}(\Omega), \psi^-_{Bp\widetilde{Neut}\,N}(\Omega)]\rangle : \Omega \in \mathfrak{M}\}$$

where $\varphi^+_{Bp\widetilde{Neut}\,N}(\Omega): \mathfrak{M} \longrightarrow [0,1]$, $\varphi^-_{Bp\widetilde{Neut}\,N}(\Omega): \mathfrak{M} \longrightarrow [-1,0]$ mean the degree of confidence, $\eta^+_{Bp\widetilde{Neut}\,N}(\Omega): \mathfrak{M} \longrightarrow [0,1]$, , $\eta^-_{Bp\widetilde{Neut}\,N}(\Omega): \mathfrak{M} \longrightarrow [-1,0]$ is the degree of hesitation and $\psi^+_{Bp\widetilde{Neut}\,N}(\Omega): \mathfrak{M} \longrightarrow [0,1]$, $\psi^-_{Bp\widetilde{Neut}\,N}(\Omega): \mathfrak{M} \longrightarrow [-1,0]$ perform the degree of falseness of the decision.

3. Single Typed Linear Neutrosophic Bipolar Pentagonal number:

Various iterations of a single-typed linear Neutrosophic Bipolar number are defined in this section. In order to assist the researchers, we created the block diagram shown in Figure 1.

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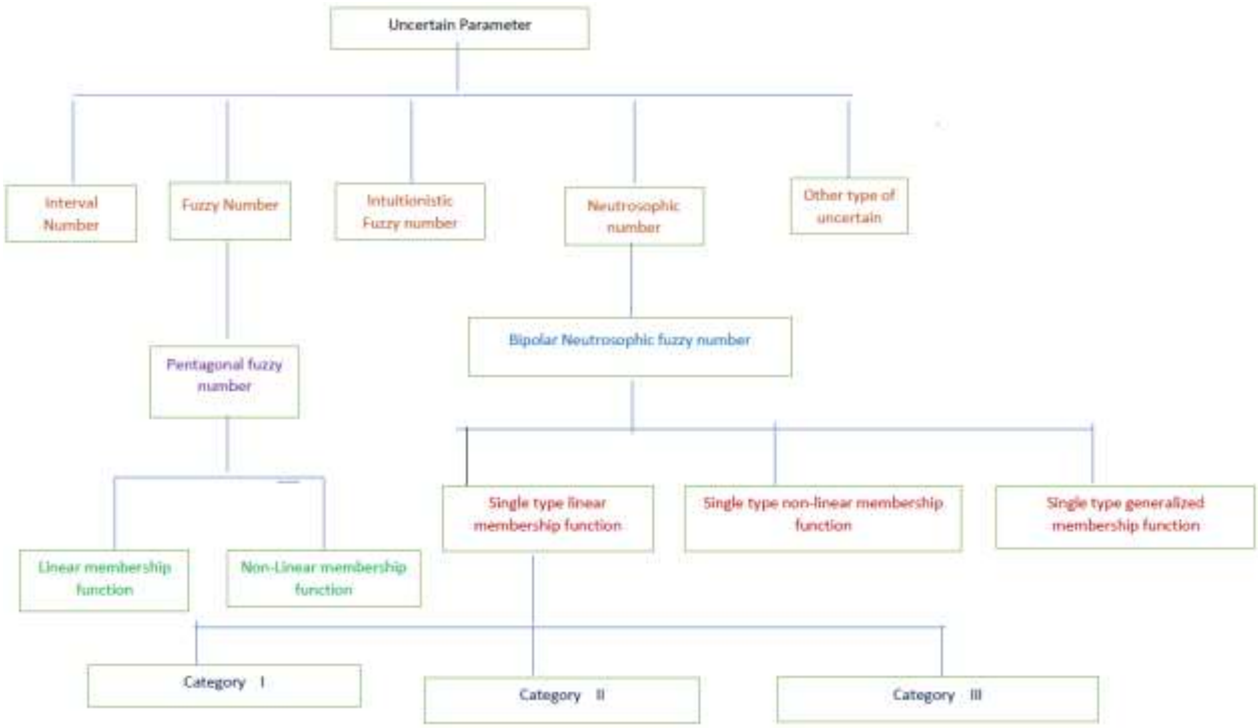

Figure 1: A block diagram shows many configurations of a single typed linear neutrosophic bipolar pentagonal number (nbpnum).

3.1 Single typed Neutrosophic Bipolar Pentagonal number of category-I

The Authenticity Hesitation and Untrue portions are separate from one another.

An illustration of a category -I neutrosophic single typed bipolar Pentgonal number is given as

$\widetilde{\mathfrak{W}}_{\text{nbpnum}} = (\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5;\ \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5;\ \mathfrak{z}_1, \mathfrak{z}_2, \mathfrak{z}_3, \mathfrak{z}_4, \mathfrak{z}_5;\ \chi\ )$, Whose memberships in terms of sincerity, hesitation, and untruth are as follows: $\mathscr{p}, \mathscr{q}, \mathscr{r}$

$$T^{+}_{\widetilde{\mathfrak{W}}_{\text{nbpnum}}}(\Omega) = \begin{cases} \chi\dfrac{\Omega - \varpi_1}{\varpi_2 - \varpi_1} & if\ \varpi_1 \le \Omega \le \varpi_2 \\ 1-(1-\chi)\dfrac{\Omega - \varpi_2}{\varpi_3 - \varpi_2} & if\ \varpi_2 \le \Omega \le \varpi_3 \\ 1 & if\ \ \Omega = \varpi_3 \\ 1-(1-\chi)\dfrac{\varpi_4 - \Omega}{\varpi_4 - \varpi_3} & if\ \varpi_3 \le \Omega \le \varpi_4 \\ \chi\dfrac{\varpi_5 - \Omega}{\varpi_5 - \varpi_4} & if\ \varpi_4 \le \Omega \le \varpi_5 \\ 0 & other\ wise \end{cases}$$

$$T^{-}_{\widetilde{\mathfrak{W}}_{\text{nbpnum}}}(\Omega) = \begin{cases} \chi\dfrac{\varpi_2 - \Omega}{\varpi_2 - \varpi_1} & if\ \varpi_1 \le \Omega \le \varpi_2 \\ 1-(1-\chi)\dfrac{\varpi_2 - \Omega}{\varpi_3 - \varpi_2} & if\ \varpi_2 \le \Omega \le \varpi_3 \\ -1 & if\ \ \Omega = \varpi_3 \\ 1-(1-\chi)\dfrac{\Omega - \varpi_4}{\varpi_4 - \varpi_3} & if\ \varpi_3 \le \Omega \le \varpi_4 \\ \chi\dfrac{\Omega - \varpi_5}{\varpi_5 - \varpi_4} & if\ \varpi_4 \le \Omega \le \varpi_5 \\ 0 & other\ wise \end{cases}$$

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and

$$I^{+}_{\widetilde{\mathfrak{W}}_{\text{nbpnum}}}(\Omega) = \begin{cases} \chi \dfrac{\varepsilon_2 - \Omega}{\varepsilon_2 - \varepsilon_1} & if\ \varepsilon_1 \le \Omega \le \varepsilon_2 \\ 1-(1-\chi)\dfrac{\varepsilon_3 - \Omega}{\varepsilon_3 - \varepsilon_2} & if\ \varepsilon_2 \le \Omega \le \varepsilon_3 \\ 0 & if\ \ \Omega = \varepsilon_3 \\ 1-(1-\chi)\dfrac{\Omega - \varepsilon_3}{\varepsilon_4 - \varepsilon_3} & if\ \varepsilon_3 \le \Omega \le \varepsilon_4 \\ \chi \dfrac{\Omega - \varepsilon_4}{\varepsilon_5 - \varepsilon_4} & if\ \varepsilon_4 \le \Omega \le \varepsilon_5 \\ 1 & other\ wise \end{cases}$$

$$I^{-}_{\widetilde{\mathfrak{W}}_{\text{nbpnum}}}(\Omega) = \begin{cases} \chi \dfrac{\Omega - \varepsilon_2}{\varepsilon_2 - \varepsilon_1} & if\ \varepsilon_1 \le \Omega \le \varepsilon_2 \\ 1-(1-\chi)\dfrac{\Omega - \varepsilon_3}{\varepsilon_3 - \varepsilon_2} & if\ \varepsilon_2 \le \Omega \le \varepsilon_3 \\ 1 & if\ \ \Omega = \varepsilon_3 \\ 1-(1-\chi)\dfrac{\varepsilon_3 - \Omega}{\varepsilon_4 - \varepsilon_3} & if\ \varepsilon_3 \le \Omega \le \varepsilon_4 \\ \chi \dfrac{\varepsilon_4 - \Omega}{\varepsilon_5 - \varepsilon_4} & if\ \varepsilon_4 \le \Omega \le \varepsilon_5 \\ -1 & other\ wise \end{cases}$$

and $F^{+}_{\widetilde{\mathfrak{W}}_{\text{nbpnum}}}(\Omega) = \begin{cases} \chi \frac{\epsilon_2 - \Omega}{\epsilon_2 - \epsilon_1} & if\ \epsilon_1 \le \Omega \le \epsilon_2 \\ 1-(1-\chi)\frac{\epsilon_3 - \Omega}{\epsilon_3 - \epsilon_2} & if\ \epsilon_2 \le \Omega \le \epsilon_3 \\ 0 & if\ \Omega = \epsilon_3 \\ 1-(1-\chi)\frac{\Omega - \epsilon_3}{\epsilon_4 - \epsilon_3} & if\ \epsilon_3 \le \Omega \le \epsilon_4 \\ \chi \frac{\Omega - \epsilon_4}{\epsilon_5 - \epsilon_4} & if\ \epsilon_4 \le \Omega \le \epsilon_5 \\ 1 & other\ wise \end{cases}$

$$F^{-}_{\widetilde{\mathfrak{W}}_{\text{nbpnum}}}(\Omega) = \begin{cases} \chi \dfrac{\Omega - \epsilon_2}{\epsilon_2 - \epsilon_1} & if\ \epsilon_1 \le \Omega \le \epsilon_2 \\ 1-(1-\chi)\dfrac{\Omega - \epsilon_3}{\epsilon_3 - \epsilon_2} & if\ \epsilon_2 \le \Omega \le \epsilon_3 \\ 0 & if\ \ \Omega = \mathfrak{z}_3 \\ 1-(1-\chi)\dfrac{\epsilon_3 - \Omega}{\epsilon_4 - \epsilon_3} & if\ \epsilon_3 \le \Omega \le \epsilon_4 \\ \chi \dfrac{\epsilon_4 - \Omega}{\epsilon_5 - \epsilon_4} & if\ \epsilon_4 \le \Omega \le \epsilon_5 \\ -1 & other\ wise \end{cases}$$

Where $-3 \le T_{\widetilde{\mathfrak{W}}_{\text{nbpnum}}}(\Omega) + I_{\widetilde{\mathfrak{W}}_{\text{nbpnum}}}(\Omega) + F_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega) \le 3,\ \Omega \in \widetilde{\mathfrak{W}}_{nbpnum}$

The aforementioned type number's parametric form is

$$\left(\widetilde{\mathfrak{W}}_{nbpnum}\right)_{(\phi,\varphi,\rho)} = \begin{bmatrix} T_{nbpnum1L}(\phi), & T_{nbpnum2L}(\phi), & T_{nbpnum1R}(\phi), & T_{nbpnum2R}(\phi) \\ I_{nbpnum1L}(\varphi), & I_{nbpnum2L}(\varphi), & I_{nbpnum1R}(\varphi), & I_{nbpnum2R}(\varphi) \\ F_{nbpnum1L}(\rho), & F_{nbpnum2L}(\rho), & F_{nbpnum1R}(\rho), & F_{nbpnum2R}(\rho) \end{bmatrix}$$

Since

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$T^{+}_{nbpnum1L}(\phi)=\varpi_1+\frac{\phi}{\chi}(\varpi_2-\varpi_1)\ for\ \phi\in[0,\chi\,], T^{+}_{nbpnum2L}(\phi)=\varpi_2-\frac{1-\phi}{1-\chi}(\varpi_3-\varpi_2)\ for\ \phi\in[\chi\,,1],$
$T^{+}_{nbpnum2R}(\phi)=\varpi_4-\frac{1-\phi}{1-\chi}(\varpi_4-\varpi_3)\ for\ \phi\in[\chi\,,1], T^{+}_{nbpnum1R}(\phi)=\varpi_5-\frac{\phi}{\chi}(\varpi_5-\varpi_4)\ for\ \phi\in[0,\chi\,],$

$T^{-}_{nbpnum1L}(\phi)=\varpi_1-\frac{\phi}{\chi}(\varpi_2-\varpi_1)\ for\ \phi\in[0,\chi\,], T^{-}_{nbpnum2L}(\phi)=\varpi_2-\frac{1-\phi}{1-\chi}(\varpi_3-\varpi_2)\ for\ \phi\in[\chi\,,1],$
$T^{-}_{nbpnum2R}(\phi)=\varpi_4+\frac{1-\phi}{1-\chi}(\varpi_4-\varpi_3)\ for\ \phi\in[\chi\,,1], T^{-}_{nbpnum1R}(\phi)=\varpi_5+\frac{\phi}{\chi}(\varpi_5-\varpi_4)\ for\ \phi\in[0,\chi\,],$

$I^{+}_{nbpnum1L}(\varphi)=\varepsilon_2-\frac{\varphi}{\chi}(\varepsilon_2-\varepsilon_1)\ for\ \varphi\in[\chi,1\,], I^{+}_{nbpnum2L}(\varphi)=\varepsilon_3-\frac{1-\varphi}{1-\chi}(\varepsilon_3-\varepsilon_2)\ for\ \varphi\in[0,\chi\,],$
$I^{+}_{nbpnum2R}(\varphi)=\varepsilon_3+\frac{1-\varphi}{1-\chi}(\varepsilon_4-\varepsilon_3)\ for\varphi\in[0,\chi\,], I^{+}_{nbpnum1R}(\varphi)=\varepsilon_4+\frac{\varphi}{\chi}(\varepsilon_5-\varepsilon_4)\ for\ \ \varphi\in$
$[\chi,1\,], I^{-}_{nbpnum1L}(\varphi)=y_2+\frac{\varphi}{\chi}(\varepsilon_2-\varepsilon_1)\ for\ \varphi\in[\chi,1\,], I^{-}_{nbpnum2L}(\varphi)=\varepsilon_3+\frac{1-\varphi}{1-\chi}(\varepsilon_3-\psi_2)\ for\ \varphi\in$
$[0,\chi\,],\qquad I^{+}_{nbpnum2R}(\varphi)=\varepsilon_3-\frac{1-\varphi}{1-\chi}(\varepsilon_4-\varepsilon_3)\ for\varphi\in[0,\chi\,], I^{+}_{nbpnum1R}(\varphi)=\varepsilon_4-\frac{\varphi}{\chi}(\varepsilon_5-\varepsilon_4)\ for\ \ \varphi\in$
$[\chi,1\,],$

$F^{+}_{nbpnum1L}(\rho)=\epsilon_2-\frac{\rho}{\chi}(\epsilon_2-\epsilon_1)\ for\ \rho\in[\chi,1\,], F^{+}_{nbpnum2L}(\rho)=\epsilon_3-\frac{1-\rho}{1-\chi}(\epsilon_3-\epsilon_2)\ for\ \rho\in[0,\chi\,],$

$F^{+}_{nbpnum2R}(\rho)=\epsilon_3+\frac{1-\rho}{1-\chi}(\epsilon_4-\epsilon_3)\ for\rho\in[0,\chi\,], F^{+}_{nbpnum1R}(\rho)=\epsilon_4+\frac{\rho}{\chi}(\epsilon_5-\epsilon_4)\ for\ \ \rho\in[\chi,1\,],$

$F^{-}_{nbpnum1L}(\rho)=\epsilon_2+\frac{\rho}{\chi}(\epsilon_2-\epsilon_1)\ for\ \rho\in[\chi,1\,], F^{-}_{nbpnum2L}(\rho)=\epsilon_3+\frac{1-\rho}{1-\chi}(\epsilon_3-\epsilon_2)\ for\ \rho\in[0,\chi\,],$

$F^{+}_{nbpnum2R}(\rho)=\epsilon_3-\frac{1-\rho}{1-\chi}(\epsilon_4-\epsilon_3)\ for\rho\in[0,\chi\,], F^{+}_{nbpnum1R}(\rho)=\epsilon_4-\frac{\rho}{\chi}(\epsilon_5-\epsilon_4)\ for\ \ \rho\in[\chi,1\,],$

Hence $-1\le\phi\le1,-1\le\varphi\le1,-1\le\rho\le1\ \ \&-3\le\phi+\varphi+\rho\le3.$

3.2 Pentagonal single Typed Neutrosophic number of category-2:

The section on hesitation and untruth are separate from one another.

The following is a description of a category-2 pentagonal single-type Neutrosophic number: $\widetilde{\mathfrak{W}}_{nbpnum}=(\varpi_1,\varpi_2,\varpi_3,\varpi_4,\varpi_5;\ \ \varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4,\varepsilon_5;\lambda_{nbpnum};\vartheta_{nbpnum}\ \ )$, whose true, hesitation and false membership functions are specified as follows.

$$T^{+}_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega)=\begin{cases}\chi\dfrac{\Omega-\varpi_1}{\varpi_2-\varpi_1} & if\ \varpi_1\le\Omega\le\varpi_2\\ 1-(1-\chi)\dfrac{\Omega-\varpi_2}{\varpi_3-\varpi_2} & if\ \varpi_2\le\Omega\le\varpi_3\\ 1 & if\ \ \Omega=\varpi_3\\ 1-(1-\chi)\dfrac{\varpi_4-\Omega}{\varpi_4-\varpi_3} & if\ \varpi_3\le\Omega\le\varpi_4\\ \chi\dfrac{\varpi_5-\Omega}{\varpi_5-\varpi_4} & if\ \varpi_4\le\Omega\le\varpi_5\\ 0 & otherwise\end{cases}$$

$$T^{-}_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega)=\begin{cases}\chi\dfrac{\varpi_1-\Omega}{\varpi_2-\varpi_1} & if\ \varpi_1\le\Omega\le\varpi_2\\ 1-(1-\chi)\dfrac{\varpi_2-\Omega}{\varpi_3-\varpi_2} & if\ \mathfrak{x}_2\le\Omega\le\varpi_3\\ -1 & if\ \ \Omega=\varpi_3\\ 1-(1-\chi)\dfrac{\varpi_4-\Omega}{\varpi_4-\varpi_3} & if\ \varpi_3\le\Omega\le\varpi_4\\ \chi\dfrac{\varpi_5-\Omega}{\varpi_5-\varpi_4} & if\ \varpi_4\le\Omega\le\varpi_5\\ 0 & other\ wise\end{cases}$$

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and

$$I^{+}_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega)=\begin{cases}\dfrac{\varepsilon_2-\Omega+\lambda_{nbpnum}(\Omega-\varepsilon_1)}{\varepsilon_2-\varepsilon_1} & if\ \varepsilon_1\le\Omega\le\varepsilon_2\\ \dfrac{\varepsilon_3-\Omega+\lambda_{nbpnum}(\Omega-\varepsilon_2)}{\varepsilon_3-\varepsilon_2} & if\ \varepsilon_2\le\Omega\le\varepsilon_3\\ \lambda & if\ \ \Omega=\varepsilon_3\\ \dfrac{\Omega-\varepsilon_3+\lambda_{nbpnum}(\varepsilon_4-\Omega)}{\varepsilon_4-\varepsilon_3} & if\ \varepsilon_3\le\Omega\le\varepsilon_4\\ \dfrac{\Omega-\varepsilon_4+\lambda_{nbpnum}(\varepsilon_5-\Omega)}{\varepsilon_5-\varepsilon_4} & if\ \varepsilon_4\le\Omega\le\varepsilon_5\\ 0 & other\ wise\end{cases}$$

$$I^{-}_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega)=\begin{cases}\dfrac{\Omega-\varepsilon_1+\lambda_{nbpnum}(\varepsilon_2-\Omega)}{\varepsilon_2-\varepsilon_1} & if\ \varepsilon_1\le\Omega\le\varepsilon_2\\ \dfrac{\Omega-\varepsilon_2+\lambda_{nbpnum}(\varepsilon_3-\Omega)}{\psi_3-\psi_2} & if\ \varepsilon_2\le\Omega\le\varepsilon_3\\ \lambda & if\ \ \Omega=\varepsilon_3\\ \dfrac{\varepsilon_3-\Omega+\lambda_{nbpnum}(\Omega-\varepsilon_3)}{\varepsilon_4-\varepsilon_3} & if\ \varepsilon_3\le\Omega\le\varepsilon_4\\ \dfrac{\varepsilon_5-\Omega+\lambda_{nbpnum}(\Omega-\varepsilon_4)}{\varepsilon_5-\varepsilon_4} & if\ \varepsilon_4\le\Omega\le\varepsilon_5\\ -1 & other\ wise\end{cases}$$

and $F^{+}_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega)=\begin{cases}\frac{\varepsilon_2-\Omega+\vartheta_{nbpnum}(\Omega-\varepsilon_1)}{\varepsilon_2-\varepsilon_1} & if\ \varepsilon_1\le\Omega\le\varepsilon_2\\ \frac{\varepsilon_3-\Omega+\vartheta_{nbpnum}(\Omega-\varepsilon_2)}{\varepsilon_3-\varepsilon_2} & if\ \varepsilon_2\le\Omega\le\varepsilon_3\\ \vartheta & if\ \ \Omega=\varepsilon_3\\ \frac{\Omega-\varepsilon_3+\vartheta_{nbpnum}(\varepsilon_4-\Omega)}{\varepsilon_4-\varepsilon_3} & if\ {}^{\varepsilon}_{\varepsilon_3}\le\Omega\le\varepsilon_4\\ \frac{\Omega-\varepsilon_4+\vartheta_{nbpnum}(\varepsilon_5-\Omega)}{\varepsilon_5-\varepsilon_4} & if\ \varepsilon_4\le\Omega\le\varepsilon_5\\ 1 & other\ wise\end{cases}$

$$F^{-}_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega)=\begin{cases}\dfrac{\Omega-\varepsilon_1+\vartheta_{nbpnum}(\varepsilon_2-\Omega)}{\varepsilon_2-\varepsilon_1} & if\ \varepsilon_1\le\Omega\le\varepsilon_2\\ \dfrac{\Omega-\varepsilon_2+\vartheta_{nbpnum}(\varepsilon_3-\Omega)}{\varepsilon_3-\varepsilon_2} & if\ \varepsilon_2\le\Omega\le\varepsilon_3\\ \vartheta & if\ \ \Omega=\varepsilon_3\\ \dfrac{\varepsilon_3-\Omega+\vartheta_{nbpnum}(\Omega-\varepsilon_3)}{y_4-y_3} & if\ \varepsilon_3\le\Omega\le\varepsilon_4\\ \dfrac{\varepsilon_5-\Omega+\vartheta_{nbpnum}(\Omega-\varepsilon_4)}{\varepsilon_5-\varepsilon_4} & if\ \varepsilon_4\le\Omega\le\varepsilon_5\\ -1 & other\ wise\end{cases}$$

Where -2$\le T_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega)+I_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega)+F_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega)\le 2,\Omega\in\widetilde{\mathfrak{W}}_{nbpnum}$

The following is the parametric representation of a category-2 number:

$$\left(\widetilde{\mathfrak{W}}_{nbpnum}\right)_{(\phi,\varphi,\rho)}=\begin{bmatrix}T_{nbpnum1L}(\phi), & T_{nbpnum2L}(\phi), & T_{nbpnum1R}(\phi), & T_{nbpnum2R}(\phi)\\ I_{nbpnum1L}(\varphi), & I_{nbpnum2L}(\varphi), & I_{phnnm1R}(\varphi), & I_{nbpnum2R}(\varphi)\\ F_{nbpnum1L}(\rho), & F_{nbpnum2L}(\rho), & F_{nbpnum1R}(\rho), & F_{nbpnum2R}(\rho)\end{bmatrix}$$

Where

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$T^{+}_{nbpnum1L}(\phi) = \varpi_1 + \frac{\phi}{\chi}(\varpi_2 - \varpi_1)\ for\ \phi \in [0,\chi\,], T^{+}_{nbpnum2L}(\phi) = \varpi_2 + \frac{1-\phi}{1-\chi}(\varpi_3 - \varpi_2)\ for\ \phi \in [\chi\,,1],$
$T^{+}_{nbpnum2R}(\phi) = \varpi_4 - \frac{1-\phi}{1-\chi}(\varpi_4 - \varpi_3)\ for\ \phi \in [\chi\,,1], T^{+}_{nbpnum1R}(\phi) = \varpi_5 - \frac{\phi}{\chi}(\varpi_5 - \varpi_4)\ for\ \phi \in [0,\chi\,],$

$T^{-}_{nbpnum1L}(\phi) = \varpi_1 - \frac{\phi}{\chi}(\varpi_2 - \varpi_1)\ for\ \phi \in [0,\chi\,], T^{-}_{nbpnum2L}(\phi) = \varpi_2 - \frac{1-\phi}{1-\chi}(\varpi_3 - \varpi_2)\ for\ \phi \in [\chi\,,1],$
$T^{-}_{nbpnum2R}(\phi) = \varpi_4 + \frac{1-\phi}{1-\chi}(\varpi_4 - \varpi_3)\ for\ \phi \in [\chi\,,1], T^{-}_{nbpnum1R}(\phi) = \varpi_5 + \frac{\phi}{\chi}(\varpi_5 - \varpi_4)\ for\ \phi \in [0,\chi\,],$

$I^{+}_{nbpnum1L}(\varphi) = \frac{\varepsilon_2 - \lambda_{nbpnum}(\varepsilon_1) - \varphi(\varepsilon_2 - \varepsilon_1)}{1-\lambda_{nbpnum}}\ for\ \varphi \in [\chi, 1\,],\ I^{+}_{nbpnum2L}(\varphi) = \frac{\varepsilon_3 - \lambda_{nbpnum}(\varepsilon_2) - \varphi(\varepsilon_3 - \varepsilon_2)}{1-\lambda_{nbpnum}}\ for\ \varphi \in [0,\chi\,],$
$I^{+}_{nbpnum2R}(\varphi) = \frac{\varepsilon_3 - \lambda_{nbpnum}(\psi_4) + \varphi(\varepsilon_4 - \varepsilon_3)}{1-\lambda_{nbpnum}}\ for \varphi \in [0,\chi\,], I^{+}_{nbpnum1R}(\varphi) = \frac{\psi_4 - \lambda_{nbpnum}(\varepsilon_5) + \varphi(\varepsilon_5 - \varepsilon_4)}{1-\lambda_{nbpnum}}\ for\ \ \varphi \in [\chi, 1\,],$

$I^{-}_{nbpnum1L}(\varphi) = \frac{\varepsilon_2 - \lambda_{nbpnum}(\varepsilon_1) + \varphi(\varepsilon_2 - \varepsilon_1)}{1-\lambda_{nbpnum}}\ for\ \varphi \in [\chi, 1\,], I^{-}_{nbpnum2L}(\varphi) = \frac{\varepsilon_3 - \lambda_{nbpnum}(\varepsilon_2) + \varphi(\varepsilon_3 - \varepsilon_2)}{1-\lambda_{nbpnum}}\ for\ \varphi \in [0,\chi\,],$
$I^{+}_{nbpnum2R}(\varphi) = \frac{\varepsilon_3 - \lambda_{nbpnum}(\varepsilon_4) - \varphi(\varepsilon_4 - \varepsilon_3)}{1-\lambda_{nbpnum}}\ for \varphi \in [0,\chi\,], I^{+}_{nbpnum1R}(\varphi) = \frac{\varepsilon_4 - \lambda_{nbpnum}(y_5) + \varphi(\varepsilon_5 - \varepsilon_4)}{1-\lambda_{nbpnum}} for\ \ \varphi \in [\chi, 1\,],$

$F^{+}_{nbpnum1L}(\rho) = \frac{\varepsilon_2 - \vartheta_{nbpnum}(\varepsilon_1) - \rho(\varepsilon_2 - \varepsilon_1)}{1-\lambda_{nbpnum}}\ for\ \rho \in [\chi, 1\,], F^{+}_{nbpnum2L}(\rho) = \frac{\varepsilon_3 - \vartheta_{nbpnum}(\varepsilon_2) - \rho(\varepsilon_3 - \varepsilon_2)}{1-\vartheta_{nbpnum}}\ for\ \rho \in [0,\chi\,],$
$F^{+}_{nbpnum2R}(\rho) = \frac{\varepsilon_3 - \vartheta_{nbpnum}(\varepsilon_4) + \rho(\varepsilon_4 - \varepsilon_3)}{1-\vartheta_{nbpnum}}\ for \rho \in [0,\chi\,], F^{+}_{nbpnum1R}(\rho) = \frac{\varepsilon_4 - \vartheta_{nbpnum}(\varepsilon_5) + \rho(\varepsilon_5 - \varepsilon_4)}{1-\vartheta_{nbpnum}}\ for\ \ \rho \in [\chi, 1\,],$

$F^{-}_{nbpnum1L}(\rho) = \frac{\varepsilon_2 - \vartheta_{nbpnum}(\varepsilon_1) + \rho(\varepsilon_2 - \varepsilon_1)}{1-\vartheta_{nbpnum}}\ for\ \rho \in [\chi, 1\,], F^{-}_{nbpnum2L}(\rho) = \frac{\varepsilon_3 - \vartheta_{nbpnum}(\varepsilon_2) + \varphi(\varepsilon_3 - \varepsilon_2)}{1-\vartheta_{nbpnum}}\ for\ \rho \in [0,\chi\,],$
$F^{+}_{nbpnum2R}(\rho) = \frac{\varepsilon_3 - \vartheta_{nbpnum}(\psi_4) - \rho(\varepsilon_4 - \varepsilon_3)}{1-\vartheta_{nbpnum}} for \rho \in [0,\chi\,], F^{+}_{nbpnum1R}(\rho) = \frac{\varepsilon_4 - \vartheta_{nbpnum}(\varepsilon_5) - \rho(\varepsilon_5 - \varepsilon_4)}{1-\vartheta_{nbpnum}}\ for\ \ \rho \in [\chi, 1\,],$

Hence $-1 \le \phi \le 1, \lambda_{nbpnum} \le \varphi \le 1, \vartheta_{nbpnum} \le \rho \le 1\ \ \& -1 \le \varphi + \rho \le 1\ and -1 \le \phi + \varphi + \rho \le 2$.

4 Arithmetic operations and Generalisation on Neutrosophic Bipolar Pentagonal number:

This part presents the fundamental operations of arithmetic and a generalization of the neutrosophic bipolar pentagonal number.

4.1 Arithmetic Operations:

Assume that, there are two neutrosophic bipolar pentagonal numbers:

$\tilde{A}_{nbpnum} = (\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5, \alpha_a, \beta_a, \gamma_a)$ and $\tilde{B}_{nbpnum} = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \alpha_b, \beta_b, \gamma_b)$ then

(i) $\tilde{A}_{nbpnum} + \tilde{B}_{nbpnum} = [\varpi_1 + \varepsilon_1, \varpi_2 + \varepsilon_2, \varpi_3 + \varepsilon_3, \varpi_4 + \varepsilon_4, \varpi_5 +\ \ \varepsilon_5; \max\{\alpha_{a,}\alpha_b\}, \min\{\beta_a, \beta_b\}, \min\{\gamma_a, \gamma_b\}]$

(ii) $\tilde{A}_{nbpnum} - \tilde{B}_{nbpnum} = [\varpi_1 - \varepsilon_1, \varpi_2 - \varepsilon_2, \varpi_3 - \varepsilon_3, \varpi_4 - \varepsilon_4, \varpi_5 -\ \ \varepsilon_5; \max\{\alpha_{a,}\alpha_b\}, \min\{\beta_a, \beta_b\}, \min\{\gamma_a, \gamma_b\}]$

(iii) $s\tilde{A}_{nbpnum} = (s\varpi_1, s\varpi_2, s\varpi_3, s\varpi_4, s\varpi_5, \alpha_a, \beta_a, \gamma_a) if\ s > 0,$

$s\tilde{A}_{nbpnum} = (s\varpi_5, s\varpi_4, s\varpi_3, s\varpi_2, s\varpi_1, \alpha_a, \beta_a, \gamma_a) if\ s < 0$

(iv)$\tilde{A}^{-1}_{nbpnum} = [\frac{1}{\varpi_5}, \frac{1}{\varpi_4}, \frac{1}{\varpi_3}, \frac{1}{\varpi_2}, \frac{1}{\varpi_1}, \alpha_a, \beta_a, \gamma_a]$

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4.2 Single-typed Generalized Neutrosophic Bipolar Pentagonal Number:

The definition of a single typed generalised nbpnum is $\widetilde{\mathfrak{W}}_{nbpnum} = (\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5;$ $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5; \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5;\ \alpha, \beta, \gamma)$and the positive, hesitation and negative membership are :

$$T^{+}_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega) = \begin{cases} \alpha\left(\chi\dfrac{\Omega-\varpi_1}{\varpi_2-\varpi_1}\right) & if\ \varpi_1 \le \Omega < \varpi_2 \\ \alpha\left(1-(1-\chi)\dfrac{\Omega-\varpi_2}{\varpi_3-\varpi_2}\right) & if\ \varpi_2 \le \Omega < \varpi_3 \\ \alpha & if\ \ \Omega = \varpi_3 \\ \alpha\left(1-(1-\chi)\dfrac{\varpi_4-\Omega}{\varpi_4-\varpi_3}\right) & if\ \varpi_3 \le \Omega < \varpi_4 \\ \alpha(\chi\dfrac{\varpi_5-\Omega}{\varpi_5-\varpi_4}) & if\ \varpi_4 \le \Omega \le \varpi_5 \\ 0 & other\ wise \end{cases}$$

$$T^{-}_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega) = \begin{cases} \alpha\left(\chi\dfrac{\varpi_2-\Omega}{\varpi_2-\varpi_1}\right) & if\ \varpi_1 \le \Omega < \varpi_2 \\ \alpha\left(1-(1-\chi)\dfrac{\varpi_2-\Omega}{\varpi_3-\varpi_2}\right) & if\ \varpi_2 \le \Omega < \varpi_3 \\ -\alpha & if\ \ \Omega = \varpi_3 \\ \alpha\left(1-(1-\chi)\dfrac{\Omega-\varpi_4}{\varpi_4-\varpi_3}\right) & if\ \varpi_3 \le \Omega < \varpi_4 \\ \alpha(\chi\dfrac{\Omega-\varpi_5}{\varpi_5-\varpi_4}) & if\ \varpi_4 \le \Omega \le \varpi_5 \\ 0 & other\ wise \end{cases}$$

and

$$I^{+}_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega) = \begin{cases} \beta\left(\chi\dfrac{\varepsilon_2-\Omega}{\varepsilon_2-\varepsilon_1}\right) & if\ \varepsilon_1 \le \Omega < \varepsilon_2 \\ \beta\left(1-(1-\chi)\dfrac{\varepsilon_3-\Omega}{\varepsilon_3-\varepsilon_2}\right) & if\ \varepsilon_2 \le \Omega < \varepsilon_3 \\ 0 & if\ \ \Omega = \varepsilon_3 \\ \beta\left(1-(1-\chi)\dfrac{\Omega-\varepsilon_3}{\varepsilon_4-\varepsilon_3}\right) & if\ \varepsilon_3 \le \Omega < \varepsilon_4 \\ \beta(\chi\dfrac{\Omega-\varepsilon_4}{\varepsilon_5-\varepsilon_4}) & if\ \varepsilon_4 \le \Omega \le \varepsilon_5 \\ \beta & other\ wise \end{cases}$$

$$I^{-}_{\widetilde{\mathfrak{W}}_{nbpnum}}(\Omega) = \begin{cases} \beta\left(\chi\dfrac{\Omega-\varepsilon_2}{\varepsilon_2-\varepsilon_1}\right) & if\ \varepsilon_1 \le \Omega < \varepsilon_2 \\ \beta\left(1-(1-\chi)\dfrac{\Omega-\varepsilon_3}{\varepsilon_3-\varepsilon_2}\right) & if\ \varepsilon_2 \le \Omega < \varepsilon_3 \\ 0 & if\ \ \Omega = \varepsilon_3 \\ \beta\left(1-(1-\chi)\dfrac{\varepsilon_3-\Omega}{\varepsilon_4-\varepsilon_3}\right) & if\ \varepsilon_3 \le \Omega < \varepsilon_4 \\ \beta(\chi\dfrac{\varepsilon_4-\Omega}{\varepsilon_5-\varepsilon_4}) & if\ \varepsilon_4 \le \Omega \le \varepsilon_5 \\ -\beta & other\ wise \end{cases}$$

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and $$F^{+}_{\mathfrak{W}_{nbpnum}}(\Omega) = \begin{cases} \gamma\left(\chi\frac{\epsilon_2-\Omega}{\epsilon_2-\epsilon_1}\right) & if\ \epsilon_1 \leq \Omega < \epsilon_2 \\ \gamma\left(1-(1-\chi)\frac{\epsilon_3-\Omega}{\epsilon_3-\epsilon_2}\right) & if\ \epsilon_2 \leq \Omega < \epsilon_3 \\ 0 & if\ \ \Omega = \epsilon_3 \\ \gamma\left(1-(1-\chi)\frac{\Omega-\epsilon_3}{\epsilon_4-\epsilon_3}\right) & if\ \epsilon_3 \leq \Omega < \epsilon_4 \\ \gamma(\chi\frac{\Omega-\epsilon_4}{\epsilon_5-\epsilon_4}) & if\ \epsilon_4 \leq \Omega \leq \epsilon_5 \\ \gamma & other\ wise \end{cases}$$

$$F^{-}_{\mathfrak{W}_{nbpnum}}(\Omega) = \begin{cases} \gamma\left(\chi\frac{\Omega-\epsilon_2}{\epsilon_2-\epsilon_1}\right) & if\ \epsilon_1 \leq \Omega < \epsilon_2 \\ \gamma\left(1-(1-\chi)\frac{\Omega-\epsilon_3}{\epsilon_3-\epsilon_2}\right) & if\ \epsilon_2 \leq \Omega < \epsilon_3 \\ 0 & if\ \ \Omega = \epsilon_3 \\ \gamma\left(1-(1-\chi)\frac{\epsilon_3-\Omega}{\epsilon_4-\epsilon_3}\right) & if\ \epsilon_3 \leq \Omega < \epsilon_4 \\ \gamma(\chi\frac{\epsilon_4-\Omega}{\epsilon_5-\epsilon_4}) & if\ \epsilon_4 \leq \Omega \leq \epsilon_5 \\ -\gamma & other\ wise \end{cases}$$

Where $T^{+}_{\mathfrak{W}_{nbpnum}}(\Omega): \mathfrak{M} \in [0,1]$, $T^{-}_{\mathfrak{W}_{nbpnum}}(\Omega): \mathfrak{M} \in [-1,0]$, $I^{+}_{\mathfrak{W}_{nbpnum}}(\Omega): \mathfrak{M} \in [0,1]$, $I^{-}_{\mathfrak{W}_{nbpnum}}(\Omega): \mathfrak{M} \in [-1,0]$, $F^{+}_{\mathfrak{W}_{nbpnum}}(\Omega): \mathfrak{M} \in [0,1]$, $F^{-}_{\mathfrak{W}_{nbpnum}}(\Omega): \mathfrak{M} \in [-1,0]$.

5. De-Formulation of a linear symmetric neutrosophic bipolar pentagonal number:

The de-formulation procedure is important for a problem for two main reasons:

*People who are unfamiliar with the fuzzy notion can relate to the outcome or solution.

* It is determined what the fuzzy solution's crispified value comprises.

The conversion process that will be appropriate and rational to convert a nbpnum into a crisp number in our nbpnum environment is of great interest to researchers. In terms of membership functions, there are three separate categories for neutrosophic bipolar pentagonal numbers. Our final proposal for converting a nbpnum into a crisp number is the "removal area technique."

*Centre of area (COA)

*BADD (Basic Defuzzification Distributions)

*CDD (Constraint Decision Defuzzification)

*EQM (Extended Quality Method)

*Bisector of area (BOA)

*COG (Centre of Gravity)

*FCD (Fuzzy Clustering Defuzzification), etc.

The appropriate way of transforming the nbpnum to a crisp number was ambiguous in this nbpnum investigation. In the neutrosophic bipolar pentagonal number, there are three unique membership functions.

The "removal area method" is suggested in this article as a way to convert neutrosophic numbers to crisp numbers.

5.1 De-Formulation using removal Area method:

For any pentagonal single typed neutrosophic number $\tilde{\tau}_{nbpnum} = (a,b,c,d,e;f,g,h,i,j;k,l,m,n,o)$

Mean is described as

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$$\mathfrak{G}_{nbpnum}(\tilde{\mathfrak{Q}},\mathfrak{d}) = \frac{\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{Q}},\mathfrak{d})+\mathfrak{G}_{nbpnumr}(\tilde{\mathfrak{Q}},\mathfrak{d})}{2}, \quad \mathfrak{G}_{nbpnum}(\tilde{\mathfrak{F}},\mathfrak{d}) = \frac{\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{F}},\mathfrak{d})+\mathfrak{G}_{nbpnumr}(\tilde{\mathfrak{F}},\mathfrak{d})}{2},$$

$$\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{A}},\mathfrak{d}) = \frac{\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{A}},\mathfrak{d})+\mathfrak{G}_{nbpnumr}(\tilde{\mathfrak{A}},\mathfrak{d})}{2}$$

Then, the specified de-bipolarization of a linear nbpnum is as follows:

$$\mathfrak{G}_{nbpnum}(\widetilde{\mathbb{U}_{nbpnum}},\mathfrak{d}) = \frac{\mathfrak{G}_{nbpnum}(\tilde{\mathfrak{Q}},\mathfrak{d})+\mathfrak{G}_{nbpnum}(\tilde{\mathfrak{F}},\mathfrak{d})+\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{A}},\mathfrak{d})}{3}, \text{ for } \mathfrak{d} = 0,$$

$$\mathfrak{G}_{nbpnum}(\tilde{\mathfrak{Q}},0) = \frac{\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{Q}},0)+\mathfrak{G}_{nbpnumr}(\tilde{\mathfrak{Q}},0)}{2}, \quad \mathfrak{G}_{nbpnum}(\tilde{\mathfrak{F}},0) = \frac{\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{F}},0)+\mathfrak{G}_{nbpnumr}(\tilde{\mathfrak{F}},0)}{2},$$

$$\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{A}},0) = \frac{\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{A}},0)+\mathfrak{G}_{nbpnumr}(\tilde{\mathfrak{A}},0)}{2}$$

Then, $\mathfrak{G}_{nbpnum}(\widetilde{\mathbb{U}_{nbpnum}},0) = \frac{\mathfrak{G}_{nbpnum}(\tilde{\mathfrak{Q}},0)+\mathfrak{G}_{nbpnum}(\tilde{\mathfrak{F}},0)+\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{A}},0)}{3}$

Let us denote $\tilde{\mathfrak{x}} = (a,b,c,d,e)$, $\tilde{\mathfrak{y}} = (f,g,h,i,j)$ and $\tilde{\mathfrak{z}} = (k,l,m,n,o)$

$$\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{Q}},0) = \frac{(a+b)\mathfrak{p}}{2} + \frac{(b+c)(1-\mathfrak{p})}{2}$$

$$\mathfrak{G}_{nbpnumr}(\tilde{\mathfrak{Q}},0) = \frac{(d+e)\mathfrak{p}}{2} + \frac{(c+d)(1-\mathfrak{p})}{2}$$

$$\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{F}},0) = \frac{(g+h)\mathfrak{p}}{2} + \frac{(f+g)(1-\mathfrak{p})}{2}$$

$$\mathfrak{G}_{nbpnumr}(\tilde{\mathfrak{F}},0) = \frac{(h+i)\mathfrak{p}}{2} + \frac{(i+j)(1-\mathfrak{p})}{2}$$

$$\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{A}},0) = \frac{(l+m)\mathfrak{p}}{2} + \frac{(k+l)(1-\mathfrak{p})}{2}$$

$$\mathfrak{G}_{nbpnumr}(\tilde{\mathfrak{A}},0) = 0o\frac{(m+n)\mathfrak{p}}{2} + \frac{(n+o)(1-\mathfrak{p})}{2}$$

Hence,

$$\mathfrak{G}_{nbpnum}(\tilde{\mathfrak{Q}},0) = \frac{\frac{(a+b)\mathfrak{p}}{2}+\frac{(b+c)(1-\mathfrak{p})}{2}+\frac{(d+e)\mathfrak{p}}{2}+\frac{(c+d)(1-\mathfrak{p})}{2}}{2},$$

$$\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{F}},0) = \frac{\frac{(g+h)\mathfrak{p}}{2}+\frac{(f+g)(1-\mathfrak{p})}{2}+\frac{(h+i)\mathfrak{p}}{2}+\frac{(i+j)(1-\mathfrak{p})}{2}}{2},$$

$$\mathfrak{G}_{nbpnuml}(\tilde{\mathfrak{A}},0) = \frac{\frac{(l+m)\mathfrak{p}}{2}+\frac{(k+l)(1-\mathfrak{p})}{2}+\frac{(m+n)\mathfrak{p}}{2}+\frac{(n+o)(1-\mathfrak{p})}{2}}{2}$$

So, $$\mathfrak{G}_{nbpnum}(\widetilde{\mathbb{U}_{nbpnum}},0) = \frac{(a+b+d+e+g+2h+i+l+2m+n)\mathfrak{p}+(b+2c+d+f+g+i+j+k+l+n+o)(1-\mathfrak{p})}{12}$$

6. Multi-Criteria Decision-Making in Neutrosophic Bipolar Pentagonal Number Environment

Here always obliged to choose between numerous items in life, and these challenges are known as M_C_D_M dilemmas. Consider the following situation:

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Consider someone who wants to purchase a new two-wheeler (i.e., Motorcycle) within their financial range. There were several motorcycle companies and various kinds of motorcycles available in the market, such as standard, touring, sport bike, cruiser, dual sport bikes, and so on, with variations in some of their core features, such as disc brakes with dual-channel ABS (Anti-lock Braking System), performance tires, liquid and oil cooling, more modern chassis, and so on.

ABS, which provides high safety in the situation of breaking at higher speeds, reducing the likelihood of skidding by preventing the wheels from locking up during braking, is one of the features offered by motorcycle manufacturers. Second, performance tires, as tires are the most important component of a bike's structural integrity, as the rubber compound of the tire, its profile, tread, and construction are all important factors. The next feature that many manufacturers offer is liquid cooling, which is a more efficient cooling method that includes two types: liquid and oil cooling. Most present-day motorcycle manufacturers then developed a more modern chassis and suspension combination allowing a fast-moving motorcycle to steer swiftly and stay composed during every journey. Forks were the main emphasis of manufacturers, as they improve the motorcycle's dynamic capabilities and stability. A motorcycle built to be aerodynamically efficient must be lightweight so that it can cut through the wind with the least amount of resistance, increasing the engine's efficiency and life. Most current motorcycles were cleverly designed to deflect the wind toward the engine, keeping the hot engine cool. Fuel Injection is a development in carburetor technology in motorcycles that is dynamically regulated by the computer and used to dynamically determine the amount of fuel that needs to be allowed inside the engine at any given time.

Most manufacturers determine what features to keep and what to remove based on the bike's price range for all of the features stated above. And different manufacturers set varying pricing for the same features, making it difficult for consumers to choose the best motorcycles available at any given time for the given price. Their mind is torn between buying a product and not buying it, so they hesitate. As a result, the problem is classified as a bipolar neutrosophic fuzzy environment problem. opinion about a product and then form their own view based on their preference for a specific feature. As a result, this turns into a M_C_D_M dilemma, and the individual must choose the best option.

6.1 M_C_D_M Scenario:

Let $R = \{R_1, R_2, \dots R_m\}$ be the distinct alternative set and $S = \{S_1, S_2, \dots S_n\}$ is the distinct characteristics set, respectively. Let $He = \{He_1, He_2, \dots He_n\}$ be the High set associated with the characteristics S, where each $He \geq 0$ and satisfies the relation $\sum_{i=1}^{n} He_i = 1$. Here consider the set of decision makers $M = \{M_1, M_2, \dots M_k\}$ associated with alternatives whose high vector is defined as $\phi = \{\phi_1, \phi_2, \dots \phi_k\}$ where each $\phi_i \geq 0$ and also satisfies the relation $\sum_{i=1}^{n} \phi_i = 1$ this high vector will be chosen based on the decision maker's judgment, knowledge, and mental power, among other factors.

6.2 Hight mean and normalization Algorithm of M_C_D_M problem:

Step 1 The creation of decision matrices entails the following steps:

To begin, they develop decision matrices for each decision maker's alternative versus characteristics functions option.

All $a_{ij}'s$ are members of the bipolar neutrosophic set because they are members of the matrices in the Bipolar neutrosophic environment. The following is the definition of the associated matrix:

$$z^p = \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \\ \vdots \\ R_m \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & \dots & S_n \end{array} \\ \begin{bmatrix} a_{11}^p & a_{12}^p & a_{13}^p & \dots & a_{1n}^p \\ a_{21}^p & a_{22}^p & a_{23}^p & \cdots & a_{2n}^p \\ a_{31}^p & a_{32}^p & a_{33}^p & \cdots & a_{3n}^p \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1}^p & a_{m2}^p & a_{m3}^p & \vdots & a_{mn}^p \end{bmatrix} \end{array}$$

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Step 2 Weighted single-decision matrix construction:

For each and every choice matrix z^i we use the operation $a'_{ij} = \{\sum_{i=1}^{k} H_{e_i}\, z^i\}$ to create a single group decision matrix, and the new matrix is as follows:

$$
z^p = \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \\ \vdots \\ R_m \end{array}
\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 \ldots & S_n \end{array} \\
\begin{bmatrix} a'^p_{11} & a'^p_{12} & a'^p_{13} \ldots & a'^p_{1n} \\ a'^p_{21} & a'^p_{22} & a'^p_{23} \cdots & a'^p_{2n} \\ a'^p_{31} & a'^p_{32} & a'^p_{33} \cdots & a'^p_{3n} \\ \vdots & \vdots & \vdots \cdots & \vdots \\ a'^p_{m1} & a'^p_{m2} & a'^p_{m3} \vdots & a'^p_{mn} \end{bmatrix} \end{array}
$$

Step:3 Using a weight vector, create a weighted priority matrix:

This process creates a decision matrix with a single column.

Consequently, $a''_{ij} = \{\sum_{i=1}^{n} \phi_i a'_{ki}, k = 1,2, \ldots m\}$ the decision matrix is as follows for each individual column. S_1

$$
z^p = \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ \vdots \\ R_m \end{array}
\begin{bmatrix} a''^p_{11} \\ a''^p_{21} \\ a''^p_{31} \\ \vdots \\ a''^p_{m1} \end{bmatrix}
$$

Step:4 Ranking

In order to assess the best alternative based on the greatest features, we will now look at the de-bipolarization value and convert the matrix (3) to crisp form. In this case, ascending score values are analyzed, and the best-fitting outcome is chosen. The best outcome is produced by the highest value and the worst by the lowest.

Flow Chart: The flowchart is given in Figure 2:

Flow Chart: The flowchart is given in Figure 2:

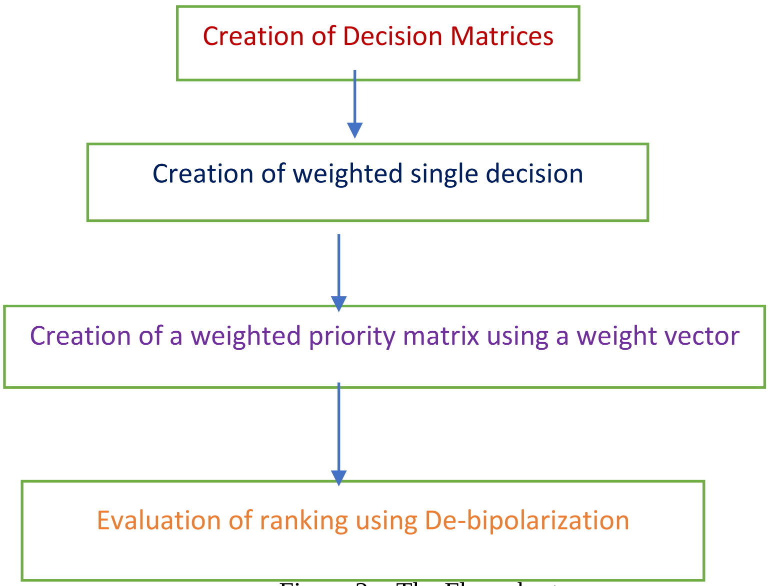

Figure 2: The Flow chart

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6.3 Example:

Consider an issue with four distinct things and their distinguishing qualities. We are aware that there are numerous things on the market, each with its own set of components and characteristics.

There are various decision-makers involved in this multi-criteria problem. The problem is described as follows:

I_1=Item 1, I_2=Item 2, I_3=Item 3, I_4=Item 4, are the alternatives; S_1=Quality, S_2=Price, S_3=Longibility,

S_4=Service are the characteristics.

Consider that there are four different categories of decision-makers: DM_1=Senior people, DM_2=Middle People, DM_3= Young People, DM_4 = New generation people having function DM= {0.40,0.20,0.35,0.37} and We also take into account the characteristics function's weight vector ϕ={ 0.25,0.35,0.10,0.30}. The designer creates a verbal matrix to help the decision maker develop their own decision matrix. Table 1 lists the verbal phrases for all of the different traits.

Table 2: List of Verbal phrases

Cases	Characteristics	Verbal Phrase
		Quantitative Characteristics
1	Quality of the product	Too High(TH), Good(G), Very Good(VG), Super(S), Nice(N), Very Nice(VN)
2	Price of the item	Too High(TH), Moderate(M), High(H), Too Low(TL), Low(L) .
3	Legibility of the item	Too High(TH), Moderate(M), High(H), Too Low(TL), Low(L) .
4	Service of the item	Too High(TH), Moderate(M), High(H), Too Low(TL), Low(L) .

Step: We analyze the matrices in terms of each decision maker's preferences for alternatives and characteristic functions. The components of the matrices are all spherical neutrosophic. As a result, these are the decision matrices:

$$DM_2 = \begin{array}{c} \begin{array}{cccc} Q_1 & Q_2 & Q_3 & Q_4 \end{array} \\ \begin{pmatrix} I_1 = \langle 1.3.5,0.5,0.6,2.1; 3.9,5,3.5,2.1; 1.5,6,3.1,9,7.5 \rangle & \langle 1.3,5,1,2,4.1; 5,0.5,1.7,2.6,3; 4.3,5,0.7,1,2.5 \rangle & \langle 2,2,3,4,5; 0.5,1.2,0.2,0.6,1.5; 2.5,1.5,1.7,3,4 \rangle & \langle 2,4,0.5,0.6,1; 3,8.5,3.5,2.5; 1.3,6,3.9,5.5 \rangle \\ I_2 = \langle 2.4,2.3,1.1,2.1; 3.4,5.5,6,7; 8,9.5,1.2,2.5,3.1 \rangle & \langle 3.4,2.5,7.1,2; 1.5,1.6,2.3,8.4; 5.9,8.2,7.2 \rangle & \langle\langle 0.5,3.5,2.4,6; 6.5,1.7,2.5,0.5,5,4,7,7.5,5,5,3.9 \rangle\rangle & \langle 3.5,4,3.1,1.2,1; 3.4,5,5,6.5,7; 8,9.5,1.2,5,3 \rangle \\ I_3 = \langle 1.4,4.1,6.5,3,2.5; 1.2,5.4,6.1,7; 9.3,5.2,2.5,1 \rangle & \langle 3.1,5,6.8,2; 1.4,3.5,4.2,6.5,2.5; 1,6.1,9,8.5,4 \rangle & \langle 3.5,5,6,6.8; 1.5,2.5,3.5,4.5,5; 1,3.5,5.6,9.1,2.5 \rangle & \langle 1.5,4.5,6.5,3.5,2.5; 1,1.5,4.6,7; 9.3,2.3,5,1 \rangle \\ I_4 = \langle 5.2,4.1,3.2,1; 0.5,1.1,2.3,5,5.5; 4.2,7.9,1.6,4 \rangle & \langle 2.1,4.2,3.5,1.5,2; 3.1,7.5,3.1; 5.5,1.5,2.5,3.2 \rangle & \langle 2.5,6,7.5,3,1; 3.5,9.5,1.5,7; 9.5,8,7.5,6,5.5 \rangle & \langle 3.5,4,3,0.5,1; 0.5,1.5,2.5,3.1.5; 4.5,4.9,6.3 \rangle \end{pmatrix} \end{array}$$

$$DM_3 = \begin{array}{c} \begin{array}{cccc} Q_1 & Q_2 & Q_3 & Q_4 \end{array} \\ \begin{pmatrix} I_1 = \langle 0.5,1.2.5,4.5; 0.6,1.5,3,0.6,1.5; 1.5,2.5,1.7,3,2 \rangle & \langle 4,1,0.5,0.6,3; 3,8.5,4.5,3.5,2.5; 1.2,6,2,8,7.5 \rangle & \langle 1.5,2,0.6,0.1,2; 4.3,1.5,5.6; 1.5,0.5,2.4,5,2 \rangle & \langle 0.5,2.5,1,3,4; 5,0.5,1.5,2.6,2; 3,3.5,0.7,1.1,5 \rangle \\ I_2 = \langle 2.3,5,1.5,4,6; 5.5,1.7,2.5,0.5,5,4,6.5,5,4.5,3.9 \rangle & \langle 4.2,3.1,1.2,1; 2,4.5,0,6.5,6; 1.9,5,4.2,5,3 \rangle & \langle 3,5,5,6.5,7,1; 2.1,2.3,4.1,6,5,1; 1.1,4,3,2.5,2 \rangle & \langle 2,4,5,5,7,1; 1.5,1.6,2,3,4,4; 5,6,7,5,7,1,1 \rangle \\ I_3 = \langle 3.5,4,7,6; 1.5,2.5,3.5,4.5,5; 1.3,5,5.6,1.1,2.4 \rangle & \langle 4,1.5,0.5, 3.5,2.5; 1.1,2,0.4,6,7; 2,7,3,2.5,1 \rangle & \langle 2,3,4,7,8; 6.4,8,3,2; 5.5,1.5,3.5,1,0.5 \rangle & \langle 2,5,6,8,3; 2.5,1.5,3.5,6.5,1.5; 0.7,6.5,2,1 \rangle \\ I_4 = \langle 2.5,6,7.5,3,1; 3.5,9.5,1.5,7; 9.5,8,7.5,6,5.5 \rangle & \langle 1,2,3,4,1.5; 1.5,2,0.6,3.5,5.5; 3.5,2.9,5,4 \rangle & \langle 1.5,3,4,6; 0.5,0.1,1.4,7,2.5; 1.2,0.8,2.5,4,2.1 \rangle & \langle 3.1,2,2,3.5,1.5,2; 4,2,8,3,0.5; 4.5,1.4,2.5,3,2 \rangle \end{pmatrix} \end{array}$$

$$DM_4 = \begin{array}{c} \begin{array}{cccc} Q_1 & Q_2 & Q_3 & Q_4 \end{array} \\ \begin{pmatrix} I_1 = \langle 1.4,0.5,0.6,2; 3,9,5,5,5,3.5,2.5; 1.5,6,3,9,7.5 \rangle & \langle 4,5,5,6,7.5,1; 2,3,2,4,1,5,6,1; 6,1,5,2,2.5,3 \rangle & \langle 1.5,4.5,6.5,3.5,2.5; 1,2,4,6,7; 9,3,2,5,1 \rangle & \langle 2.9,4,3.1,1.2,1; 3,4.5,5,6.5,7; 8,9.5,1,2.5,3 \rangle \\ I_2 = \langle 0.5,3,0.7,0.1,2; 4.5,1.5,7,6; 1.7,2.3,4.5,6 \rangle & \langle 2.5,6,7.5,3,1; 3.5,9,5,1.5,7; 9.5,8,7.5,6,5.5 \rangle & \langle 1,3.5,2.4,6; 6.5,1.7,2.5,0.5,5,4,7,7.5,5,5,3,9 \rangle & \langle 6,1,3,4,5; 0.5,0.8,1.5,7,2.5; 1.7,2.8,3.5,4,5.1 \rangle \\ I_3 = \langle 3,4.5,5,7,2; 1.5,1.8,2.3,8,4; 5,9,8.5,7,2.1 \rangle & \langle 1.5,3.5,1.2,4; 5,0.5,1.7,2.6,3; 4,3.5,0.7,1.2,5 \rangle & \langle 2,1,3,4,5; 0.6,1.2,0.3,0.7,1.5; 2.5,1.5,1.7,3,4 \rangle & \langle 5,4,3,2,1; 0.5,1.5,2.5,3.5,5.5; 4.5,7,9,6,4 \rangle \\ I_4 = \langle 6.1,4.2,3.5,1.5,2; 4.7,8,3,1; 5.5,1.5,2.5,3.2 \rangle & \langle 1,4,3,7,9; 6,5,4,3,2; 5.5,2.5,4.5,1,1.5 \rangle & \langle 3,5,6,8,2; 1.5,3.5,4.5,6.5,2.5; 1.5,6,9,8,4 \rangle & \langle 4,5,6,7,8; 1.5,2.5,3.5,4.5,5; 1,3.5,5.6,9,1.2.5 \rangle \end{pmatrix} \end{array}$$

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$$
\begin{array}{c}
\qquad\qquad Q_1 \qquad\qquad\qquad\qquad Q_2 \qquad\qquad\qquad\qquad Q_3 \qquad\qquad\qquad\qquad Q_4 \\
DN_1 = \begin{pmatrix}
I_1 = \langle 2,1,3,4,5; 0.6,1.2,0.3,0.7,1.5; 2.5,1.5,1.7,3,4\rangle & \langle 1,4,0.5,0.6,2; 3,0.5,5.5,3.5,2.5; 1.5,6,3,9,7.5\rangle & \langle 0.5,3,0.7,0.1,2; 4.5,1.5,7,6; 1.7,2,3,4.5,6\rangle & \langle 1.5,3.5,1,2,4; 5,0.5,1.7,2.6,3; 4.3.5,0.7,1,2.5\rangle \\
I_2 = \langle 1,3.5,2,4,6; 6.5,1.7,2.5,0.5,5,4.7,7.5,5.5,3.9\rangle & \langle 2.9,4,3.1,1.2,1; 3,4.5,5,6.5,7; 8,9.5,1,2.5,3\rangle & \langle 4,5.5,6,7.5,1; 2,3.2,4.1,5,6.1; 6,1.5,2,2.5,3\rangle & \langle 3,4.5,5,7,2; 1.5,1.8,2,3.8,4; 5,9,8.5,7,2.1\rangle \\
I_3 = \langle 4,5,6,7,8; 1.5,2.5,3.5,4.5,5; 1,3.5,5.6,9,1.2,5\rangle & \langle 1.5,4.5,6.5,3.5,2.5; 1,2,4,6,7; 0.3,2.5,1\rangle & \langle 1,4,3,7,9; 6.5,4,3,2; 5.5,2.5,4.5,1,1.5\rangle & \langle 3,5,6,8,2; 1.5,3.5,4.5,6.5,2.5; 1.5,6,9,8,4\rangle \\
I_4 = \langle 2.5,6,7.5,3,1; 3.5,9.5,1.5,7; 9.5,8,7.5,6.5,5\rangle & \langle 5,4,3,2,1; 0.5,1.5,2.5,3.5,5.5; 4.5,7,9,6,4\rangle & \langle 6,1,3,4,5; 0.5,0.8,1.5,7,2.5; 1.7,2,8.3,5,4.5,1\rangle & \langle 6,1,4,2,3.5,1.5,2; 4,7,8,3,1; 5.5,1.5,2.5,3,2\rangle
\end{pmatrix}
\end{array}
$$

Step:2 Weighted single-decision matrix construction:

$$
DN_1 = \begin{pmatrix}
I_1 = \langle [illegible] \rangle & \langle [illegible] \rangle & \langle [illegible] \rangle & \langle [illegible] \rangle \\
I_2 = \langle [illegible] \rangle & \langle [illegible] \rangle & \langle [illegible] \rangle & \langle [illegible] \rangle \\
I_3 = \langle [illegible] \rangle & \langle [illegible] \rangle & \langle [illegible] \rangle & \langle [illegible] \rangle \\
I_4 = \langle [illegible] \rangle & \langle [illegible] \rangle & \langle [illegible] \rangle & \langle [illegible] \rangle
\end{pmatrix}
$$

Step-3 Using a weight vector, create a weighted priority matrix:

$$
DM=\begin{pmatrix}
\langle 2.45,4.14,2.52,3.05,3.71; 3.92,5.90,4.51,4.58,4.51; 4.27,5.68,2.47,6.15,6.57\rangle \\
\langle 3.61,5,4.89,4.84,3.27; 3.97,4.52,4.32,5.63,7.05; 5.96,9.15,6.62,6.08,4.61\rangle \\
\langle 3.59,5.42,5.82,6.87,4.75; 2.55,2.86,4.11,6.42,5.85; 5.01,6.41,6.52,6.38,3.26\rangle \\
\langle 4.54,5.38,5.53,4.39,3.97; 3.56,5.75,5.79,4.41,4.69; 6.53,5.12,7.65,6.26,4.25\rangle
\end{pmatrix}
$$

Step:4 Ranking

We now transform the neutrosophic bipolar pentagonal numbers using the De-bipolarization value given in section 5.1. As a result, we get the following final optimal decision matrix.

$$
DM=\begin{pmatrix} 4.31 \\ 5.44 \\ 5.28 \\ 5.34 \end{pmatrix}
$$

After you've arranged the numbers in ascending order, we obtain 4.31<5.28<5.34<5.44.

as a result, the ranking of the best alternatives is $I_2 > I_4 > I_3 > I_1$.

The coding for the above decision-making problem to select the optimal product is given below:

Implementation in Octave:

```
x = csvread ("DataSet.csv")
a=x(1:4,:)
b=x(5:8,:)
c=x(9:12,:)
d=x(13:16,:)

row=1
col=1

data= input("Enter Data value 1: ");
data1= input("Enter Data value 2: ");
data2= input("Enter Data value 3: ");
data3= input("Enter Data value 4: ");

a=[a(:,1:15)*data a(:,16:30)*data1 a(:,31:45)*data2 a(:,46:60)*data3]
b=[b(:,1:15)*data b(:,16:30)*data1 b(:,31:45)*data2 b(:,46:60)*data3]
c=[c(:,1:15)*data c(:,16:30)*data1 c(:,31:45)*data2 c(:,46:60)*data3]
d=[d(:,1:15)*data d(:,16:30)*data1 d(:,31:45)*data2 d(:,46:60)*data3]

res=a+b+c+d
```

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```
dataset= input("Enter Data 1: ");
dataset1= input("Enter Data 2: ");
dataset2= input("Enter Data 3: ");
dataset3= input("Enter Data 4: ");

a=[res(1:1,1:15)*dataset+res(1:1,16:30)*dataset2+res(1:1,31:45)*dataset3+res(1:1,46:60)*dataset4]
b=[res(2:2,1:15)*dataset+res(2:2,16:30)*dataset2+res(2:2,31:45)*dataset3+res(2:2,46:60)*dataset4]
c=[res(3:3,1:15)*dataset+res(3:3,16:30)*dataset2+res(3:3,31:45)*dataset3+res(3:3,46:60)*dataset4]
d=[res(4:4,1:15)*dataset+res(4:4,16:30)*dataset2+res(4:4,31:45)*dataset3+res(4:4,46:60)*dataset4]

res=[a' b' c' d']'
```

7 Concluding Remarks

In [9] to [15] the authors interpreted several types of new approaches, for instance, triangular, trapezoidal, bipolar pentagonal neutrosophic, and cylindrical neutrosophic numbers in multi-criteria decision-making problems. The present strategy stated above can be used to tackle any type of M_C_D_M problem. In this study, we looked at the theory of a bipolar neutrosophic number from a range of viewpoints. When the membership functions of a nbpnum were associated with one another, and then applied the idea of linear and nonlinear form with the truth, hesitant and false membership functions. When there is a need to identify the optimum solution to a variety of decision-making situations with a finite number of choices and qualities and multiple decision-makers, the concept of de-formulation proves to be very helpful. Many products with interesting features are available in the current situation. Choosing a suitable product is very difficult among the best brands of two-wheelers with great features. Different types of approaches have been made in the decision-making process by the various researchers to select the product for the comfortable ride. This work describes selecting the appropriate product based on the optimistic features in an interesting manner and gives finer results than the other works.

Future Scope

In the future, the theory of neutrosophic bipolar pentagonal fuzzy set can be applied in the following sections:

- Multi-attribute decision-making models
- Extension of algebraic sub-structures
- Extension of Topological systems

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