On the Conditions of Imperfect Neutrosophic Duplets and Imperfect Neutrosophic Triplets

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Abstract:

In any neutrosophic ring R(I), an imperfect neutrosophic duplet consists of two elements x, y with a condition $xy = yx = x$ and an imperfect neutrosophic triplet consists of three elements x, y, z with condition $xy = yx = x, yz = zy = z,\ and\ xz = zx = y$. The objective of this paper is to determine the necessary and sufficient conditions for neutrosophic duplets and triplets in any neutrosophic ring R(I), and to determine all triplets in Z(I).

Keywords: Neutrosophic duplet; neutrosophic imperfect triplet; neutrosophic ring.

1. Introduction

Neutrosophy is a new kind of generalized philosophy founded by F. Smarandache to deal with uncertainty in the real world problems. Neutrosophic sets were used widely in decision making, optimization, computer science and number theory [16].

Recently, there are many efforts in neutrosophical algebraic studies, where algebraic concepts such as neutrosophic rings [7,17], and modules [9,10,15], were defined and studied widely. We can see classification theorems for equations [22], homomorphisms [18], and structures of generalized subspaces [8,14,19].

Special elements in neutrosophic ring theory were studied on a wide range in the last few years such as idempotents [20], semi idempotents [21], and symmetric elements [23].

If R is a ring, then the corresponding neutrosophic ring R(I) is defined as $R(I) = R + RI = \{a + bI;\ a, b \in R\}$. Duplets were defined firstly in [11], and studied in neutrosophic number rings $Z(I), Q(I), R(I)$ in [13]. On the other hand, we find many different triplets in those rings in [12].

The Neutrosophic Duplets and the Neutrosophic Duplet Algebraic Structures were introduced by Florentin Smarandache [1, 2, 3], in 2016.

Let *U* be a universe of discourse, and a set *D* included in *U*, with a well-defined law #.

We say that <*a*, *neut(a)*>, where *a*, and its neutral *neut(a)* belong to D, is a neutrosophic duplet if:

1) *neut(a)* is different from the unitary element of *D* with respect to the law # (if any);

2) *a#neut(a) = neut(a)#a = a;*

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3) there is no opposite *anti(a)* belonging to *D* for which *a#anti(a) = anti(a)#a = neut(a)*. For example in *(Z_8, #)*, the set of integers with respect to the regular multiplicationmodulo 8, one has thefollowing neutrosophic duplets:

<2, 5 >, <4, 3>, <4, 5>, <4, 7>, and <6, 5>.

Neutrosophic Triplets [4,5,6,3] were introduced by F. Smarandache & M. Ali in 2014 – 2016:

Let *U* be a universe of discourse, and *(N, *)* a set included in it, endowed with a well-defined

binary law *.

A neutrosophic triplet is an object of the form *<x, neut(x), anti(x)>*, for *x*∈ *N*, where

neut(x)∈ *N* is the neutral of *x*, different from the classical algebraic unitary element if any, such that:

$$x*neut(x) = neut(x)*x = x$$

and *anti(x)*∈N is the opposite of *x* such that:

$$x*anti(x) = anti(x)*x = neut(x).$$

In general, an element *x* may have more *neut*'s and more *anti*'s.

A neutrosophic triplet (a, b, c) is called perfect neutrosophic triplet if (c, b, a) and (b, b, b) are also neutrosophic triplets.

A neutrosophic triplet (a, b, c) is called imperfect neutrosophic triplet if at least one of (c, b, a) or (b, b, b) is not a neutrosophic triplet.

An imperfect neutrosophic duplet consists of two elements x, y with condition $xy = yx = x$, an imperfect neutrosophic triplet consists of three elements x, y, z with condition $xy = yx = x, yz = zy = z, xz = zx = y$. Many properties of these elements can be found in [11,12,13].

In this paper, we try to describe the necessary and sufficient condition for imperfect duplets and imperfect triplets in any neutrosophic ring R(I), then we use our algorithm to describe all imperfect duplets and triplets in neutrosophic rings of numbers $R(I), Z(I), Q(I)$.

This work is considered as a continuation of significant efforts released by Smarandache and Kandasamy in [11,12,13]. They solved the problem of determining triplets and duplets partially in some special cases of Z(I), Q(I), R(I). We aim to extend their work into any neutrosophic ring R(I), even when it is not commutative.

The motivation of this work is to provide a general description of the structure of imperfect duplets and triplets in any neutrosophic ring.

We want to refer that the structure of duplets in $Z(I), Q(I), R(I)$ has been determined in [13], here we introduce another new proof for this known fact.

All duplets and triplets through this paper are considered imperfect (with invertibility condition).

2. Preliminaries

Definition 2.1: [7]

Let R be any ring, then $R(I) = \{R + RI = \{a + bI; a, b \in R\}$ is called the neutrosophic ring generated by R and I, where $I^2 = I$.

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Definition 2.2: [11]

Let R be any ring, x, y are two arbitrary elements in R. We call them a duplet with y acts as an identity if and only if

$xy = yx = x$.

Definition 2.3: [12]

Let R be any ring, x, y, z three arbitrary elements in R. We call them a triplet with y acts as an identity if and only if

$xy = yx = x, zy = yz = z, xz = zx = y$.

Theorem 2.4: [13]

Non trivial Triplets in Q(I), R(I) are :

$$\left(b, I, \frac{1}{b}I\right); b \neq 0, \left(a - aI, 1 - I, \frac{1}{a} - \frac{1}{a}I\right); a \neq 0.$$

Remark 2.5: [12,13]

$(a)(0, x), (1, y)$is a trivial duplet at all.

(b) If R has a unity 1, and x is invertible, then $(x, 1, x^{-1})$ is a trivial duplet.

3. Main discussion

-The following theorem describes the condition of duplets in any neutrosophic ring R(I), not only the commutative case [12,13].

Theorem 3.1:

Let $x = a + bI, y = c + dI$ be any two elements in R(I), then (x, y) is a duplet with y acts as an identity if and only if $(a, c), (a + b, c + d)$ are duplets in the classical ring R with $c, c + d$ act as identities.

Proof:

We suppose that $(a, c), (a + b, c + d)$ are duplets in the classical ring R, with $c, c + d$ act as identities , then $xy = ac + (ad + bc + bd)I = ac + I[(a + b)(c + d) - ac] = ca + I[(c + d)(a + b) - ca] = yx = a + I[(a + b) - a] = a + bI = x$. Thus (x, y) is a neutrosophic duplet.

Conversely, we assume that (x, y) is a neutrosophic duplet, then we have $xy = yx = x$. This implies that: $xy = ac + I[(a + b)(c + d) - ac] = ca + I[(c + d)(a + b) - ca] = a + bI, hence ac = ca = a, (a + b)(c + d) = (c + d)(a + b) = a + b, so\ that (a, c), (a + b, c + d)$ are duplets in the classical ring R, with$c, c + d$act as identities .

-The following three theorems are considered as new proofs for well known results in [12,13].

Theorem 3.2:

The set of all non trivial duplets in the neutrosophic ring of integers Z(I) is

$D = \{(a - aI, 1 + [d]I), (bI, c + [1 - c]I)\}$.

Proof:

Consider$x = a + bI, y = c + dI$ as a duplet with y acts as an identity, then we have, $ac = a, (a + b)(c + d) = a + b$.

If $a \neq 0, b \neq -a$ then $c = 1, c + d = 1, hence\ d = 0, which\ is\ a\ trivial\ duplet$.

If $a = 0, b \neq -a = 0$, then c is arbitrary and $c + d = 1$, thus $d = 1 - c$, thus the duplet's structure is $(bI, c + [1 - c]I)$.

If $a \neq 0, b = -a$, then $ac = a$ implies $c = 1, and\ (a + b)(c + d) = 0 = a + b$, which means that $c + d$is arbitrary, thus d is arbitrary, this implies that the structure of the duplet is $(a - aI, 1 + [d]I)$.

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If $a = 0, b = -a = 0$, we get a trivial duplet.

Theorem 3.3:

The set of all non-trivial duplets in the neutrosophic ring of rational numbers Q(I) is

$D = \{(a - aI, 1 + [d]I), (bI, c + [1 - c]I)\}$.

The proof is similar to Theorem 3.2.

Theorem 3.4:

The set of all non-trivial duplets in the neutrosophic ring of real numbers R(I) is

$D = \{(a - aI, 1 + [d]I), (bI, c + [1 - c]I)\}; b, d, a \neq 0$.

The proof is similar to Theorem 3.2.

Example 3.5:

Consider the neutrosophic ring Q(I), then it is clear that $(2 - 2I, 1 + 4I)$ is a duplet, with $1 + 4I$ acts as an identity.

The following Theorem describes the condition for triplets in any neutrosophic ring R(I), not only the commutative case as [11,12,13].

Theorem 3.6:

Let $x = a + bI, y = c + dI,\ z = m + nI$ be any three elements in R(I) with unity 1, then (x, y, z) is a triplet with y acts as an identity if and only if $(a, c, m), (a + b, c + d, m + n)$ are triplets in the classical ring R with $c, c + d$ acting as identities.

Proof:

First of all, we assume that (x, y, z) is a triplet with y acts as an identity, then $(x, y), (z, y)$ are duplets.

Under the assumption that $xz = zx = y$, we get $xz = am + I[(a + b)(m + n) - am] = ma + I[(m + n)(a + b) - ma] = c + dI, hence\ ma = am = c, (a + b)(m + n) = (m + n)(a + b) = c + d.$ Thus $(a, c, m), (a + b, c + d, m + n)$ are triplets in the classical ring R with $c, c + d$ act as identities.

Conversely, we suppose that $(a, c, m), (a + b, c + d, m + n)$ are triplets in the classical ring R with $c, c + d$ act as identities. By using Theorem 3.2, we get that $(x, y), (z, y)$ are duplets. On the other hand, we have:

$xz = am + [(a + b)(m + n) - am]I = ma + [(m + n)(a + b) - ma] = c + [c + d - c]I = c + dI = y$.Thus (x, y, z) is a triplet.

-In [12], we find a tiny mistake, where authors claimed that there are no triplets in Z(I). The following theorem finds that Z(I) has exactly four different non trivial triplets.

Theorem 3.7:

The set of all non-trivial triplets in the neutrosophic ring of integers Z(I) is$D = \{(1 - I, 1 - I, 1 - I), (-1 + I, 1 - I, -1 + I), (I, I, I), (-I, I, -I)\}$.

Proof:

Let $x = a + bI, y = c + dI,\ z = m + nI$ be a triplet in Z(I) with y acts as an identity, then we have $(a, c, m), (a + b, c + d, m + n)$ are triplets in Z. From this point of view, we are searching for triplets in the ring $Z \times Z$. Every classical triplet $[(f, k), (g, l), (h, s)]$ is equivalent to a neutrosophic triplet $a + bI, c + dI, m + nI, where\ a = f, a + b = k, c = g, c + d = l, h = m, m + n = s.$

The set of all non zero triplets (trivial and non trivial) in $Z \times Z$ is $\{[(1,0), (1,0), (1,0)], [(0,1), (0,1), (0,1)], [(-1,0), (1,0), (-1,0)], [(0, -1), (0,1), (0, -1)],$

$[(1, -1), (1,1), (1, -1)], [(-1,1), (1,1), (-1,1)], [(-1, -1), (1,1), (-1, -1)]$, thus the triplets in Z(I) are

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$$\{(1-I,1-I,1-I),(-1+I,1-I,-1+I),(I,I,I),(-I,I,-I),(1-2I,1,1-2I),(-1+2I,1,-1+2I)\}.$$

On the other hand, we find that -1+2I, 1-2I are invertible elements, hence their triplet is considered as trivial one. Thus D is the set of all non trivial triplets in Z(I).

Remark 3.8:

The cyclic group G generated by the triple$(-1+I,1-I,-1+I)$ with respect to multiplication in Z(I) is isomorphic to $(Z_2,+)$. The same thing for the cyclic group generated by $(-I,I,-I)$.

Remark 3.9:

The structure of triplets in Q(I),R(I) has been analyzed in [12,13] completely.

Conclusion

In this paper, we have determined a general description of the neutrosophic duplets and triplets in any neutrosophic ring R(I). Also, we have determined all triplets in Z(I). This result can be considered as a correction of the claim proposed in [12].

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