



Different Types of Operations on Neutrosophic Graphs

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Abstract

The fuzzy graph theory uses a substantial and important role in modelling and structuring many optimization problems. Different type of uncertainties exist in most of the optimization problems in real life scenarios due to indeterminate and incomplete information and it is a challenging task for the expert to design those optimization problems applying fuzzy graph. To design the incomplete, uncertainty and vagueness in graphical optimization problems, several extensions of graph theoretical ideas are proposed. The idea of neutrosophic graph plays an important role to manage the uncertainty, linked with the indeterminate and incomplete data/information of any optimization problem. In this manuscript, we present the idea of regular neutrosophic graph, strong neutrosophic graph, bipartite neutrosophic graph, regular neutrosophic graph, and regular strong neutrosophic graph. We also introduce six different operations on neutrosophic graph, viz., cartesian product, composition, join, direct product, lexicographic and strong product.

Keywords: Neutrosophic; uncertainty; neutrosophic graph; bipartite neutrosophic graph.

1 Introduction

Graph theory has many real-life applications to problems in computer applications, systems analysis, computer networking, transportation, operations research, and economics. A graph is essentially a relational model and is used to represent a real-world problem consisting of a relationship between objects. The vertices and edges of the graph are used to represent the objects and relationships between the object of the problem. In many optimization problems, the available information is usually imprecise or imprecise for various reasons such as information loss, lack of evidence, imperfect statistical data, and insufficient information. In general, uncertainty in real-life problems can appear in the problem information that defines the problem.

Classical graph theory uses the concept of classical set theory introduced by Cantor. In a classical graph, there are two possibilities for any vertex or edge: either it is in the graph or it is not a graph. Thus, a classical graph cannot model an uncertain optimization problem. Because real-life problems are often uncertain and it is very difficult to model uncertain real-life problems using a classical graph. A fuzzy set¹ is an extended version of a classical set, where objects have different degrees of membership. A fuzzy set gives its objects different degrees of membership between 0 and 1. Degree of membership is not the same as probability; rather, it describes membership in vaguely defined sets. Atanassov used a new component that finds the degree of

non-membership in a fuzzy set. Fuzzy set uses degree of membership while intuitionistic fuzzy set uses both degree of membership and degree of non-membership.

Smarandache² presented the concept of neutrosophic set in 1995, by altering the concept of fuzzy set and its extension. The idea of neutrosophic set can manage with indeterminate, uncertain, vague, and inconsistent data of any uncertain optimization real world problem. It is mainly modified idea of classical (crisp) set, type 1 and intuitionistic fuzzy set. It is presented by of three membership degree: true, indeterminate and false membership value. All the 3 different membership degrees are independent to each other and the values lie between the non-standard unit interval] 0, 1[. Neutrosophic graph³ can handle inconsistent information about several types of real life problem. Recently, several researchers have more actively researched on neutrosophic graph theory, for instance, Yang et al.,⁴ Ye,⁵ Broumi,⁶ Naz et al.,⁷ and Arkam.⁸⁻¹¹

2 Preliminaries

In this portion, we will discuss about single valued neutrosophic graph, adjacent node, path, isolated node, strength of a path, strong neutrosophic graph, complement neutrosophic graph and complete neutrosophic graph, which is efficient for the present work.

Definition 2.1. Let $G = (P, R)$ be a neutrosophic graph^{12,13} where P and R are represented two neutrosophic sets on V and E respectively which satisfy the following.

$$\begin{aligned} \mathcal{T}_P(w, x) &\leq \min(\mathcal{T}_R(w), \mathcal{T}_R(x)) \\ \mathcal{I}_P(w, x) &\geq \max(\mathcal{I}_R(w), \mathcal{I}_R(x)) \\ \mathcal{F}_P(w, x) &\geq \max(\mathcal{F}_R(w), \mathcal{F}_R(x)) \end{aligned}$$

Here, w and x are two vertices of G and $(w, x) \in E$ is an edge of G .

Definition 2.2. The two nodes w and x of a neutrosophic graph $G = (P, R)$ are defined as adjacent node in G If and only If $(\mathcal{T}_P(w, x), \mathcal{I}_P(w, x), \mathcal{F}_P(w, x)) > 0$. The two nodes w and x are called neighbor node and (w, x) is incident at w and x .

Definition 2.3. Let $G = (P, R)$ be a neutrosophic graph and P be a path of G . P is a collection of different nodes, $w_0, w_1, w_2, \dots, w_n$ such that $(\mathcal{T}_P(w_{i-1}, w_i), \mathcal{I}_P(w_{i-1}, w_i), \mathcal{F}_P(w_{i-1}, w_i)) > 0$ for $0 \leq i \leq n$. Here, n represents the neutrosophic length of the path P . A single neutrosophic vertex, i.e., w_i in G is also assumed as a path. The path length of a single node w_i is $(0, 0, 0)$. We define the order pair (w_{i-1}, w_i) as the edge of the path. P is said to be a neutrosophic cycle If $w_0 = w_n$ and $n \geq 3$.

Definition 2.4. A node $w_i \in V$ of a neutrosophic graph G is called the isolated node If there exists no incident arc to the vertex w_i .

Definition 2.5. Let $G = (P, R)$ be a neutrosophic graph. G having a path P of path length¹² k from node w to node x in G such as $P = \{a = w_1, (w_1, w_2), w_2, \dots, w_{k-1}(w_{k-1}, w_k), w_k = b\}$ then $T_M^k(w, x)$, $I_M^k(w, x)$ and $F_M^k(w, x)$ are described as follows.

$$\begin{aligned} T_M^k(w, x) &= \sup(\mathcal{T}_P(a, w_1) \wedge \mathcal{T}_P(w_1, w_2) \wedge \dots \wedge \mathcal{T}_R(w_{k-1}, w_k)) \\ I_M^k(w, x) &= \inf(\mathcal{I}_P(a, w_1) \vee \mathcal{I}_P(w_1, w_2) \vee \dots \vee \mathcal{I}_P(w_{k-1}, w_k)) \\ F_M^k(w, x) &= \inf(\mathcal{F}_P(a, w_1) \vee \mathcal{F}_P(w_1, w_2) \vee \dots \vee \mathcal{F}_P(w_{k-1}, w_k)) \end{aligned}$$

Definition 2.6. Let $G = (P, R)$ is a neutrosophic graph. The strength of connection of a path P between two nodes w and x is defined by $(\mathcal{T}_M^\infty(w, x), \mathcal{I}_M^\infty(w, x), \mathcal{F}_M^\infty(w, x))$ where

$$\mathcal{T}_M^\infty(w, x) = \sup\{T_M^k(w, x) | k = 1, 2, \dots\}$$

$$\mathcal{I}_M^\infty(w, x) = \inf\{I_M^k(w, x) \mid k = 1, 2, \dots\}$$

$$\mathcal{F}_M^\infty(w, x) = \inf\{F_M^k(w, x) \mid k = 1, 2, \dots\}$$

Definition 2.7. Let $G = (P, R)$ is a neutrosophic graph. G is said to be connected neutrosophic graph¹² If there exists no isolated node is in G .

Definition 2.8. Let $G = (P, R)$ be a neutrosophic graph and $a \in V$ is node of G . The degree of node w is the sum of the truth membership values, sum of the indeterminacy membership values and sum of the membership values of falsity of all the arcs which are adjacent to the node w . The degree of node^{7,11,14} w is denoted by $d(w) = (d_{\mathcal{T}}(w), d_{\mathcal{I}}(w), d_{\mathcal{F}}(w))$ where

$$d_{\mathcal{T}}(w) = \sum_{\substack{w \in V \\ w \neq x}} \mathcal{T}_P(w, x), d_{\mathcal{I}}(w) = \sum_{\substack{w \in V \\ w \neq x}} \mathcal{I}_P(w, x), d_{\mathcal{F}}(w) = \sum_{\substack{w \in V \\ w \neq x}} \mathcal{F}_P(w, x)$$

Here, $d_{\mathcal{T}}(w)$, $d_{\mathcal{I}}(w)$ and $d_{\mathcal{F}}(w)$ are the degree of truth membership value, degree of indeterminacy membership value and degree falsity membership value respectively of the vertex w .

Definition 2.9. Let $G = (P, R)$ be a neutrosophic graph. Then the order of G is denoted by $O(G) = (O_{\mathcal{T}}(G), O_{\mathcal{I}}(G), O_{\mathcal{F}}(G))$ where

$$O_{\mathcal{T}}(G) = \sum_{a \in V} \mathcal{T}_R(w), O_{\mathcal{I}}(G) = \sum_{a \in V} \mathcal{I}_R(w), O_{\mathcal{F}}(G) = \sum_{a \in V} \mathcal{F}_R(w)$$

Here, $O_{\mathcal{T}}(G)$, $O_{\mathcal{I}}(G)$ and $O_{\mathcal{F}}(G)$ are the order of the membership degree of truth value, indeterminacy value and falsity value of G respectively.

Definition 2.10. Let $G = (P, R)$ be a neutrosophic graph and the size graph of G is described as $S(G) = (S_{\mathcal{T}}(G), S_{\mathcal{I}}(G), S_{\mathcal{F}}(G))$ where

$$S_{\mathcal{T}}(G) = \sum_{\substack{w, x \in V \\ a \neq b}} \mathcal{T}_P(w, x), S_{\mathcal{I}}(G) = \sum_{\substack{w, x \in V \\ a \neq b}} \mathcal{I}_P(w, x), S_{\mathcal{F}}(G) = \sum_{\substack{w, x \in V \\ a \neq b}} \mathcal{F}_P(w, x)$$

Here, $S_{\mathcal{T}}(G)$, $S_{\mathcal{I}}(G)$ and $S_{\mathcal{F}}(G)$ are respectively the order of the membership degree of truth, indeterminacy and falsity of G .

Definition 2.11. Let $G = (P, R)$ is a neutrosophic graph. G is said to be a strong neutrosophic graph^{7,11,14} If

$$\mathcal{T}_P(w, x) = \min(\mathcal{T}_R(w), \mathcal{T}_R(x))$$

$$\mathcal{I}_P(w, x) = \max(\mathcal{I}_R(w), \mathcal{I}_R(x))$$

$$\mathcal{F}_P(w, x) = \max(\mathcal{F}_R(w), \mathcal{F}_R(x)), \forall (w, x) \in E$$

Definition 2.12. Let $G = (P, R)$ be a neutrosophic graph and G is said to be a complete neutrosophic graph^{7,11,14} If

$$\mathcal{T}_P(w, x) = \min(\mathcal{T}_R(w), \mathcal{T}_R(x))$$

$$\mathcal{I}_P(w, x) = \max(\mathcal{I}_R(w), \mathcal{I}_R(x))$$

$$\mathcal{F}_P(w, x) = \max(\mathcal{F}_R(w), \mathcal{F}_R(x)), \forall w, x \in V$$

Definition 2.13. Let $G = (P, R)$ be a neutrosophic graph. The $G^c = (P^c, R^c)$ is the complement of a neutrosophic graph^{7,11,14} If $P^c = P$ and R^c is computed as below.

$$\mathcal{T}_P^c(w, x) = \min(\mathcal{T}_R(w), \mathcal{T}_R(x)) - \mathcal{T}_P(w, x)$$

$$\mathcal{I}_P^c(w, x) = \max(\mathcal{I}_R(w), \mathcal{I}_R(x)) - \mathcal{I}_P(w, x)$$

$$\mathcal{F}_P^c(w, x) = \max(\mathcal{F}_R(w), \mathcal{F}_R(x)) - \mathcal{F}_P(w, x), \forall w, x \in V$$

Here, $\mathcal{T}_P^c(w, x)$, $\mathcal{I}_P^c(w, x)$ and $\mathcal{F}_P^c(w, x)$ are denoted the true, intermediate and false membership degree for edge (w, x) of G^c .

3 Operations on neutrosophic graph

In this section, we introduce six operations on neutrosophic graph, viz., cartesian product, composition, join, direct product, lexicographic and strong product.

Definition 3.1. Let $G_1 = (P_1, R_1)$ and $G_2 = (P_2, R_2)$ are two neutrosophic graph of $G_1^* = (\mathcal{V}_1, \mathcal{E}_1)$ and $G_2^* = (\mathcal{V}_2, \mathcal{E}_2)$ respectively. The cartesian product $G_1 \times G_2$ of neutrosophic graph G_1 and G_2 is defined by (A, B) , where $A = (\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P)$ and $B = (\mathcal{T}_R, \mathcal{I}_R, \mathcal{F}_R)$ are two neutrosophic sets on $V=(V_1 \times V_2)$, and $E = \{(x, w_2), (x, x_2) | x \in \mathcal{V}_1, w_2x_2 \in \mathcal{E}_2\} \cup \{(w_1, z), (x_1, z) | z \in \mathcal{V}_2, w_1x_1 \in \mathcal{E}_1\}$ respectively which satisfies the followings:

- (i) $\forall (w_1, w_2) \in V_1 \times V_2$,
 - (a) $\mathcal{T}_P((w_1, w_2)) = \mathcal{T}_{P_1}(w_1) \wedge \mathcal{T}_{P_2}(w_2)$
 - (b) $\mathcal{I}_P((w_1, w_2)) = \mathcal{I}_{P_1}(w_1) \wedge \mathcal{I}_{P_2}(w_2)$
 - (c) $\mathcal{F}_P((w_1, w_2)) = \mathcal{F}_{P_1}(w_1) \vee \mathcal{F}_{P_2}(w_2)$
- (ii) $\forall x \in V_1$ and $\forall (w_2, x_2) \in E_2$,
 - (a) $\mathcal{T}_R((x, w_2)(x, x_2)) = \mathcal{T}_{P_1}(w_1) \wedge \mathcal{T}_{R_2}(w_2x_2)$
 - (b) $\mathcal{I}_R((x, w_2)(x, x_2)) = \mathcal{I}_{P_1}(w_1) \vee \mathcal{I}_{R_2}(w_2x_2)$
 - (c) $\mathcal{F}_R((x, w_2)(x, x_2)) = \mathcal{F}_{P_1}(w_1) \vee \mathcal{F}_{R_2}(w_2x_2)$
- (iii) $\forall z \in V_2$ and $\forall (w_1, w_2) \in E_1$,
 - (a) $\mathcal{T}_R((w_1, z)(x_1, z)) = \mathcal{T}_{R_1}(w_1x_1) \wedge \mathcal{T}_{P_2}(z)$
 - (b) $\mathcal{I}_R((w_1, z)(x_1, z)) = \mathcal{I}_{R_1}(w_1x_1) \vee \mathcal{I}_{P_2}(z)$
 - (c) $\mathcal{F}_R((w_1, z)(x_1, z)) = \mathcal{F}_{R_1}(w_1x_1) \vee \mathcal{F}_{P_2}(z)$

Definition 3.2. The composition G_1G_2 of two neutrosophic graph $G_1 = (P_1, R_1)$ and $G_2 = (P_2, R_2)$ defined as a pair (A, B) , where $A = (\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P)$ and $B = (\mathcal{T}_R, \mathcal{I}_R, \mathcal{F}_R)$ are two neutrosophic sets on $V=(V_1 \times V_2)$, and $E = \{(x, w_2), (x, x_2) | x \in \mathcal{V}_1, w_2x_2 \in \mathcal{E}_2\} \cup \{(w_1, z), (x_1, z) | z \in \mathcal{V}_2, w_1x_1 \in \mathcal{E}_1\} \cup \{(w_1, w_2), (x_1, x_2) | w_2x_2 \in \mathcal{V}_2 \neq x_2, w_1x_1 \in \mathcal{E}_1\}$ respectively, which satisfies the followings:

- (i) $\forall (w_1, w_2) \in V_1 \times V_2$,
 - (a) $\mathcal{T}_P((w_1, w_2)) = \mathcal{T}_{P_1}(w_1) \wedge \mathcal{T}_{P_2}(w_2)$
 - (b) $\mathcal{I}_P((w_1, w_2)) = \mathcal{I}_{P_1}(w_1) \vee \mathcal{I}_{P_2}(w_2)$
 - (c) $\mathcal{F}_P((w_1, w_2)) = \mathcal{F}_{P_1}(w_1) \vee \mathcal{F}_{P_2}(w_2)$
- (ii) $\forall x \in V_1$ and $\forall (w_2, x_2) \in E_2$,
 - (a) $\mathcal{T}_R((x, w_2)(x, x_2)) = \mathcal{T}_{P_1}(w_1) \wedge \mathcal{T}_{R_2}(w_2x_2)$
 - (b) $\mathcal{I}_R((x, w_2)(x, x_2)) = \mathcal{I}_{P_1}(w_1) \vee \mathcal{I}_{R_2}(w_2x_2)$
 - (c) $\mathcal{F}_R((x, w_2)(x, x_2)) = \mathcal{F}_{P_1}(w_1) \vee \mathcal{F}_{R_2}(w_2x_2)$

(iii) $\forall z \in V_2$ and $\forall (w_1, w_2) \in E_1$,

$$(a) \mathcal{T}_R((w_1, z)(x_1, z)) = \mathcal{T}_{R_1}(w_1x_1) \wedge \mathcal{T}_{P_2}(z)$$

$$(b) \mathcal{I}_R((w_1, z)(x_1, z)) = \mathcal{I}_{R_1}(w_1x_1) \vee \mathcal{I}_{P_2}(z)$$

$$(c) \mathcal{F}_R((w_1, z)(x_1, z)) = \mathcal{F}_{R_1}(w_1x_1) \vee \mathcal{F}_{P_2}(z)$$

(iv) $\forall w_2x_2 \in V_2, w_2 \neq x_2$ and $\forall (w_1x_1) \in E_1$,

$$(a) \mathcal{T}_R((w_1, w_2)(x_1, x_2)) = \mathcal{T}_{P_2}(w_2) \wedge \mathcal{T}_{P_2}(x_2) \wedge \mathcal{T}_{R_1}(w_1x_1)$$

$$(b) \mathcal{I}_R((w_1, w_2)(x_1, x_2)) = \mathcal{I}_{P_2}(w_2) \wedge \mathcal{I}_{P_2}(x_2) \wedge \mathcal{I}_{R_1}(w_1x_1)$$

$$(c) \mathcal{F}_R((w_1, w_2)(x_1, x_2)) = \mathcal{F}_{P_2}(w_2) \wedge \mathcal{F}_{P_2}(x_2) \wedge \mathcal{F}_{R_1}(w_1x_1)$$

Definition 3.3. The union $G_1 \cup G_2$ of two neutrosophic graph $G_1 = (P_1, R_1)$ and $G_2 = (P_2, R_2)$ is defined as (A, B) , where $A = (\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P)$ is a neutrosophic set on $V = \mathcal{V}_1 \cup \mathcal{V}_2$ and $B = (\mathcal{T}_R, \mathcal{I}_R, \mathcal{F}_R)$ is an another neutrosophic set on $E = \mathcal{E}_1 \cup \mathcal{E}_2$, which satisfies the following:

$$(i) \quad (a) \mathcal{T}_P(x) = \mathcal{T}_{P_1}(x) \quad \text{If } x \in \mathcal{V}_1 \text{ and } x \notin \mathcal{V}_2$$

$$(b) \mathcal{T}_P(x) = \mathcal{T}_{P_2}(x) \quad \text{If } x \in \mathcal{V}_2 \text{ and } x \notin \mathcal{V}_1$$

$$(c) \mathcal{T}_P(x) = \mathcal{T}_{P_1}(x) \wedge \mathcal{T}_{P_2}(x) \quad \text{If } x \in \mathcal{V}_1 \cap \mathcal{V}_2$$

$$(ii) \quad (a) \mathcal{I}_P(x) = \mathcal{I}_{P_1}(x) \quad \text{If } x \in \mathcal{V}_1 \text{ and } x \notin \mathcal{V}_2$$

$$(b) \mathcal{I}_P(x) = \mathcal{I}_{P_2}(x) \quad \text{If } x \in \mathcal{V}_2 \text{ and } x \notin \mathcal{V}_1$$

$$(c) \mathcal{I}_P(x) = \mathcal{I}_{P_1}(x) \wedge \mathcal{I}_{P_2}(x) \quad \text{If } x \in \mathcal{V}_1 \cap \mathcal{V}_2$$

$$(iii) \quad (a) \mathcal{F}_P(x) = \mathcal{F}_{P_1}(x) \quad \text{If } x \in \mathcal{V}_1 \text{ and } x \notin \mathcal{V}_2$$

$$(b) \mathcal{F}_P(x) = \mathcal{F}_{P_2}(x) \quad \text{If } x \in \mathcal{V}_2 \text{ and } x \notin \mathcal{V}_1$$

$$(c) \mathcal{F}_P(x) = \mathcal{F}_{P_1}(x) \wedge \mathcal{I}_{P_2}(x) \quad \text{If } x \in \mathcal{V}_1 \cap \mathcal{V}_2$$

$$(iv) \quad (a) \mathcal{T}_R(xy) = \mathcal{T}_{R_1}(xy) \quad \text{If } xy \in \mathcal{E}_1 \text{ and } xy \notin \mathcal{E}_2$$

$$(b) \mathcal{T}_R(xy) = \mathcal{T}_{R_2}(xy) \quad \text{If } xy \in \mathcal{E}_2 \text{ and } xy \notin \mathcal{E}_1$$

$$(c) \mathcal{T}_R(xy) = \mathcal{T}_{R_1}(xy) \wedge \mathcal{T}_{R_2}(xy) \quad \text{If } xy \in \mathcal{E}_1 \cap \mathcal{E}_2$$

$$(v) \quad (a) \mathcal{I}_R(xy) = \mathcal{I}_{R_1}(xy) \quad \text{If } xy \in \mathcal{E}_1 \text{ and } xy \notin \mathcal{E}_2$$

$$(b) \mathcal{I}_R(xy) = \mathcal{I}_{R_2}(xy) \quad \text{If } xy \in \mathcal{E}_2 \text{ and } xy \notin \mathcal{E}_1$$

$$(c) \mathcal{I}_R(xy) = \mathcal{I}_{R_1}(xy) \wedge \mathcal{I}_{R_2}(xy) \quad \text{If } xy \in \mathcal{E}_1 \cap \mathcal{E}_2$$

- (vi) (a) $\mathcal{F}_R(xy) = \mathcal{F}_{R_1}(xy)$ If $xy \in \mathcal{E}_1$ and $xy \notin \mathcal{E}_2$
- (b) $\mathcal{F}_R(xy) = \mathcal{F}_{R_2}(xy)$ If $xy \in \mathcal{E}_2$ and $xy \notin \mathcal{E}_1$
- (c) $\mathcal{F}_R(xy) = \mathcal{F}_{R_1}(xy) \wedge \mathcal{F}_{R_2}(xy)$ If $xy \in \mathcal{E}_1 \cap \mathcal{E}_2$

Definition 3.4. The join $G_1 + G_2$ of two neutrosophic graph $G_1 = (P_1, R_1)$ and $G_2 = (P_2, R_2)$ is defined as (A, B) , where $A = (\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P)$ is a neutrosophic set on $V = \mathcal{V}_1 \cup \mathcal{V}_2$ and $B = (\mathcal{T}_R, \mathcal{I}_R, \mathcal{F}_R)$ is another neutrosophic set on $E = \mathcal{E}_1 \cup \mathcal{E}_2 \cup \mathcal{E}'$ (\mathcal{E}' represents all edges joining the vertex of \mathcal{V}_1 and \mathcal{V}_2), which satisfies the following:

- (i) (a) $\mathcal{T}_P(x) = \mathcal{T}_{P_1}(x)$ If $x \in \mathcal{V}_1$ and $x \notin \mathcal{V}_2$
- (b) $\mathcal{T}_P(x) = \mathcal{T}_{P_2}(x)$ If $x \in \mathcal{V}_2$ and $x \notin \mathcal{V}_1$
- (c) $\mathcal{T}_P(x) = \mathcal{T}_{P_1}(x) \wedge \mathcal{T}_{P_2}(x)$ If $x \in \mathcal{V}_1 \cap \mathcal{V}_2$
- (ii) (a) $\mathcal{I}_P(x) = \mathcal{I}_{P_1}(x)$ If $x \in \mathcal{V}_1$ and $x \notin \mathcal{V}_2$
- (b) $\mathcal{I}_P(x) = \mathcal{I}_{P_2}(x)$ If $x \in \mathcal{V}_2$ and $x \notin \mathcal{V}_1$
- (c) $\mathcal{I}_P(x) = \mathcal{I}_{P_1}(x) \wedge \mathcal{I}_{P_2}(x)$ If $x \in \mathcal{V}_1 \cap \mathcal{V}_2$
- (iii) (a) $\mathcal{F}_P(x) = \mathcal{F}_{P_1}(x)$ If $x \in \mathcal{V}_1$ and $x \notin \mathcal{V}_2$
- (b) $\mathcal{F}_P(x) = \mathcal{F}_{P_2}(x)$ If $x \in \mathcal{V}_2$ and $x \notin \mathcal{V}_1$
- (c) $\mathcal{F}_P(x) = \mathcal{F}_{P_1}(x) \wedge \mathcal{F}_{P_2}(x)$ If $x \in \mathcal{V}_1 \cap \mathcal{V}_2$
- (iv) (a) $\mathcal{T}_R(xy) = \mathcal{T}_{R_1}(xy)$ If $xy \in \mathcal{E}_1$ and $xy \notin \mathcal{E}_2$
- (b) $\mathcal{T}_R(xy) = \mathcal{T}_{R_2}(xy)$ If $xy \in \mathcal{E}_2$ and $xy \notin \mathcal{E}_1$
- (c) $\mathcal{T}_R(xy) = \mathcal{T}_{R_1}(xy) \wedge \mathcal{T}_{R_2}(xy)$ If $xy \in \mathcal{E}_1 \cap \mathcal{E}_2$
- (v) (a) $\mathcal{I}_R(xy) = \mathcal{I}_{R_1}(xy)$ If $xy \in \mathcal{E}_1$ and $xy \notin \mathcal{E}_2$
- (b) $\mathcal{I}_R(xy) = \mathcal{I}_{R_2}(xy)$ If $xy \in \mathcal{E}_2$ and $xy \notin \mathcal{E}_1$
- (c) $\mathcal{I}_R(xy) = \mathcal{I}_{R_1}(xy) \wedge \mathcal{I}_{R_2}(xy)$ If $xy \in \mathcal{E}_1 \cap \mathcal{E}_2$
- (vi) (a) $\mathcal{F}_R(xy) = \mathcal{F}_{R_1}(xy)$ If $xy \in \mathcal{E}_1$ and $xy \notin \mathcal{E}_2$
- (b) $\mathcal{F}_R(xy) = \mathcal{F}_{R_2}(xy)$ If $xy \in \mathcal{E}_2$ and $xy \notin \mathcal{E}_1$
- (c) $\mathcal{F}_R(xy) = \mathcal{F}_{R_1}(xy) \wedge \mathcal{F}_{R_2}(xy)$ If $xy \in \mathcal{E}_1 \cap \mathcal{E}_2$
- (vii) (a) $\mathcal{T}_R(xy) = \mathcal{T}_{R_1}(x) \vee \mathcal{T}_{R_2}(y)$ If $xy \in \mathcal{E}'$

$$(b) \mathcal{I}_R(xy) = \mathcal{I}_{R_1}(x) \wedge \mathcal{I}_{R_2}(y) \quad \text{If } xy \in \mathcal{E}'$$

$$(c) \mathcal{F}_R(xy) = \mathcal{F}_{R_1}(x) \wedge \mathcal{F}_{R_2}(y) \quad \text{If } xy \in \mathcal{E}'$$

Definition 3.5. The direct product $G_1 * G_2$ of two neutrosophic graph G_1 and G_2 is defined as a pair (A, B) , where $A = (\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P)$ is a neutrosophic set on $V = \mathcal{V}_1 \times \mathcal{V}_2$ and $B = (\mathcal{T}_R, \mathcal{I}_R, \mathcal{F}_R)$ is an another neutrosophic set on $E = \{(w_1, w_2)(x_1, x_2) | w_1x_1 \in \mathcal{E}_1, w_2x_2 \in \mathcal{E}_2\}$, which satisfies the followings:

$$(i) \forall (w_1, w_2) \in \mathcal{V}_1 \times$$

$$(a) \mathcal{T}_P(w_1, w_2) = \mathcal{T}_{P_1}(w_1) \vee \mathcal{T}_{P_2}(w_2)$$

$$(b) \mathcal{I}_P(w_1, w_2) = \mathcal{I}_{P_1}(w_1) \wedge \mathcal{I}_{P_2}(w_2)$$

$$(c) \mathcal{F}_P(w_1, w_2) = \mathcal{F}_{P_1}(w_1) \wedge \mathcal{F}_{P_2}(w_2)$$

$$(ii) \forall (w_1x_1) \in \mathcal{E}_1, \forall (w_2x_2) \in \mathcal{E}_2$$

$$(a) \mathcal{T}_R(w_1, w_2)(x_1, x_2) = \mathcal{T}_{R_1}(w_1x_1) \vee \mathcal{T}_{R_2}(w_2x_2)$$

$$(b) \mathcal{I}_R(w_1, w_2)(x_1, x_2) = \mathcal{I}_{R_1}(w_1x_1) \wedge \mathcal{I}_{R_2}(w_2x_2)$$

$$(c) \mathcal{F}_R(w_1, w_2)(x_1, x_2) = \mathcal{F}_{R_1}(w_1x_1) \wedge \mathcal{F}_{R_2}(w_2x_2)$$

4 Conclusion

The idea of graph/network theory has applied to several types to the problems in the area of computer network, operations research, economics, wireless networking, computer systems analysis, transportation and traffic planning. Uncertainty exists in almost every graph theory problem in real life scenarios. Neutrosophic set is a popular and useful method to manage the uncertainty. To model the complex real life problems, a number of extension of neutrosophic graph have been studied. The main objective of this manuscript is to present the idea of different type of neutrosophic graph and its different operation. We also present some different types of neutrosophic graph such as regular picture fuzzy graph, strong picture fuzzy graph, complete picture fuzzy graph, and complement picture fuzzy graph. Five different operations on neutrosophic graph, viz. cartesian product, composition, join, direct product, lexicographic and strong product are described in this paper. .

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