Interval Valued Neutrosophic Subbisemirings of Bisemirings

M. Palanikumar [1], K. Arulmozhi[2], Aiyared Iampan[3] *
[1]Department of Advanced Mathematical Science, Saveetha School of Engineering, Saveetha University, Saveetha Institute of Medical and Technical Sciences, Chennai-602105, India
[2]Department of Mathematics, Bharath Institute of Higher Education and Research, Tamil Nadu, Chennai-600073, India
[3]Fuzzy Algebras and Decision-Making Problems Research Unit, Department of Mathematics, School of Science, University of Phayao, Mae Ka, Mueang, Phayao 56000, Thailand
Emails: palanimaths86@gmail.com[1], arulmozhiems@gmail.com[2], aiyared.ia@up.ac.th[3]

Abstract

We introduce the notion of interval valued neutrosophic subbisemirings (IVNSBSs), level sets of IVNSBSs and interval valued neutrosophic normal subbisemirings (IVNNSBSs) of bisemirings. Also, we introduce an approach to (α, β)-IVNSBSs and IVNNSBSs over bisemirings. Let $\widetilde{A}$ be an interval valued neutrosophic set (IVN set) in a bisemiring $\mathcal{S}$. We have proved that $\widetilde{\varsigma} = (\widetilde{\varsigma_A^{\mathcal{T}}}, \widetilde{\varsigma_A^{\mathcal{I}}}, \widetilde{\varsigma_A^{\mathcal{F}}})$ is an IVNSBS of $\mathcal{S}$ if and only if all non-void level set $\widetilde{\varsigma^{(t,s)}}$ is a subbisemiring of $\mathcal{S}$ for $t, s \in [[0,1]]$. Let $\widetilde{A}$ be an IVNSBS of a bisemiring $\mathcal{S}$ and $\widetilde{V}$ be the strongest interval valued neutrosophic relation (SIVNR) of $\mathcal{S}$. Prove that $\widetilde{A}$ is an IVNSBS of $\mathcal{S}$ if and only if $\widetilde{V}$ is an IVNSBS of $\mathcal{S} \times \mathcal{S}$. We illustrate homomorphic image of IVNSBS is an IVNSBS. We find that homomorphic preimage of IVNSBS is an IVNSBS. Examples are provided to illustrate our results.

Keywords: IVNSBS; IVNNSBS; SIVNR; homomorphism

1 Introduction

Dedekinds engagement with the theories of commutative rings led to the opening of the study of semirings. Later, semirings were investigated by Vandever in 1934. In essence, it was the generalization of distributive lattices and rings. The idea of semirings, however, has been developing since 1950. The important work from 1965, Zadeh, was put out by the fuzzy set (FS) theory.[22] This definition of a FS states that it is a function that is described by a membership value. Degrees are measured in real unit intervals. Numerous uncertain theories, such as FS logic,[22] intuitionistic fuzzy set (IFS) logic,[5] Pythagorean fuzzy set (PFS) logic[21] and spherical fuzzy set (SFS) logic[4] are proposed in order to deal with the uncertainties. A FS is a set having degrees of belongingness between 0 and 1; these grades are referred to as an elements membership value in the particular set. Atanassov later introduced the idea of an IFS, which is categorized by the sum of its membership grade (MG) and non-membership grade (NMG) value that does not exceed 1.[5] When using a decision-making approach, we occasionally convey a single issue where the total value for MGs and NMGs exceeds 1. In order to generalize IFS, Yager[21] developed the new concept of PFS logic, which is defined by the square sum of its MGs and NMGs and whose value does not exceed 1. But these hypotheses are unable to demonstrate the neutral state (neither favor nor disfavor). Cuong and Kreinovich,[6] who created the idea of image FS logic, used three pointers: positive MG, neutral MG, and negative MG, with the sum of these three grades not exceeding 1. Finally, it has more benefits than IFS and PFS for a few applications. The neutrosophic

https://doi.org/10.54216/IJNS.190109

Received: March 08, 2022 Accepted: September 16, 2022

set (NSS), in which the truth MG, indeterminacy MG, and falsity MG are each independently represented, is a new generalization of the FS and IFS.

Smarandache[20] developed neutrosophy to address the problems of unclear and inconsistent information because this set contains numerous application related challenges. Recently, a brand new theory called neutrosophic logic and sets has been proposed. Neutosophy is the study of neutral cognition, and the major difference between FS and IFS is represented by this neutral. Smarandache is credited with introducing neutrosophic logic.[20] It is a logic in which the degrees of truth, indeterminacy, and falsity of each assertion are judged. Each component of the universe has a degree of truth, indeterminacy, and falsity, which ranges from $[0, 1]$ in NSS theory. From a philosophical perspective, a NSS generalizes a classical set, FS, interval valued FS, etc. The study of NSS and the expansion of this concept has continued, for example, Al-Tahan et al.[2] defined the notion of single valued neutrosophic sets in ordered groupoids in 2020. A year later, Jagadeeswari et al.[11] initiated Certain Kinds of Bipolar interval valued nentrosphic graphs. In 2022, Iampan et al.[9,10] introduced the concept of IVN sets to ideals/subalgebras of Hilbert algebras. A semiring $(S, +, \cdot)$ is a non-void set in which $(S, +)$ and $(S, \cdot)$ are semigroups such that " $\cdot$ " is distributive over " $+$ ". The concept of fuzzy semirings was first suggested by Ahsan et al.[1] in 1993. Sen and Ghosh[19] presented the idea of bisemirings in 2001. An algebraic structure $(\mathcal{S}, \diamondsuit_1, \diamondsuit_2, \diamondsuit_3)$ is a bisemiring,[19] if $(\mathcal{S}, \diamondsuit_1, \diamondsuit_2)$ and $(\mathcal{S}, \diamondsuit_2, \diamondsuit_3)$ are semirings, that is $(\mathcal{S}, \diamondsuit_1), (\mathcal{S}, \diamondsuit_2)$ and $(\mathcal{S}, \diamondsuit_3)$ are semigroups and (1) $z_1\diamondsuit_2(z_2\diamondsuit_1 z_3) = (z_1\diamondsuit_2 z_2)\diamondsuit_1(z_1\diamondsuit_2 z_3)$, (2) $(z_2\diamondsuit_1 z_3)\diamondsuit_2 z_1 = (z_2\diamondsuit_2 z_1)\diamondsuit_1(z_3\diamondsuit_2 z_1)$, (3) $z_1\diamondsuit_3(z_2\diamondsuit_2 z_3) = (z_1\diamondsuit_3 z_2)\diamondsuit_2(z_1\diamondsuit_3 z_3)$, and (4) $(z_2\diamondsuit_2 z_3)\diamondsuit_3 z_1 = (z_2\diamondsuit_3 z_1)\diamondsuit_2(z_3\diamondsuit_3 z_1)$ for all $z_1, z_2, z_3 \in \mathcal{S}$. A non-void subset A of a bisemiring $(\mathcal{S}, \diamondsuit_1, \diamondsuit_2, \diamondsuit_3)$ is a subbisemiring[8] in case if $z_1\diamondsuit_1 z_2 \in A, z_1\diamondsuit_2 z_2 \in A$, and $z_1\diamondsuit_3 z_2 \in A$ for all $z_1, z_2 \in A$. Golan, Palanikumar, and Arulmozhi discussed various ideals structures such semirings, semigroups, bisemirings.[3,7,12–18]

The goal of this paper is to look at various facets of the subbisemiring theory to IVNSBS idea and offer findings. The article is made up of the next five components. The introduction is in Section 1, and the semiring and SBS preparation information is in Section 2. Its attributes are presented in Section 3 of IVNSBS. The concept of (α, β)-IVNSBS homomorphism is introduced along with an analysis of its features in Section 4. In Section 5, the concept of (α, β)-IVNNSBS homomorphism is presented. Give some numerical examples while evaluating the IVNSBS and IVNNSBS.

2 Preliminaries

We will go over the ideas of semirings and bisemirings in this part to make our case as fully as possible and to make the debate that follows easier to follow.

An IVN set $\widetilde{A}$ in a universe X is an object having the form

$$\widetilde{A} = \Big\{\Big(x, \widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{I}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(x)\Big) : x \in X\Big\},$$

where $\widetilde{\varsigma_A^{\mathcal{T}}}(x) = [\varsigma_A^{\mathcal{T}L}, \varsigma_A^{\mathcal{T}U}], \widetilde{\varsigma_A^{\mathcal{I}}}(x) = [\varsigma_A^{\mathcal{I}L}, \varsigma_A^{\mathcal{I}U}], \widetilde{\varsigma_A^{\mathcal{F}}}(x) = [\varsigma_A^{\mathcal{F}L}, \varsigma_A^{\mathcal{F}U}]$, and $\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{I}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(x) : X \to [[0, 1]]$ represents the truth, indeterminacy and falsity-membership mapping respectively. For simplicity the symbol $\Big(\widetilde{\varsigma_A^{\mathcal{T}}}, \widetilde{\varsigma_A^{\mathcal{I}}}, \widetilde{\varsigma_A^{\mathcal{F}}}\Big)$ is used for the IVN set $\widetilde{A} = \Big\{\Big(x, \widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{I}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(x)\Big) : x \in X\Big\}$.

Let $\widetilde{A} = \Big(\widetilde{\varsigma_A^{\mathcal{T}}}, \widetilde{\varsigma_A^{\mathcal{I}}}, \widetilde{\varsigma_A^{\mathcal{F}}}\Big)$ and $\widetilde{B} = \Big(\widetilde{\varsigma_B^{\mathcal{T}}}, \widetilde{\varsigma_B^{\mathcal{I}}}, \widetilde{\varsigma_B^{\mathcal{F}}}\Big)$ be two IVN sets of a set X. We define two IVN sets of X as follows:

(1) $\widetilde{A} \cap \widetilde{B} = \Big\{\Big(x, \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_B^{\mathcal{T}}}(x)\}, \min\{\widetilde{\varsigma_A^{\mathcal{I}}}(x), \widetilde{\varsigma_B^{\mathcal{I}}}(x)\}, \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_B^{\mathcal{F}}}(x)\}\Big) : x \in X\Big\}$.

(2) $\widetilde{A} \cup \widetilde{B} = \Big\{\Big(x, \max\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_B^{\mathcal{T}}}(x)\}, \max\{\widetilde{\varsigma_A^{\mathcal{I}}}(x), \widetilde{\varsigma_B^{\mathcal{I}}}(x)\}, \min\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_B^{\mathcal{F}}}(x)\}\Big) : x \in X\Big\}$.

For any IVN set $\widetilde{A} = \Big(\widetilde{\varsigma_A^{\mathcal{T}}}, \widetilde{\varsigma_A^{\mathcal{I}}}, \widetilde{\varsigma_A^{\mathcal{F}}}\Big)$ of a set X, we defined an (α, β)-cut of as the crisp subset $\Big\{x \in X : \widetilde{\varsigma_A^{\mathcal{T}}}(x) \geq \widetilde{\alpha}, \widetilde{\varsigma_A^{\mathcal{I}}}(x) \geq \widetilde{\alpha}, \widetilde{\varsigma_A^{\mathcal{F}}}(x) \leq \widetilde{\beta}\Big\}$ of X.

https://doi.org/10.54216/IJNS.190109

Received: March 08, 2022 Accepted: September 16, 2022

Let $\widetilde{A}$ and $\widetilde{B}$ be be two IVN sets of a set X. The Cartesian product of $\widetilde{A}$ and $\widetilde{B}$ denoted by $\widetilde{A\times B}$ is defined as $\widetilde{A\times B}=\left(\widetilde{\varsigma^{\mathcal{T}}_{A\times B}},\widetilde{\varsigma^{\mathcal{I}}_{A\times B}},\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}\right)$ of $X\times X$, where for each $x,y\in X$,

$$\begin{cases}\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x,y)=\min\left\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(x),\widetilde{\varsigma^{\mathcal{T}}_{B}}(y)\right\}\\ \widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x,y)=\frac{\widetilde{\varsigma^{\mathcal{I}}_{A}}(x)+\widetilde{\varsigma^{\mathcal{I}}_{B}}(y)}{2}\\ \widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x,y)=\max\left\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x),\widetilde{\varsigma^{\mathcal{F}}_{B}}(y)\right\}.\end{cases}$$

[13] A FS A of a bisemiring $(\mathcal{S},\diamondsuit_1,\diamondsuit_2,\diamondsuit_3)$ is called a *fuzzy subbisemiring* of $\mathcal{S}$ if for each $x,y\in\mathcal{S}$,

$$\begin{cases}\varsigma_A(x\diamondsuit_1 y)\geq\min\{\varsigma_A(x),\varsigma_A(y)\}\\ \varsigma_A(x\diamondsuit_2 y)\geq\min\{\varsigma_A(x),\varsigma_A(y)\}\\ \varsigma_A(x\diamondsuit_3 y)\geq\min\{\varsigma_A(x),\varsigma_A(y)\}.\end{cases}$$

[13] A FS A of a bisemiring $(\mathcal{S},\diamondsuit_1,\diamondsuit_2,\diamondsuit_3)$ is called a *fuzzy normal subbisemiring* of $\mathcal{S}$ if for each $x,y\in\mathcal{S}$,

$$\begin{cases}\varsigma_A(x\diamondsuit_1 y)=\varsigma_A(y\diamondsuit_1 x)\\ \varsigma_A(x\diamondsuit_2 y)=\varsigma_A(y\diamondsuit_2 x)\\ \varsigma_A(x\diamondsuit_3 y)=\varsigma_A(y\diamondsuit_3 x).\end{cases}$$

[8] Let $(\mathcal{S},+,\cdot,\times)$ and $(\mathcal{T},\circledast,\circ,\otimes)$ be two bisemirings. A function $\varphi:\mathcal{S}\to\mathcal{T}$ is called a *homomorphism* if for each $x,y\in\mathcal{S}$,

$$\begin{cases}\varphi(x+y)=\varphi(x)\circledast\varphi(y)\\ \varphi(x\cdot y)=\varphi(x)\circ\varphi(y)\\ \varphi(x\times y)=\varphi(x)\otimes\varphi(y).\end{cases}$$

3 Interval Valued Neutrosophic Subbisemirings

Unless otherwise indicated, let $\mathcal{S}$ signify a bisemiring in the following.

An IVN subset $\widetilde{A}$ of $\mathcal{S}$ is called an IVNSBS of $\mathcal{S}$ if for each $x,y\in\mathcal{S}$,

$$\begin{cases}\widetilde{\varsigma^{\mathcal{T}}_{A}}(x\diamondsuit_1 y)\geq\min\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(x),\widetilde{\varsigma^{\mathcal{T}}_{A}}(y)\}\\ \widetilde{\varsigma^{\mathcal{T}}_{A}}(x\diamondsuit_2 y)\geq\min\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(x),\widetilde{\varsigma^{\mathcal{T}}_{A}}(y)\}\\ \widetilde{\varsigma^{\mathcal{T}}_{A}}(x\diamondsuit_3 y)\geq\min\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(x),\widetilde{\varsigma^{\mathcal{T}}_{A}}(y)\}\end{cases}
\quad
\begin{cases}\widetilde{\varsigma^{\mathcal{I}}_{A}}(x\diamondsuit_1 y)\geq\frac{\widetilde{\varsigma^{\mathcal{I}}_{A}}(x)+\widetilde{\varsigma^{\mathcal{I}}_{A}}(y)}{2}\\ \text{or}\\ \widetilde{\varsigma^{\mathcal{I}}_{A}}(x\diamondsuit_2 y)\geq\frac{\widetilde{\varsigma^{\mathcal{I}}_{A}}(x)+\widetilde{\varsigma^{\mathcal{I}}_{A}}(y)}{2}\\ \text{or}\\ \widetilde{\varsigma^{\mathcal{I}}_{A}}(x\diamondsuit_3 y)\geq\frac{\widetilde{\varsigma^{\mathcal{I}}_{A}}(x)+\widetilde{\varsigma^{\mathcal{I}}_{A}}(y)}{2}\end{cases}$$

$$\begin{cases}\widetilde{\varsigma^{\mathcal{F}}_{A}}(x\diamondsuit_1 y)\leq\max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x),\widetilde{\varsigma^{\mathcal{F}}_{A}}(y)\}\\ \widetilde{\varsigma^{\mathcal{F}}_{A}}(x\diamondsuit_2 y)\leq\max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x),\widetilde{\varsigma^{\mathcal{F}}_{A}}(y)\}\\ \widetilde{\varsigma^{\mathcal{F}}_{A}}(x\diamondsuit_3 y)\leq\max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x),\widetilde{\varsigma^{\mathcal{F}}_{A}}(y)\}.\end{cases}$$

Let $\mathcal{S}=\{\mu,\nu,\psi,\omega\}$ be a bisemiring with the following Cayley table:

$\diamondsuit_1$	μ	ν	ψ	ω
μ	μ	μ	μ	μ
ν	μ	ν	μ	ν
ψ	μ	μ	ψ	ψ
ω	μ	ν	ψ	ω

$\diamondsuit_2$	μ	ν	ψ	ω
μ	μ	ν	ψ	ω
ν	ν	ν	ω	ω
ψ	ψ	ω	ψ	ω
ω	ω	ω	ω	ω

$\diamondsuit_3$	μ	ν	ψ	ω
μ	μ	μ	μ	μ
ν	μ	ν	ψ	ω
ψ	ω	ω	ω	ω
ω	ω	ω	ω	ω

https://doi.org/10.54216/IJNS.190109
Received: March 08, 2022 Accepted: September 16, 2022

	$l=\mu$	$l=\nu$	$l=\psi$	$l=\omega$
$\widetilde{\varsigma_A^{\mathcal{T}}}(l)$	$[0.75, 0.85]$	$[0.65, 0.75]$	$[0.35, 0.45]$	$[0.55, 0.65]$
$\widetilde{\varsigma_A^{\mathcal{I}}}(l)$	$[0.45, 0.55]$	$[0.35, 0.45]$	$[0.15, 0.25]$	$[0.25, 0.35]$
$\widetilde{\varsigma_A^{\mathcal{F}}}(l)$	$[0.55, 0.65]$	$[0.65, 0.75]$	$[0.90, 0.95]$	$[0.80, 0.85]$

Then $\widetilde{A}$ is an IVNSBS of $\mathcal{S}$.

The intersection of a collection of IVNSBSs of $\mathcal{S}$ is an IVNSBS of $\mathcal{S}$.

Proof. Let $\{\widetilde{V}_i : i \in I\}$ be a collection of IVNSBSs of $\mathcal{S}$ and $\widetilde{A} = \bigcap_{i\in I}\widetilde{V}_i$. Let x and y in $\mathcal{S}$. Then

$$\begin{aligned}\widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_1 y) &= \inf_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{T}}}(x\diamondsuit_1 y)\}\\ &\geq \inf_{i\in I}\{\min\{\widetilde{\varsigma_{V_i}^{\mathcal{T}}}(x), \widetilde{\varsigma_{V_i}^{\mathcal{T}}}(y)\}\}\\ &= \min\left\{\inf_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{T}}}(x)\}, \inf_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{T}}}(y)\}\right\}\\ &= \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y)\}.\end{aligned}$$

Similarly, $\widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_2 y) \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y)\}$ and $\widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_3 y) \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y)\}$. Now,

$$\begin{aligned}\widetilde{\varsigma_A^{\mathcal{I}}}(x\diamondsuit_1 y) &= \inf_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{I}}}(x\diamondsuit_1 y)\}\\ &\geq \inf_{i\in I}\left\{\frac{\widetilde{\varsigma_{V_i}^{\mathcal{I}}}(x) + \widetilde{\varsigma_{V_i}^{\mathcal{I}}}(y)}{2}\right\}\\ &= \frac{\inf_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{I}}}(x)\} + \inf_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{I}}}(y)\}}{2}\\ &= \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x) + \widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2}.\end{aligned}$$

Similarly, $\widetilde{\varsigma_A^{\mathcal{I}}}(x\diamondsuit_2 y) \geq \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x)+\widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2}$ and $\widetilde{\varsigma_A^{\mathcal{I}}}(x\diamondsuit_3 y) \geq \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x)+\widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2}$. Now,

$$\begin{aligned}\widetilde{\varsigma_A^{\mathcal{F}}}(x\diamondsuit_1 y) &= \sup_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{F}}}(x\diamondsuit_1 y)\}\\ &\leq \sup_{i\in I}\{\max\{\widetilde{\varsigma_{V_i}^{\mathcal{F}}}(x), \widetilde{\varsigma_{V_i}^{\mathcal{F}}}(y)\}\}\\ &= \max\left\{\sup_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{F}}}(x)\}, \sup_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{F}}}(y)\}\right\}\\ &= \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y)\}.\end{aligned}$$

Similarly, $\widetilde{\varsigma_A^{\mathcal{F}}}(x\diamondsuit_2 y) \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y)\}$ and $\widetilde{\varsigma_A^{\mathcal{F}}}(x\diamondsuit_3 y) \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y)\}$. Hence $\widetilde{A}$ is an IVNSBS of $\mathcal{S}$. □

If $\widetilde{A}$ and $\widetilde{B}$ are IVNSBSs of bisemirings $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively, then $\widetilde{A\times B}$ is an IVNSBS of $\mathcal{S}_1 \times \mathcal{S}_2$.

Proof. Let $\widetilde{A}$ and $\widetilde{B}$ be two IVNSBSs of $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively. Let $(x_1, y_1), (x_2, y_2) \in \mathcal{S}_1 \times \mathcal{S}_2$. Then

$$\begin{aligned}\widetilde{\varsigma_{A\times B}^{\mathcal{T}}}[(x_1, y_1)\diamondsuit_1(x_2, y_2)] &= \widetilde{\varsigma_{A\times B}^{\mathcal{T}}}(x_1\diamondsuit_1 x_2, y_1\diamondsuit_1 y_2)\\ &= \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x_1\diamondsuit_1 x_2), \widetilde{\varsigma_B^{\mathcal{T}}}(y_1\diamondsuit_1 y_2)\}\\ &\geq \min\{\min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x_1), \widetilde{\varsigma_A^{\mathcal{T}}}(x_2)\}, \min\{\widetilde{\varsigma_B^{\mathcal{T}}}(y_1), \widetilde{\varsigma_B^{\mathcal{T}}}(y_2)\}\}\\ &= \min\{\min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x_1), \widetilde{\varsigma_B^{\mathcal{T}}}(y_1)\}, \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x_2), \widetilde{\varsigma_B^{\mathcal{T}}}(y_2)\}\}\\ &= \min\{\widetilde{\varsigma_{A\times B}^{\mathcal{T}}}(x_1, y_1), \widetilde{\varsigma_{A\times B}^{\mathcal{T}}}(x_2, y_2)\}.\end{aligned}$$

https://doi.org/10.54216/IJNS.190109

Received: March 08, 2022 Accepted: September 16, 2022

Also,

$$\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}[(x_1,y_1)\diamondsuit_2(x_2,y_2)] \geq \min\{\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x_1,y_1), \widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x_2,y_2)\}$$

and

$$\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}[(x_1,y_1)\diamondsuit_3(x_2,y_2)] \geq \min\{\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x_1,y_1), \widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x_2,y_2)\}.$$

Now,

$$\begin{aligned}
\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}[(x_1,y_1)\diamondsuit_1(x_2,y_2)] &= \widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_1\diamondsuit_1 x_2, y_1\diamondsuit_1 y_2)\\
&= \frac{\widetilde{\varsigma^{\mathcal{I}}_{A}}(x_1\diamondsuit_1 x_2) + \widetilde{\varsigma^{\mathcal{I}}_{B}}(y_1\diamondsuit_1 y_2)}{2}\\
&\geq \frac{1}{2}\Big\{\frac{\widetilde{\varsigma^{\mathcal{I}}_{A}}(x_1) + \widetilde{\varsigma^{\mathcal{I}}_{A}}(x_2)}{2} + \frac{\widetilde{\varsigma^{\mathcal{I}}_{B}}(y_1) + \widetilde{\varsigma^{\mathcal{I}}_{B}}(y_2)}{2}\Big\}\\
&= \frac{1}{2}\Big\{\frac{\widetilde{\varsigma^{\mathcal{I}}_{A}}(x_1) + \widetilde{\varsigma^{\mathcal{I}}_{B}}(y_1)}{2} + \frac{\widetilde{\varsigma^{\mathcal{I}}_{A}}(x_2) + \widetilde{\varsigma^{\mathcal{I}}_{B}}(y_2)}{2}\Big\}\\
&= \frac{1}{2}\Big[\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_1,y_1) + \widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_2,y_2)\Big].
\end{aligned}$$

Also,

$$\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}[(x_1,y_1)\diamondsuit_2(x_2,y_2)] \geq \frac{1}{2}\Big[\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_1,y_1) + \widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_2,y_2)\Big]$$

and

$$\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}[(x_1,y_1)\diamondsuit_3(x_2,y_2)] \geq \frac{1}{2}\Big\{\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_1,y_1) + \widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_2,y_2)\Big\}.$$

Now,

$$\begin{aligned}
\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}[(x_1,y_1)\diamondsuit_1(x_2,y_2)] &= \widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_1\diamondsuit_1 x_2, y_1\diamondsuit_1 y_2)\\
&= \max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x_1\diamondsuit_1 x_2), \widetilde{\varsigma^{\mathcal{F}}_{B}}(y_1\diamondsuit_1 y_2)\}\\
&\leq \max\{\max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x_1), \widetilde{\varsigma^{\mathcal{F}}_{A}}(x_2)\}, \max\{\widetilde{\varsigma^{\mathcal{F}}_{B}}(y_1), \widetilde{\varsigma^{\mathcal{F}}_{B}}(y_2)\}\}\\
&= \max\{\max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x_1), \widetilde{\varsigma^{\mathcal{F}}_{B}}(y_1)\}, \max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x_2), \widetilde{\varsigma^{\mathcal{F}}_{B}}(y_2)\}\}\\
&= \max\{\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_1,y_1), \widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_2,y_2)\}.
\end{aligned}$$

Also,

$$\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}[(x_1,y_1)\diamondsuit_2(x_2,y_2)] \leq \max\{\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_1,y_1), \widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_2,y_2)\}$$

and

$$\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}[(x_1,y_1)\diamondsuit_3(x_2,y_2)] \leq \max\{\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_1,y_1), \widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_2,y_2)\}.$$

Hence $\widetilde{A\times B}$ is an IVNSBS of $\mathcal{S}_1\times\mathcal{S}_2$. □

If $\widetilde{A_1}, \widetilde{A_2}, ..., \widetilde{A_n}$ are IVNSBSs of bisemirings $\mathcal{S}_1, \mathcal{S}_2, ..., \mathcal{S}_n$ respectively, then $\widetilde{A_1\times A_2\times ...\times A_n}$ is an IVNSBS of $\mathcal{S}_1\times\mathcal{S}_2\times ...\times\mathcal{S}_n$.

Let $\widetilde{A}$ be an IVN subset in $\mathcal{S}$, the SIVNR on $\mathcal{S}$, that is an IVNR on $\widetilde{A}$ is $\widetilde{V}$ given by

$$\begin{cases}
\widetilde{\varsigma^{\mathcal{T}}_{V}}(x,y) = \min\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(x), \widetilde{\varsigma^{\mathcal{T}}_{A}}(y)\}\\
\widetilde{\varsigma^{\mathcal{I}}_{V}}(x,y) = \frac{\widetilde{\varsigma^{\mathcal{I}}_{A}}(x)+\widetilde{\varsigma^{\mathcal{I}}_{A}}(y)}{2}\\
\widetilde{\varsigma^{\mathcal{F}}_{V}}(x,y) = \max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x), \widetilde{\varsigma^{\mathcal{F}}_{A}}(y)\}.
\end{cases}$$

Let $\widetilde{A}$ be an IVNSBS of $\mathcal{S}$ and $\widetilde{V}$ be the SIVNR of $\mathcal{S}$. Then $\widetilde{A}$ is an IVNSBS of $\mathcal{S}$ if and only if $\widetilde{V}$ is an IVNSBS of $\mathcal{S}\times\mathcal{S}$.

Proof. Suppose that $\widetilde{A}$ is an IVNSBS of $\mathcal{S}$ and $\widetilde{V}$ is the SIVNR of $\mathcal{S}$. Let $x=(x_1,x_2), y=(y_1,y_2)\in\mathcal{S}\times\mathcal{S}$. Then

$$\begin{aligned}
\widetilde{\varsigma_{V}^{\mathcal{T}}}(x\diamondsuit_1 y) &= \widetilde{\varsigma_{V}^{\mathcal{T}}}[((x_1,x_2)\diamondsuit_1(y_1,y_2)]\\
&= \widetilde{\varsigma_{V}^{\mathcal{T}}}(x_1\diamondsuit_1 y_1, x_2\diamondsuit_1 y_2)\\
&= \min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1\diamondsuit_1 y_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2\diamondsuit_1 y_2)\}\\
&\geq \min\{\min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(y_1)\}, \min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2), \widetilde{\varsigma_{A}^{\mathcal{T}}}(y_2)\}\}\\
&= \min\{\min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2)\}, \min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(y_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(y_2)\}\}\\
&= \min\{\widetilde{\varsigma_{V}^{\mathcal{T}}}(x_1,x_2), \widetilde{\varsigma_{V}^{\mathcal{T}}}(y_1,y_2)\}\\
&= \min\{\widetilde{\varsigma_{V}^{\mathcal{T}}}(x), \widetilde{\varsigma_{V}^{\mathcal{T}}}(y)\}.
\end{aligned}$$

Also, $\widetilde{\varsigma_{V}^{\mathcal{T}}}(x\diamondsuit_2 y)\geq\min\{\widetilde{\varsigma_{V}^{\mathcal{T}}}(x)$ and $\widetilde{\varsigma_{V}^{\mathcal{T}}}(y)\}, \widetilde{\varsigma_{V}^{\mathcal{T}}}(x\diamondsuit_3 y)\geq\min\{\widetilde{\varsigma_{V}^{\mathcal{T}}}(x), \widetilde{\varsigma_{V}^{\mathcal{T}}}(y)\}$. Now,

$$\begin{aligned}
\widetilde{\varsigma_{V}^{\mathcal{I}}}(x\diamondsuit_1 y) &= \widetilde{\varsigma_{V}^{\mathcal{I}}}[((x_1,x_2)\diamondsuit_1(y_1,y_2)]\\
&= \widetilde{\varsigma_{V}^{\mathcal{I}}}(x_1\diamondsuit_1 y_1, x_2\diamondsuit_1 y_2)\\
&= \frac{\widetilde{\varsigma_{A}^{\mathcal{I}}}(x_1\diamondsuit_1 y_1)+\widetilde{\varsigma_{A}^{\mathcal{I}}}(x_2\diamondsuit_1 y_2)}{2}\\
&\geq \frac{1}{2}\Big\{\frac{\widetilde{\varsigma_{A}^{\mathcal{I}}}(x_1)+\widetilde{\varsigma_{A}^{\mathcal{I}}}(y_1)}{2}+\frac{\widetilde{\varsigma_{A}^{\mathcal{I}}}(x_2)+\widetilde{\varsigma_{A}^{\mathcal{I}}}(y_2)}{2}\Big\}\\
&= \frac{1}{2}\Big\{\frac{\widetilde{\varsigma_{A}^{\mathcal{I}}}(x_1)+\widetilde{\varsigma_{A}^{\mathcal{I}}}(x_2)}{2}+\frac{\widetilde{\varsigma_{A}^{\mathcal{I}}}(y_1)+\widetilde{\varsigma_{A}^{\mathcal{I}}}(y_2)}{2}\Big\}\\
&= \frac{\widetilde{\varsigma_{V}^{\mathcal{I}}}(x_1,x_2)+\widetilde{\varsigma_{V}^{\mathcal{I}}}(y_1,y_2)}{2}\\
&= \frac{\widetilde{\varsigma_{V}^{\mathcal{I}}}(x)+\widetilde{\varsigma_{V}^{\mathcal{I}}}(y)}{2}.
\end{aligned}$$

Also, $\widetilde{\varsigma_{V}^{\mathcal{I}}}(x\diamondsuit_2 y)\geq\frac{\widetilde{\varsigma_{V}^{\mathcal{I}}}(x)+\widetilde{\varsigma_{V}^{\mathcal{I}}}(y)}{2}$ and $\widetilde{\varsigma_{V}^{\mathcal{I}}}(x\diamondsuit_3 y)\geq\frac{\widetilde{\varsigma_{V}^{\mathcal{I}}}(x)+\widetilde{\varsigma_{V}^{\mathcal{I}}}(y)}{2}$. Similarly, $\widetilde{\varsigma_{V}^{\mathcal{F}}}(x\diamondsuit_1 y)\leq\max\{\widetilde{\varsigma_{V}^{\mathcal{F}}}(x), \widetilde{\varsigma_{V}^{\mathcal{F}}}(y)\}$, $\widetilde{\varsigma_{V}^{\mathcal{F}}}(x\diamondsuit_2 y)\leq\max\{\widetilde{\varsigma_{V}^{\mathcal{F}}}(x), \widetilde{\varsigma_{V}^{\mathcal{F}}}(y)\}$, and $\widetilde{\varsigma_{V}^{\mathcal{F}}}(x\diamondsuit_3 y)\leq\max\{\widetilde{\varsigma_{V}^{\mathcal{F}}}(x), \widetilde{\varsigma_{V}^{\mathcal{F}}}(y)\}$. Hence $\widetilde{V}$ is an IVNSBS of $\mathcal{S}\times\mathcal{S}$.

Conversely, assume that $\widetilde{V}$ is an IVNSBS of $\mathcal{S}\times\mathcal{S}$. Let $x=(x_1,x_2)$ and $y=(y_1,y_2)$ be in $\mathcal{S}\times\mathcal{S}$. Then

$$\begin{aligned}
\min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1\diamondsuit_1 y_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2\diamondsuit_1 y_2)\} &= \widetilde{\varsigma_{V}^{\mathcal{T}}}(x_1\diamondsuit_1 y_1, x_2\diamondsuit_1 y_2)\\
&= \widetilde{\varsigma_{V}^{\mathcal{T}}}[(x_1,x_2)\diamondsuit_1(y_1,y_2)]\\
&= \widetilde{\varsigma_{V}^{\mathcal{T}}}(x\diamondsuit_1 y)\\
&\geq \min\{\widetilde{\varsigma_{V}^{\mathcal{T}}}(x), \widetilde{\varsigma_{V}^{\mathcal{T}}}(y)\}\\
&= \min\{\widetilde{\varsigma_{V}^{\mathcal{T}}}(x_1,x_2)\}, \widetilde{\varsigma_{V}^{\mathcal{T}}}(y_1,y_2)\}\\
&= \min\{\min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2)\}, \min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(y_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(y_2)\}\}.
\end{aligned}$$

If $\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1\diamondsuit_1 y_1)\leq\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2\diamondsuit_1 y_2)$, then $\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1)\leq\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2)$ and $\widetilde{\varsigma_{A}^{\mathcal{T}}}(y_1)\leq\widetilde{\varsigma_{A}^{\mathcal{T}}}(y_2)$. We get $\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1\diamondsuit_1 y_1)\geq\min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(y_1)\}$, and

$$\min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1\diamondsuit_2 y_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2\diamondsuit_2 y_2)\}\geq\min\{\min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2)\}, \min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(y_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(y_2)\}\}.$$

If $\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1\diamondsuit_2 y_1)\leq\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2\diamondsuit_2 y_2)$, then $\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1\diamondsuit_2 y_1)\geq\min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(y_1)\}$ and

$$\min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1\diamondsuit_3 y_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2\diamondsuit_3 y_2)\}\geq\min\{\min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(x_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(x_2)\}, \min\{\widetilde{\varsigma_{A}^{\mathcal{T}}}(y_1), \widetilde{\varsigma_{A}^{\mathcal{T}}}(y_2)\}\}.$$

https://doi.org/10.54216/IJNS.190109
Received: March 08, 2022 Accepted: September 16, 2022

If $\widetilde{\varsigma_A^{\mathcal{T}}}(x_1\diamondsuit_3 y_1) \leq \widetilde{\varsigma_A^{\mathcal{T}}}(x_2\diamondsuit_3 y_2)$, then $\widetilde{\varsigma_A^{\mathcal{T}}}(x_1\diamondsuit_3 y_1) \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x_1), \widetilde{\varsigma_A^{\mathcal{T}}}(y_1)\}$. Now,

$$\begin{aligned}
\frac{1}{2}\Big\{\widetilde{\varsigma_A^{\mathcal{I}}}(x_1\diamondsuit_1 y_1) + \widetilde{\varsigma_A^{\mathcal{I}}}(x_2\diamondsuit_1 y_2)\Big\} &= \widetilde{\varsigma_V^{\mathcal{I}}}(x_1\diamondsuit_1 y_1, x_2\diamondsuit_1 y_2)\\
&= \widetilde{\varsigma_V^{\mathcal{I}}}[(x_1,x_2)\diamondsuit_1(y_1,y_2)]\\
&= \widetilde{\varsigma_V^{\mathcal{I}}}(x\diamondsuit_1 y)\\
&\geq \frac{\widetilde{\varsigma_V^{\mathcal{I}}}(x) + \widetilde{\varsigma_V^{\mathcal{I}}}(y)}{2}\\
&= \frac{\widetilde{\varsigma_V^{\mathcal{I}}}(x_1,x_2) + \widetilde{\varsigma_V^{\mathcal{I}}}(y_1,y_2)}{2}\\
&= \frac{1}{2}\Big\{\frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x_1) + \widetilde{\varsigma_A^{\mathcal{I}}}(x_2)}{2} + \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(y_1) + \widetilde{\varsigma_A^{\mathcal{I}}}(y_2)}{2}\Big\}.
\end{aligned}$$

If $\widetilde{\varsigma_A^{\mathcal{I}}}(x_1\diamondsuit_1 y_1) \leq \widetilde{\varsigma_A^{\mathcal{I}}}(x_2\diamondsuit_1 y_2)$, then $\widetilde{\varsigma_A^{\mathcal{I}}}(x_1) \leq \widetilde{\varsigma_A^{\mathcal{I}}}(x_2)$ and $\widetilde{\varsigma_A^{\mathcal{I}}}(y_1) \leq \widetilde{\varsigma_A^{\mathcal{I}}}(y_2)$. We get $\widetilde{\varsigma_A^{\mathcal{I}}}(x_1\diamondsuit_1 y_1) \geq \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x_1)+\widetilde{\varsigma_A^{\mathcal{I}}}(y_1)}{2}$. Similarly, $\widetilde{\varsigma_A^{\mathcal{I}}}(x_1\diamondsuit_2 y_1) \geq \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x_1)+\widetilde{\varsigma_A^{\mathcal{I}}}(y_1)}{2}$ and $\widetilde{\varsigma_A^{\mathcal{I}}}(x_1\diamondsuit_3 y_1) \geq \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x_1)+\widetilde{\varsigma_A^{\mathcal{I}}}(y_1)}{2}$. Similarly to prove that $\max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x_1\diamondsuit_1 y_1), \widetilde{\varsigma_A^{\mathcal{F}}}(x_2\diamondsuit_1 y_2)\} \leq \max\{\max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x_1), \widetilde{\varsigma_A^{\mathcal{F}}}(x_2)\}, \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(y_1), \widetilde{\varsigma_A^{\mathcal{F}}}(y_2)\}\}$.

If $\widetilde{\varsigma_A^{\mathcal{F}}}(x_1\diamondsuit_1 y_1) \geq \widetilde{\varsigma_A^{\mathcal{F}}}(x_2\diamondsuit_1 y_2)$, then $\widetilde{\varsigma_A^{\mathcal{F}}}(x_1) \geq \widetilde{\varsigma_A^{\mathcal{F}}}(x_2)$ and $\widetilde{\varsigma_A^{\mathcal{F}}}(y_1) \geq \widetilde{\varsigma_A^{\mathcal{F}}}(y_2)$. We get $\widetilde{\varsigma_A^{\mathcal{F}}}(x_1\diamondsuit_1 y_1) \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x_1), \widetilde{\varsigma_A^{\mathcal{F}}}(y_1)\}$ and

$$\max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x_1\diamondsuit_2 y_1), \widetilde{\varsigma_A^{\mathcal{F}}}(x_2\diamondsuit_2 y_2)\} \leq \max\{\max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x_1), \widetilde{\varsigma_A^{\mathcal{F}}}(x_2)\}, \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(y_1), \widetilde{\varsigma_A^{\mathcal{F}}}(y_2)\}\}.$$

If $\widetilde{\varsigma_A^{\mathcal{F}}}(x_1\diamondsuit_2 y_1) \geq \widetilde{\varsigma_A^{\mathcal{F}}}(x_2\diamondsuit_2 y_2)$, then $\widetilde{\varsigma_A^{\mathcal{F}}}(x_1\diamondsuit_2 y_1) \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x_1), \widetilde{\varsigma_A^{\mathcal{F}}}(y_1)\}$ and

$$\max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x_1\diamondsuit_3 y_1), \widetilde{\varsigma_A^{\mathcal{F}}}(x_2\diamondsuit_3 y_2)\} \leq \max\{\max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x_1), \widetilde{\varsigma_A^{\mathcal{F}}}(x_2)\}, \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(y_1), \widetilde{\varsigma_A^{\mathcal{F}}}(y_2)\}\}.$$

If $\widetilde{\varsigma_A^{\mathcal{F}}}(x_1\diamondsuit_3 y_1) \geq \widetilde{\varsigma_A^{\mathcal{F}}}(x_2\diamondsuit_3 y_2)$, then $\widetilde{\varsigma_A^{\mathcal{F}}}(x_1\diamondsuit_3 y_1) \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x_1), \widetilde{\varsigma_A^{\mathcal{F}}}(y_1)\}$.

Hence $\widetilde{A}$ is an IVNSBS of $\mathcal{S}$. □

Let $\widetilde{A}$ be an IVN subset in $\mathcal{S}$. Then $\widetilde{\varsigma} = (\widetilde{\varsigma_A^{\mathcal{T}}}, \widetilde{\varsigma_A^{\mathcal{I}}}, \widetilde{\varsigma_A^{\mathcal{F}}})$ is an IVNSBS of $\mathcal{S}$ if and only if all non-void level set $\widetilde{\varsigma^{(t,s)}}$ is a subbisemiring of $\mathcal{S}$ for $t, s \in [[0,1]]$.

Proof. Assume that $\widetilde{\varsigma}$ is an IVNSBS of $\mathcal{S}$. For each $t, s \in [[0,1]]$ and $a_1, a_2 \in \widetilde{\varsigma^{(t,s)}}$. We have $\widetilde{\varsigma_A^{\mathcal{T}}}(a_1) \geq \widetilde{t}, \widetilde{\varsigma_A^{\mathcal{T}}}(a_2) \geq \widetilde{t}$ and $\widetilde{\varsigma_A^{\mathcal{I}}}(a_1) \geq \widetilde{t}, \widetilde{\varsigma_A^{\mathcal{I}}}(a_2) \geq \widetilde{t}$ and $\widetilde{\varsigma_A^{\mathcal{F}}}(a_1) \leq \widetilde{s}, \widetilde{\varsigma_A^{\mathcal{F}}}(a_2) \leq \widetilde{s}$. Now, $\widetilde{\varsigma_A^{\mathcal{T}}}(a_1\diamondsuit_1 a_2) \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(a_1), \widetilde{\varsigma_A^{\mathcal{T}}}(a_2)\} \geq \widetilde{t}$ and $\widetilde{\varsigma_A^{\mathcal{I}}}(a_1\diamondsuit_1 a_2) \geq \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(a_1)+\widetilde{\varsigma_A^{\mathcal{I}}}(a_2)}{2} \geq \frac{\widetilde{t}+\widetilde{t}}{2} = \widetilde{t}$ and $\widetilde{\varsigma_A^{\mathcal{F}}}(a_1\diamondsuit_1 a_2) \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(a_1), \widetilde{\varsigma_A^{\mathcal{F}}}(a_2)\} \leq \widetilde{s}$. This implies that $a_1\diamondsuit_1 a_2 \in \widetilde{\varsigma^{(t,s)}}$. Similarly, $a_1\diamondsuit_2 a_2 \in \widetilde{\varsigma^{(t,s)}}$ and $a_1\diamondsuit_3 a_2 \in \widetilde{\varsigma^{(t,s)}}$. Therefore $\widetilde{\varsigma^{(t,s)}}$ is a subbisemiring of $\mathcal{S}$ for each $t, s \in [[0,1]]$.

Conversely, assume that $\widetilde{\varsigma^{(t,s)}}$ is a subbisemiring of $\mathcal{S}$ for each $t, s \in [[0,1]]$. Suppose if there exist $a_1, a_2 \in \mathcal{S}$ such that $\widetilde{\varsigma_A^{\mathcal{T}}}(a_1\diamondsuit_1 a_2) < \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(a_1), \widetilde{\varsigma_A^{\mathcal{T}}}(a_2)\}, \widetilde{\varsigma_A^{\mathcal{I}}}(a_1\diamondsuit_1 a_2) < \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(a_1)+\widetilde{\varsigma_A^{\mathcal{I}}}(a_2)}{2}$ and $\widetilde{\varsigma_A^{\mathcal{F}}}(a_1\diamondsuit_1 a_2) > \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(a_1), \widetilde{\varsigma_A^{\mathcal{F}}}(a_2)\}$. Select $t, s \in [[0,1]]$ such that $\widetilde{\varsigma_A^{\mathcal{T}}}(a_1\diamondsuit_1 a_2) < \widetilde{t} \leq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(a_1), \widetilde{\varsigma_A^{\mathcal{T}}}(a_2)\}$, $\widetilde{\varsigma_A^{\mathcal{I}}}(a_1\diamondsuit_1 a_2) < \widetilde{t} \leq \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(a_1)+\widetilde{\varsigma_A^{\mathcal{I}}}(a_2)}{2}$ and $\widetilde{\varsigma_A^{\mathcal{F}}}(a_1\diamondsuit_1 a_2) > \widetilde{s} \geq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(a_1), \widetilde{\varsigma_A^{\mathcal{F}}}(a_2)\}$. Then $a_1, a_2 \in \widetilde{\varsigma^{(t,s)}}$, but $a_1\diamondsuit_1 a_2 \notin \widetilde{\varsigma^{(t,s)}}$. This contradicts to that $\widetilde{\varsigma^{(t,s)}}$ is a subbisemiring of $\mathcal{S}$. Hence $\widetilde{\varsigma_A^{\mathcal{T}}}(a_1\diamondsuit_1 a_2) \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(a_1), \widetilde{\varsigma_A^{\mathcal{T}}}(a_2)\}, \widetilde{\varsigma_A^{\mathcal{I}}}(a_1\diamondsuit_1 a_2) \geq \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(a_1)+\widetilde{\varsigma_A^{\mathcal{I}}}(a_2)}{2}$ and $\widetilde{\varsigma_A^{\mathcal{F}}}(a_1\diamondsuit_1 a_2) \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(a_1), \widetilde{\varsigma_A^{\mathcal{F}}}(a_2)\}$. Similarly, $\diamondsuit_2$ and $\diamondsuit_3$ cases. Hence $\widetilde{\varsigma} = (\widetilde{\varsigma_A^{\mathcal{T}}}, \widetilde{\varsigma_A^{\mathcal{I}}}, \widetilde{\varsigma_A^{\mathcal{F}}})$ is an IVNSBS of $\mathcal{S}$. □

Received: March 08, 2022 Accepted: September 16, 2022

Let $(\mathcal{S}_1, \circledast_1, \circledast_2, \circledast_3)$ and $(\mathcal{S}_2, \circledcirc_1, \circledcirc_2, \circledcirc_3)$ be bisemirings. Let $\Xi : \mathcal{S}_1 \to \mathcal{S}_2$ be any function and $\widetilde{A}$ be any IVNSBS in $\mathcal{S}_1$, $\widetilde{V}$ be any IVNSBS in $\Xi(\mathcal{S}_1) = \mathcal{S}_2$. If $\widetilde{\varsigma_A} = [\widetilde{\varsigma_A^{\mathcal{T}}}, \widetilde{\varsigma_A^{\mathcal{I}}}, \widetilde{\varsigma_A^{\mathcal{F}}}]$ is an IVN set in $\mathcal{S}_1$, then $\widetilde{\varsigma_V}$ is an IVN set in $\mathcal{S}_2$, defined by

$$\widetilde{\varsigma_V^{\mathcal{T}}}(y) = \begin{cases} \sup\{\widetilde{\varsigma_A^{\mathcal{T}}}(x)\} & \text{if } x \in \Xi^{-1}y \\ 0 & \text{otherwise} \end{cases} \qquad \widetilde{\varsigma_V^{\mathcal{I}}}(y) = \begin{cases} \sup\{\widetilde{\varsigma_A^{\mathcal{I}}}(x)\} & \text{if } x \in \Xi^{-1}y \\ 0 & \text{otherwise} \end{cases}$$

$$\widetilde{\varsigma_V^{\mathcal{F}}}(y) = \begin{cases} \inf\{\widetilde{\varsigma_A^{\mathcal{F}}}(x)\} & \text{if } x \in \Xi^{-1}y \\ 1 & \text{otherwise} \end{cases}$$

for all $x \in \mathcal{S}_1$ and $y \in \mathcal{S}_2$ is called the image of ς_A under Ξ. Similarly, If $\widetilde{\varsigma_V} = [\widetilde{\varsigma_V^{\mathcal{T}}}, \widetilde{\varsigma_V^{\mathcal{I}}}, \widetilde{\varsigma_V^{\mathcal{F}}}]$ is an IVN set in $\mathcal{S}_2$, then IVN set $\widetilde{\varsigma_A} = \Xi \circ \widetilde{\varsigma_V}$ in $\mathcal{S}_1$ [i.e., the IVN set defined by $\widetilde{\varsigma_A}(x) = \widetilde{\varsigma_V}(\Xi(x))$] is called the preimage of $\widetilde{\varsigma_V}$ under Ξ.

Let $(\mathcal{S}_1, \circledast_1, \circledast_2, \circledast_3)$ and $(\mathcal{S}_2, \circledcirc_1, \circledcirc_2, \circledcirc_3)$ be bisemirings. The homomorphic image of IVNSBS of $\mathcal{S}_1$ is an IVNSBS of $\mathcal{S}_2$.

Proof. Let $\Xi : \mathcal{S}_1 \to \mathcal{S}_2$ be any homomorphism. Then $\Xi(x \circledast_1 y) = \Xi(x) \circledcirc_1 \Xi(y), \Xi(x \circledast_2 y) = \Xi(x) \circledcirc_2 \Xi(y)$ and $\Xi(x \circledast_3 y) = \Xi(x) \circledcirc_3 \Xi(y)$ for all $x, y \in \mathcal{S}_1$. Let $\widetilde{V} = \Xi(\widetilde{A})$, $\widetilde{A}$ is any IVNSBS of $\mathcal{S}_1$. Let $\Xi(x), \Xi(y) \in \mathcal{S}_2$. Let $x \in \Xi^{-1}(\Xi(x))$ and $y \in \Xi^{-1}(\Xi(y))$ be such that $\widetilde{\varsigma_A^{\mathcal{T}}}(x) = \sup_{z \in \Xi^{-1}(\Xi(x))} \{\widetilde{\varsigma_A^{\mathcal{T}}}(z)\}$ and $\widetilde{\varsigma_A^{\mathcal{T}}}(y) = \sup_{z \in \Xi^{-1}(\Xi(y))} \{\widetilde{\varsigma_A^{\mathcal{T}}}(z)\}$. Now,

$$\begin{aligned} \widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x) \circledcirc_1 \Xi(y)) &= \sup_{z' \in \Xi^{-1}(\Xi(x) \circledcirc_1 \Xi(y))} \{\widetilde{\varsigma_A^{\mathcal{T}}}(z')\} \\ &= \sup_{z' \in \Xi^{-1}(\Xi(x \circledast_1 y)} \{\widetilde{\varsigma_A^{\mathcal{T}}}(z')\} \\ &= \widetilde{\varsigma_A^{\mathcal{T}}}(x \circledast_1 y) \\ &\geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y)\} \\ &= \min\{\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{T}}}\Xi(y)\}. \end{aligned}$$

Thus, $\widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x) \circledcirc_1 \Xi(y)) \geq \min\{\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{T}}}\Xi(y)\}$. Similarly, $\widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x) \circledcirc_2 \Xi(y)) \geq \min\{\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{T}}}\Xi(y)\}$ and $\widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x) \circledcirc_3 \Xi(y)) \geq \min\{\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{T}}}\Xi(y)\}$. Let $x \in \Xi^{-1}(\Xi(x))$ and $y \in \Xi^{-1}(\Xi(y))$ be such that $\widetilde{\varsigma_A^{\mathcal{I}}}(x) = \sup_{z \in \Xi^{-1}(\Xi(x))} \{\widetilde{\varsigma_A^{\mathcal{I}}}(z)\}$ and $\widetilde{\varsigma_A^{\mathcal{I}}}(y) = \sup_{z \in \Xi^{-1}(\Xi(y))} \{\widetilde{\varsigma_A^{\mathcal{I}}}(z)\}$. Now,

$$\begin{aligned} \widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x) \circledcirc_1 \Xi(y)) &= \sup_{z' \in \Xi^{-1}(\Xi(x) \circledcirc_1 \Xi(y))} \{\widetilde{\varsigma_A^{\mathcal{I}}}(z')\} \\ &= \sup_{z' \in \Xi^{-1}(\Xi(x \circledast_1 y))} \{\widetilde{\varsigma_A^{\mathcal{I}}}(z')\} \\ &= \widetilde{\varsigma_A^{\mathcal{I}}}(x \circledast_1 y) \\ &\geq \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x) + \widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2} \\ &= \frac{\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(x) + \widetilde{\varsigma_V^{\mathcal{I}}}\Xi(y)}{2}. \end{aligned}$$

Thus, $\widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x) \circledcirc_1 \Xi(y)) \geq \frac{\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(x) + \widetilde{\varsigma_V^{\mathcal{I}}}\Xi(y)}{2}$. Similarly, $\widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x) \circledcirc_2 \Xi(y)) \geq \frac{\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(x) + \widetilde{\varsigma_V^{\mathcal{I}}}\Xi(y)}{2}$ and $\widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x) \circledcirc_3 \Xi(y)) \geq \frac{\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(x) + \widetilde{\varsigma_V^{\mathcal{I}}}\Xi(y)}{2}$. Let $\Xi(x), \Xi(y) \in \mathcal{S}_2$. Let $x \in \Xi^{-1}(\Xi(x))$ and $y \in \Xi^{-1}(\Xi(y))$ be such that

https://doi.org/10.54216/IJNS.190109

Received: March 08, 2022 Accepted: September 16, 2022

$\widetilde{\varsigma_A^{\mathcal{F}}}(x) = \inf_{z \in \Xi^{-1}(\Xi(x))} \{\widetilde{\varsigma_A^{\mathcal{F}}}(z)\}$ and $\widetilde{\varsigma_A^{\mathcal{F}}}(y) = \inf_{z \in \Xi^{-1}(\Xi(y))} \{\widetilde{\varsigma_A^{\mathcal{F}}}(z)\}$. Now,

$$\begin{aligned}
\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x) \circledcirc_1 \Xi(y)) &= \inf_{z^{'} \in \Xi^{-1}(\Xi(x) \circledcirc_1 \Xi(y))} \{\widetilde{\varsigma_A^{\mathcal{F}}}(z^{'})\} \\
&= \inf_{z^{'} \in \Xi^{-1}(\Xi(x \circledast_1 y)} \{\widetilde{\varsigma_A^{\mathcal{F}}}(z^{'})\} \\
&= \widetilde{\varsigma_A^{\mathcal{F}}}(x \circledast_1 y) \\
&\leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y)\} \\
&= \max\{\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{F}}}\Xi(y)\}.
\end{aligned}$$

Thus, $\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x) \circledcirc_1 \Xi(y)) \leq \max\{\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{F}}}\Xi(y)\}$. Similarly, $\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x) \circledcirc_2 \Xi(y)) \leq \max\{\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{F}}}\Xi(y)\}$ and $\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x) \circledcirc_3 \Xi(y)) \leq \max\{\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{F}}}\Xi(y)\}$. Hence $\widetilde{V}$ is an IVNSBS of $\mathcal{S}_2$. □

Let $(\mathcal{S}_1, \circledast_1, \circledast_2, \circledast_3)$ and $(\mathcal{S}_2, \circledcirc_1, \circledcirc_2, \circledcirc_3)$ be bisemirings. The homomorphic preimage of IVNSBS of $\mathcal{S}_2$ is an IVNSBS of $\mathcal{S}_1$.

Proof. Let $\Xi : \mathcal{S}_1 \to \mathcal{S}_2$ be any homomorphism. Then $\Xi(x \circledast_1 y) = \Xi(x) \circledcirc_1 \Xi(y), \Xi(x \circledast_2 y) = \Xi(x) \circledcirc_2 \Xi(y)$, and $\Xi(x \circledast_3 y) = \Xi(x) \circledcirc_3 \Xi(y)$ for all $x, y \in \mathcal{S}_1$. Let $\widetilde{V} = \Xi(\widetilde{A})$, where $\widetilde{V}$ is any IVNSBS of $\mathcal{S}_2$. Let $x, y \in \mathcal{S}_1$. Now, $\widetilde{\varsigma_A^{\mathcal{T}}}(x \circledast_1 y) = \widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x \circledast_1 y)) = \widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x) \circledcirc_1 \Xi(y)) \geq \min\{\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{T}}}\Xi(y)\} = \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y)\}$. Thus, $\widetilde{\varsigma_A^{\mathcal{T}}}(x \circledast_1 y) \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y)\}$. Now, $\widetilde{\varsigma_A^{\mathcal{I}}}(x \circledast_1 y) = \widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x \circledast_1 y)) = \widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x) \circledcirc_1 \Xi(y)) \geq \frac{\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(x) + \widetilde{\varsigma_V^{\mathcal{I}}}\Xi(y)}{2} = \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x) + \widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2}$. Thus, $\widetilde{\varsigma_A^{\mathcal{I}}}(x \circledast_1 y) \geq \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x) + \widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2}$. Now, $\widetilde{\varsigma_A^{\mathcal{F}}}(x \circledast_1 y) = \widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x \circledast_1 y)) = \widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x) \circledcirc_1 \Xi(y)) \leq \max\{\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{F}}}\Xi(y)\} = \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y)\}$. Thus, $\widetilde{\varsigma_A^{\mathcal{F}}}(x \circledast_1 y) \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y)\}$. Similarly other operations, $\widetilde{A}$ is an IVNSBS of $\mathcal{S}_1$. □

Let $(\mathcal{S}_1, \circledast_1, \circledast_2, \circledast_3)$ and $(\mathcal{S}_2, \circledcirc_1, \circledcirc_2, \circledcirc_3)$ be bisemirings. If $\Xi : \mathcal{S}_1 \to \mathcal{S}_2$ is a homomorphism, then $\Xi(\widetilde{A_{(t,s)}})$ is a level subbisemiring of IVNSBS $\widetilde{V}$ of $\mathcal{S}_2$.

Proof. Let $\Xi : \mathcal{S}_1 \to \mathcal{S}_2$ be any homomorphism. Then $\Xi(x \circledast_1 y) = \Xi(x) \circledcirc_1 \Xi(y), \Xi(x \circledast_2 y) = \Xi(x) \circledcirc_2 \Xi(y)$, and $\Xi(x \circledast_3 y) = \Xi(x) \circledcirc_3 \Xi(y)$ for all $x, y \in \mathcal{S}_1$. Let $\widetilde{V} = \Xi(\widetilde{A})$, $\widetilde{A}$ is an IVNSBS of $\mathcal{S}_1$. By Theorem 3, $\widetilde{V}$ is an IVNSBS of $\mathcal{S}_2$. Let $\widetilde{A_{(t,s)}}$ be any level subbisemiring of $\widetilde{A}$. Suppose that $x, y \in \widetilde{A_{(t,s)}}$. Then $\Xi(x \circledast_1 y), \Xi(x \circledast_2 y)$ and $\Xi(x \circledast_3 y) \in \widetilde{A_{(t,s)}}$. Now, $\widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x)) = \widetilde{\varsigma_A^{\mathcal{T}}}(x) \geq \widetilde{t}, \widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(y)) = \widetilde{\varsigma_A^{\mathcal{T}}}(y) \geq \widetilde{t}$. Thus, $\widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x) \circledcirc_1 \Xi(y)) \geq \widetilde{\varsigma_A^{\mathcal{T}}}(x \circledast_1 y) \geq \widetilde{t}$. Now, $\widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x)) = \widetilde{\varsigma_A^{\mathcal{I}}}(x) \geq \widetilde{t}, \widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(y)) = \widetilde{\varsigma_A^{\mathcal{I}}}(y) \geq \widetilde{t}$. Thus, $\widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x) \circledcirc_1 \Xi(y)) \geq \widetilde{\varsigma_A^{\mathcal{I}}}(x \circledast_1 y) \geq \widetilde{t}$. Now, $\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x)) = \widetilde{\varsigma_A^{\mathcal{F}}}(x) \leq \widetilde{s}, \widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(y)) = \widetilde{\varsigma_A^{\mathcal{F}}}(y) \leq \widetilde{s}$. Thus, $\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x) \circledcirc_1 \Xi(y)) \leq \widetilde{\varsigma_A^{\mathcal{F}}}(x \circledast_1 y) \leq \widetilde{s}$, for all $\Xi(x), \Xi(y) \in \mathcal{S}_2$. Similarly other operations, $\Xi(\widetilde{A_{(t,s)}})$ is a level subbisemiring of IVNSBS $\widetilde{V}$ of $\mathcal{S}_2$. □

Let $(\mathcal{S}_1, \circledast_1, \circledast_2, \circledast_3)$ and $(\mathcal{S}_2, \circledcirc_1, \circledcirc_2, \circledcirc_3)$ be bisemirings. If $\Xi : \mathcal{S}_1 \to \mathcal{S}_2$ is any homomorphism, then $\widetilde{A_{(t,s)}}$ is a level subbisemiring of IVNSBS $\widetilde{A}$ of $\mathcal{S}_1$.

Proof. Let $\Xi : \mathcal{S}_1 \to \mathcal{S}_2$ be any homomorphism. Then $\Xi(x \circledast_1 y) = \Xi(x) \circledcirc_1 \Xi(y), \Xi(x \circledast_2 y) = \Xi(x) \circledcirc_2 \Xi(y)$, and $\Xi(x \circledast_3 y) = \Xi(x) \circledcirc_3 \Xi(y)$ for all $x, y \in \mathcal{S}_1$. Let $\widetilde{V} = \Xi(\widetilde{A})$, $\widetilde{V}$ is an IVNSBS of $\mathcal{S}_2$. By Theorem 3, $\widetilde{A}$ is an IVNSBS of $\mathcal{S}_1$. Let $\Xi(\widetilde{A_{(t,s)}})$ be a level subbisemiring of $\widetilde{V}$. Suppose that $\Xi(x), \Xi(y) \in \Xi(\widetilde{A_{(t,s)}})$. Then $\Xi(x \circledast_1 y), \Xi(x \circledast_2 y)$ and $\Xi(x \circledast_3 y) \in \Xi(\widetilde{A_{(t,s)}})$. Now, $\widetilde{\varsigma_A^{\mathcal{T}}}(x) = \widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x)) \geq \widetilde{t}, \widetilde{\varsigma_A^{\mathcal{T}}}(y) = \widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(y)) \geq \widetilde{t}$. Thus, $\widetilde{\varsigma_A^{\mathcal{T}}}(x \circledast_1 y) \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y)\} \geq \widetilde{t}$. Now, $\widetilde{\varsigma_A^{\mathcal{I}}}(x) = \widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x)) \geq \widetilde{t}, \widetilde{\varsigma_A^{\mathcal{I}}}(y) = \widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(y)) \geq \widetilde{t}$. Thus, $\widetilde{\varsigma_A^{\mathcal{I}}}(x \circledast_1 y) \geq \frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x) + \widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2} \geq \widetilde{t}$. Now, $\widetilde{\varsigma_A^{\mathcal{F}}}(x) = \widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x)) \leq \widetilde{s}, \widetilde{\varsigma_A^{\mathcal{F}}}(y) = \widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(y)) \leq \widetilde{s}$. Thus, $\widetilde{\varsigma_A^{\mathcal{F}}}(x \circledast_1 y) = \widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x) \circledcirc_1 \Xi(y)) \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y)\} \leq \widetilde{s}$, for all $x, y \in \mathcal{S}_1$. Similarly other operations, $\widetilde{A_{(t,s)}}$ is a level subbisemiring of IVNSBS $\widetilde{A}$ of $\mathcal{S}_1$. □

4 (α, β)-IVNSBSs

In what follows that, $(\widetilde{\alpha}, \widetilde{\beta}) \in [[0,1]]$ be such that $0 \leq \widetilde{\alpha} < \widetilde{\beta} \leq 1$.

Let $\widetilde{A}$ be any IVN subset of $\mathcal{S}$ is called an (α, β)-IVNSBS of $\mathcal{S}$ if for each $x, y \in \mathcal{S}$,

$$\begin{cases} \max\{\widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_1 y), \widetilde{\alpha}\} \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y), \widetilde{\beta}\} \\ \max\{\widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_2 y), \widetilde{\alpha}\} \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y), \widetilde{\beta}\} \\ \max\{\widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_3 y), \widetilde{\alpha}\} \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y), \widetilde{\beta}\} \end{cases} \quad \begin{cases} \max\{\widetilde{\varsigma_A^{\mathcal{I}}}(x\diamondsuit_1 y), \widetilde{\alpha}\} \geq \min\left\{\frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x)+\widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2}, \widetilde{\beta}\right\} \\ \text{or} \\ \max\{\widetilde{\varsigma_A^{\mathcal{I}}}(x\diamondsuit_2 y), \widetilde{\alpha}\} \geq \min\left\{\frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x)+\widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2}, \widetilde{\beta}\right\} \\ \text{or} \\ \max\{\widetilde{\varsigma_A^{\mathcal{I}}}(x\diamondsuit_3 y), \widetilde{\alpha}\} \geq \min\left\{\frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x)+\widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2}, \widetilde{\beta}\right\} \end{cases}$$

$$\begin{cases} \min\{\widetilde{\varsigma_A^{\mathcal{F}}}(x\diamondsuit_1 y), \widetilde{\alpha}\} \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y), \widetilde{\beta}\} \\ \min\{\widetilde{\varsigma_A^{\mathcal{F}}}(x\diamondsuit_2 y), \widetilde{\alpha}\} \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y), \widetilde{\beta}\} \\ \min\{\widetilde{\varsigma_A^{\mathcal{F}}}(x\diamondsuit_3 y), \widetilde{\alpha}\} \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y), \widetilde{\beta}\}. \end{cases}$$

By Example 3, we get

	$l = \mu$	$l = \nu$	$l = \psi$	$l = \omega$
$\widetilde{\varsigma_A^{\mathcal{T}}}(l)$	$[0.80, 0.85]$	$[0.75, 0.80]$	$[0.55, 0.60]$	$[0.70, 0.75]$
$\widetilde{\varsigma_A^{\mathcal{I}}}(l)$	$[0.75, 0.80]$	$[0.70, 0.75]$	$[0.62, 0.64]$	$[0.65, 0.70]$
$\widetilde{\varsigma_A^{\mathcal{F}}}(l)$	$[0.30, 0.35]$	$[0.60, 0.65]$	$[0.75, 0.80]$	$[0.65, 0.70]$

Then $\widetilde{A}$ is a $([0.40, 0.45], [0.55, 0.60])$-IVNSBS of $\mathcal{S}$.

The intersection of a collection of (α, β)-IVNSBSs of $\mathcal{S}$ is an (α, β)-IVNSBS of $\mathcal{S}$.

Proof. Let $\{\widetilde{V}_i : i \in I\}$ be a collection of (α, β)-IVNSBSs of $\mathcal{S}$ and $\widetilde{A} = \bigcap_{i\in I} \widetilde{V}_i$. Let $x, y \in \mathcal{S}$. Then

$$\begin{aligned} \max\{\widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_1 y), \widetilde{\alpha}\} &= \inf_{i\in I}\{\max\{\widetilde{\varsigma_{V_i}^{\mathcal{T}}}(x\diamondsuit_1 y), \widetilde{\alpha}\}\} \\ &\geq \inf_{i\in I}\{\min\{\widetilde{\varsigma_{V_i}^{\mathcal{T}}}(x), \widetilde{\varsigma_{V_i}^{\mathcal{T}}}(y), \widetilde{\beta}\}\} \\ &= \min\left\{\inf_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{T}}}(x)\}, \inf_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{T}}}(y)\}, \widetilde{\beta}\right\} \\ &= \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y), \widetilde{\beta}\}. \end{aligned}$$

Similarly, $\max\{\widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_2 y), \widetilde{\alpha}\} \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y), \widetilde{\beta}\}$ and $\max\{\widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_3 y), \widetilde{\alpha}\} \geq \min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x), \widetilde{\varsigma_A^{\mathcal{T}}}(y), \widetilde{\beta}\}$. Now,

$$\begin{aligned} \max\{\widetilde{\varsigma_A^{\mathcal{I}}}(x\diamondsuit_1 y), \widetilde{\alpha}\} &= \inf_{i\in I}\{\max\{\widetilde{\varsigma_{V_i}^{\mathcal{I}}}(x\diamondsuit_1 y), \widetilde{\alpha}\}\} \\ &\geq \inf_{i\in I}\left\{\min\left\{\frac{\widetilde{\varsigma_{V_i}^{\mathcal{I}}}(x) + \widetilde{\varsigma_{V_i}^{\mathcal{I}}}(y)}{2}, \widetilde{\beta}\right\}\right\} \\ &= \min\left\{\frac{\inf_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{I}}}(x)\} + \inf_{i\in I}\{\widetilde{\varsigma_{V_i}^{\mathcal{I}}}(y)\}}{2}, \widetilde{\beta}\right\} \\ &= \min\left\{\frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x) + \widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2}, \widetilde{\beta}\right\}. \end{aligned}$$

https://doi.org/10.54216/IJNS.190109
Received: March 08, 2022 Accepted: September 16, 2022

Similarly, $\max\{\widetilde{\varsigma^{\mathcal{I}}_A}(x\diamondsuit_2 y),\widetilde{\alpha}\}\geq \min\left\{\frac{\widetilde{\varsigma^{\mathcal{I}}_A}(x)+\widetilde{\varsigma^{\mathcal{I}}_A}(y)}{2},\widetilde{\beta}\right\}$ and $\max\{\widetilde{\varsigma^{\mathcal{I}}_A}(x\diamondsuit_3 y),\widetilde{\alpha}\}\geq \min\left\{\frac{\widetilde{\varsigma^{\mathcal{I}}_A}(x)+\widetilde{\varsigma^{\mathcal{I}}_A}(y)}{2},\widetilde{\beta}\right\}$. Now,

$$\begin{aligned}\min\{\widetilde{\varsigma^{\mathcal{F}}_A}(x\diamondsuit_1 y),\widetilde{\alpha}\}&=\sup_{i\in I}\{\min\{\widetilde{\varsigma^{\mathcal{F}}_{V_i}}(x\diamondsuit_1 y),\widetilde{\alpha}\}\}\\&\leq \sup_{i\in I}\{\max\{\widetilde{\varsigma^{\mathcal{F}}_{V_i}}(x),\widetilde{\varsigma^{\mathcal{F}}_{V_i}}(y),\widetilde{\beta}\}\}\\&=\max\left\{\sup_{i\in I}\{\widetilde{\varsigma^{\mathcal{F}}_{V_i}}(x)\},\sup_{i\in I}\{\widetilde{\varsigma^{\mathcal{F}}_{V_i}}(y)\},\widetilde{\beta}\right\}\\&=\max\{\widetilde{\varsigma^{\mathcal{F}}_A}(x),\widetilde{\varsigma^{\mathcal{F}}_A}(y),\widetilde{\beta}\}.\end{aligned}$$

Similarly, $\min\{\widetilde{\varsigma^{\mathcal{F}}_A}(x\diamondsuit_2 y),\widetilde{\alpha}\}\leq\max\{\widetilde{\varsigma^{\mathcal{F}}_A}(x),\widetilde{\varsigma^{\mathcal{F}}_A}(y),\widetilde{\beta}\}$ and $\min\{\widetilde{\varsigma^{\mathcal{F}}_A}(x\diamondsuit_3 y),\widetilde{\alpha}\}\leq\max\{\widetilde{\varsigma^{\mathcal{F}}_A}(x),\widetilde{\varsigma^{\mathcal{F}}_A}(y),\widetilde{\beta}\}$. Hence $\widetilde{A}$ is an (α,β)-IVNSBS of $\mathcal{S}$. □

If $\widetilde{A}$ and $\widetilde{B}$ are (α,β)-IVNSBSs of bisemirings $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively, then $\widetilde{A\times B}$ is an (α,β)-IVNSBS of $\mathcal{S}_1\times\mathcal{S}_2$.

Proof. Let $\widetilde{A}$ and $\widetilde{B}$ be two (α,β)-IVNSBSs of $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively. Let $(x_1,y_1),(x_2,y_2)\in\mathcal{S}_1\times\mathcal{S}_2$. Now,

$$\begin{aligned}\max\left\{\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}[(x_1,y_1)\diamondsuit_1(x_2,y_2)],\widetilde{\alpha}\right\}&=\max\left\{\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x_1\diamondsuit_1 x_2,y_1\diamondsuit_1 y_2),\widetilde{\alpha}\right\}\\&=\min\Big\{\max\{\widetilde{\varsigma^{\mathcal{T}}_A}(x_1\diamondsuit_1 x_2),\widetilde{\alpha}\},\max\{\widetilde{\varsigma^{\mathcal{T}}_B}(y_1\diamondsuit_1 y_2),\widetilde{\alpha}\}\Big\}\\&\geq\min\Big\{\min\{\widetilde{\varsigma^{\mathcal{T}}_A}(x_1),\widetilde{\varsigma^{\mathcal{T}}_A}(x_2),\widetilde{\beta}\},\min\{\widetilde{\varsigma^{\mathcal{T}}_B}(y_1),\widetilde{\varsigma^{\mathcal{T}}_B}(y_2),\widetilde{\beta}\}\Big\}\\&=\min\Big\{\{\min\{\widetilde{\varsigma^{\mathcal{T}}_A}(x_1),\widetilde{\varsigma^{\mathcal{T}}_B}(y_1)\},\min\{\widetilde{\varsigma^{\mathcal{T}}_A}(x_2),\widetilde{\varsigma^{\mathcal{T}}_B}(y_2)\}\},\widetilde{\beta}\Big\}\\&=\min\left\{\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x_1,y_1),\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x_2,y_2),\widetilde{\beta}\right\}.\end{aligned}$$

Also,

$$\max\left\{\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}[(x_1,y_1)\diamondsuit_2(x_2,y_2)],\widetilde{\alpha}\right\}\geq\min\left\{\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x_1,y_1),\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x_2,y_2),\widetilde{\beta}\right\}$$

and

$$\max\left\{\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}[(x_1,y_1)\diamondsuit_3(x_2,y_2)],\widetilde{\alpha}\right\}\geq\min\left\{\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x_1,y_1),\widetilde{\varsigma^{\mathcal{T}}_{A\times B}}(x_2,y_2),\widetilde{\beta}\right\}.$$

Now,

$$\begin{aligned}\max\left\{\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}[(x_1,y_1)\diamondsuit_1(x_2,y_2)],\widetilde{\alpha}\right\}&=\max\left\{\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_1\diamondsuit_1 x_2,y_1\diamondsuit_1 y_2),\widetilde{\alpha}\right\}\\&=\frac{1}{2}\Big\{\max\left\{\widetilde{\varsigma^{\mathcal{I}}_A}(x_1\diamondsuit_1 x_2),\widetilde{\alpha}\right\}+\max\left\{\widetilde{\varsigma^{\mathcal{I}}_B}(y_1\diamondsuit_1 y_2),\widetilde{\alpha}\right\}\Big\}\\&\geq\frac{1}{2}\Big\{\min\left\{\frac{\widetilde{\varsigma^{\mathcal{I}}_A}(x_1)+\widetilde{\varsigma^{\mathcal{I}}_A}(x_2)}{2},\widetilde{\beta}\right\}+\min\left\{\frac{\widetilde{\varsigma^{\mathcal{I}}_B}(y_1)+\widetilde{\varsigma^{\mathcal{I}}_B}(y_2)}{2},\widetilde{\beta}\right\}\Big\}\\&=\min\left\{\frac{1}{2}\Big\{\frac{\widetilde{\varsigma^{\mathcal{I}}_A}(x_1)+\widetilde{\varsigma^{\mathcal{I}}_B}(y_1)}{2}+\frac{\widetilde{\varsigma^{\mathcal{I}}_A}(x_2)+\widetilde{\varsigma^{\mathcal{I}}_B}(y_2)}{2}\Big\},\widetilde{\beta}\right\}\\&=\min\left\{\frac{\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_1,y_1)+\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_2,y_2)}{2},\widetilde{\beta}\right\}.\end{aligned}$$

Also,

$$\max\left\{\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}[(x_1,y_1)\diamondsuit_2(x_2,y_2)],\widetilde{\alpha}\right\}\geq\min\left\{\frac{\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_1,y_1)+\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_2,y_2)}{2},\widetilde{\beta}\right\}$$

and

$$\max\left\{\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}[(x_1,y_1)\diamondsuit_3(x_2,y_2)],\widetilde{\alpha}\right\}\geq\min\left\{\frac{\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_1,y_1)+\widetilde{\varsigma^{\mathcal{I}}_{A\times B}}(x_2,y_2)}{2},\widetilde{\beta}\right\}.$$

https://doi.org/10.54216/IJNS.190109

Received: March 08, 2022 Accepted: September 16, 2022

Similarly,

$$\begin{aligned}\min\left\{\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}[(x_1,y_1)\diamondsuit_1(x_2,y_2)],\widetilde{\alpha}\right\} &= \min\left\{\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_1\diamondsuit_1 x_2, y_1\diamondsuit_1 y_2),\widetilde{\alpha}\right\}\\ &= \max\left\{\min\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x_1\diamondsuit_1 x_2),\widetilde{\alpha}\},\min\{\widetilde{\varsigma^{\mathcal{F}}_{B}}(y_1\diamondsuit_1 y_2),\widetilde{\alpha}\}\right\}\\ &\leq \max\left\{\max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x_1),\widetilde{\varsigma^{\mathcal{F}}_{A}}(x_2),\widetilde{\beta}\},\max\{\widetilde{\varsigma^{\mathcal{F}}_{B}}(y_1),\widetilde{\varsigma^{\mathcal{F}}_{B}}(y_2),\widetilde{\beta}\}\right\}\\ &= \max\left\{\{\max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x_1),\widetilde{\varsigma^{\mathcal{F}}_{B}}(y_1)\},\max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x_2),\widetilde{\varsigma^{\mathcal{F}}_{B}}(y_2)\}\},\widetilde{\beta}\right\}\\ &= \max\left\{\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_1,y_1),\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_2,y_2),\widetilde{\beta}\right\}.\end{aligned}$$

Also,

$$\min\left\{\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}[(x_1,y_1)\diamondsuit_2(x_2,y_2)],\widetilde{\alpha}\right\} \leq \max\left\{\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_1,y_1),\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_2,y_2),\widetilde{\beta}\right\}$$

and

$$\min\left\{\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}[(x_1,y_1)\diamondsuit_3(x_2,y_2)],\widetilde{\alpha}\right\} \leq \max\left\{\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_1,y_1),\widetilde{\varsigma^{\mathcal{F}}_{A\times B}}(x_2,y_2),\widetilde{\beta}\right\}.$$

Hence $\widetilde{A\times B}$ is an (α,β)-IVNSBS s of $\mathcal{S}$. □

If $\widetilde{A_1},\widetilde{A_2},...,\widetilde{A_n}$ are (α,β)-IVNSBSs of bisemirings $\mathcal{S}_1,\mathcal{S}_2,...,\mathcal{S}_n$ respectively, then $\widetilde{A_1\times A_2\times ...\times A_n}$ is an (α,β)-IVNSBS of $\mathcal{S}_1\times\mathcal{S}_2\times ...\times\mathcal{S}_n$.

Let $\widetilde{A}$ be an (α,β)-IVN subset in $\mathcal{S}$, the (α,β)-SIVNR on $\mathcal{S}$, that is an (α,β)-IVNR on $\widetilde{A}$ is $\widetilde{V}$ given by

$$\begin{cases}\max\{\widetilde{\varsigma^{\mathcal{T}}_{V}}(x,y),\widetilde{\alpha}\} = \min\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(x),\widetilde{\varsigma^{\mathcal{T}}_{A}}(y),\widetilde{\beta}\}\\ \max\{\widetilde{\varsigma^{\mathcal{I}}_{V}}(x,y),\widetilde{\alpha}\} = \min\{\widetilde{\varsigma^{\mathcal{I}}_{A}}(x),\widetilde{\varsigma^{\mathcal{I}}_{A}}(y),\widetilde{\beta}\}\\ \min\{\widetilde{\varsigma^{\mathcal{F}}_{V}}(x,y),\widetilde{\alpha}\} = \max\{\widetilde{\varsigma^{\mathcal{F}}_{A}}(x),\widetilde{\varsigma^{\mathcal{F}}_{A}}(y),\widetilde{\beta}\}.\end{cases}$$

Let $\widetilde{A}$ be an (α,β)-IVNSBS of $\mathcal{S}$ and $\widetilde{V}$ be the (α,β)-SIVNR of $\mathcal{S}$. Then $\widetilde{A}$ is an (α,β)-IVNSBS of $\mathcal{S}$ if and only if $\widetilde{V}$ is an (α,β)-IVNSBS of $\mathcal{S}\times\mathcal{S}$.

Proof. The proof is similar to Theorem 3. □

Let $(\mathcal{S}_1,\circledast_1,\circledast_2,\circledast_3)$ and $(\mathcal{S}_2,\circledcirc_1,\circledcirc_2,\circledcirc_3)$ be bisemirings. The homomorphic image of (α,β)-IVNSBS of $\mathcal{S}_1$ is an (α,β)-IVNSBS of $\mathcal{S}_2$.

Proof. Let $\Xi:\mathcal{S}_1\rightarrow\mathcal{S}_2$ be any homomorphism. Then $\Xi(x\circledast_1 y)=\Xi(x)\circledcirc_1\Xi(y),\Xi(x\circledast_2 y)=\Xi(x)\circledcirc_2\Xi(y)$, and $\Xi(x\circledast_3 y)=\Xi(x)\circledcirc_3\Xi(y)$ for all $x,y\in\mathcal{S}_1$. Let $\widetilde{V}=\Xi(\widetilde{A})$, $\widetilde{A}$ be any (α,β)-IVNSBS of $\mathcal{S}_1$. Let $\Xi(x),\Xi(y)\in\mathcal{S}_2$. Let $x\in\Xi^{-1}(\Xi(x))$ and $y\in\Xi^{-1}(\Xi(y))$ be such that $\widetilde{\varsigma^{\mathcal{T}}_{A}}(x)=\sup\limits_{z\in\Xi^{-1}(\Xi(x))}\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(z)\}$ and $\widetilde{\varsigma^{\mathcal{T}}_{A}}(y)=\sup\limits_{z\in\Xi^{-1}(\Xi(y))}\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(z)\}$. Now,

$$\begin{aligned}\max\left\{\widetilde{\varsigma^{\mathcal{T}}_{V}}(\Xi(x)\circledcirc_1\Xi(y)),\widetilde{\alpha}\right\} &= \max\left\{\sup_{z'\in\Xi^{-1}(\Xi(x)\circledcirc_1\Xi(y))}\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(z')\},\widetilde{\alpha}\right\}\\ &= \max\left\{\sup_{z'\in\Xi^{-1}(\Xi(x\circledast_1 y))}\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(z')\},\widetilde{\alpha}\right\}\\ &= \max\left\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(x\circledast_1 y),\widetilde{\alpha}\right\}\\ &\geq \min\left\{\widetilde{\varsigma^{\mathcal{T}}_{A}}(x),\widetilde{\varsigma^{\mathcal{T}}_{A}}(y),\widetilde{\beta}\right\}\\ &= \min\left\{\widetilde{\varsigma^{\mathcal{T}}_{V}}\Xi(x),\widetilde{\varsigma^{\mathcal{T}}_{V}}\Xi(y),\widetilde{\beta}\right\}.\end{aligned}$$

https://doi.org/10.54216/IJNS.190109
Received: March 08, 2022 Accepted: September 16, 2022

Thus $\max\left\{\widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x)\circledcirc_1\Xi(y)),\widetilde{\alpha}\right\}\geq\min\left\{\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(x),\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(y),\widetilde{\beta}\right\}$. Similarly, $\max\left\{\widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x)\circledcirc_2\Xi(y)),\widetilde{\alpha}\right\}\geq\min\left\{\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(x),\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(y),\widetilde{\beta}\right\}$ and $\max\left\{\widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x)\circledcirc_3\Xi(y)),\widetilde{\alpha}\right\}\geq\min\left\{\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(x),\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(y),\widetilde{\beta}\right\}$. Let $\Xi(x),\Xi(y)\in\mathcal{S}_2$. Let $x\in\Xi^{-1}(\Xi(x))$ and $y\in\Xi^{-1}(\Xi(y))$ be such that $\widetilde{\varsigma_A^{\mathcal{I}}}(x)=\sup_{z\in\Xi^{-1}(\Xi(x))}\{\widetilde{\varsigma_A^{\mathcal{I}}}(z)\}$ and $\widetilde{\varsigma_A^{\mathcal{I}}}(y)=\sup_{z\in\Xi^{-1}(\Xi(y))}\{\widetilde{\varsigma_A^{\mathcal{I}}}(z)\}$. Now,

$$\begin{aligned}\max\left\{\widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x)\circledcirc_1\Xi(y)),\widetilde{\alpha}\right\}&=\max\Big\{\sup_{z'\in\Xi^{-1}(\Xi(x)\circledcirc_1\Xi(y))}\{\widetilde{\varsigma_A^{\mathcal{I}}}(z')\},\widetilde{\alpha}\Big\}\\&=\max\Big\{\sup_{z'\in\Xi^{-1}(\Xi(x\circledast_1 y))}\{\widetilde{\varsigma_A^{\mathcal{I}}}(z')\},\widetilde{\alpha}\Big\}\\&=\max\left\{\widetilde{\varsigma_A^{\mathcal{I}}}(x\circledast_1 y),\widetilde{\alpha}\right\}\\&\geq\min\left\{\frac{\widetilde{\varsigma_A^{\mathcal{I}}}(x)+\widetilde{\varsigma_A^{\mathcal{I}}}(y)}{2},\widetilde{\beta}\right\}\\&=\min\left\{\frac{\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(x)+\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(y)}{2},\widetilde{\beta}\right\}.\end{aligned}$$

Thus $\max\left\{\widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x)\circledcirc_1\Xi(y)),\widetilde{\alpha}\right\}\geq\min\left\{\frac{\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(x)+\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(y)}{2},\widetilde{\beta}\right\}$. Similarly,

$$\max\left\{\widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x)\circledcirc_2\Xi(y)),\widetilde{\alpha}\right\}\geq\min\left\{\frac{\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(x)+\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(y)}{2},\widetilde{\beta}\right\}$$

and

$$\max\left\{\widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x)\circledcirc_3\Xi(y)),\widetilde{\alpha}\right\}\geq\min\left\{\frac{\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(x)+\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(y)}{2},\widetilde{\beta}\right\}.$$

Let $x\in\Xi^{-1}(\Xi(x))$ and $y\in\Xi^{-1}(\Xi(y))$ be such that $\widetilde{\varsigma_A^{\mathcal{F}}}(x)=\inf_{z\in\Xi^{-1}(\Xi(x))}\{\widetilde{\varsigma_A^{\mathcal{F}}}(z)\}$ and $\widetilde{\varsigma_A^{\mathcal{F}}}(y)=\inf_{z\in\Xi^{-1}(\Xi(y))}\{\widetilde{\varsigma_A^{\mathcal{F}}}(z)\}$. Then

$$\begin{aligned}\min\left\{\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x)\circledcirc_1\Xi(y)),\widetilde{\alpha}\right\}&=\min\Big\{\inf_{z'\in\Xi^{-1}(\Xi(x)\circledcirc_1\Xi(y))}\{\widetilde{\varsigma_A^{\mathcal{F}}}(z')\},\widetilde{\alpha}\Big\}\\&=\min\Big\{\inf_{z'\in\Xi^{-1}(\Xi(x\circledast_1 y))}\{\widetilde{\varsigma_A^{\mathcal{F}}}(z')\},\widetilde{\alpha}\Big\}\\&=\min\left\{\widetilde{\varsigma_A^{\mathcal{F}}}(x\circledast_1 y),\widetilde{\alpha}\right\}\\&\leq\max\left\{\widetilde{\varsigma_A^{\mathcal{F}}}(x),\widetilde{\varsigma_A^{\mathcal{F}}}(y),\widetilde{\beta}\right\}\\&=\max\left\{\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(x),\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(y),\widetilde{\beta}\right\}.\end{aligned}$$

Thus $\min\left\{\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x)\circledcirc_1\Xi(y)),\widetilde{\alpha}\right\}\leq\max\left\{\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(x),\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(y),\widetilde{\beta}\right\}$. Similarly, $\min\left\{\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x)\circledcirc_2\Xi(y)),\widetilde{\alpha}\right\}\leq\max\left\{\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(x),\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(y),\widetilde{\beta}\right\}$ and $\min\left\{\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x)\circledcirc_3\Xi(y)),\widetilde{\alpha}\right\}\leq\max\left\{\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(x),\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(y),\widetilde{\beta}\right\}$. Hence $\widetilde{V}$ is an (α,β)-IVNSBS of $\mathcal{S}_2$. □

Let $(\mathcal{S}_1,\circledast_1,\circledast_2,\circledast_3)$ and $(\mathcal{S}_2,\circledcirc_1,\circledcirc_2,\circledcirc_3)$ be bisemirings. The homomorphic preimage of (α,β)-IVNSBS of $\mathcal{S}_2$ is an (α,β)-IVNSBS of $\mathcal{S}_1$.

Proof. Let $\Xi:\mathcal{S}_1\to\mathcal{S}_2$ be any homomorphism. Then $\Xi(x\circledast_1 y)=\Xi(x)\circledcirc_1\Xi(y),\Xi(x\circledast_2 y)=\Xi(x)\circledcirc_2\Xi(y)$, and $\Xi(x\circledast_3 y)=\Xi(x)\circledcirc_3\Xi(y)$ for all $x,y\in\mathcal{S}_1$. Let $\widetilde{V}=\Xi(\widetilde{A})$, where $\widetilde{V}$ is any (α,β)-IVNSBS of $\mathcal{S}_2$. Let $x,y\in\mathcal{S}_1$. Then $\max\{\widetilde{\varsigma_A^{\mathcal{T}}}(x\circledast_1 y),\widetilde{\alpha}\}=\max\{\widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x\circledast_1 y)),\widetilde{\alpha}\}=\max\{\widetilde{\varsigma_V^{\mathcal{T}}}(\Xi(x)\circledcirc_1\Xi(y)),\widetilde{\alpha}\}\geq\min\{\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(x),\widetilde{\varsigma_V^{\mathcal{T}}}\Xi(y),\widetilde{\beta}\}=\min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x),\widetilde{\varsigma_A^{\mathcal{T}}}(y),\widetilde{\beta}\}$. Thus,

$$\max\{\widetilde{\varsigma_A^{\mathcal{T}}}(x\circledast_1 y),\widetilde{\alpha}\}\geq\min\{\widetilde{\varsigma_A^{\mathcal{T}}}(x),\widetilde{\varsigma_A^{\mathcal{T}}}(y),\widetilde{\beta}\}.$$

Received: March 08, 2022 Accepted: September 16, 2022

Now, $\max\{\widetilde{\varsigma_A^{\mathcal{I}}}(x \circledast_1 y), \widetilde{\alpha}\} = \max\{\widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x \circledast_1 y)), \widetilde{\alpha}\} = \max\{\widetilde{\varsigma_V^{\mathcal{I}}}(\Xi(x) \circledcirc_1 \Xi(y)), \widetilde{\alpha}\} \geq \min\{\widetilde{\varsigma_V^{\mathcal{I}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{I}}}\Xi(y), \widetilde{\beta}\} = \min\{\widetilde{\varsigma_A^{\mathcal{I}}}(x), \widetilde{\varsigma_A^{\mathcal{I}}}(y), \widetilde{\beta}\}$. Thus,

$$\max\{\widetilde{\varsigma_A^{\mathcal{I}}}(x \circledast_1 y), \widetilde{\alpha}\} \geq \min\{\widetilde{\varsigma_A^{\mathcal{I}}}(x), \widetilde{\varsigma_A^{\mathcal{I}}}(y), \widetilde{\beta}\}.$$

Now, $\min\{\widetilde{\varsigma_A^{\mathcal{F}}}(x \circledast_1 y), \widetilde{\alpha}\} = \min\{\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x \circledast_1 y)), \widetilde{\alpha}\} = \min\{\widetilde{\varsigma_V^{\mathcal{F}}}(\Xi(x) \circledcirc_1 \Xi(y)), \widetilde{\alpha}\} \leq \max\{\widetilde{\varsigma_V^{\mathcal{F}}}\Xi(x), \widetilde{\varsigma_V^{\mathcal{F}}}\Xi(y), \widetilde{\beta}\} = \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y), \widetilde{\beta}\}$. Thus,

$$\min\{\widetilde{\varsigma_A^{\mathcal{F}}}(x \circledast_1 y), \widetilde{\alpha}\} \leq \max\{\widetilde{\varsigma_A^{\mathcal{F}}}(x), \widetilde{\varsigma_A^{\mathcal{F}}}(y), \widetilde{\beta}\}.$$

In a similar manner for the other two operations. Hence $\widetilde{A}$ is an (α, β)-IVNSBS of $\mathcal{S}_1$. □

5 (α, β)-IVNNSBSs

Let $\widetilde{A}$ be any IVN subset of $\mathcal{S}$ is called an IVNNSBS of $\mathcal{S}$ if for each $x, y \in \mathcal{S}$,

$$\begin{cases} \widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_1 y) = \widetilde{\varsigma_A^{\mathcal{T}}}(y\diamondsuit_1 x) \\ \widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_2 y) = \widetilde{\varsigma_A^{\mathcal{T}}}(y\diamondsuit_2 x) \\ \widetilde{\varsigma_A^{\mathcal{T}}}(x\diamondsuit_3 y) = \widetilde{\varsigma_A^{\mathcal{T}}}(y\diamondsuit_3 x) \end{cases} \begin{cases} \widetilde{\varsigma_A^{\mathcal{I}}}(x\diamondsuit_1 y) = \widetilde{\varsigma_A^{\mathcal{I}}}(y\diamondsuit_1 x) \\ \text{or} \\ \widetilde{\varsigma_A^{\mathcal{I}}}(x\diamondsuit_2 y) = \widetilde{\varsigma_A^{\mathcal{I}}}(y\diamondsuit_2 x) \\ \text{or} \\ \widetilde{\varsigma_A^{\mathcal{I}}}(x\diamondsuit_3 y) = \widetilde{\varsigma_A^{\mathcal{I}}}(y\diamondsuit_3 x) \end{cases} \begin{cases} \widetilde{\varsigma_A^{\mathcal{F}}}(x\diamondsuit_1 y) = \widetilde{\varsigma_A^{\mathcal{F}}}(y\diamondsuit_1 x) \\ \widetilde{\varsigma_A^{\mathcal{F}}}(x\diamondsuit_2 y) = \widetilde{\varsigma_A^{\mathcal{F}}}(y\diamondsuit_2 x) \\ \widetilde{\varsigma_A^{\mathcal{F}}}(x\diamondsuit_3 y) = \widetilde{\varsigma_A^{\mathcal{F}}}(y\diamondsuit_3 x). \end{cases}$$

(1) The intersection of a collection of IVNNSBSs of $\mathcal{S}$ is an IVNNSBSs of $\mathcal{S}$.

(2) The intersection of a collection of (α, β)-IVNNSBSs of $\mathcal{S}$ is an (α, β)-IVNNSBS s of $\mathcal{S}$.

Let $\mathcal{S}_1, \mathcal{S}_2, ..., \mathcal{S}_n$ be bisemirings.

(1) If $\widetilde{A_1}, \widetilde{A_2}, ..., \widetilde{A_n}$ are IVNNSBSs of $\mathcal{S}_1, \mathcal{S}_2, ..., \mathcal{S}_n$ respectively, then $A_1 \times \widetilde{A_2 \times} ... \times A_n$ is a IVNNSBS of $\mathcal{S}_1 \times \mathcal{S}_2 \times ... \times \mathcal{S}_n$.

(2) If $\widetilde{A_1}, \widetilde{A_2}, ..., \widetilde{A_n}$ are (α, β)-IVNNSBSs of $\mathcal{S}_1, \mathcal{S}_2, ..., \mathcal{S}_n$ respectively, then $A_1 \times \widetilde{A_2 \times} ... \times A_n$ is an (α, β)-IVNNSBS of $\mathcal{S}_1 \times \mathcal{S}_2 \times ... \times \mathcal{S}_n$.

(1) Let $\widetilde{A}$ be any IVNNSBS of $\mathcal{S}$ and $\widetilde{V}$ be the SIVNR of $\mathcal{S}$. Then $\widetilde{A}$ is a IVNNSBS of $\mathcal{S}$ if and only if $\widetilde{V}$ is a IVNNSBS of $\mathcal{S} \times \mathcal{S}$.

(2) Let $\widetilde{A}$ be any (α, β)-IVNNSBS of $\mathcal{S}$ and $\widetilde{V}$ be the (α, β)-SIVNR of $\mathcal{S}$. Then $\widetilde{A}$ is an (α, β)-IVNNSBS of $\mathcal{S}$ if and only if $\widetilde{V}$ is an (α, β)-IVNNSBS of $\mathcal{S} \times \mathcal{S}$.

Let $(\mathcal{S}_1, \circledast_1, \circledast_2, \circledast_3)$ and $(\mathcal{S}_2, \circledcirc_1, \circledcirc_2, \circledcirc_3)$ be bisemirings.

(1) The homomorphic image of any IVNNSBS of $\mathcal{S}_1$ is a IVNNSBS of $\mathcal{S}_2$.

(2) The homomorphic image of any (α, β)-IVNNSBS of $\mathcal{S}_1$ is an (α, β)-IVNNSBS of $\mathcal{S}_2$.

Let $(\mathcal{S}_1, \circledast_1, \circledast_2, \circledast_3)$ and $(\mathcal{S}_2, \circledcirc_1, \circledcirc_2, \circledcirc_3)$ be bisemirings.

(1) The homomorphic preimage of any IVNNSBS of $\mathcal{S}_2$ is a IVNNSBS of $\mathcal{S}_1$.

(2) The homomorphic preimage of any (α, β)-IVNNSBS of $\mathcal{S}_2$ is an (α, β)-IVNNSBS of $\mathcal{S}_1$.

https://doi.org/10.54216/IJNS.190109
Received: March 08, 2022 Accepted: September 16, 2022

6 Conclusion

We developed the concepts of level sets of an IVNSBS and an IVNNSBS. We introduced an approach for (α, β)-IVNSBSs and IVNNSBSs over bisemirings. Presenting a homomorphism fuzzy subbisemirings of subbisemirings to IVNNSBSs of subbisemirings is the major objective of this work. Therefore, in the future, we should think about the applications of cubic subbisemirings and interval valued soft neutrosophic subbisemirings.

Acknowledgments: We, the authors, would like to thank the reviewers for the many suggestions and it is important that the ideas presented in this article are consistent. This work was supported by the revenue budget in 2022, School of Science, University of Phayao, Thailand.

Conflicts of Interest: The authors declare no conflict of interest.

References

[1] J. Ahsan, K. Saifullah, F. Khan, Fuzzy semirings, Fuzzy Sets and systems, vol. 60, pp. 309–320, 1993.

[2] M. Al-Tahan, B. Davvaz, M. Parimala, A note on single valued neutrosophic sets in ordered groupoids, International Journal of Neutrosophic Science, vol. 10, no. 2, pp. 73–83, 2020.

[3] K. Arulmozhi, The algebraic theory of semigroups and semirings, Lap Lambert Academic Publishing, Mauritius, 2019.

[4] S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani, T. Mahmood, Spherical fuzzy sets and their applications in multi-attribute decision making problems, Journal of Intelligent and Fuzzy Systems, vol. 36, pp. 2829–2844, 2019.

[5] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 20, no. 1, pp. 87–96, 1986.

[6] B. C. Cuong, V. Kreinovich, Picture fuzzy sets a new concept for computational intelligence problems, Proceedings of 2013 Third World Congress on Information and Communication Technologies (WICT 2013), IEEE, pp. 1–6, 2013.

[7] S. J. Golan, Semirings and their applications, Kluwer Academic Publishers, London, 1999.

[8] F. Hussian, R. M. Hashism, A. Khan, M. Naeem, Generalization of bisemirings, International Journal of Computer Science and Information Security, vol. 14, no. 9, pp. 275–289, 2016.

[9] A. Iampan, P. Jayaraman, S. D. Sudha, N. Rajesh, Interval-valued neutrosophic ideals of Hilbert algebras, International Journal of Neutrosophic Science, vol. 18, no. 4, pp. 223–237, 2022.

[10] A. Iampan, P. Jayaraman, S. D. Sudha, N. Rajesh, Interval-valued neutrosophic subalgebras of Hilbert algebras, Asia Pacific Journal of Mathematics, vol. 9, Article no. 16, 2022.

[11] L. Jagadeeswari, V. J. Sudhakar, V. Navaneethakumar, S. Broumi, Certain kinds of bipolar interval valued neutrosophic graphs, International Journal of Neutrosophic Science, vol. 16, no. 1, pp. 49–61, 2021.

[12] M. Palanikumar, K. Arulmozhi, On various ideals and its applications of bisemirings, Gedrag and Organisatie Review, vol. 33, no, 2, pp. 522–533, 2020.

[13] M. Palanikumar, K. Arulmozhi, On intuitionistic fuzzy normal subbisemirings of bisemirings, Nonlinear Studies, vol. 28, no. 3, pp. 717–721, 2021.

[14] M. Palanikumar, K. Arulmozhi, On new ways of various ideals in ternary semigroups, Matrix Science Mathematic, vol. 4, no. 1, pp. 6–9, 2020.

[15] M. Palanikumar, K. Arulmozhi, (α, β)-Neutrosophic subbisemiring of bisemiring, Neutrosophic Sets and Systems, vol. 48, pp. 368–385, 2022.

https://doi.org/10.54216/IJNS.190109
Received: March 08, 2022 Accepted: September 16, 2022

[16] M. Palanikumar, K. Arulmozhi, On various tri-ideals in ternary semirings, Bulletin of the International Mathematical Virtual Institute, vol. 11, no. 1, pp. 79–90, 2021.

[17] M. Palanikumar, K. Arulmozhi, On Pythagorean normal subbisemiring of bisemiring, Annals of Communications in Mathematics, vol. 4, no. 1, pp. 63–72, 2021.

[18] M. Palanikumar, K. Arulmozhi, On various almost ideals of semirings, Annals of Communications in Mathematics, vol. 4, no. 1, pp. 17–25, 2021.

[19] M. K. Sen, S. Ghosh, An introduction to bisemirings, Asian Bulletin of Mathematics, vol. 28, no. 3, pp. 547–559, 2001.

[20] F. Smarandache, A unifying field in logics. Neutrosophy: neutrosophic probability, set and logic, American Research Press, Rehoboth, 1999.

[21] R. R. Yager, Pythagorean membership grades in multi criteria decision-making, IEEE Transactions on Fuzzy Systems, vol. 22, no. 4, pp. 958–965, 2014.

[22] L. A. Zadeh, Fuzzy sets, Information and Control, vol. 8, no. 3, pp. 338–353, 1965.

https://doi.org/10.54216/IJNS.190109

Received: March 08, 2022 Accepted: September 16, 2022