On Some Novel Results About Neutrosophic Square Complex Matrices

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Abstract

The objective of this paper is to study algebraic properties of complex neutrosophic matrices, where a necessary and sufficient condition for the invertibility of a complex square neutrosophic matrix is presented by defining the complex neutrosophic determinant. On the other hand, this work introduces the concept of neutrosophic characteristic polynomial and neutrosophic Cayley-Hamilton theorem for the complex case.

Keywords: Neutrosophic complex matrix; neutrosophic real number; neutrosophic determinant; neutrosophic inverse

1. Introduction

2. Preliminaries

Definition 2.1 [28]: Classical neutrosophic number has the form $a + bI$ where a, b are real or complex numbers and I is the indeterminacy such that $0 \cdot I = 0$ and $I^2 = I$ which results that $I^n = I$ for all positive integers n.

Definition 2.2 [29]: Let $w_1 = a_1 + b_1 I$, $w_2 = a_2 + b_2 I$ Then we have:

$$\frac{w_1}{w_2} = \frac{a_1}{a_2} + \frac{a_1 b_2 - a_2 b_1}{a_2(a_2 + b_2)}$$

Definition 2.3 [10]: Let Kbe a field, the neutrosophic file generated by $\langle K \cup I \rangle$ which is denoted by $K(I) = \langle K \cup I \rangle$.

Definition 2.4 (Neutrosophic complex matrix) [16]. Let $M_{m\times n} = \{(a_{ij}): a_{ij} \in K(I)\}$,where $K(I)$ is the neutrosophic complex field. We call to be the neutrosophic complex matrix.

Definition 2.4 : Let $M_{m\times n}$ is a neutrosophiccomplex matrix. We call to be the neutrosophic square complex matrix if $m = n$.

Now a neutrosophic n square complex matrix is defined by form $M = A + BI$where A and B are two$n$ squares complex matrices.

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3. Main discussion

Definition 3.1:

Let $M = A + BI$ be a neutrosophic n squarecomplex matrix. The determinant of M is defined as

$detM = detA + I[det(A + B) - detA]$.

Definition 3.2:

Let $M = A + BI$ a neutrosophic square $n \times n$ matrix, where , B are two squares $n \times n$complex matrices, then M is invertible if and only if$A$ and $A + B$ are invertible matrices and

$M^{-1} = A^{-1} + I[(A + B)^{-1} - A^{-1}]$.

Theorem 3.3:

M is invertible matrix if and only if $detM\ is\ invertible$.

Proof:

From **Definition 3.2** we find that M is invertible matrix if and only if $A + B, A$ are two invertible matrices, hence $det[A + B] \neq 0, detA \neq 0$ which means

$detM = detA + I[\det(A + B) - detA]\ is\ invertible$.

Example 3.4:

Consider the following neutrosophic complex matrix

$M = A + BI = \begin{pmatrix} i & -1 \\ 0 & 1 - i \end{pmatrix} + I \begin{pmatrix} 0 & i \\ -i & -1 + i \end{pmatrix}$.

(a)$detA = 1 + i, A + B = \begin{pmatrix} i & -1 + i \\ -i & 0 \end{pmatrix}, \det(A + B) = -1 - i, detM = 1 + i + (-2 - 2i)I \neq 0,$ hence M is invertible.

(b) We have:

$$A^{-1} = \begin{pmatrix} -i & \frac{1}{2} - \frac{i}{2} \\ 0 & \frac{1}{2} + \frac{i}{2} \end{pmatrix}, (A + B)^{-1} = \begin{pmatrix} 0 & i \\ -\frac{1}{2} - \frac{i}{2} & -\frac{1}{2} - \frac{i}{2} \end{pmatrix},$$

$$thus\ M^{-1} = (A^{-1}) + I[(A + B)^{-1} - A^{-1}] = \begin{pmatrix} -i & \frac{1}{2} - \frac{i}{2} \\ 0 & \frac{1}{2} + \frac{i}{2} \end{pmatrix} + I \begin{pmatrix} i & -\frac{1}{2} + \frac{3i}{2} \\ -\frac{1}{2} - \frac{i}{2} & -1 - i \end{pmatrix}.$$

(c) We can compute $MM^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = U_{2\times 2}$.

Definition 3.5:

Let $M = A + BI$ be a neutrosophic n square complex matrix, were A and B are two n square complex matrices, then

$M^T = A^T + I[(A + B)^T - A^T]$.

Definition 3.6:

Let $M = A + BI$ be a neutrosophic n squarecomplex matrix, were A and B are two n square complexmatrices, then

$M^r = A^r + I[(A + B)^r - A^r]$.

Remark 3.7:

Let $M = A + BI$ and $N = C + DI$ be two neutrosophic n squarecomplexmatrices, then

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(3.7.1) $det(M \cdot N) = detM \cdot detN$.

(3.7.2) $\det(M^{-1}) = (detM)^{-1}$.

(3.7.3) $detM = detM^T$.

Remark: The result in the section (c) can be generalized easily to the following fact:

$detM = detA$ if and only if $detA = det(A + B)$.

Definition 3.8:

Let $M = A + BI$ be a neutrosophic n square complex matrix, where A and B are two n square complex matrices, And $Z = X + YI$. We define the neutrosophic characteristic polynomial by the form:

$$\varphi(Z) = det[ZU_{n\times n} - M] = det[ZU_{n\times n} - (A + BI)] = det[(ZU_{n\times n} - A) + (-B)I]$$

$$\varphi(Z) = det(ZU_{n\times n} - A) + I\left[det\left(ZU_{n\times n} - (A + B)\right) - det(ZU_{n\times n} - A)\right]$$

$$\varphi(Z) = \alpha(Z) + I\left[\beta(Z) - \alpha(Z)\right].$$

Where:

$$\alpha(Z) = det(ZU_{n\times n} - A)\,, \beta(Z) = det\left(ZU_{n\times n} - (A + B)\right)$$

Example 3.9:

Consider the following neutrosophic complex matrix

$M = A + BI = \begin{pmatrix} i & -1 \\ 0 & 1-i \end{pmatrix} + I\begin{pmatrix} 0 & i \\ -i & -1+i \end{pmatrix}, A + B = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$. Then.

$$\varphi(Z) = \alpha(Z) + I\left[\beta(Z) - \alpha(Z)\right]$$

$$\alpha(Z) = det(ZU_{2\times 2} - A) = \begin{vmatrix} Z-i & -1 \\ 0 & Z-(1-i) \end{vmatrix}$$

$$\alpha(Z) = Z^2 - (1-i)Z - iZ + 1 + i = Z^2 - Z + (1+i)$$

$$\beta(Z) = Z^2 - iZ - i - 1$$

Then.

$$\varphi(Z) = \alpha(Z) + I\left[\beta(Z) - \alpha(Z)\right] = Z^2 - Z + (1+i) + I[(1-i)Z - 2 - 2i]$$

Theorem 3.10:

A neutrosophic characteristic polynomial of neutrosophic square complex matrix is equal a neutrosophic characteristic polynomial of its transpose.

Proof:

Let $M = A + BI$ be a neutrosophic n squarecomplex matrix, where A and B are two n squarecomplexmatrices.

Let$\varphi(Z) = \alpha(Z) + I\left[\beta(Z) - \alpha(Z)\right]$a neutrosophic characteristic polynomial for $M$and $M^T$is transpose for M.

Let $\psi(Z)$ be the neutrosophic characteristic polynomial of M^T, then

$$\varphi(Z) = det[ZU_{n\times n} - M]$$

$$\varphi(Z) = det(ZU_{n\times n} - A) + I\left[det\left(ZU_{n\times n} - (A + B)\right) - det(ZU_{n\times n} - A)\right]$$

Now we have.

$$\psi(Z) = det[ZU_{n\times n} - M]^T = det[(ZU_{n\times n} - A) + (-B)I]^T$$

$$\psi(Z) = \det\left[(ZU_{n\times n} - A)^T + I\left[\left(ZU_{n\times n} - (A + B)\right)^T - (ZU_{n\times n} - A)^T\right]\right]$$

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$$\psi(Z) = \det(ZU_{n\times n} - A)^T + I\left[det\left(ZU_{n\times n} - (A+B)\right)^T - det(ZU_{n\times n} - A)^T\right]$$

By **Remark 3.7**we have.

$$[det(ZU_{n\times n} - A)]^T = det(ZU_{n\times n} - A)$$

$$det\left(ZU_{n\times n} - (A+B)\right)^T = det\left(ZU_{n\times n} - (A+B)\right)$$

Then.

$$\psi(Z) = det(ZU_{n\times n} - A) + I\left[det\left(ZU_{n\times n} - (A+B)\right) - det(ZU_{n\times n} - A)\right]$$

Then.

$$\varphi(Z) = \psi(Z)$$

Example 3.11:

Consider the neutrosophic matrix defined in Example 3.9, we have:

$\varphi(Z) = Z^2 - Z + (1+i) + I[(1-i)Z - 2 - 2i]$

Now.

$A^T = \begin{pmatrix} i & 0 \\ -1 & 1-i \end{pmatrix}, B^T = \begin{pmatrix} 0 & -i \\ i & -1+i \end{pmatrix}$ Then.

$$\psi(Z) = \alpha^*(Z) + I\left[\beta^*(Z) - \alpha^*(Z)\right]$$

$$\alpha^*(Z) = det(ZU_{2\times 2} - A^T) = \begin{vmatrix} Z-i & 0 \\ -1 & Z-(1-i) \end{vmatrix}$$

$$\alpha^*(Z) = Z^2 - Z + (1+i)$$

$$\beta^*(Z) = det(ZU_{2\times 2} - (A+B)^T) = \begin{vmatrix} Z & i \\ -i & Z-(-1+i) \end{vmatrix}$$

$$\beta^*(Z) = Z^2 - iZ - i - 1$$

Then.

$$\varphi(Z) = \psi(Z) = Z^2 - Z + (1+i) + I[(1-i)Z - 2 - 2i]$$

Theorem 3.12:(Neutrosophic complex Cayely-Hamilton): Any neutrosophic square complex matrix is a root of its a neutrosophic characteristic polynomial.

Example 3.13:

Consider the neutrosophic matrix defined in Example 3.9, we have:

$\varphi(Z) = Z^2 - Z + (1+i) + I[(1-i)Z - 2 - 2i]$

Now we find $\varphi(M)$.

$$\varphi(M) = M^2 - M + (1+i)U_{2\times 2} + (1-i)MI + (-2-2i)U_{2\times 2}I$$

$$M^2 = A^2 + I[(A+B)^2 - A^2] = \begin{pmatrix} -1 & -1 \\ 0 & -2i \end{pmatrix} + I\begin{pmatrix} i+1 & -i \\ 1 & 1+3i \end{pmatrix}$$

$$\varphi(M) = \begin{pmatrix} -1 & -1 \\ 0 & -2i \end{pmatrix} + I\begin{pmatrix} i+1 & -i \\ 1 & 1+3i \end{pmatrix} + \begin{pmatrix} -i & 1 \\ 0 & -1+i \end{pmatrix} + I\begin{pmatrix} 0 & -i \\ i & 1-i \end{pmatrix} + \begin{pmatrix} i+1 & 0 \\ 0 & i+1 \end{pmatrix}$$
$$+ I\begin{pmatrix} 1+i & 2i \\ -1-i & 0 \end{pmatrix} + I\begin{pmatrix} -2-2i & 0 \\ 0 & -2-2i \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + I\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\varphi(M) = 0$

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4. Refined neutrosophic matrix

Definition 4.1: The structure of refined neutrosophic numbers is taken as $a + bI_1 + cI_2$ instead of (a, bI_1, cI_2).

Definition 4.2: $I_1{}^2 = I_1$, $I_2{}^2 = I_2$, $I_1.I_2 = I_2.I_1 = I_1$

Definition 4.3: (**Refined neutrosophic complex matrix).**

Let $A = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix}$ be an $m \times n$ matrix: if $a_{ij} = a + bI_1 + cI_2 \in C_2(I)$, then it is called an refined neutrosophic complex matrix, where $C_2(I)$ is an refined neutrosophic complex field.

Example 4.4: Let $A = \begin{pmatrix} 1 + i + (-1 - i)I_1 & (1 + i)I_1 - iI_2 \\ 3 - iI_1 & (2 - i)I_2 \end{pmatrix}$ as a 2×2 refined neutrosophiccomplex matrix.

Theorem 4.5: Let $M = A + BI_1 + CI_2$ be a square $n \times n$refined neutrosophiccomplex matrix; then it is invertible if only of $A, A + C$ and $A + B + C$ are invertible. The inverse of M is

$$M^{-1} = A^{-1} + ((A + B + C)^{-1} - (A + C)^{-1})I_1 + ((A + C)^{-1} - A^{-1})I_2$$

Proof: The proof holds as a special case of invertible elements in refined neutrosophic rings [30].

Definition 4.6:

Let $M = A + BI_1 + CI_2$ be a refinedneutrosophic n square complex matrix, where A, B and c are n square complex matrices, then.

$$M^T = A^T + [(A + B + C)^T - (A + C)^T]I_1 + [(A + C)^T - A^T]I_2.$$

Definition 4.7:

Let $M = A + BI_1 + CI_2$ be a refinedneutrosophic n square complex matrix, where A, B and C are n square complex matrices, then

$$detM = det(A + BI_1 + CI_2) = detA + [det(A + B + C) - det(A + C)]I_1 + [det(A + C) - detA]I_2.$$

Remark 4.8:

(a).If $A$is an$m \times n$matrix, then it can be represented as an element of the refined neutrosophic ring of matrices such as the following: $M = A + BI_1 + CI_2$, where A, B and $C$are complex matrices with elements from ring $C$and from size $m \times n$.

For example, $M = \begin{pmatrix} (-1 - i) + I_1 + 3iI_2 & 1 - (1 - i)I_1 - I_2 \\ 3 + (1 + i)I_2 & 1 + (2 + i)I_1 \end{pmatrix} = \begin{pmatrix} -1 - i & 1 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} 1 & -1 + i \\ 0 & 2 + i \end{pmatrix} I_1 + \begin{pmatrix} 3i & -1 \\ (1 + i) & 0 \end{pmatrix} I_2.$

(b). Multiplication can be defined by using the same representation as a special case multiplication on refined neutrosophic rings as follows:

$$(A + BI_1 + CI_2)(X + YI_1 + ZI_2) = (AX) + (AY + BX + BY + BZ + CY)I_1 + (AZ + CZ + CX)I_2$$

Theorem 4.9:

Let $M = A + BI_1 + CI_2$ be a neutrosophic n square complex matrix, where A, B and C are two n square complex matrices, And $Z = X + YI_1 + TI_2$. We define the neutrosophic characteristic polynomial by form:

$$\varphi(z) = det[ZU_{n\times n} - M] = det[ZU_{n\times n} - (A + BI_1 + CI_2)] = det[(ZU_{n\times n} - A) + (-B)I_1 + (-C)I_2]$$

$$\varphi(z) = det(ZU_{n\times n} - A) + \left[det\left(ZU_{n\times n} - (A + B + C)\right) - det\left(ZU_{n\times n} - (A + C)\right)\right]I_1 + \left[det\left(ZU_{n\times n} - (A + C)\right) - det(ZU_{n\times n} - A)\right]I_2$$

$$\varphi(z) = \alpha(Z) + \left[\beta(Z) - \gamma(Z)\right]I_1 + \left[\gamma(Z) - \alpha(Z)\right]I_2.$$

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Where:

$$\alpha(Z) = det(ZU_{n\times n} - A)\,,\beta(Z) = det\big(ZU_{n\times n} - (A + B + C)\big), \gamma(Z) = det\big(ZU_{n\times n} - (A + C)\big)$$

Example 4.10:

Consider the following neutrosophic matrix

$M = A + BI_1 + CI_2$. Where $A = \begin{pmatrix} 2+i & 1 \\ -i & -1+i \end{pmatrix}$, $B = \begin{pmatrix} 1 & -i \\ 0 & 2i \end{pmatrix}$, $C = \begin{pmatrix} 1-i & -1 \\ i & 0 \end{pmatrix}$

$A + B + C = \begin{pmatrix} 4 & -i \\ 0 & -1+3i \end{pmatrix}$, $A + C = \begin{pmatrix} 3 & 0 \\ 0 & -1+i \end{pmatrix}$.

Then.

$$\varphi(z) = \alpha(Z) + \big[\beta(Z) - \gamma(Z)\big]I_1 + \big[\gamma(Z) - \alpha(Z)\big]I_2$$

$$\alpha(Z) = det(ZU_{n\times n} - A) = \begin{vmatrix} Z - (2+i) & -1 \\ i & Z + (1-3i) \end{vmatrix}$$

$$\alpha(Z) = Z^2 - (1+2i)Z - 3 + 2i$$

$$\beta(Z) = det\big(ZU_{n\times n} - (A + B + C)\big) = \begin{vmatrix} Z - 4 & i \\ 0 & Z + (1-3i) \end{vmatrix}$$

$$\beta(Z) = Z^2 - (1+4i)Z - (5 - 5i)$$

$$\gamma(Z) = det\big(ZU_{n\times n} - (A + C)\big) = \begin{vmatrix} Z - 3 & 0 \\ 0 & Z + (1-i) \end{vmatrix}$$

$$\gamma(Z) = Z^2 - (2+i)Z + (-3 + 3i)$$

Then.

$$\varphi(Z) = \alpha(Z) + \big[\beta(Z) - \gamma(Z)\big]I_1 + \big[\gamma(Z) - \alpha(Z)\big]I_2$$

$$\varphi(Z) = Z^2 - (1+2i)Z - 3 + 2i + [(1-3i)Z + (-2+2i)]I_1 + [(-1+i)Z + i]I_2$$

Conclusion

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References

1] Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 6 , pp. 80-86. 2020.

[2] Abobala, M.,. "A Study of AH-Substructures in *n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.

[3] Sankari, H., and Abobala, M., "Neutrosophic Linear Diophantine Equations With two Variables", Neutrosophic Sets and Systems, Vol. 38, pp. 22-30, 2020.

[4] Sankari, H., and Abobala, M." *n*-Refined Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 36, pp. 1-11. 2020.

[5] Abobala, M., Hatip, A., Bal,M.," A Study Of Some Neutrosophic Clean Rings", International journal of neutrosophic science, 2022.

DOI: https://doi.org/10.54216/JNFS.040103
Received: April 13, 2022 Accepted: August 15, 2022

[6]Smarandache, F., " A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press. Rehoboth, 2003.

[7] Suresh, R., and S. Palaniammal,. "Neutrosophic Weakly Generalized open and Closed Sets", Neutrosophic Sets and Systems, Vol. 33, pp. 67-77,. 2020.

[8] Olgun, N., and Hatip, A., "The Effect Of The Neutrosophic Logic On The Decision Making, in Quadruple Neutrosophic Theory And Applications", Belgium, EU, Pons Editions Brussels,pp. 238-253. 2020.

[9] Hatip, A., Alhamido, R., and Abobala, M., "A Contribution to Neutrosophic Groups", International Journal of Neutrosophic Science", Vol. 0, pp. 67-76 . 2019.

[10] Abobala, M., " *n*-Refined Neutrosophic Groups I", International Journal of Neutrosophic Science, Vol. 0, pp. 27-34. 2020.

[11] Abobala, M., "Classical Homomorphisms Between n-refined Neutrosophic Rings", International Journal of Neutrosophic Science", Vol. 7, pp. 74-78. 2020.

[12] Smarandache, F., and Abobala, M., n-Refined neutrosophic Rings, International Journal of Neutrosophic Science, Vol. 5 , pp. 83-90, 2020.

[13] Abobala, M., On Some Special Substructures of Neutrosophic Rings and Their Properties, International Journal of Neutrosophic Science", Vol. 4 , pp. 72-81, 2020.

[14] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 59-66. 2020.

[15] Sankari, H., and Abobala, M.," AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 38, pp. 101-112, 2020.

[16] Smarandache, F., and Kandasamy, V.W.B., " Finite Neutrosophic Complex Numbers",·Source: arXiv. 2011.

[17] Agboola, A.A.A,. Akwu, A.D,. and Oyebo, Y.T., " Neutrosophic Groups and Subgroups", International .J .Math. Combin, Vol. 3, pp. 1-9. 2012.

[18] Smarandache, F., " *n*-Valued Refined Neutrosophic Logic and Its Applications in Physics", Progress in Physics, 143-146, Vol. 4, 2013.

[19] Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.

[20] Hatip, A., and Abobala, M., "AH-Substructures In Strong Refined Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 9, pp. 110-116 . 2020.

[21] Smarandache F., and Abobala, M., "*n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54. 2020.

[22] Sankari, H., and Abobala, M., "Solving Three Conjectures About Neutrosophic Quadruple Vector Spaces", Neutrosophic Sets and Systems, Vol. 38, pp. 70-77. 2020.

[23] Adeleke, E.O., Agboola, A.A.A., and Smarandache, F., "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science, Vol. 2(2), pp. 89-94. 2020.

[24] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021

DOI: https://doi.org/10.54216/JNFS.040103
Received: April 13, 2022 Accepted: August 15, 2022

[25] Abobala, M., A Study of Maximal and Minimal Ideals of n-Refined Neutrosophic Rings, Journal of Fuzzy Extension and Applications, Vol. 2, pp. 16-22, 2021.

[26] Abobala, M., " Semi Homomorphisms and Algebraic Relations Between Strong Refined Neutrosophic Modules and Strong Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 39, 2021.

[27] Giorgio, N, Mehmood, A., and Broumi, S.," Single Valued neutrosophic Filter", International Journal of Neutrosophic Science, Vol. 6, 2020.

[28] Chellamani, P., and Ajay, D., "Pythagorean neutrosophic Fuzzy Graphs", International Journal of Neutrosophic Science, Vol. 11, 2021.

[29] Milles, S, Barakat, M, and Latrech, A., " Completeness and Compactness In Standard Single Valued neutrosophic Metric Spaces", International Journal of Neutrosophic Science, Vol.12 , 2021.

[30] Abobala, M., "On Some Neutrosophic Algebraic Equations", Journal of New Theory, Vol. 33, 2020.

[31] Abobala, M., On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations, Journal of Mathematics, Hindawi, 2021.

[32] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021

[33] Kandasamy V, Smarandache F., and Kandasamy I., Special Fuzzy Matrices for Social Scientists . Printed in the United States of America,2007, book, 99 pages.

[34] Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 45, 2021.

[35] Abobala, M., Partial Foundation of Neutrosophic Number Theory, Neutrosophic Sets and Systems, Vol. 39 , 2021.

[36] F. Smarandache, *Neutrosophic Theory and Applications*, Le Quy Don Technical University, Faculty of Information technology, Hanoi, Vietnam, 17th May 2016.

[37] Ibrahim, M.A., Agboola, A.A.A, Badmus, B.S. and Akinleye, S.A., "On refined Neutrosophic Vector Spaces I", International Journal of Neutrosophic Science, Vol. 7, pp. 97-109. 2020.

[38] Ibrahim, M.A., Agboola, A.A.A, Badmus, B.S., and Akinleye, S.A., "On refined Neutrosophic Vector Spaces II", International Journal of Neutrosophic Science, Vol. 9, pp. 22-36. 2020.

[39] Abobala, M, "*n*-Cyclic Refined Neutrosophic Algebraic Systems Of Sub-Indeterminacies, An Application To Rings and Modules", International Journal of Neutrosophic Science, Vol. 12, pp. 81-95 . 2020.

[40] Smarandache, F., "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets", Inter. J. Pure Appl. Math., pp. 287-297. 2005.

[41] Abobala, M., "On Some Special Elements In Neutrosophic Rings and Refined Neutrosophic Rings", Journal of New Theory, vol. 33, 2020.

[42] Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., The algebraic creativity In The Neutrosophic Square Matrices, Neutrosophic Sets and Systems, Vol. 40, pp. 1-11, 2021.

[43] Aswad, F, M., " A Study of Neutrosophic Complex Number and Applications", Neutrosophic Knowledge, Vol. 1, 2020.

DOI: https://doi.org/10.54216/JNFS.040103
Received: April 13, 2022 Accepted: August 15, 2022

[44] Aswad. F, M., " A Study of neutrosophic Bi Matrix", Neutrosophic Knowledge, Vol. 2, 2021.

[45] Abobala, M., "Neutrosophic Real Inner Product Spaces", Neutrosophic Sets and Systems, vol. 43, 2021.

[46] Aswad, M., " A Study of The Integration Of Neutrosophic Thick Function", International journal of neutrosophic Science, 2020.

[47] Abobala, M., "A Study Of Nil Ideals and Kothe's Conjecture In Neutrosophic Rings", International Journal of Mathematics and Mathematical Sciences, hindawi, 2021.

[48] Abobala, M., Bal, M., Aswad, M., "A Short Note On Some Novel Applications of Semi Module Homomorphisms", International journal of neutrosophic science, 2022.

[49] Abobala, M., and Hatip, A., "An Algebraic Approach to Neutrosophic Euclidean Geometry", Neutrosophic Sets and Systems, Vol. 43, 2021.

[50] Aswad, M., " A Study Of neutrosophic Differential Equation By using A Neutrosophic Thick Function", neutrosophic knowledge, Vol. 1, 2020.

[51] Olgun, N., Hatip, A., Bal, M., and Abobala, M., " A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined Neutrosophic Matrices", International Journal of Neutrosophic Science, Vol. 16, pp. 72-79, 2021.

[52] Abobala, M., Bal, M., and Hatip, A.," A Review On Recent Advantages In Algebraic Theory Of Neutrosophic Matrices**",** International Journal of Neutrosophic Science, Vol. 17, 2021.

[53] Singh, P,K., " Data With Turiyam Set for Fourth Dimension Quantum Information Processing", Journal of Neutrosophic and Fuzzy Systems, vol.1, 2022.

[54] Singh, P, K., Ahmad, K., Bal, M., Aswad, M.," On The Symbolic Turiyam Rings", Journal of Neutrosophic and Fuzzy Systems, 2022.

[55] Ibrahim, M., and Abobala, M., "An Introduction To Refined Neutrosophic Number Theory", Neutrosophic Sets and Systems, Vol. 45, 2021.

[56] Bal, M.,, Singh, P., Ahmad, K., and Aswad, M., " A Short Introduction To The Concept Of Symbolic Turiyam Matrix", Neutrosophic and Fuzzy Systems, 2022.

[57] Bal, M., Singh, P., and Ahmad, K., " An Introduction To The Symbolic Turiyam R-Modules and Turiyam Modulo Integers", Neutrosophic and Fuzzy Systems, 2022.

[58] Bal, M., Singh, P., and Ahmad, K., " An Introduction To The Symbolic Turiyam Vector Spaces and Complex Numbers", Neutrosophic and Fuzzy Systems, 2022.

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