An Introduction to The Symbolic Turiyam Groups and AH-Substructures

A. Alrida Basheer [*1], Katy D. Ahmad[2], Rozina Ali[3]

[1] Imam Kadhum College, Iraq
[2] Islamic University Of Gaza, Palestine
[3] Cairo University, Egypt

Emails: basheerabdalrida66n@gmail.com ; katyon765@gmail.com ; rozyyy123n@gmail.com

Abstract

The aim of This paper is to define for the first time the concept of symbolic Turiyam group. This work is devoted to study some elementary properties of symbolic Turiyam groups and to establish the algebraic basis of this structure such as symbolic Turiyam subgroups, symbolic Turiyam homomorphisms, and symbolic Turiyam isomorphisms.

Keywords: symbolic Turiyam group; symbolic Turiyam subgroup; symbolic Turiyam homomorphism.

1. Introduction

Turiyam set [77] is a new interesting concept aims to generalize the neutrosophic ideas [1-3]. The symbolic Turiyam set was supposed in [78], with an algebraic application by the symbolic Turiyam ring.

In [80-82], we can see many algebraic structures build over the idea of symbolic Turiyam set such as Turiyam modules, spaces, and matrices to deal the data with Non-Euclidean Geometry [83-84].

In the literature, many authors around the world have studied the algebra of neutrosophic sets, where we find many generalizations of classical algebraic structures such as neutrosophic matrices, neutrosophic groups\rings, and spaces [4-8,11-25,30-42,50-70].

In this work, we put the theoretical basis of the symbolic Turiyam group in a similar way of n-refined neutrosophic groups defined in [10, 27]. Also, we define some related AH-substructures to make this new algebraic structure more understandable.

DOI: https://doi.org/10.54216/JNFS.030205
Received: April 05, 2022 Accepted: July 16, 2022

Main results

Definition:

Let $(G,*)$ be a group, we define the corresponding symbolic Turiyam group $T(G)$ as follows:

$T(G) = (< G \cup \{T, F, I, Y\} >, *) = \{(a_0, a_1T, a_2F, a_3I, a_4Y);\ a_i \in G\}$.

It is easy to see that $T(G)$ is closed under $*$, and it is a semi group but not a group since T has no inverse with respect to $*$ in general.

Remark :

If (G,+) is an additive abelian group, then addition on $T(G)$ can be described as follows:

Consider $x = (a_0, a_1T, a_2F, a_3I, a_4Y), y = (b_0, b_1T, b_2F, b_3I, b_4Y)$, we have

$x + y = (a_0 + b_0, [a_1 + b_1]T, [a_2 + b_2]F, [a_3 + b_{31}]I, [a_4 + b_4]Y)$. In this case $(T(G), +)$ is a classical abelian group.

The identity element is (0,0,0,0,0).

It is easy to see that $T(G) \cong G \times G \times .. \times G$ $(5 -\ times)$ in the case of abelian additive group G.

Example:

Let $G = Z_2$ be the additive group of integers modulo 2, the corresponding symbolic Turiyam group is $T(G) = \{(0,0,0,0,0), (1,0,0,0,0), (1, T, 0,0,0), (1, T, F, 0,0), (1, T, F, I, 0), (1, T, F, I, Y)$

$, (0, T, 0,0,0), (0, T, F, 0,0), (0, T, F, I, 0), (0, T, F, I, Y) \ldots etc\}$.

Remark:

If G is a multiplicative group, then group product on $T(G)$ can be described as follows:

Consider $x = (a_0, a_1T, a_2F, a_3I, a_4Y), y = (b_0, b_1T, b_2F, b_3I, b_4Y)$, we have

و$xy = (t_0, t_1T, t_2F, t_3I, t_4\ Y)$; $\mathrm{t}_0 = a_0b_0, t_1 = a_0b_1a_1b_1$

$t_2 = a_0b_2a_1b_2a_2b_0a_2b_1a_2b_2, t_3 = a_0b_3a_1b_3a_2b_3a_3b_3a_4b_3a_3b_0a_3b_1a_3b_2a_3b_4,$

$t_4 = a_0b_4a_1b_4a_2b_4a_4b_4a_4b_0a_4b_1a_4b_2.$

The identity element is $(e_G, e_GI_1, \ldots, e_GI_n)$.

In this case $T(G)$ is not isomorphic to the direct product of 5 copies of G, since it is not a classical group in this case.

Definition:

(a) Let $T(G)$ be a symbolic Turiyam group. It is called abelian if $x * y = y * x$ for all $x, y \in T(G)$.

(b) The subset $Z(T(G)) = \{y \in T(G); y * x = x * y\ for\ all\ x \in T(G)\}$ is called symbolic Turiyam center.

Theorem :

Let $T(G)$ be a symbolic Turiyam group. Then

(a) If G is abelian, T is abelian.

(b) $T(G)$ is abelian if and only if $T(G) = Z(T(G))$.

Definition:

Let $T(G)$ be a symbolic Turiyam group, H be a nonempty subset of $T(G)$, we say that H is a symbolic Turiyam subgroup if H contains a subgroup of G.

DOI: https://doi.org/10.54216/JNFS.030205
Received: April 05, 2022 Accepted: July 16, 2022

Example:

Let $G = Z_2$ be the additive group of integers modulo 2, the corresponding symbolic Turiyam group is $T(G)$. The set $H = \{(0,0,0,0,0), (1,0,0,0,0), (1, T, 0,0,0), (1,0, F, I, Y)\}$ is symbolic Turiyam subgroup of $T(G)$, since it contains $G = \{(0,0,0,0,0), (1,0,0,0,0)\}$ which is isomorphic to a subgroup of G. (We can consider it as a subgroup of G).

By previous example, we can see that Lagrange's theorem is not true in general in the case of finite n symbolic Turiyam group.

Definition:

Let $T(G)$ be an symbolic Turiyam group, we denote to the number of elements in $T(G)$ by

O($T(G)$). If $T(G)$ is finite, then $O\big(T(G)\big) = m$, elsewhere $O\big(T(G)\big) = \infty$.

$O\big(T(G)\big)$ is called the order of n symbolic Turiyam group $T(G)$.

Theorem:

Let G be a finite group, $T(G)$ be its corresponding symbolic Turiyam group. Then if $O(G) = m$, we have $O\big(T(G)\big) = m^5$.

Definition:

Let $T(G), T(K)$ be two symbolic Turiyam groups, $f: T(G) \to T(K)$ be a well defined map, we say that f is a symbolic Turiyam homomorphism if:

(a) $f(xy) = f(x)f(y)$ for all $x, y \in T(G)$.

(b) $f(0, T, 0,0,0) = (0, T, 0,0,0), f(0,0, F, 0,0) = (0,0, F, 0,0), f(0,0,0, I, 0) = (0,0,0, I, 0), f(0,0,0,0, Y) = (0,0,0,0, Y)$.

Example:

Let
$G = Z, K = Z_4, f: T(G) \to T(K); f(x, yT, zF, pI, qY) =$
$((x\ mod 4), (y\ mod 4)T, (z\ mod 4)F, (p\ mod\ 4)I, (q\ mod\ 4)Y)$

, where $x, y, z, p, q \in Z$.

Let $m = (x, yT, zF, pI, qY), n = (a, bT, cF, dI, eY)$ be two arbitrary elements in $T(G)$, it is clear that

$f(m + n) = f(m) + f(n)$.

$f(0, T, 0,0,0) = (0, T, 0,0,0), f(0,0, F, 0,0) = (0,0, F, 0,0), f(0,0,0, I, 0) = (0,0,0, I, 0), f(0,0,0,0, Y) = (0,0,0,0, Y)$. Thus f is a symbolic Turiyam homomorphism.

Definition:

Let $T(G), T(K)$ be two symbolic Turiyam groups, $f: T(G) \to T(K)$ be a symbolic Turiyam homomorphism, we define:

(a) $Ker(f) = \{x \in T(G); f(x) = e_{T(K)}\}$.

(b) $Im(f) = \{y \in T(K);\ \exists\ x \in T(G): f(x) = y\}$.

Theorem:

Let $T(G), T(K)$ be two symbolic Turiyam groups, $f: N_n(G) \to N_n(K)$ be a symbolic Turiyam homomorphism, we have:

(a) $Ker(f)$ is a symbolic Turiyam subgroup of $T(G)$.

(b) $Im(f)$ is a symbolic Turiyam subgroup of $T(K)$.

DOI: https://doi.org/10.54216/JNFS.030205

Received: April 05, 2022 Accepted: July 16, 2022

Example:

Let
$G = Z, K = Z_4, f: T(G) \to T(K); f(x, yT, zF, pI, qY) =$
$\big((x\ mod4), (y\ mod4)T, (z\ mod4)F, (p\ mod\ 4)I, (q\ mod\ 4)Y\big)$.

$Ker(f) = (4Z, 4ZT, 4ZF, 4ZI, 4ZY) = \{(4x, 4yT, 4zF, 4pI, 4qY); x, y, z, p, q \in Z\}$, which is symbolic Turiyam subgroup, since it contains $L = (4Z, 0,0,0,0)$.

Definition:

Let $T(G), T(K)$ be symbolic Turiyam groups, $f: T(G) \to T(K)$ be a symbolic Turiyam, we call it a symbolic Turiyam isomorphism if it is bijective.

Example:

Let $G = Z$ be the group of integers with normal addition, $T(G) = \{(a, bT, cF, dI, pY); a, b, c, d, p \in Z\}$ be its corresponding symbolic Turiyam group. We define

$f: T(G) \to T(G); f(a, bT, cF, dI, pY) = (-a, bT, cF, dI, pY)$, it is clear that f is a bijective symbolic Turiyam homomorphism, thus it is a symbolic Turiyam isomorphism.

Definition:

Let $T(G) = \{(a_0, a_1T, a_2F, a_3I, a_4Y);\ a_i \in G\}$ be a symbolic Turiyam group,

$T(H) = \{(b_0, b_1T, b_2F, b_3I, b_4Y);\ b_i \in H_i;\ H_i$ is a subgroup of $G\ for\ all\ i\}$ is called a Turiyam AH-subgroup of $T(G)$.

If $H_i \cong H_j\ for\ all\ i \neq j$, then it is called an AHS-subgroup.

The AH-subgroup $T(H)$ is called AH-abelian if H_i is abelian for all i. Also, it is called AH-cyclic if H_i is cyclic for all i.

Example:

Let $G = S_3$ be the non abelian symmetric group of order 6, there are two non isomorphic subgroups of G,

$K \cong Z_2, S \cong Z_3$, consider the corresponding symbolic Turiyam group $T(G)$, we have:

$N_3(H) = (K, ST, KF, SI, SY) = \{(a, bT, cF, dI, pY); a, c \in K\ and\ b, d, p \in S\}$ is an AH-subgroup of $T(G)$.

$T(H)$ is an AH-cyclic, since K, S are cyclic.

Definition:

Let G be any group, $T(G)$ be its corresponding symbolic Turiyam group,
$T(H) = (H_0, H_1T, H_2F, H_3I, H_4Y);\ H_i \leq G$ be an AH-subgroup of $T(G)$. We say

(a) $T(H)$ is AH-normal subgroup if H_i is normal for all i.

(b) $T(H)$ is AH-nilpotent subgroup if H_i is nilpotent for all i.

(c) $T(H)$ is AH-solvable subgroup if H_i is solvable for all i.

(d) $T(H)$ is AH-meta abelian subgroup if H_i is meta abelian for all i.

(e) $T(H)$ is AH-simple subgroup if H_i is simple for all i.

Example :

Consider the symmetric group of order 6 ($G = S_3$), it has one normal subgroup $H \cong Z_3$, and three 2-Sylow subgroups $K \cong S \cong L \cong Z_2$.

Let $T(G)$ be the corresponding symbolic Turiyam group, we have

(a) $M_1 = (K, HT, KF, KI, HY)$ is an AH-subgroup of $T(G)$.

DOI: https://doi.org/10.54216/JNFS.030205
Received: April 05, 2022 Accepted: July 16, 2022

(b) M_1 is an AH-nilpotent/AH-solvable, since K,H are nilpotent, and solvable subgroups of G.

(c) M_1 is an AH-simple, since K,H are simple.

(d) M_1 is not an AH-normal, since K is not normal.

(e) M_1 is an AH-meta abelian, since H,K are meta abelian subgroups.

Remark:

If G is an additive abelian group, then any AH-subgroup is a classical subgroup. This is because $T(G)$ is a classical group and isomorphic to the direct product of G with itself (5 times).

Definition:

Let G,K be any two groups, $T(G), T(K)$ be their corresponding symbolic Turiyam groups,

$f_i: G \to K; 0 \leq i \leq 4$ be a classical homomorphism for all i. We say

(a) $f: T(G) \to T(K); f(a_0, a_1T, a_2F, a_3I, a_4Y) = (f_0(a_0), f_1(a_1)T, f_2(a_2)F, f_3(a_3)T, f_4(a_4)Y)$ is an AH-homomorphism.

(b) If $f_i: G \to K$ is an isomorphism for all i, then $f: T(G) \to T(K)$ is called an AH-isomorphism.

(c) If $f_i = f_j; i \neq j$, then $f: T(G) \to T(K)$ is called an AHS-homomorphism.

(d) The AH-kernel is defined as follows: $AH - Ker(f) = (Ker(f_0), Ker(f_1)T, Ker(f_2)F, Ker(f_3)I, Ker(f_4)Y)$.

(e) The AH-image is defined as: $AH - Im(f) = (im(f_0), im(f_1)T, im(f_2)F, im(f_3)I, im(f_4)Y)$.

We denote to the AH-homomorphism $: N_n(G) \to N_n(K); f(a_0, a_1T, a_2F, a_3I, a_4Y) = (f_0(a_0), f_1(a_1)T, f_2(a_2)F, f_3(a_3)T, f_4(a_4)Y)$ by

$f = (f_0, f_1T, f_2F, f_3I, f_4Y)$.

Definition :

Let G be any group, $T(G)$ be its corresponding symbolic Turiyam group,

$T(H) = (H_0, H_1T, H_2F, H_3I, H_4Y), T(K) = (K_0, K_1T, K_2F, K_3I, K_4Y);\ H_i, K_i \leq G$ be any two AH-subgroups of $T(G)$.

(a) We define the intersection as follows: $T(H) \cap T(K) = (H_0 \cap K_0, (H_1 \cap K_1)T, (H_2 \cap K_2)F, (H_3 \cap K_3)I, (H_4 \cap K_4)Y)$.

(b) We define the product as follows: $T(H).T(K) = (H_0.K_0, (H_1.K_1)T, (H_2.K_2)F, (H_3.K_3)I, (H_4.K_4)Y)$.

(c) We define the direct product as follows: $T(H) \times T(K) = (H_0 \times K_0, (H_1 \times K_1)T, (H_2 \times K_2)F, (H_3 \times K_3)I, (H_4 \times K_4)Y)$.

Example :

Let $G = (Z, +), K = (Z_6, +)$ be two groups, $T(G), T(K)$ be the corresponding symbolic Turiyam groups, we have

(a) $f_0: G \to K;\ f_0(a) = a\, mod6, f_1: G \to K;\ f_1(a) = 2a\, mod6$ are two classical homomorphisms.

(b) $f: T(G) \to T(K); f(a_0, a_1T, a_2F, a_1I, a_1Y) = (f_0(a_0), f_0(a_1)T, f_1(a_2)F, f_0(a_3)I, f_1(a_4)Y) = (a_0\, mod6, (a_1\, mod6\,)T, (2a_2\, mod6)F, (a_3\, mod6)I, (2a_4\, mod\ 6)Y)$ is an AH-homomorphism.

(c) $AH - Ker(f) = (Ker(f_0), Ker(f_0)T, Ker(f_1)F, Ker(f_0)I, Ker(f_1)Y) = (6Z, 6Z\,T, 3Z\,F, 3Z\,I, 6Z\,Y)$.

Theorem :

Let G be any group, $T(G)$ be its corresponding symbolic Turiyam group,

$T(H) = (H_0, H_1T, H_2F, H_3I, H_4Y), T(K) = (K_0, K_1T, K_2F, K_3I, K_4Y);\ H_i, K_i \leq G$ be any two AH-subgroups of $T(G)$. We have:

(a) $T(H) \cap T(K)$ is an AH-subgroup of $T(G)$.

(b) If $T(H), T(K)$ are AH-normal, then $T(H).T(K)$ is AH-normal.

(c) $T(H) \times T(K)$ is an AH-subgroup of $T(G) \times T(G)$.

(d) If $T(H), T(K)$ are AH-abelian, then $T(H) \cap T(K), T(G) \times T(G)$ are AH-abelian.

(e) If $T(H), T(K)$ are AH-cyclic, then $T(H) \cap T(K)$ is AH-cycilc.

(f) If $T(H), T(K)$ are AH-nilpotent, then $T(H) \cap T(K), T(G) \times T(G)$are AH-nilpotent.

(g) If $T(H), T(K)$ are AH-solvable, then $T(H).T(K), T(G) \times T(G), T(G) \cap T(G)$ are AH-solvable.

(h) If $T(H), T(K)$ are AH- meta abelian, then $T(H) \cap T(K), T(G) \times T(G)$are AH-meta abelian.

Theorem :

Let G,K be any two groups, $T(G), T(K)$ be their corresponding symbolic Turiyam groups,

$T(H) = (H_0, H_1T, H_2F, H_3I, H_4Y);\ H_i \leq G$ be an AH-subgroup, $f_i: G \to K; 0 \leq i \leq n$ be a classical homomorphism for all $i, f: T(G) \to T(K); f(a_0, a_1T, a_2F, a_1I, a_1Y) = (f_0(a_0), f_1(a_1)T, f_2(a_2)F, f_3(a_3)T, f_4(a_4)Y)$ be an AH-homomorphism, we have:

(a) If $T(H)$ is AH-cyclic/AH-abelian, then $f(T(H))$ is AH-cyclic/AH-abelian.

(b) If $T(H)$ is AH-nilpotent/AH-solvable, then $f(T(H))$ is AH-nilpotent/AH-solvable.

(c) If $T(H)$ is AH-meta abelian, then $f(T(H))$ is AH-meta abelian.

(d) If $T(H)$ is AH-normal, then $f(T(H))$ is AH-normal.

(e) AH-Ker(f) is an AH-normal subgroup of $T(G)$.

(f) AH-Im(f) is an AHS-subgroup of $T(K)$.

Conclusion

In this article we have defined the concept of symbolic Turiyam group for the first time. Also, we have introduced some corresponding notions such as symbolic Turiyam subgroup, symbolic Turiyam homomorphism, and symbolic Turiyam isomorphism. Many examples were constructed to clarify these concepts.

Acknowledgements: Author thanks the anonymous reviewers and editor for their valuable time and suggestions.

Funding: Author declares that there is no funding for this paper.

Conflict of Interest: Author declares that there is no conflict of interest for this paper.

References

[1] Smarandache, F., " A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press. Rehoboth, 2003.

[2] Alhamido, R., and Gharibah, T., "Neutrosophic Crisp Tri-Topological Spaces", Journal of New Theory, Vol. 23 , pp.13-21. 2018.

[3] Edalatpanah. S.A., "Systems of Neutrosophic Linear Equations", Neutrosophic Sets and Systems, Vol. 33, pp. 92-104. 2020.

[4] Sankari, H., and Abobala, M., "Neutrosophic Linear Diophantine Equations With two Variables", Neutrosophic Sets and Systems, Vol. 38, pp. 22-30, 2020.

DOI: https://doi.org/10.54216/JNFS.030205

Received: April 05, 2022 Accepted: July 16, 2022

[5] Sankari, H., and Abobala, M." *n*-Refined Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 36, pp. 1-11. 2020.

[6] Alhamido, R., and Abobala, M., "AH-Substructures in Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 7, pp. 79-86 . 2020.

[7] Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 6 , pp. 80-86. 2020.

[8] Abobala, M.,. "A Study of AH-Substructures in *n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.

[9] Hatip, A., Alhamido, R., and Abobala, M., "A Contribution to Neutrosophic Groups", International Journal of Neutrosophic Science", Vol. 0, pp. 67-76 . 2019.

[10] Abobala, M., " *n*-Refined Neutrosophic Groups I", International Journal of Neutrosophic Science, Vol. 0, pp. 27-34. 2020.

[11] Kandasamy, V.W.B., and Smarandache, F., "Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures", Hexis, Phonex, Arizona, 2006.

[12] Agboola, A.A.A., Akinola, A.D., and Oyebola, O.Y., " Neutrosophic Rings I" , International J.Mathcombin, Vol 4,pp 1-14. 2011

[13] Agboola, A.A.A., "On Refined Neutrosophic Algebraic Structures," Neutrosophic Sets and Systems,Vol.10, pp. 99-101. 2015.

[14] Abobala, M., "Classical Homomorphisms Between Refined Neutrosophic Rings and Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 72-75. 2020.

[15] Smarandache, F., and Abobala, M., n-Refined neutrosophic Rings, International Journal of Neutrosophic Science, Vol. 5 , pp. 83-90, 2020.

[16] Kandasamy, I., Kandasamy, V., and Smarandache, F., "Algebraic structure of Neutrosophic Duplets in Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 18, pp. 85-95. 2018.

[17] Yingcang, Ma., Xiaohong Zhang ., Smarandache, F., and Juanjuan, Z., "The Structure of Idempotents in Neutrosophic Rings and Neutrosophic Quadruple Rings", Symmetry Journal (MDPI), Vol. 11. 2019.

[18] Kandasamy, V. W. B,. Ilanthenral, K., and Smarandache, F., "Semi-Idempotents in Neutrosophic Rings", Mathematics Journal (MDPI), Vol. 7. 2019.

[19] Abobala, M., On Some Special Substructures of Neutrosophic Rings and Their Properties, International Journal of Neutrosophic Science", Vol. 4 , pp. 72-81, 2020.

[20] Smarandache, F., " An Introduction To neutrosophic Genetics", International Journal of neutrosophic Science, Vol.13, 2021.

[21] Martin, N, Smarandache, F, and Broumi, S., " Covid 19 Decision Making using Extended Plithogenic hypersoft Sets With Dual Dominent Attributes", International Journal of neutrosophic Science, Vol. 13, 2021.

[22]Agboola, A.A., "Introduction To Neutro groups", International Journal of neutrosophic Science, Vol. 6, 2020.

[23] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 59-66. 2020.

[24] Smarandache, F., and Ali, M., "Neutrosophic Triplet Group", Neural. Compute. Appl. 2019.

[25] Sankari, H., and Abobala, M.," AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 38, pp. 101-112, 2020.

[26] Smarandache, F., and Kandasamy, V.W.B., " Finite Neutrosophic Complex Numbers",·Source: arXiv. 2011.

[27]. Abobala, M., " *n*-Refined Neutrosophic Groups II", International Journal of Neutrosophic Science, Vol. 0, 2020.

DOI: https://doi.org/10.54216/JNFS.030205

Received: April 05, 2022 Accepted: July 16, 2022

[28] Olgun, N., Hatip, A., Bal, M., and Abobala, M., " A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined Neutrosophic Matrices", International Journal of Neutrosophic Science, Vol. 16, pp. 72-79, 2021.

[29] Aswad, F, M., " A Study of Neutrosophic Complex Number and Applications", Neutrosophic Knowledge, Vol. 1, 2020.

[30] Aswad, M., " A Study of The Integration Of Neutrosophic Thick Function", International journal of neutrosophic Science, 2020.

[31] Abobala, M, "*n*-Cyclic Refined Neutrosophic Algebraic Systems Of Sub-Indeterminacies, An Application To Rings and Modules", International Journal of Neutrosophic Science, Vol. 12, pp. 81-95 . 2020.

[32] Smarandache, F., "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets", Inter. J. Pure Appl. Math., pp. 287-297. 2005.

[33] M. Ali, F. Smarandache, M. Shabir and L. Vladareanu., "Generalization of Neutrosophic Rings and Neutrosophic Fields", Neutrosophic Sets and Systems, vol. 5, pp. 9-14, 2014.

[34] Anuradha V. S., "Neutrosophic Fuzzy Hierarchical Clustering for Dengue Analysis in Sri Lanka", Neutrosophic Sets and Systems, vol. 31, pp. 179-199. 2020.

[35] Olgun, N., and Hatip, A., "The Effect Of The Neutrosophic Logic On The Decision Making, in Quadruple Neutrosophic Theory And Applications", Belgium, EU, Pons Editions Brussels,pp. 238-253. 2020.

[36] Abobala, M., Bal, M., and Hatip, A.," A Review On Recent Advantages In Algebraic Theory Of Neutrosophic Matrices**",** International Journal of Neutrosophic Science, Vol. 17, 2021.

[37] Turksen, I., "Interval valued fuzzy sets based on normal forms", Fuzzy Sets and Systems, 20, pp.191-210, 1986. 1986.

[38] Chalapathi, T., and Madhavi, L., "Neutrosophic Boolean Rings", Neutrosophic Sets and Systems, Vol. 33, pp. 57-66. 2020.

[39] Abobala, M., "Classical Homomorphisms Between n-refined Neutrosophic Rings", International Journal of Neutrosophic Science", Vol. 7, pp. 74-78. 2020.

[40] Agboola, A.A.A,. Akwu, A.D,. and Oyebo, Y.T., " Neutrosophic Groups and Subgroups", International .J .Math. Combin, Vol. 3, pp. 1-9. 2012.

[41] Smarandache, F., " *n*-Valued Refined Neutrosophic Logic and Its Applications in Physics", Progress in Physics, 143-146, Vol. 4, 2013.

[42] Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.

[43] Abobala, M., Bal, M., Aswad, M., "A Short Note On Some Novel Applications of Semi Module Homomorphisms", International journal of neutrosophic science, 2022.

[44] Hatip, A., and Olgun, N., "On Refined Neutrosophic R-Module", International Journal of Neutrosophic Science, Vol. 7, pp.87-96. 2020.

[45] Bal, M., and Abobala, M., "On The Representation Of Winning Strategies Of Finite Games By Groups and Neutrosophic Groups", Journal Of Neutrosophic and Fuzzy Systems, 2022.

[46] Smarandache F., and Abobala, M., "*n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54. 2020.

[47] Sankari, H., and Abobala, M., "Solving Three Conjectures About Neutrosophic Quadruple Vector Spaces", Neutrosophic Sets and Systems, Vol. 38, pp. 70-77. 2020.

[48] Adeleke, E.O., Agboola, A.A.A., and Smarandache, F., "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science, Vol. 2(2), pp. 89-94. 2020.

DOI: https://doi.org/10.54216/JNFS.030205
Received: April 05, 2022 Accepted: July 16, 2022

[49] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021

[50] Abobala, M., A Study of Maximal and Minimal Ideals of n-Refined Neutrosophic Rings, Journal of Fuzzy Extension and Applications, Vol. 2, pp. 16-22, 2021.

[51] Abobala, M., " Semi Homomorphisms and Algebraic Relations Between Strong Refined Neutrosophic Modules and Strong Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 39, 2021.

[52] Abobala, M., "On Some Neutrosophic Algebraic Equations", Journal of New Theory, Vol. 33, 2020.

[53] Abobala, M., On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations, Journal of Mathematics, Hindawi, 2021.

[54] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021

[55] Kandasamy V, Smarandache F., and Kandasamy I., Special Fuzzy Matrices for Social Scientists . Printed in the United States of America,2007, book, 99 pages.

[56] Khaled, H., and Younus, A., and Mohammad, A., " The Rectangle Neutrosophic Fuzzy Matrices", Faculty of Education Journal Vol. 15, 2019. (Arabic version).

[57] Abobala, M., Partial Foundation of Neutrosophic Number Theory, Neutrosophic Sets and Systems, Vol. 39 , 2021.

[58] F. Smarandache, *Neutrosophic Theory and Applications,* Le Quy Don Technical University, Faculty of Information technology, Hanoi, Vietnam, 17th May 2016.

[59] Sankari, H, and Abobala, M., " On A New Criterion For The Solvability of non Simple Finite Groups and m-Abelian Solvability, Journal of Mathematics, Hindawi, 2021.

[60] Giorgio, N, Mehmood, A., and Broumi, S.," Single Valued neutrosophic Filter", International Journal of Neutrosophic Science, Vol. 6, 2020.

[61] Abobala, M., "A Study Of Nil Ideals and Kothe's Conjecture In Neutrosophic Rings", International Journal of Mathematics and Mathematical Sciences, hindawi, 2021

[62] Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., The algebraic creativity In The Neutrosophic Square Matrices, Neutrosophic Sets and Systems, Vol. 40, pp. 1-11, 2021.

[63]Alhamido, K., R., "A New Approach of neutrosophic Topological Spaces", International Journal of neutrosophic Science, Vol.7, 2020.

[64] Abobala, M., "Neutrosophic Real Inner Product Spaces", Neutrosophic Sets and Systems, Vol. 43, 2021.

[65] Abobala, M., "On Some Special Elements In Neutrosophic Rings and Refined Neutrosophic Rings", Journal of New Theory, vol. 33, 2020.

[66] Abobala, M., and Hatip, A., "An Algebraic Approch To Neutrosophic Euclidean Geometry", Neutrosophic Sets and Systems, Vol. 43, 2021.

[67] Sundar, J., Vadivel, A., " New operators Using Neutrosophic ϑ −Open Sets", Journal Of Neutrosophic and Fuzzy Systems, 2022.

[68] Sankari, H, and Abobala, M, " A Contribution to m-Power Closed Groups", UMM-Alqura University Journal for Applied Sciences, KSA, 2020.

[69] Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 45, 2021.

[70] Chellamani, P., and Ajay, D., "Pythagorean neutrosophic Fuzzy Graphs", International Journal of Neutrosophic Science, Vol. 11, 2021.

[71] Singh P. K., “Fourth dimension data representation and its analysis using Turiyam Context”, Journal of Computer and Communications, 2021, Vol. 9, no. 6, pp. 222-229,DOI: 10.4236/jcc.2021.96014, https://www.scirp.org/journal/paperinformation.aspx?paperid=110694

DOI: https://doi.org/10.54216/JNFS.030205

Received: April 05, 2022 Accepted: July 16, 2022

[72] Singh P. K., "NeutroAlgebra and NeutroGeometry for Dealing Heteroclinic Patterns". In: Theory and Applications of NeutroAlgebras as Generalizations of Classical Algebras, IGI Global Publishers, April 2022, Chapter 6, DOI: 10.4018/978-1-6684-3495-6, ISBN13: 9781668434956

[73] Singh P. K., "Three-way n-valued neutrosophic concept lattice at different granulation", International Journal of Machine Learning and Cybernetics, Vol 9, Issue 11, pp. 1839-1855, 2018.

[74] Abobala, M., Hatip, A., Bal,M.," A Study Of Some Neutrosophic Clean Rings", International journal of neutrosophic science, 2022.

[75] Singh P. K., "AntiGeometry and NeutroGeometry Characterization of Non-Euclidean Data Sets", Journal of Neutrosophic and Fuzzy Systems, Vol 1, Issue 1, pp. 24-33, 2021,
, DOI: https://doi.org/10.54216/JNFS.0101012

[76] Singh, P,K., " Anti-geometry and NeutroGeometry Characterization of Non-Euclidean Data", Journal of Neutrosophic and Fuzzy Systems, Volume 1, Issue 1, pp. 24-33, 2021.

[77] Singh, P,K., " Data With Turiyam Set for Fourth Dimension Quantum Information Processing", Journal of Neutrosophic and Fuzzy Systems, Volume 1, Issue 1, pp. 9-23, 2021.

[78] Singh, P, K., Ahmad, K., Bal, M., Aswad, M.," On The Symbolic Turiyam Rings", Journal of Neutrosophic and Fuzzy Systems, Vol. 1 , No. 2 , pp. 80-88 , 2021

[79] Ibrahim, M., and Abobala, M., "An Introduction To Refined Neutrosophic Number Theory", Neutrosophic Sets and Systems, Vol. 45, 2021.

[80] Bal, M.,, Singh, P. K., Ahmad, K., and Aswad, M., " A Short Introduction To The Concept Of Symbolic Turiyam Matrix", Neutrosophic and Fuzzy Systems, Vol. 2 , No. 1 , pp. 88-99 , 2022.

[81] Bal, M., Singh, P. K, and Ahmad, K., " An Introduction To The Symbolic Turiyam R-Modules and Turiyam Modulo Integers", Neutrosophic and Fuzzy Systems, Vol. 2, No. 2 , pp. 8-19,2022.

[82] Bal, M., Singh, P. K, and Ahmad, K., " An Introduction To The Symbolic Turiyam Vector Spaces and Complex Numbers", Neutrosophic and Fuzzy Systems, Vol. 2 , No. 1 , pp. 76-87 , Mar 2022.

[83] Singh P. K., "Non-Euclidean, Anti-Geometry and NeutroGeometry Characterization", International Journal of Neutrosophic Sciences, Vol 18, Issue 3, pp. 8-19, 2022.

[84] Singh P. K., "Data with Non-Euclidean Geometry and its Characterization". Journal of Artificial Intelligence and Technology, Vol 2, Issue 1, pp. 3-8, 2022, DOI: 10.37965/jait.2021.12001

DOI: https://doi.org/10.54216/JNFS.030205
Received: April 05, 2022 Accepted: July 16, 2022