Bipolar neutrosophic soft contra generalized pre-continuous and contra generalized α-continuous mappings

P. Arulpandy[1] *, . **Trinita Pricilla**[2]**M**
[1]Department of Mathematics, KPR Institute of Engineering and Technology, Coimbatore, India
[2]Department of Mathematics, Nirmala College for Women, Coimbatore, India
Emails; arulpandy002@gmail.com[1], abishai_kennet@yahoo.in[2]

Abstract

In this paper, we introduce and investigate the classes of continuous mappings in bipolar neutrosophic soft topological spaces such as bipolar neutrosophic soft contra generalized pre-continuous mappings and bipolar neutrosophic soft contra generalized α-continuous mappings. Further, we investigate some of its properties via theorems.

Keywords; Bipolar neutrosophic soft set; BNSCGP-continuous; BNSCG-closed set; BNSCGα-continuous; BNSS-topology.)

1 Introduction

Zadeh[5] introduced the concept of fuzzy sets in 1965. Initially, fuzzy sets were used in decision making problems; but nowadays it is applied in many science and engineering fields. Later, fuzzy topological spaces was introduced by Cheng[9] in 1965. Atanassov[10] developed some concepts in fuzzy sets and proposed intiutionistic fuzzy sets which is the extension of conventional Fuzzy sets. Smarandache[1,2] introduced a theory of neutrosophic sets as the extension and development of fuzzy sets which includes three membership degrees namely, truth, indeterminacy and falsify memberships. Deli et al.[8] extended the neutrosophic set and proposed bipolar neutrosophic sets. Later, many researchers were investigated and proposed various neutrosophic sets.[4,7] Molodtsov[3] introduced soft set theory in 1999. A.A.Salama et al.[13] developed a new concept neutrosophic topology, i.e. the topology concepts were introduced for neutrosophic sets. Norman Levine[11,12] introduced generalized closed set and some continuity mappings in point set topology in 1970. In this study, we have proposed bipolar neutrosophic soft contra generalized pre-continuous mapping and bipolar neutrosophic soft contra generalized α-continuous mappings.

This paper is organized as follows: Section 2 consists the required preliminary definitions for main concept. Section 3 consists bipolar neutrosophic soft generalized homeomorphism and pre-homeomorphism. Section 4 consists bipolar neutrosophic soft contra generalized pre-continuous mapping. Section 5 consists bipolar neutrosophic soft contra generalized α-continuous mapping. Each section deals with theorems related to the proposed mappings and examples. Section 6 concludes the proposed work.

https://doi.org/10.54216/IJNS.180112
Received: September 05, 2021 Accepted: January 22, 2022

2 Preliminaries

Definition 2.1. [1] Let X be a universe set. For every $x \in X$, the components $u(x), v(x)$ and $w(x)$ are truth, indeterminate and false degrees of x. Then the Neutrosophic set (NS) over X be defined as follows.
$N = \{u(x), v(x), w(x) : x \in X\}$
Here, $u(x), v(x), w(x)$ ranges in the non-standard interval $]^{-}0, 1^{+}[$ and their sum $^{-}0 \leq u + v + w \leq 3^{+}$. For scientific problems, we prefer standard interval $[0, 1]$ instead of non-standard interval and it is called single-valued neutrosophic set.

Definition 2.2. [4] For the universe set X and positive member values $u^{+}, v^{+}, w^{+} : E \to [0, 1]$, negative member values $u^{-}, v^{-}, w^{-} : E \to [-1, 0]$, A bipolar neutrosophic set (BNS) is defined by

$$B = \left\{ \langle x, u^{+}(x), v^{+}(x), w^{+}(x), u^{-}(x), v^{-}(x), w^{-}(x) \rangle : x \in X \right\}$$

Definition 2.3. [3] A sot set is a function which maps a parameter set to the power set of X. It is denoted by (f, E) and is defined by
$f : E \to P(x)$
Each member of X is parametrized with the parameter set E by the function f.

Definition 2.4. [4] A bipolar neutrosophic soft set (BNSS) is the fusion of soft set and bipolar neutrosophic set and is defined as follows.

$$BNS = (f_A, E) = \{\langle e, f_A(x) \rangle : e \in A \subset E, f_A(x) \in BNS(X)\}$$

Here $f_A(x) = \left\{ \left\langle x, u^{+}_{f_A(e)}(x), v^{+}_{f_A(e)}(x), w^{+}_{f_A(e)}(x), u^{-}_{f_A(e)}(x), v^{-}_{f_A(e)}(x), w^{-}_{f_A(e)}(x) \right\rangle : x \in X \right\}$.

Definition 2.5. [4] Let B_1 and B_2 be two $BNSS$s. Then for every $x \in X, e \in E$,

1. (subset) $B_1 \subseteq B_2$ if and only if

$$\{B_1 \subseteq B_2\} = \left\{ \begin{array}{c} \left[u^{+}_{B_1}(x) \leq u^{+}_{B_2}(x)\right], \left[v^{+}_{B_1}(x) \geq v^{+}_{B_2}(x)\right], \left[w^{+}_{B_1}(x) \geq w^{+}_{B_2}(x)\right] \\ \left[u^{-}_{B_1}(x) \geq u^{-}_{B_2}(x)\right], \left[v^{-}_{B_1}(x) \leq v^{-}_{B_2}(x)\right], \left[w^{-}_{B_1}(x) \leq w^{-}_{B_2}(x)\right] \end{array} \right\}$$

.

2. (equal) $B_1 = B_2$ if and only if $B_1 \subseteq B_2$ and $B_2 \subseteq B_1$.

3. (complement)

$$B^{c} = \left\{ \left\langle e, w^{+}_{f}(e), 1 - v^{+}_{f}(e), u^{+}_{f}(e), w^{-}_{f}(e), -1 - v^{-}_{f}(e), u^{-}_{f}(e) \right\rangle \right\}$$

.

4. (union) $B_1 \cup B_2 = \left\{ \left\langle e, \cup_i f^{(i)}(e) \right\rangle \right\}$. Here

$$\bigcup_i f^{(i)}(e) = \Big\{ \langle x, max\left[u^{+}_{f^i}(e)(x)\right], min\left[v^{+}_{f^i}(e)(x)\right], min\left[w^{+}_{f^i}(e)(x)\right], \\ min\left[u^{-}_{f^i}(e)(x)\right], max\left[v^{-}_{f^i}(e)(x)\right], max\left[w^{-}_{f^i}(e)(x)\right] \rangle \Big\}$$

https://doi.org/10.54216/IJNS.180112

Received: September 05, 2021 Accepted: January 22, 2022

5. (intersection) $B_1 \cap B_2 = \left\{ \langle e, \cap_i f^{(i)}(e) \rangle \right\}$. Here

$$\bigcap_i f^{(i)}(e) = \left\{ \langle x, min\left[u^+_{f^i}(e)(x)\right], max\left[v^+_{f^i}(e)(x)\right], max\left[w^+_{f^i}(e)(x)\right], max\left[u^-_{f^i}(e)(x)\right], min\left[v^-_{f^i}(e)(x)\right], min\left[w^-_{f^i}(e)(x)\right] \rangle \right\}$$

6. (null) $\varphi_{\mathbb{B}} = \{ \langle e_i, \{x_i, 0, 1, 1, 0, -1, -1\} \rangle : x \in X, e \in E \}$
7. (complete) $1_{\mathbb{B}} = \{ \langle e_i, \{x_i, 1, 0, 0, -1, 0, 0\} \rangle : x \in X, e \in E \}$

Definition 2.6. A bipolar neutrosophic soft topology (BNST) on X is a collection of bipolar neutrosophic soft sets (BNSS) in X satisfying the following conditions:

1. $\varphi_{\mathbb{B}}, 1_{\mathbb{B}} \in_{\mathbb{B}}$
2. $\bigcup_{i \in n} \mathbb{B}_i \in_{\mathbb{B}}$ for each $\mathbb{B}_i \in_{\mathbb{B}}$
3. $\mathbb{B}_i \cap \mathbb{B}_j \in_{\mathbb{B}}$ for any $\mathbb{B}_i, \mathbb{B}_j \in_{\mathbb{B}}$

The pair $(X,_{\mathbb{B}})$ is called $\mathbb{BNSS}$-topological space. The members of $_{\mathbb{B}}$ are called bipolar neutrosophic soft open sets (BNOS) and their complements are called bipolar neutrosophic soft closed sets (BNCS).

Definition 2.7. Let $(X,_{\mathbb{B}})$ be a $BNST$ and $B = \left\{ \langle e, f(x) \rangle : e \in E, f(x) \in BNS(X) \right\}$ be $BNSS$ in X. Then the bipolar neutrosophic soft interior and bipolar neutrosophic soft closure are defined by

$BNint(B) = \bigcup \left\{ U \ : \ U \ is \ a \ BNOS \ in \ U \subseteq B \right\}$

$BNcl(B) = \bigcap \left\{ V \ : \ V \ is \ a \ BNCS \ in \ B \subseteq V \right\}$

Remark 2.8. Let B be BNS of a $BNTS(X,)$. Then

1. $BN\alpha cl(B) = B \cup BNcl(BNint(BNcl(B)))$
2. $BN\alpha int(B) = B \cap BNint(BNcl(BNint(B)))$

Remark 2.9. Following relations hold for any BNS set $B \in (X,)$.

1. $BNcl(B^c) = (BNint(B))^c$ and $BNint(B^c) = (BNcl(B))^c$.
2. $BNcl(B)$ is a $BNCS$ and $BNint(B)$ is a $BNOS$ in X.
3. B is $BNCS$ in X if and only if $BNcl(B) = B$.
4. B is $BNOS$ in X if and only if $BNint(B) = B$.

Definition 2.10. A BNSset B in $BNSTS(X,)$ is said to be

1. Bipolar neutrosophic soft semi closed set (BNSCS) if $BNint(BNcl(B)) \subseteq B$,
2. Bipolar neutrosophic soft pre-closed set (BNPCS) if $BNcl(BNint(B)) \subseteq B$,

https://doi.org/10.54216/IJNS.180112

Received: September 05, 2021 Accepted: January 22, 2022

3. Bipolar neutrosophic soft α-closed set (BNαCS) if $BNcl(BNint(BNcl(B))) \subseteq B$,
4. Bipolar neutrosophic soft regular closed set (BNRCS) if $B = BNcl(BNint(B))$

Definition 2.11. Let $(X,_{\mathbb{B}}^{1}, E)$ and $(Y,_{\mathbb{B}}^{2}, E')$ be two bipolar neutrosophic soft topological spaces. For every bipolar neutrosophic closed set B of $(Y,_{\mathbb{B}}^{2}, E')$, a map $\psi : ((X,_{\mathbb{B}}^{1}, E) \to (Y,_{\mathbb{B}}^{2}, E')$ is said to be,

1. Bipolar neutrosophic soft continuous (BNS-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft closed set in $(X,_{\mathbb{B}}^{1}, E)$.
2. Bipolar neutrosophic soft semi-continuous (BNSS-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft semi-closed set in $(X,_{\mathbb{B}}^{1}, E)$.
3. Bipolar neutrosophic soft pre-continuous (BNSP-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft pre-closed set in $(X,_{\mathbb{B}}^{1}, E)$.
4. Bipolar neutrosophic soft α-continuous (BNSα-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft α-closed set in $(X,_{\mathbb{B}}^{1}, E)$.
5. Bipolar neutrosophic soft regular continuous (BNSR-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft regular closed set in $(X,_{\mathbb{B}}^{1}, E)$.
6. Bipolar neutrosophic soft generalized continuous (BNSG-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft generalized closed set in $(X,_{\mathbb{B}}^{1}, E)$.
7. Bipolar neutrosophic soft generalized semi-continuous (BNSGS-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft generalized semi-closed set in $(X,_{\mathbb{B}}^{1}, E)$.
8. Bipolar neutrosophic soft generalized α-continuous (BNSα-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft generalized α-closed set in $(X,_{\mathbb{B}}^{1}, E)$.

Definition 2.12. A map $\psi : ((X,_{\mathbb{B}}^{1}, E) \to (Y,_{\mathbb{B}}^{2}, E')$ is said to be bipolar neutrosophic soft generalized pre irresolute (BNSGP-irresolute) mapping if $\psi^{-1}(B)$ is a BNSGPCS in $(X,_{\mathbb{B}}^{1}, E)$ for every BNSGPCS B in $(Y,_{\mathbb{B}}^{2}, E)$.

Definition 2.13. Let $(X,_{\mathbb{B}}, E)$ be a BNSGT. The bipolar neutrosophic soft generalized pre closure $(BNSGPcl(B))$ for any BNS B is defined as follows.
$BNSGPcl(B) = \cap \{K |\ K\ is\ a\ BNSGPCS\ in\ X\ and\ B \in K\}$.
If B is BNSGPCS, then $BNSGPcl(B) = B$.

Definition 2.14. Let $(X,_1)$ and $(Y,_2)$ be any two topological spaces. A map $f : (X,_1) \to (Y,_2)$ is said to be contra continuous if $f^{-1}(V)$ is closed set in $(X,_1)$ for every open set V in $(Y,_2)$.

Definition 2.15. Let f be a bijective mapping from a topological space $(X,_1)$ into a topological space $(Y,_2)$. Then f is said to be generalized homeomorphism if f and f^{-1} are generalized continuous mappings.

Definition 2.16. A map $f : (X,_1) \to (Y,_2)$ is said to be $generalized\ pre\ closed$ if $f(V)$ is generalized pre closed set in $(Y,_2)$ for every closed set V in $(X,_1)$.

3 Bipolar neutrosophic soft generalized homeomorphism and pre homeomorphism in topological spaces

Definition 3.1. Let ψ be a bijective mapping from a BNST $(X,_1, E)$ into a BNST $(Y,_2, E)$. Then ψ is said to be

https://doi.org/10.54216/IJNS.180112

Received: September 05, 2021 Accepted: January 22, 2022

1. Bipolar neutrosophic soft homeomorphism (BNS-homeomorphism) if ψ and ψ^{-1} are BNS-continuous mapping.

2. Bipolar neutrosophic soft pre homeomorphism (BNSP-homeomorphism) if ψ and ψ^{-1} are BNSP-continuous mapping.

3. Bipolar neutrosophic soft generalized homeomorphism (BNSG-homeomorphism) if ψ and ψ^{-1} are BNSG-continuous mapping.

Definition 3.2. A map $\psi : (X,_1, E) \to (Y,_2, E)$ is said to be Bipolar neutrosophic soft generalized pre closed if $\psi(V)$ is bipolar neutrosophic soft generalized pre closed set in $(Y,_2, E)$ for every BNS-closed set V in $(X,_1, E)$.

Definition 3.3. A bijection mapping $\psi : (X,_1, E) \to (Y,_2, E)$ is called a Bipolar neutrosophic soft generalized pre-homeomorphism (BNSGP-homeomorphism) if ψ and ψ^{-1} are BNSGP-continuous mapping.

Example 3.4. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.5, 0.6, 0.8, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.5, -0.4, -0.3\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.6, 0.3, -0.4, -0.7, -0.2\rangle, \langle x_2,\ 0.7, 0.2, 0.1, -0.3, -0.5, -0.7\rangle\} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.3, 0.1, 0.7, -0.5, -0.6, -0.3\rangle, \langle x_2,\ 0, 1, 1, -0.7, 0, -1\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.5, 0.7, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.3, -0.1, -0.6, -0.3\rangle\} \right\rangle \end{array} \right\}$$

Then $_1 = \{0, B_1, 1\}$ and $_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X,_1, E) \to (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$ with . Then ψ is a BNSGP-continuous mapping and ψ^{-1} is also a BNSGP-continuous mapping. Therefore ψ is a BNSGP homeomorphism.

Theorem 3.5. *Every BNS-homeomorphism is a BNSGP-homeomorphism but not conversely.*

Proof. Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a BNS-homeomorphism. Then ψ and ψ^{-1} are BNS-continuous mapping. This gives, ψ and ψ^{-1} are BNSGP-continuous mapping. Therefore, ψ is a BNSGP-homeomorphism. □

Example 3.6. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.5, 0.6, 0.8, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.5, -0.4, -0.3\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.6, 0.3, -0.4, -0.7, -0.2\rangle, \langle x_2,\ 0.7, 0.2, 0.1, -0.3, -0.5, -0.7\rangle\} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.3, 0.1, 0.7, -0.5, -0.6, -0.3\rangle, \langle x_2,\ 0, 1, 1, -0.7, 0, -1\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.5, 0.7, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.3, -0.1, -0.6, -0.3\rangle\} \right\rangle \end{array} \right\}$$

Then $_1 = \{0, B_1, 1\}$ and $_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X,_1, E) \to (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSGP-homeomorphism but not BNS-homeomorphism sinceψ and ψ^{-1} are not BNS-continuous mapping.

Theorem 3.7. *Every BNSP-homeomorphism is a BNSGP-homeomorphism but not conversely.*

https://doi.org/10.54216/IJNS.180112
Received: September 05, 2021 Accepted: January 22, 2022

Proof. Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a BNSP-homeomorphism. Then ψ and ψ^{-1} are BNSP-continuous mapping. This gives, ψ and ψ^{-1} are BNSGP-continuous mapping. Therefore, ψ is a BNSGP-homeomorphism. □

Example 3.8. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{c} \Big\langle e_1, \{\langle x_1,\ 0.5, 0.6, 0.8, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.5, -0.4, -0.3\rangle\}\Big\rangle, \\ \Big\langle e_2, \{\langle x_1,\ 0.4, 0.6, 0.3, -0.4, -0.7, -0.2\rangle, \langle x_2,\ 0.7, 0.2, 0.1, -0.3, -0.5, -0.7\rangle\}\Big\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} \Big\langle e_1, \{\langle x_1,\ 0.3, 0.1, 0.7, -0.5, -0.6, -0.3\rangle, \langle x_2,\ 0, 1, 1, -0.7, 0, -1\rangle\}\Big\rangle, \\ \Big\langle e_2, \{\langle x_1,\ 0.2, 0.5, 0.7, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.3, -0.1, -0.6, -0.3\rangle\}\Big\rangle \end{array} \right\}$$

Then $_1 = \{0, B_1, 1\}$ and $_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X,_1, E) \to (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSGP-homeomorphism but not BNSP-homeomorphism since ψ and ψ^{-1} are not BNSP-continuous mapping.

Theorem 3.9. *Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a bijective mapping. If ψ is a BNSGP continuous mapping, then the following statements are equivalent.*

1. *ψ is a BNSGP closed mapping.*
2. *ψ is a BNSGP open mapping.*
3. *ψ is a BNSGP homeomorphism.*

Proof. (1) $\to$ (2): Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a bijective mapping and let ψ be a BNSGP-closed mapping. This gives, $\psi^{-1} : (Y,_2, E) \to (X,_1, E)$ is a BNSGP-continuous mapping. This means, every BNOS in X is a BNSGPOS in Y. Hence ψ is a BNSGP-open mapping.

(2) $\to$ (3): Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a bijective mapping and let ψ be a BNSGP-open mapping. This gives, $\psi^{-1} : (Y,_2, E) \to (X,_1, E)$ is a BNSGP-continuous mapping. Hence ψ and ψ^{-1} are BNSGP-homeomorphism.

(3) $\to$ (1): Let ψ is a BNSGP homeomorphism. This means, ψ and ψ^{-1} are BNSGP continuous mapping. Since every BNCS in X is a $BNSGPCS$ in Y, then ψ is a BNSGP-closed mapping. □

Remark 3.10. The composition of two BNSGP homeomorphism need not be a BNSGP homeomorphism in general.

Example 3.11. Let $X = \{a, b\}, Y = \{u, v\}, Z = \{p, q\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{c} \Big\langle e_1, \{\langle x_1,\ 1, 0, 1, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.5, -0.4, -0.3\rangle\}\Big\rangle, \\ \Big\langle e_2, \{\langle x_1,\ 0.4, 0.6, 0.3, -0.4, -0.7, -0.2\rangle, \langle x_2,\ 0.7, 0.2, 0.1, -0.3, -0.5, -0.7\rangle\}\Big\rangle \end{array} \right\}$$

https://doi.org/10.54216/IJNS.180112

Received: September 05, 2021 Accepted: January 22, 2022

$$B_2 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.3, 0.1, 0.7, -0.5, -0.6, -0.3\rangle, \langle x_2,\ 0, 1, 1, -0.7, 0, -1\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.5, 0.7, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.3, -0.1, -0.6, -0.3\rangle\} \right\rangle \end{array} \right\}$$

$$B_3 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 1, 0, 0.7, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.7, 0, -0.3\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.5, 0.3, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.1, -0.3, -0.5, -0.3\rangle\} \right\rangle \end{array} \right\}$$

Let $_1 = \{0, B_1, 1\}$, $_2 = \{0, B_2, 1\}$ and $_3 = \{0, B_3, 1\}$ are BNSTs on X, Y and Z respectively. Now we define a bijective mapping $\psi : (X,_1, E) \to (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$ and $\varphi : (Y,_2, E) \to (Z,_3, E)$ by $\varphi(x) = p$ and $\varphi(y) = q$. Then ψ and ψ^{-1} are BNSGP-continuous mappings, also φ and φ^{-1} are BNSGP-continuous mappings. Hence ψ and φ are BNSGP-homeomorphism. But the mapping $\psi \circ \varphi : (X,_1, E) \to (Z,_3, E)$ is not a BNSGP-homeomorphism since $\psi \circ \varphi$ is not a BNSGP-continuous mapping.

4 Bipolar neutrosophic soft contra generalized pre continuous mappings

Definition 4.1. A map $\psi : (X,_1, E) \to (Y,_2, E)$ is said to be a Bipolar neutrosophic soft contra generalized pre-continuous (BNSCGP-continuous) mapping if $\psi^{-1}(B)$ is a BNSGPCS in $(X,_1, E)$ for every BNOS B in $(Y,_2, E)$.

Example 4.2. Let $X = \{a, b\}$, $Y = \{u, v\}$, $E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.5, 0.6, 0.8, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.5, -0.4, -0.3\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.6, 0.3, -0.4, -0.7, -0.2\rangle, \langle x_2,\ 0.7, 0.2, 0.1, -0.3, -0.5, -0.7\rangle\} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.3, 0.1, 0.7, -0.5, -0.6, -0.3\rangle, \langle x_2,\ 0, 1, 1, -0.7, 0, -1\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.5, 0.7, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.3, -0.1, -0.6, -0.3\rangle\} \right\rangle \end{array} \right\}$$

Then $_1 = \{0, B_1, 1\}$ and $_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X,_1, E) \to (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSCGP-continuous mapping.

Theorem 4.3. *Every BNSC-continuous mapping is a BNSCGP-continuous mapping but not conversely.*

Proof. Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a BNSC-continuous mapping. Let B be a BNOS in Y. Then $\psi^{-1}(B)$ is a BNCS in X. Since every BNCS is a BNSGPCS, $\psi^{-1}(B)$ is a BNSGPCS in X. Hence, ψ is a BNSCGP continuous mapping. □

Example 4.4. Let $X = \{a, b\}$, $Y = \{u, v\}$, $E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.5, 0.6, 0.8, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.5, -0.4, -0.3\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.6, 0.3, -0.4, -0.7, -0.2\rangle, \langle x_2,\ 0.7, 0.2, 0.1, -0.3, -0.5, -0.7\rangle\} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.3, 0.1, 0.7, -0.5, -0.6, -0.3\rangle, \langle x_2,\ 0, 1, 1, -0.7, 0, -1\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.5, 0.7, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.3, -0.1, -0.6, -0.3\rangle\} \right\rangle \end{array} \right\}$$

https://doi.org/10.54216/IJNS.180112
Received: September 05, 2021 Accepted: January 22, 2022

Then $_1 = \{0, B_1, 1\}$ and $_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X,_1, E) \to (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSCGP-continuous mapping but not a BNSC-continuous mapping.

Theorem 4.5. *Every BNSCα-continuous mapping is a BNSCGP continuous mapping but not conversely.*

Proof. Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a BNSCα-continuous mapping. Let B be a BNOS in Y. Then $\psi^{-1}(B)$ is a BNSαCS in X. Since every BNSαCS is a BNGPCS, $\psi^{-1}(B)$ is a BNGPCS in X. Hence, ψ is a BNSCGP-continuous mapping. $\square$

Example 4.6. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.5, 0.6, 0.8, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.5, -0.4, -0.3\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.6, 0.3, -0.4, -0.7, -0.2\rangle, \langle x_2,\ 0.7, 0.2, 0.1, -0.3, -0.5, -0.7\rangle\}\right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.3, 0.1, 0.7, -0.5, -0.6, -0.3\rangle, \langle x_2,\ 0, 1, 1, -0.7, 0, -1\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.5, 0.7, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.3, -0.1, -0.6, -0.3\rangle\}\right\rangle \end{array} \right\}$$

Then $_1 = \{0, B_1, 1\}$ and $_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X,_1, E) \to (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSCGP-continuous mapping but not a BNSCα-continuous mapping.

Theorem 4.7. *Every BNSCP-continuous mapping is a BNSCGP continuous mapping but not conversely.*

Proof. Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a BNSCP-continuous mapping. Let B be a BNOS in Y. Then $\psi^{-1}(B)$ is a BNSPCS in X. Since every BNSPCS is a BNSGPCS, $\psi^{-1}(B)$ is a BNSGPCS in X. Hence, ψ is a BNSCGP-continuous mapping. $\square$

Example 4.8. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.5, 0.6, 0.8, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.5, -0.4, -0.3\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.6, 0.3, -0.4, -0.7, -0.2\rangle, \langle x_2,\ 0.7, 0.2, 0.1, -0.3, -0.5, -0.7\rangle\}\right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.3, 0.1, 0.7, -0.5, -0.6, -0.3\rangle, \langle x_2,\ 0, 1, 1, -0.7, 0, -1\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.5, 0.7, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.3, -0.1, -0.6, -0.3\rangle\}\right\rangle \end{array} \right\}$$

Then $_1 = \{0, B_1, 1\}$ and $_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X,_1, E) \to (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSCGP-continuous mapping but not a BNSCP-continuous mapping.

Theorem 4.9. *Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a mapping. Then the following statements are equivalent.*

1. *ψ is a BNSCGP continuous mapping.*
2. *$\psi^{-1}(B)$ is a BNSGPOS in X for every BNCS B in Y.*

https://doi.org/10.54216/IJNS.180112

Received: September 05, 2021 Accepted: January 22, 2022

Proof. $(1) \to (2)$: Let B be a BNCS in Y. Then B^c is a BNOS in Y. By statement, $\psi^{-1}(B^c)$ is a BNSGPCS in X. Hence $\psi^{-1}(B)$ is a BNSGPOS in X.

$(2) \to (1)$: Let B be a BNOS in Y. Then B^c is a BNCS in Y. By statement, $\psi^{-1}(B^c)$ is a BNSGPOS in X. Hence $\psi^{-1}(B)$ is a BNSGPCS in X. Thus ψ is a BNSCGP-continuous mapping. □

Theorem 4.10. *Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a mapping. If one of the following properties hold:*

1. $\psi(BNSPcl(A)) \subseteq BNSint(\psi(A))$ *for each A in X.*
2. $BNSPcl(\psi^{-1}(B)) \subseteq \psi^{-1}(BNSint(B))$ *for each B in Y.*
3. $\psi^{-1}(BNScl(B)) \subseteq BNSPint(\psi^{-1}(B))$ *for each B in Y.*

Then ψ is a BNSCGP-continuous mapping.

Proof. $(1) \to (2)$: Let B be a BNS in Y. Then $\psi^{-1}(B)$ is a BNS in X. By statement, we have $\psi(BNSPcl(\psi^{-1}(B))) \subseteq BNSint(\psi(\psi^{-1}(B))) \subseteq BNSint(B)$. Now $BNSPcl(\psi^{-1}(B)) \subseteq \psi^{-1}(\psi(BNSPcl(\psi^{-1}(B)))) \subseteq \psi^{-1}(BNSint(B))$.

$(2) \to (1)$: By taking the complement in (2). Suppose that (3) holds. Let B be a BNCS in Y. Then $BNcl(B) = B$. By the assumption, $\psi^{-1}(B) = \psi^{-1}(BNcl(B)) \subseteq BNSPint(\psi^{-1}(B))$. But $BNSPint(\psi^{-1}(B)) \subseteq \psi^{-1}(B)$, hence $BNSPint(\psi^{-1}(B)) = \psi^{-1}(B)$. This implies $\psi^{-1}(B)$ is a BNSPOS in X and hence $\psi^{-1}(B)$ is a BNSGPOS in X. Thus ψ is a BNSCGP-continuous mapping. □

Theorem 4.11. *Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a bijective mapping. If one of the following properties hold:*

1. $\psi^{-1}(BNcl(B)) \subseteq BNint(BNSPcl(\psi^{-1}(B)))$ *for each B in Y.*
2. $BNcl(BNSPint(\psi^{-1}(B))) \subseteq \psi^{-1}(BNint(B))$ *for each B in Y.*
3. $\psi(BNcl(BNSPint(A))) \subseteq BNint(\psi(A))$ *for each A in X.*
4. $\psi(BNcl(A)) \subseteq BNint(\psi(A))$ *for each BNPOS A in X.*

Then ψ is a BNSCGP-continuous mapping.

Proof. $(1) \to (2)$: It is obvious, by taking the complement in (1).

$(2) \to (3)$: Let A be a BNS in X. Put $B = \psi(A)$ in Y. This implies $A = \psi^{-1}(\psi(A)) = \psi^{-1}(B)$ in X. Now $BNcl(BNSPint(A)) = BNcl(BNSPint(\psi^{-1}(B))) \subseteq \psi^{-1}(BNint(B))$ by the hypothesis. Therefore, $\psi(BNcl(BNSPint(A))) \subseteq \psi(\psi^{-1}(BNint(B))) = BNint(B) = BNint(\psi(A))$.

$(3) \to (4)$: Let A be a BNSPOS in X. Then $BNSPint(A) = A$. By hypothesis,
$\psi(BNcl(BNSPint(A))) \subseteq BNint(\psi(A))$.
Therefore, $\psi(BNcl(A)) = \psi(BNcl(BNSPint(A))) \subseteq BNint(\psi(A))$.

Suppose (4) holds: Let A be a BNOS in Y. Then $\psi^{-1}(A)$ is a BNOS in X and $BNSPint(\psi^{-1}(A))$ is a BNSPOS in X. Hence, by hypothesis,

Received: September 05, 2021 Accepted: January 22, 2022

$\psi(BNcl(BNSPint(\psi^{-1}(A)))) \subseteq BNint(\psi(BNSPint(\psi^{-1}(A)))) \subseteq BNint(\psi(\psi^{-1}(A))) = BNint(A) \subseteq A$.
Therefore $BNcl(BNSPint(\psi^{-1}(A))) = \psi^{-1}(\psi(BNcl(BNSPint(\psi^{-1}(A))))) \subseteq \psi^{-1}(A)$. Now, $BNcl(BNint(\psi^{-1}(A)))$ $BNcl(BNSPint(\psi^{-1}(A))) \subseteq \psi^{-1}(A)$. This implies $\psi^{-1}(A)$ is a BNSPCS in X and hence a BNSGPCS in X. Thus ψ is a BNSCGP-continuous mapping. □

Theorem 4.12. *Let $\psi : (X,_{1}, E) \to (Y,_{2}, E)$ be a bijective mapping. Then ψ is a BNSCGP continuous mapping if $BNcl(\psi(A)) \subseteq \psi(BNSPint(A))$ for every BNS A in X.*

Proof. Let A be a BNCS in Y. Then $BNcl(A) = A$ and $\psi^{-1}(A)$ is a BNS in X. By hypothesis, $BNcl(\psi(\psi^{-1}(A))) \subseteq \psi(BNSPint(\psi^{-1}(A)))$. Since ψ is onto, $\psi(\psi^{-1}(A)) = A$. Therefore $A = BNcl(A) = BNcl(\psi(psi^{-1}(A))) \subseteq \psi(BNSPint(\psi^{-1}(A)))$.
Now $\psi^{-1}(A) \subseteq \psi^{-1}(\psi(BNSPint(\psi^{-1}(A)))) = BNSPint(\psi^{-1}(A)) \subseteq \psi^{-1}(A)$. Hence $\psi^{-1}(A)$ is a BNSPOS in X and hence BNSGPOS in X. Thus ψ is a BNSCGP-continuous mapping. □

Theorem 4.13.
1. *If $\psi : (X,_{1}, E) \to (Y,_{2}, E)$ is a BNSCGP-continuous mapping and $\varphi : (Y,_{2}, E) \to (Z,_{3}, E)$ is a continuous mapping, then $\varphi \circ \psi : (X,_{1}, E) \to (Z,_{3}, E)$ is a BNSCGP-continuous mapping.*
2. *If $\psi : (X,_{1}, E) \to (Y,_{2}, E)$ is a BNSCGP-continuous mapping and $\varphi : (Y,_{2}, E) \to (Z,_{3}, E)$ is a BNSC-continuous mapping, then $\varphi \circ \psi : (X,_{1}, E) \to (Z,_{3}, E)$ is a BNSGP-continuous mapping.*
3. *If $\psi : (X,_{1}, E) \to (Y,_{2}, E)$ is a BNSGP-irresolute mapping and $\varphi : (Y,_{2}, E) \to (Z,_{3}, E)$ is a BNSCGP-continuous mapping, then $\varphi \circ \psi : (X,_{1}, E) \to (Z,_{3}, E)$ is a BNSCGP-continuous mapping.*

Proof.
1. Let A be BNOS in Z. Then $\varphi^{-1}(A)$ is a BNOS in Y, by hypothesis. Since ψ is a BNSCGP-continuous mapping, $\psi^{-1}(\varphi^{-1}(A))$ is a BNSGPCS in X. Hence $\varphi \circ \psi$ is a BNSCGP-continuous mapping.
2. Let A be BNOS in Z. Then $\varphi^{-1}(A)$ is a BNCS in Y, by hypothesis. Since ψ is a BNSCGP-continuous mapping, $\psi^{-1}(\varphi^{-1}(A))$ is a BNSGPOS in X. Hence $\varphi \circ \psi$ is a BNSGP-continuous mapping.
3. Let A be BNOS in Z. Then $\varphi^{-1}(A)$ is a BNSGPCS in Y, by hypothesis. Since ψ is a BNSGP-continuous mapping, $\psi^{-1}(\varphi^{-1}(A))$ is a BNSGPCS in X. Hence $\varphi \circ \psi$ is a BNSCGP-continuous mapping.

□

Theorem 4.14. *A mapping $\psi : (X,_{1}, E) \to (Y,_{2}, E)$ is a BNSCGP-continuous mapping if $\psi^{-1}(BNSPcl(B)) \subseteq BNint(\psi^{-1}(B))$ for every BNS B in Y.*

Proof. Let B be a BNCS in Y. Then $BNcl(B) = B$. Since every BNCS is a BNSPCS, this implies $BNSPcl(B) = B$. By hypothesis, $\psi^{-1}(B) = \psi^{-1}(BNSPcl(B)) \subseteq BNint(\psi^{-1}(B)) \subseteq \psi^{-1}(B)$. This implies $\psi^{-1}(B)$ is a BNOS in X. Therefore ψ is a BNSC-continuous mapping, since every BNSC-continuous mapping is a BNSCGP-continuous mapping, ψ is a BNSCGP-continuous mapping. □

Theorem 4.15. *A BNS-continuous mapping, $\psi : (X,_{1}, E) \to (Y,_{2}, E)$ is a BNSCGP continuous mapping if $BNSGPO(X) = BNSGPC(X)$.*

Proof. Let B be a BNOS in Y. By hypothesis, $\psi^{-1}(B)$ is a BNOS in X and hence is a BNSGPOS in X. Since BNSGPO(X)=BNSGPC(X), $\psi^{-1}(B)$ is a BNSGPCS in X. Therefore, ψ is a BNSCGP-continuous mapping. □

https://doi.org/10.54216/IJNS.180112
Received: September 05, 2021 Accepted: January 22, 2022

5 Bipolar neutrosophic soft contra generalized α-continuous mapping

in this section we introduce bipolar neutrosophic soft contra generalized α-continuous mapping and investigate some of its properties.

Definition 5.1. A bipolar neutrosophic soft set B in $(X, , E)$ is said to be a bipolar neutrosophic soft generalized α-closed set (BNSGαCS) if $BN\alpha cl(B) \subseteq U$ whenever $B \subseteq U$ is a BNαOS in $(X, , E)$.

Definition 5.2. A mapping $\psi : (X,_1, E) \rightarrow (Y,_2, E)$ is said to be a bipolar neutrosophic soft contra generalized α-continuous (BNSCGα-continuous) if $\psi^{-1}(B)$ is a bipolar neutrosophic soft generalized α-closed set in $(x,_1, E)$ for every bipolar neutrosophic soft open set B in $(Y,_2, E)$.

Definition 5.3. A mapping $\psi : (X,_1, E) \rightarrow (Y,_2, E)$ is said to be a bipolar neutrosophic soft strongly generalized α-continuous (BNSSGα-continuous) if $\psi^{-1}(B)$ is a bipolar neutrosophic soft open set in $(x,_1, E)$ for every bipolar neutrosophic soft generalized α-open set B in $(Y,_2, E)$.

Definition 5.4. A mapping $\psi : (X,_1, E) \rightarrow (Y,_2, E)$ is said to be a bipolar neutrosophic soft contra strongly generalized α-continuous (BNSCSGα-continuous) if $\psi^{-1}(B)$ is a bipolar neutrosophic soft closed set in $(x,_1, E)$ for every bipolar neutrosophic soft generalized α-open set B in $(Y,_2, E)$.

Definition 5.5. A mapping $\psi : (X,_1, E) \rightarrow (Y,_2, E)$ is said to be a bipolar neutrosophic soft contra generalized α-irresolute if $\psi^{-1}(B)$ is a bipolar neutrosophic soft generalized α-closed set in $(x,_1, E)$ for every bipolar neutrosophic soft generalized α-open set B in $(Y,_2, E)$.

Theorem 5.6. *Every BNSC-continuous mapping is a BNSCGα-continuous mapping but converse not true.*

Proof. Let $\psi : (X,_1, E) \rightarrow (Y,_2, E)$ be a BNSC-continuous mapping and let B be BNOS in Y. Then $\psi^{-1}(B)$ is a BNCS in X. Since every BNCS is a BNSGαCS, $\psi^{-1}(B)$ is a BNSGαCS in $(X,_1, E)$. Hence ψ is a BNSGα-continuous mapping. □

Example 5.7. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{\langle x_1,\ 0.6, 0.6, 0.6, -0.5, -0.4, -0.6\rangle, \langle x_2,\ 0.4, 0.4, 0.4, -0.2, -0.4, -0.3\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.7, 0.5, 0.4, -0.6, -0.7, -0.2\rangle, \langle x_2,\ 0.6, 0.2, 0.5, -0.3, -0.5, -0.5\rangle\}\right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \left\langle e_1, \{\langle x_1,\ 0.2, 0.2, 0.3, -0.4, -0.2, -0.6\rangle, \langle x_2,\ 0.6, 0.4, 0.6, -0.7, -0.4, -0.3\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.7, 0.7, 0.8, -0.4, -0.5, -0.2\rangle, \langle x_2,\ 0.6, 0.7, 0.3, -0.4, -0.3, -0.2\rangle\}\right\rangle \end{array} \right\}$$

Then $_1 = \{0, B_1, 1\}$ and $_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X,_1, E) \rightarrow (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSCGα-continuous mapping but not a BNSC-continuous mapping.

Theorem 5.8. *Every BNSCα-continuous mapping is a BNSGα-continuous mapping but converse not true.*

Proof. Let $\psi : (X,_1, E) \rightarrow (Y,_2, E)$ be a BNSCα-continuous mapping and let B be BNOS in Y. Then $\psi^{-1}(B)$ is a BNαCS in X. Since every BNαCS is a BNSGαCS, $\psi^{-1}(B)$ is a BNSGαCS in $(X,_1, E)$. Hence ψ is a BNSGα-continuous mapping. □

https://doi.org/10.54216/IJNS.180112
Received: September 05, 2021 Accepted: January 22, 2022

Example 5.9. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{\langle x_1,\ 0.5, 0.5, 0.5, -0.4, -0.4, -0.4\rangle, \langle x_2,\ 0.4, 0.4, 0.4, -0.5, -0.5, -0.5\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.4, 0.4, -0.5, -0.5, -0.5\rangle, \langle x_2,\ 0.5, 0.5, 0.5, -0.4, -0.4, -0.4\rangle\}\right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \left\langle e_1, \{\langle x_1,\ 0.6, 0.6, 0.6, -0.4, -0.4, -0.4\rangle, \langle x_2,\ 0.4, 0.4, 0.4, -0.6, -0.6, -0.6\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.4, 0.4, -0.6, -0.6, -0.6\rangle, \langle x_2,\ 0.6, 0.6, 0.6, -0.4, -0.4, -0.4\rangle\}\right\rangle \end{array} \right\}$$

Then $_1 = \{0, B_1, 1\}$ and $_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X,_1, E) \to (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSGα-continuous mapping but not a BNSCα-continuous mapping.

Theorem 5.10. *Every BNSCSGα-continuous mapping is a BNSCGα-continuous mapping but converse not true.*

Proof. Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a BNSCSGα-continuous mapping and let B be BNOS in Y. Since every BNOS is a BNSGαOS, B is a BNSGαOS in Y. Also, since ψ is a BNSCSGα-continuous mapping, $\psi^{-1}(B)$ is a BNCS in X. Since every BNCS is a BNSGαCS, $\psi^{-1}(B)$ is a BNSGαCS in $(X,_1, E)$. Hence ψ is a BNSGα-continuous mapping. □

Example 5.11. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{\langle x_1,\ 0.3, 0.3, 0.3, -0.4, -0.4, -0.4\rangle, \langle x_2,\ 0.4, 0.4, 0.4, -0.5, -0.5, -0.5\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.2, 0.4, -0.3, -0.3, -0.5\rangle, \langle x_2,\ 0.3, 0.3, 0.3, -0.4, -0.4, -0.5\rangle\}\right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \left\langle e_1, \{\langle x_1,\ 0.8, 0.8, 0.7, -0.3, -0.3, -0.4\rangle, \langle x_2,\ 0.6, 0.6, 0.4, -0.7, -0.7, -0.4\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.4, 0.4, -0.6, -0.6, -0.6\rangle, \langle x_2,\ 0.6, 0.6, 0.6, -0.4, -0.4, -0.4\rangle\}\right\rangle \end{array} \right\}$$

Then $_1 = \{0, B_1, 1\}$ and $_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X,_1, E) \to (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSCGα-continuous mapping but not a BNSCSGα-continuous mapping.

Theorem 5.12. *Every BNSCSGα-continuous mapping is a BNSC-continuous mapping but converse not true.*

Proof. Let $\psi : (X,_1, E) \to (Y,_2, E)$ be a BNSCSGα-continuous mapping and let B be BNOS in Y. Since every BNOS is a BNSGαOS, B is a BNSGαOS in Y. Also, since ψ is a BNSCSGα-continuous mapping, $\psi^{-1}(B)$ is a BNCS in X. Hence ψ is a BNSC-continuous mapping. □

Example 5.13. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{\langle x_1,\ 0.3, 0.3, 0.3, -0.4, -0.4, -0.4\rangle, \langle x_2,\ 0.4, 0.4, 0.4, -0.5, -0.5, -0.5\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.2, 0.4, -0.3, -0.3, -0.5\rangle, \langle x_2,\ 0.3, 0.3, 0.3, -0.4, -0.4, -0.5\rangle\}\right\rangle \end{array} \right\}$$

https://doi.org/10.54216/IJNS.180112

Received: September 05, 2021 Accepted: January 22, 2022

$$B_2 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.8, 0.8, 0.7, -0.3, -0.3, -0.4\rangle, \langle x_2,\ 0.6, 0.6, 0.4, -0.7, -0.7, -0.4\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.4, 0.4, -0.6, -0.6, -0.6\rangle, \langle x_2,\ 0.6, 0.6, 0.6, -0.4, -0.4, -0.4\rangle\} \right\rangle \end{array} \right\}$$

Then $_1 = \{0, B_1, 1\}$ and $_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X,_1, E) \to (Y,_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSC-continuous mapping but not a BNSCSGα-continuous mapping.

Theorem 5.14. *1. If $\psi : (X,_1, E) \to (Y,_2, E)$ is a BNSCGα-continuous mapping and $\varphi : (Y,_2, E) \to (Z,_3, E)$ is a BNS-continuous mapping, then $\varphi \circ \psi : (X,_1, E) \to (Z,_3, E)$ is a BNSCGα-continuous mapping.*

2. If $\psi : (X,_1, E) \to (Y,_2, E)$ is a BNSCGα-continuous mapping and $\varphi : (Y,_2, E) \to (Z,_3, E)$ is a BNSCα-continuous mapping, then $\varphi \circ \psi : (X,_1, E) \to (Z,_3, E)$ is a BNSGα-continuous mapping.

3. If $\psi : (X,_1, E) \to (Y,_2, E)$ is a BNSCGα-irresolute mapping and $\varphi : (Y,_2, E) \to (Z,_3, E)$ is a BNSCGα-continuous mapping, then $\varphi \circ \psi : (X,_1, E) \to (Z,_3, E)$ is a BNSGα-continuous mapping.

4. If $\psi : (X,_1, E) \to (Y,_2, E)$ is a BNSGα-irresolute mapping and $\varphi : (Y,_2, E) \to (Z,_3, E)$ is a BNSCGα-continuous mapping, then $\varphi \circ \psi : (X,_1, E) \to (Z,_3, E)$ is a BNSCGα-continuous mapping.

Proof. 1. Let B be a BNOS in Z. Since φ is a BNS-continuous mapping, $\varphi^{-1}(B)$ is BNOS in Y. Since ψ is a BNSCα-continuous mapping, $\psi^{-1}(\varphi^{-1}(B))$ is a BNSGαCS in X. Hence, $\varphi \circ \psi$ is a BNSCGα-continuous mapping.

2. Let B be a BNOS in Z. Since φ is a BNSC-continuous mapping, $\varphi^{-1}(B)$ is BNCS in Y. Since ψ is a BNSCGα-continuous mapping, $\psi^{-1}(\varphi^{-1}(B))$ is a BNSGαOS in X. Hence, $\varphi \circ \psi$ is a BNSGα-continuous mapping.

3. Let B be a BNOS in Z. Since φ is a BNSCGα-continuous mapping, $\varphi^{-1}(B)$ is BNSGαCS in Y. Since ψ is a BNSCGα-irresolute mapping, $\psi^{-1}(\varphi^{-1}(B))$ is a BNSGαOS in X. Hence, $\varphi \circ \psi$ is a BNSGα-continuous mapping.

4. Let B be a BNOS in Z. Since φ is a BNSCGα-continuous mapping, $\varphi^{-1}(B)$ is BNSGαCS in Y. Since ψ is a BNSGα-irresolute mapping, $\psi^{-1}(\varphi^{-1}(B))$ is a BNSGαCS in X. Hence, $\varphi \circ \psi$ is a BNSCGα-continuous mapping.

□

6 Conclusion

In this paper, we have introduced bipolar neutrosophic soft contra generalized pre-continuous mapping and bipolar neutrosophic soft contra generalized α-continuous mapping. Results are in this paper shows that preservation of topological structures such as closeness and openness by various continuity mappings.

References

[1] Florentin Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, Rehoboth, American Research Press, 1998.

https://doi.org/10.54216/IJNS.180112

Received: September 05, 2021 Accepted: January 22, 2022

[2] Florentin Smarandache. Neutrosophy: A new branch of philosophy, Multiple valued logic: An international journal, 8(2002), 297-384.

[3] D. Molodtsov. Soft set theory - First results, Computers and Mathematics with applications, 37(1999), 4-5, 19-31.

[4] P. Arulpandy and M. Trinita Pricilla. Some similarity and entropy measurements of bipolar neutrosophic soft sets, Neutrosophic sets and systems, 25(2019), 1, 174-194.

[5] Zadeh LA; Fuzzy Sets. Information and Control 1965; 8 (3), 338-353.

[6] Atanassov K.; Intuitionistic fuzzy sets. Fuzzy set and systems 1999; 20, 87-96.

[7] Tuhin Bera and Nirmal Kumar Mahapatra. Introduction to neutrosophic soft topological space, OPSEARCH, 54(2017), 841-867.

[8] Irfan Deli; Mumtaz Ali and Florentin Smarandache. Bipolar Neutosophic sets and Their applications based on multi-criteria decision making problems, Proceedings of Int. Conf. on Advanced Mechatronic systems, 2015, 249-254.

[9] Chang C.L.; Fuzzy topological spaces. Journal of Mathematical Analysis and Applications, 21(1968), 182-190.

[10] Atanassov, K. Intuitionistic fuzzy sets. In VII ITKR's Session; Publishing House: Sofia, Bulgaria, 1983.

[11] Norman Levine, Generalized closed sets in topology. Rend. Circ. Mat. Palermo 1970, 19, 89-96.

[12] Norman Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.

[13] A.A.Salama and S.A.Alblowi. Neutrosophic Set and Neutrosophic Topological Spaces, IOSR Journal of Mathematics, 3(2012), 4, 31-35.

https://doi.org/10.54216/IJNS.180112

Received: September 05, 2021 Accepted: January 22, 2022