The Structure of Imperfect Triplets In Several Refined Neutrosophic Rings

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Abstract

This paper solves the imperfect triplets problem in refined neutrosophic rings, where it presents the necessary and sufficient conditions for a triple (x, y, z) to be an imperfect triplet in any refined neutrosophic ring. Also, this work introduces a full description of the structure of imperfect triplets in numerical refined neutrosophic rings such as refined neutrosophic ring of integers $Z(I_1, I_2)$, refined neutrosophic ring of rationales $Q(I_1, I_2)$, and refined neutrosophic ring or real numbers $R(I_1, I_2)$.
Keywords: Refined Neutrosophic Ring, imperfect Duplet, Imperfect triplet

1. Introduction

Neutrosophy is a generalization of intuitionistic fuzzy logic founded by F.Smarandache to deal with indeterminacy in science and real life problems.

Neutrosophic algebra began with the efforts of Kandasamy and Smarandache where the concept of neutrosophic ring was presented in [7] as a generalization of classical rings. These rings were handled by many authors uch as [1,2,9,11-19].

Recently, there is an increasing interest in the generalizations of neutrosophic rings, where refined neutrosophic rings were defined by Agboola et.al [5,6].

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If (R,+,.) is a ring, then the corresponding refined neutrosophic ring $R(I_1, I_2)$ is defined as follows:

$R(I_1, I_2) = \{(a, bI_1, cI_2); a, b, c \in R\}$. The operations on $R(I_1, I_2)$ are defined as follows:

Addition: $(a, bI_1, cI_2)+(x, yI_1, zI_2)=(a + x, (b + y)I_1, (c + z)I_2)$.

Multiplication: $(a, bI_1, cI_2).(x, yI_1, zI_2)=(ax, (ay + bx + by + bz + cy)I_1, (az + cz + cx)I_2)$

The notion of neutrosophic duplets and neutrosophic triplets was defined and handled by Smarandache et.al in [3,8,10], where they opened an interesting research direction about finding these elements in rings.

The structure of imperfect refined neutrosophic duplets was discussed in [38].

Through this paper, we extend the previous efforts to solve the problem of triplets into the case of refined neutrosophic rings, where we present the condition of imperfect triplets in any refined neutrosophic ring even when it is not commutative. In particular, we determine all possible imperfect triplets in the refined neutrosophic ring of integers $Z(I_1, I_2)$, refined neutrosophic ring of rationales $Q(I_1, I_2)$, and refined neutrosophic ring of real numbers $R(I_1, I_2)$.

2. Preliminaries

Definition 2.1: [5]

(a) If X is a set then $X(I_1, I_2) = \{(a, bI_1, cI_2): a, b, c \in X\}$ is called the refined neutrosophic set generated by X , I_1, I_2.

(b) Let (R,+,×) be a ring, $(R(I_1, I_2), +, \times)$ is called the refined neutrosophic ring generated by R ,I_1, I_2.

Definition 2.2: [12]

Let R be any ring, x, y are two arbitrary elements in R. We call them a duplet with y acts as an identity if and only if

$xy = yx = x$.

Definition 2.3: [10]

Let R be any ring, x, y, z three arbitrary elements in R. We call them a triplet with y acts as an identity if and only if

$xy = yx = x, zy = yz = z, xz = zx = y$.

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Theorem 2.4: [38]

Let $x = (x_0, x_1I_1, x_2I_2), y = (y_0, y_1I_1, y_2I_2)$ be any two elements in $R(I_1, I_2)$, then (x, y) is an imperfect refined neutrosophic duplet with y acts as an identity if and only if

$(x_0, y_0), (x_0 + x_2, y_0 + y_2), (x_0 + x_1 + x_2, y_0 + y_1 + y_2)$ are three imperfect duplets in the classical ring R with $y_0, y_0 + y_2, y_0 + y_1 + y_2$ acts as identities.

For the structure of refined neutrosophic imperfect duplets. See [38].

3. Main discussion :

Theorem 3.1:

Let $Q(I_1, I_2)$, $R(I_1, I_2)$ be the neutrosophic ring of rationales and reals respectively. Consider the following two forms of imperfect duplets

1-)$\{(0, x_1I_1, 0), (y_0, y_1I_1, y_2I_2);\ y_0 + y_1 + y_2 = 1, x_1 \neq 0\}$,

2-)$\{(0, x_1I_1, -x_1I_2), (y_0, y_1I_1, y_2I_2); y_0 + y_2 = 1\ and\ x_1 \neq 0\}$.

The corresponding imperfect triplets derived from the previous forms are

(a-)$\{(0, x_1I_1, 0), (0, I_1, 0), (0, \frac{1}{x_1}I_1, 0);\ x_1 \neq 0\}$,

(b-) $\{(0, x_1I_1, -x_1I_2), (0, -I_1, I_2), (0, \frac{1}{x_1}I_1, -\frac{1}{x_1}I_2);\ x_1 \neq 0\}$.

Proof:

Suppose that (x, y, z) is an imperfect triplet, then $xy = yx = x, xz = zx = y, yz = zy = z$. It is clear that $(x, y), (y, z)$ are imperfect duplets with y acts as an identity.

From the form (1), we have $x = (0, x_1I_1, 0)$, $y =(y_0, y_1I_1, y_2I_2)$,$z =(0, z_1I_1, 0)$ with $y_0 + y_1 + y_2 = 1, x_1 \neq 0$. From the equation $xz = y$, we get that $y_0 = y_2 = 0, y_1 = 1$ and $z_1 = \frac{1}{x_1}$, thus the corresponding imperfect triplet is (a).

From the form (2), we have $x =(0, x_1I_1, -x_1I_2)$, $y =(y_0, y_1I_1, y_2I_2)$, $z =(0, z_1I_1, -z_1I_2)$ with $y_0 + y_2 = 1$. From the equation $xz = y$, we get that $y_2 = 1, y_1 = -1, y_0 = 0, z_1 = \frac{1}{x_1}$

And the corresponding imperfect triplet is (b).

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Example 3.2:

We construct an example about an imperfect triplet with form (b).

Put $x_1 = \frac{1}{3}$. The corresponding triplet is $x = \left(0, \frac{1}{3} I_1, -\frac{1}{3} I_2\right), y = (0, -I_1, I_2), z = (0, 3 I_1, -3I_2)$.

Theorem 3.3:

Let $Q(I_1, I_2)$, $R(I_1, I_2)$ be the neutrosophic ring of rationales and reals respectively. Consider the following two forms of imperfect duplets

3-)$\{(0, x_1 I_1, x_2 I_1), (y_0, 0, y_2 I_2); y_0 + y_2 = 1\}$,

4-)$\{(x_0, 0, -x_0 I_2), (1, y_1 I_1, y_2 I_2)\}$,

The corresponding imperfect triplets has the forms

(c)$\{(0, x_1 I_1, x_2 I_2), (0,0, I_2), \left(0, \frac{-x_1}{x_2(x_1+x_2)} I_1, \frac{1}{x_2} I_2\right); x_2, x_1 + x_2 \neq 0\}$

(d) $\{(x_0, 0, -x_0 I_2), (1,0, -I_2), (\frac{1}{x_0}, 0, \frac{-1}{x_0} I_2); x_0 \neq 0\}$.

Proof:

Suppose that (x, y, z) is an imperfect triplet, then $xy = yx = x, xz = zx = y, yz = zy = z$. It is clear that $(x, y), (y, z)$ are imperfect duplets with y acts as an identity.

From the form (3), we have x=$(0, x_1 I_1, x_2 I_1)$, y=$(y_0, 0, y_2 I_2)$, z=$(0, z_1 I_1, z_2 I_1)$ with $y_0 + y_2 = 1$. From the equation $xz = y$, we get:

$y_0 = 0, y_2 = 1, z_2 = \frac{1}{x_2}, z_1 = \frac{-x_1}{x_2(x_1+x_2)}$. Thus the corresponding imperfect triplet is (c) .

From the form (4), we have x=$(x_0, 0, -x_0 I_2)$, y=$(1, y_1 I_1, y_2 I_2)$, z=$(z_0, 0, -z_0 I_2)$. From equation $xz = y$, we get $z_0 = \frac{1}{x_0}, y_2 = -1, y_1 = 0$. Thus we get the imperfect triplet (d).

Example 3.4:

We construct an example about an imperfect triplet with form (c).

Put $x_1 = \frac{1}{2}, x_2 = \frac{5}{2}$,

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We get the following triplet $x = \left(0, \frac{1}{2} I_1, \frac{5}{2} I_1\right), y = (0,0,I_2), z = \left(0, -\frac{1}{15} I_1, \frac{2}{5} I_1\right)$

Theorem 3.5:

Let $Q(I_1, I_2)$, $R(I_1, I_2)$ be the neutrosophic ring of rationales and reals respectively. Consider the following two forms of imperfect duplets

5-)$\{(x_0, x_1 I_1, -x_0 I_2), (1, y_1 I_1, -y_1 I_2); x_0 \neq 0\}$,

6-)$\{(x_0, x_1 I_1, x_2 I_2), (1, y_1 I_1, 0); x_0 + x_1 + x_2 = 0\}$.

The corresponding imperfect triplets are:

(e) $\{(x_0, x_1 I_1, -x_0 I_2), (1, I_1, -I_2), (\frac{1}{x_0}, \frac{1}{x_1} I_1, \frac{-1}{x_0} I_2); x_0, x_1 \neq 0\}$,

(f)$\{(x_0, x_1 I_1, x_2 I_2), (1, -I_1, 0), (\frac{1}{x_0}, z_1 I_1, \frac{-x_2}{x_0(x_0+x_2)} I_2); x_0 + x_1 + x_2 = 0, x_0, x_0 + x_2 \neq 0, z_1 \text{ is arbitrary}\}$.

Proof:

The proof holds by a similar discussion.

Example 3.6:

We construct an example about an imperfect triplet with form (e).

Put $x_0 = 1, x_1 = \frac{1}{2}$. Thus we get the following triplet

$$x = \left(1, \frac{1}{2} I_1, -I_2\right), y = (1, I_1, -I_2), z = (1, 2I_1, -I_2).$$

Theorem 3.7:

Let $Q(I_1, I_2)$, $R(I_1, I_2)$ be the neutrosophic ring of rationales and reals respectively. Imperfect triplets has the following 6 forms (a), (b), (c), (d), (e), (f).

The proof holds directly from previous Theorems .

Now, we check the existence of imperfect triplets in the refined neutrosophic ring of integers $Z(I_1, I_2)$.

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Remark 3.8:

A triple (x, y, z) is an imperfect triplet in $Z(I_1, I_2)$ if it can be represented by one of the forms (a-f) in Theorem 3.14 as a result of the inclusion between $Z(I_1, I_2)$ and $Q(I_1, I_2)$.

This means that the all imperfect triplets in $Z(I_1, I_2)$ are:

(a-)$\{(0, x_1I_1, 0), (0, I_1, 0I_2), \left(0, \frac{1}{x_1}I_1, 0\right);\ x_1 \in \{-1,1\}\},$

(b-) $\{(0, x_1I_1, -x_1I_1), (0, -I_1, I_2), (0, \frac{1}{x_1}I_1, -\frac{1}{x_1}I_1);\ x_1 \in \{-1,1\}\}.$

(c)$\{(0, x_1I_1, x_2I_1), (0,0, I_2), \left(0, \frac{-x_1}{x_2(x_1+x_2)}I_1, \frac{1}{x_2}I_2\right); x_2 \in \{-1,1\}\ and\ \frac{-x_1}{x_2(x_1+x_2)} \in Z\}$

(d) $\{(x_0, 0, -x_0I_2), (1,0, -I_2), (\frac{1}{x_0}, 0, \frac{-1}{x_0}I_2); x_0 \in \{-1,1\}\}.$

(e) $\{(x_0, x_1I_1, -x_0I_2), (1, I_1, -I_2), (\frac{1}{x_0}, \frac{1}{x_1}I_1, \frac{-1}{x_0}I_2); x_0, x_1 \in \{-1,1\}\},$

(f)$\{(x_0, x_1I_1, x_2I_2), (1, -I_1, 0), (\frac{1}{x_0}, z_1I_1, \frac{-x_2}{x_0(x_0+x_2)}I_2); x_0 + x_1 + x_2 = 0, x_0 \in$ $\{-1,1\}\ and\ z_1, \frac{-x_2}{x_0(x_0+x_2)} \in Z\}.$

Example 3.9:

The forms (a),(b),(d),(e) are clear. We illustrate an example about an imperfect triplet of form (c).

We put $x_2 = 1$, $x_1 = -2$, so that we get the following triplet

$x = (0, -2I_1, I_2), y = (0,0, I_2), z = (0, -2I_1, I_2).$

Another example about imperfect triplets with form (f).

Put $x_0 = x_1 = 1, x_2 = -2, z_1 = 5$, we get the following triplet

$x = (1, I_1, -2I_2), y = (1, -I_1, 0), z = (1,5I_1, -2I_2).$

Conclusion

In this paper, we have studied the problem of finding imperfect triplets in a refined neutrosophic ring. Where we have determined a general condition for imperfect duplets in such rings. In particular, we have

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presented the structure of all imperfect triplets in the refined neutrosophic rings of integers, reals, and rationales respectively.

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