A Short Note on the Kernel Subgroup of Intuitionistic Fuzzy Groups

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Abstract: This paper defines the concept of kernel subgroup of an intuitionistic fuzzy group. Also, it proves that this kernel is a group in the ordinary algebraic meaning as a direct application of the concept of kernel in fuzzy and anti-fuzzy groups. Also, we derive some properties of intuitionistic fuzzy groups.

Keywords: intuitionistic fuzzy group, intuitionistic fuzzy kernel, fuzzy subgroup

1.Introduction

Fuzzy set theory began with the work of Zadeh [1], were he has defined fuzzy subsets and relations.

These ideas have been used by many authors to study the algebra of fuzzy sets such as fuzzy groups [2,3], anti-fuzzy groups [20], intuitionistic fuzzy algebras [11] and some other interesting generalizations such as neutrosophic structures [9, 12-20].

The concept of intuitionistic fuzzy group was firstly defined in [2], and studied on a wide range in [4-7,15-19], where we find concepts such as intuitionistic fuzzy abelian subgroups, intuitionistic fuzzy normality, and many other algebraic concepts applied to fuzzy set theory.

In [25], the concept of fuzzy kernel and anti-fuzzy kernel was presented as a special subgroup of a fuzzy group and anti-fuzzy group.

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In this work, we built over the ideas presented in [25], to define the kernel of an intuitionistic fuzzy group, and to prove that it is a subgroup in the ordinary algebraic meaning.

Main discussion

Definition 1

1. Let be a group, $: \rightarrow [0,1]$. , then is called a fuzzy group if:

2. $(-1)= ()$ for all , ∈

Definition 2

1. Let be a group, $: \rightarrow [0,1]$. , then is called anti-fuzzy group if:

2. $(\quad -1 \quad)= ()$ for all , ∈

Let is called an intuitionstic-fuzzy group if:

Definition 3

1.( ) ≥ $\diamondsuit$ ( ), ( )$\diamondsuit$, (−1) = ().

For all , ∈ −1

4:

= { ∈ ; ()= ()}

Theorem 5:

Let is a subgroup of . we define the anti fuzzy kernel of with respect to as follows:

Definition 6:

= { ∈ ; ()= ()}

Theorem 7:

is a subgroup of .

Definition 8: { ∈ ;()= () ()= ()} .

be an intuitionistic fuzzy group, we define the intuitionistic fuzzy kernel of G as follows:

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Define: $= 5 = \{1,2,3,4\}.$.

Example 9:

It is $= 1, (2) = (3) = (4)$ $= 0.5, (1) = (4) = 0.1, (2) = (3) = 0.2$

Consider the multiplicative group of integers modulo 5,

Theorem $= \{1\},$ $= \{1,4\},$ $= \{1\}.$

clear that G is an intuitionistic fuzzy subgroup.

We have :

Proof: **10:**

is a subgroup of G.

The proof holds directly from the fact that the intersection of two subgroups is a subgroup.

The

Theorem 11:

Let be an intuitionistic fuzzy group, if , hence .

proof is clear. $= \{\}$

Definition 12:

Theorem 13:

Let be an intuitionistic fuzzy group, G is called intuitionisticly simple if and only if .

(„)

Let be an intuitionisticly simple fuzzy group, if the derivative subgroup is normal closed factor with respect to and , then G is abelian.

, so that G $\leq \leq , \ h \leq = \{\} =$

Assume that $'$ is a normal closed factor, hence $' \quad ' \quad ' \quad '$, thus $'$ („) is abelian.

Theorem 14:

Proof:

Let be an intuitionisticly simple fuzzy group, if the second derivative $''$ subgroup is normal closed factor with respect to and , then G is meta-abelian.

, so that G is meta-abelian. $\leq \leq , \ h \leq = \{\} =$

Assume that $''$ is a normal closed factor, hence $'' \quad '' \quad ''$, thus $''$

(a)

Definition 15:

(b) G is called intuitionisticly cyclic if and only if .

Let be an intuitionistic fuzzy group,

(c) G is called intuitionisticly solvable if and only

G is called intuitionisticly abelian if and only if

if .

Theorem 16:

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Let (, ,) be an intuitionistic fuzzy group, then:

(a) If G is abelian then its intuitionisticly abelian.

(b) If G is cyclic then its intuitionisticly cyclic.

(c) If G is solvable then its intuitionisticly solvable.

(d) If G is intuitionsticly abelian then its intuitionisticly solvable.

Proof:

is cyclic.

(a) It is known that any subgroup of abelian group is abelian, hence is abelian.

(b) It is known that any subgroup of cyclic group is cyclic, hence

hence is solvable.

(c) It is known that any subgroup of solvable group is solvable,

(d) Since every abelian group is solvable, hence is solvable.

Conclusion

In this work, we used the concept of fuzzy and anti-fuzzy kernel of a fuzzy group to build the concept of intuitionistic fuzzy kernel subgroup of an intuitionistic fuzzy group. Also, we have studied many interesting properties appear in this reach algebraic structure.

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