# On the Representation of Winning Strategies of Finite Games by Groups and Neutrosophic Groups

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**Abstract:** In this paper, we show that for a finite game with two players A , B:

Each winning strategy of the first player A can be represented by a neutrosophic subgroup of the neutrosophic group $(Z_2 \times .. \times Z_2)(I)$, and each winning strategy of the second player B can be represented by an elementary abelian group $Z_2 \times .. \times Z_2$.

Also, we introduce the concept of algebraically relative games and present some examples on it.

**Keywords:** Group , Neutrosophic group , Winning strategy

## 1.Introduction

Groups are always very useful in representations of algebraic structures, and finite games as a finite steps can be considered.

Neutrosophy as a branch of philosophy introduced by F.Smarandache has many applications in the real world and the mathematical concepts. The concept of neutrosophic group had been defined in[2] as a generalization of classical groups, subgroups and normal subgroups also were defined and studied.

The most useful understanding of neutrosophic group has been written in [3], we consider N(G) as a union of G and GI i.e N(G)=$\{x_1, x_2, \ldots x_1 I, x_2 I, \ldots . ; x_i \in G\}$.

We will use a neutrosophic subgroup to represent every winning strategy of player A, and a classical group to represent every winning strategy of player B.

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This research maybe very useful in the progression of game theory by algebraic views.

## 2. Preliminaries :

**Definition 2.1** :[2]

Let (G,*) be a group . Then the neutrosophic group is generated by G and I under * denoted by N(G)={$< G \cup I >, *$}.

I is called the indeterminate element (neutrosophic element) with the property $I^2 = I$.

The most useful understanding of this definition has been written in [3], we consider N(G) as a union of G and GI i.e N(G)={$x_1, x_2, \dots x_1 I, x_2 I, \dots . ; x_i \in G$}.

**Definition 2.2** :[2]

Let N(G) be a neutrosophic group, then a neutrosophic subgroup is a subset of N(G) contains a proper subgroup of G.

**Remark2.3** :[2]

Neutrosophic subgroup is not a group but contains a group.

**Definition 2.4 :**[5]

An abelian group G is called elementary abelian if it is isomorphic to $Z_n \times \dots \times Z_n$ for such a positive integer n.

For concepts like game, analyzing game, and combinatorial game see [4].

## 3. Main results and discussion :

Suppose that G is a game with finite steps. Two players A , B play this game, they make their steps alternately, i.e (their choices) from a finite set of objects S={$x_1, \dots x_n$}.

If we reach to a position which A cannot chose any object then B is the winner, and conversely A is the winner.

Without affecting the generality we can suppose that the alternating choices of two players can be realized as :

| A | B |
|---|---|
| $x_1$ | $x_2$ |
| $x_3$ | $x_4$ |
| …. | . … |

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We say that a step i is complete if both players were able to chose objects without being losers.

For each complete step, we can represent it by a bijective map f: $S \rightarrow S$ wich permutes the chosen objectives in this step an fixes the rest of unchosen objectives, i.e if the player A chooses the element $x_i$ and B chooses $x_j$, then we represent this complete step by the map : f: $S \rightarrow S$ with f($x_i$)= $x_j$ and f($x_j$)=$x_i$ and f($x_t$)= $x_t$ for each $t \neq i, j$ , we can use algebraic symbol as: f = $\binom{x_i \; x_j \; \ldots \; x_n}{x_j \; x_i \; \ldots \; x_n}$ .

**Theorem 3.1** :

Let $f_i$ be the representation of the complete step i, then ${f_i}^2$=I (of order 2) , where I is the identity map on S.

Proof :

It is easy to see that $f_i o f_i(x_i) = f_i(x_j) = x_i$.

We represent the beginning position of the game by I (identity map).

**Theorem 3.2 :**

Each winning strategy of second player B can be represented by a group with type

$$(Z_2)^k = Z_2 \times \ldots Z_2$$

Proof :

If B has a winning strategy, then we will reach to a position that B can choose and player A cannot, as follows :

| A | B |
|---|---|
| $x_1$ | $x_2$ |
| $x_3$ | $x_4$ |
| . ................................ | |
| .............................. | |
| $x_{m-1}$ | $x_m$ |

We assume that the number of steps is k, we remark that all steps are complete and each step's representation is a bijective with order 2, so the group generated by all representations is $(Z_2)^k$.

We call the previous group by a strategy representation.

**Definition 3.3** :

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If we reach to a position which A can chose and B cannot, we represent it by the indeterminate map J, which it means that A can pick an object and B cannot.

Remark: The indeterminate map J has the property $J^2 = J$ , we mean by this property that if we reach to a winning position of player A , then the next position is the same.

**Theorem 3.4** :

Each winning strategy of first player A can be represented by a neutrosophic subgroup with type .

$$(Z_2)^k \cup \{J\}$$

Proof :

If A has a winning strategy then we will reach to a position that A can choose and then B cannot , as the following :

| A | B |
|---|---|
| $x_1$ | $x_2$ |
| $x_3$ | $x_4$ |
| . ……………………………… ………………………….. | |
| $x_m$ | |

We assume that the number of steps is k+1 , we remark that all steps are complete unless the last step. The group generated by all steps unless the last one is $(Z_2)^k$.

For the last step we can represent it by the indeterminate J, thus the strategy representation is the neutrosophic subgroup of N($(Z_2)^k$) which is set :

$$(Z_2)^k \cup \{J\}$$

**Result 3.5** :

If A is the winner then the strategy representation is a neutrosophic group , and if B is the winner then the strategy representation is a classical group

Definition 3.6 :

(a) If the player B has a winning strategy, then the winning strategy with minimum representation group order is called the perfect strategy of B.

(b) If the player A has a winning strategy , then the winning strategy with minimum representation group order is called the perfect strategy of A.

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**Definition 3.7** :

If H , K are two finite games , we say that H is algebraically relative or (H-ar-K), if there is a perfect strategy of the Player A in both games with the same representation neutrosophic group, or a perfect strategy of the player B in both games with same representation group .

Remark : The essential meaning of algebraically relative games is that they have winning strategies with the same number of steps.

**Example 3.8** :

Suppose that we have two players A , B which they are playing Wythoff game with (3,2) as a beginning position , A at least needs two steps to win , we can clarify it by the following example:

| A | B |
|---|---|
| (1,2) (after the choice of A ) | (1,1) |
| A chooses (1,1) and wins | |

The representation neutrosophic subgroup is $Z_2 \cup \{J\}$

Let the same players play the HIM-Game defined in [4]. The beginning position is

(2,4,5,10) , A has a perfect strategy as

| A | B |
|---|---|
| (2,4,2,2) after A choice | (2,4) after B choice |
| A chooses (2,4) and wins | |

The representation neutrosophic subgroup is $Z_2 \cup \{J\}$ , thus the previous two games are algebraically relative.

## 4. Conclusion

In this research, we have introduced a representation of winning strategies of finite alternating games by groups and neutrosophic groups. Also, we have introduced the notion of algebraically relative games and gave many examples.

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