



On The Symbolic Turiyam Rings

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Abstract: In this paper, we define for the first time the concept of symbolic Turiyam ring as a direct application of Turiyam symbolic set and as a new generalization of neutrosophic rings. Also, we study many of essential properties and related concepts of these rings such as AH-ideals and subrings.

On the other hand, we illustrate many examples to clarify the validity of our work.

Key words: turiyam symbolic set, turyiam symbolic ring, AH-ideal, AH-subring.

1. Introduction

After the work of Zadeh [54], many authors tend to study algebraic structures of fuzzy sets [2], and their generalizations such as neutrosophic sets [1].

In the literature, we find many algebraic studies of neutrosophic algebraic structures such as rings [12-17], modules [4-5], spaces [6-7, 20], matrices [31,35,41-48], and groups [8,9,16].

The concept of Turiyam set was supposed in [55], as a new generalization of neutrosophic set was studied in [38].

In this work, we aim to study the algebraic structure related with turyiam concepts in a similar way of neutrosophic algebraic structures. From this point of view, we present the concept of symbolic Turyiam set, and we use this concept to define symbolic tyriam ring as a new generalization of neutrosophic rings. Also, we generalize neutrosophic AH-ideals to turyiam AH-ideals and AH-subrings.

This work may be used to define many turyiam symbolic algebraic structures.

Main Concepts and Discussion

Definition 1:

We define the Turiyam symbolic set as follows:

$T_u = \{F, T, I, Y\}$, Where (F) denotes to falsity, (T) denotes to truth, (I) denotes to Indeterminacy and (Y) denotes to fourth case of awareness or (Turiyamic) property.

We define an algebraic operation between symbols in T_u as follows:

$$T \times T = T^2 = T, F^2 = F, I^2 = I, Y^2 = Y, T \times Y = Y \times T = Y, T \times F = F \times T = F$$

$$T \times I = I \times T = I, I \times Y = Y \times I = I, F \times Y = Y \times F = Y, F \times I = I \times F = I$$

Definition 2

Let R be a ring, we define the symbolic Turiyam ring as follows:

$$T_R = \{a + bT + cF + dI + eY; a, b, c, d, e \in R\}.$$

We denote it by (STR).

If R a field, then T_R is called symbolic Turiyam field (STF).

Example 3

Let $R = Z_2 = \{0,1\}$ be the ring of integers module 2.

The corresponding (STR) is:

$$T_{Z_2} = \{0,1,T,F,I,Y,1+T,1+F,1+I,1+Y,T+F,T+Y,T+I,F+I,F+Y,I+Y,T+F+I,T+F+Y,F+I+Y,T+I+Y,T+F+I+Y,1+T+F,1+T+I,1+T+Y,1+F+I,1+F+Y,1+I+Y,1+F+T+I+Y,1+T+F+I,1+T+F+Y,1+I+F+Y,1+T+I+Y\}.$$

Definition 4: (Algebraic operations)

Let T_R be a (STR), $x = x_0 + x_1T + x_2F + x_3I + x_4Y$, $y = y_0 + y_1T + y_2F + y_3I + y_4Y$ be two arbitrary elements in T_R , we define:

Addition:

$$x + y = (x_0 + y_0) + (x_1 + y_1)T + (x_2 + y_2)F + (x_3 + y_3)I + (x_4 + y_4)Y.$$

Multiplication:

$$x \cdot y = x_0 \cdot y_0 + x_0 \cdot y_1 T + x_0 \cdot y_2 F + x_0 \cdot y_3 I + x_0 \cdot y_4 Y + x_1 \cdot y_0 T + x_1 \cdot y_2 TF + x_1 \cdot y_3 TI + x_1 \cdot y_4 TY + x_1 \cdot y_1 T^2 + x_2 \cdot y_0 F + x_2 \cdot y_1 FT + x_2 \cdot y_2 F^2 + x_2 \cdot y_3 FI + x_2 \cdot y_4 FY + x_3 \cdot y_0 I + x_3 \cdot y_1 IT + x_3 \cdot y_2 IF + x_3 \cdot y_3 I^2 + x_3 \cdot y_4 IY + x_4 \cdot y_0 Y + x_4 \cdot y_1 YT + x_4 \cdot y_2 YF + x_4 \cdot y_3 YI + x_4 \cdot y_4 Y^2.$$

$$x \cdot y = x_0 \cdot y_0 + T(x_0 \cdot y_1 + x_1 \cdot y_0 + x_1 \cdot y_1) + F(x_0 \cdot y_2 + x_1 \cdot y_2 + x_2 \cdot y_0 + x_2 \cdot y_2 + x_2 \cdot y_1) + I(x_0 \cdot y_3 + x_1 \cdot y_3 + x_2 \cdot y_3 + x_3 \cdot y_3 + x_3 \cdot y_1 + x_3 \cdot y_2 + x_3 \cdot y_3 + x_3 \cdot y_4 + x_4 \cdot y_3) + Y(x_0 \cdot y_4 + x_1 \cdot y_4 + x_2 \cdot y_4 + x_3 \cdot y_4 + x_4 \cdot y_1 + x_4 \cdot y_2 + x_4 \cdot y_4).$$

Example 5:

Consider the $(STR)T_{Z_2}$, take:

$$x = I + Y, y = 1 + F.$$

$$x + y = 1 + I + F + Y$$

$$x \cdot y = I + IF + Y + YF = I + I + Y + Y = 0 + 0 = 0$$

Remark 6

T_R is a ring in the algebraic ordinary meaning i.e. $(T_R, +)$ is abelian group.

The multiplication is associative and distributive with respect to addition.

Example 7

Consider the ring of integers Z , Let T_Z be the corresponding (STR) , take:

$$x = 1 + T + 2Y, y = 2T + 3F - I, z = 1 + F$$

$$x \cdot y = 2T + 3F - I + 2T + 3F - I + 4Y + 6Y - 2I = 4T + 6F - 4I + 10Y$$

$$(x \cdot y) \cdot z = (4T + 6F - 4I + 10Y) \cdot (1 + F) = 4T + 4F + 6F + 6F - 4I - 4I + 10Y + 10Y = 4T + 16F - 8I + 20Y.$$

$$y \cdot z = (2T + 3F) \cdot (1 + F) = 2T + 2F + 3F + 3F - I - I = 2T + 8F - 2I$$

$$x \cdot (y \cdot z) = (1 + T + 2Y)(2T + 8F - 2I) = 2T + 8F - 2I + 2T + 8F - 2I + 4Y + 16Y - 4I = 4T + 16F - 8I + 20Y.$$

It is clear that $x \cdot (y \cdot z) = (x \cdot y) \cdot z$.

On other hand:

$$x \cdot (y + z) = (1 + T + 2Y)(1 + 2T + 4F - I) = 1 + 2T + 4F - I + T + 2T + 4F - I + 2Y + 4Y + 8Y - 2I = 1 + 5T + 8F - 4I + 14Y.$$

$$x \cdot z = (1 + T + 2Y)(1 + F) = 1 + F + T + F + 2Y + 2Y = 1 + T + 2F + 4Y$$

$$x \cdot y + x \cdot z = 1 + 5T + 8F - 4I + 14Y, \text{ easily we see that } x \cdot (y + z) = x \cdot y + x \cdot z$$

Definition 8:

Let T_R be a (STR), we define the Turiyam AHS-ideal as follows:

$$K = \{a + bF + cT + dI + eY; a, b, c, d, e \in I, \text{ where } I \text{ is an ideal of } R\}.$$

K is called AH-ideal if $a \in I_1, b \in I_2, c \in I_3, d \in I_4, e \in I_5$, where I_i is an ideal of R for all $i = 1, \dots, 5$

Example 9:

Let $Z_4 = \{0,1,2,3\}$ be the ring of integers module 5, consider the following ideals:

$$I_2 = \{Z_4\}, I_3 = \{0,2\}$$

The set $K = \{a + bF + cT + dI + eY; a, c \in I_2, b, d, e \in I_3\}$ is a Turiyam AH-ideal.

Definition 10:

If I_i is a subring of R , hence K is called an AH-subring.

If $I_1 = I_2 = I_3 = I_4 = I_5$, then K is called an AHS-subring.

Theorem 11

Let T_R be a (STR), then it contains a neutrosophic ring $R(I) = \{a + bI; a, b \in R\}$.

Proof.

It is clear that $R(I) \subseteq T_R$, this means that the (STR) is a generalization of neutrosophic rings.

Remark 12

Every neutrosophic AH-ideal can be considered as a Turiyam AH-ideal.

Theorem 13

Let R be a finite ring with $|R| = n$, hence $|T_R| = n^5$.

The proof is obvious.

Example 14

a). If $R = Z_2$, hence $|T_{Z_2}| = 2^5 = 32$.

b). If $R = Z_4$, hence $|T_{Z_4}| = 4^5 = 1024$.

Conclusion

In this paper, we have defined the concept of symbolic turiyam ring (STR) as a new generalization of neutrosophic rings and an application of turiyam sets with four dimensions.

In The future, we aim to study symbolic Turiyam modules and spaces by using (STR) concept.

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