Pre-separation Axioms in Neutrosophic Topological Spaces

Sudeep Dey [1,2], **Gautam Chandra Ray**[2*]
[1]Department of Mathematics, Science College, Kokrajhar, Assam, India
[2]Department of Mathematics, Central Institute of Technology, Kokrajhar, Assam, India
Emails: sudeep.dey.1976@gmail.com, gautomofcit@gmail.com

Abstract

In this article, we first establish a few results based on neutrosophic interior, neutrosophic closure, neutrosophic pre-open sets, and neutrosophic pre-closed sets. Afterward, we define neutrosophic pre-T_0 space, neutrosophic pre-T_1 space, neutrosophic pre-T_2 space, and study their various properties along with the interconnections between the spaces. We also establish some results using neutrosophic pre-irresolute function, neutrosophic pre*-continuous function.

Keywords: Neutrosophic pre-T_0 space ; Neutrosophic pre-T_1 space ; Neutrosophic pre-T_2 space ; Neutrosophic pre-open set ; Neutrosophic pre-closed set ; Neutrosophic pre-open function ; Neutrosophic pre-continuous function ; Neutrosophic pre-irresolute function ; Neutrosophic pre*-continuous function.

1 Introduction

The thought of a fuzzy set was revealed by L.A. Zadeh[35] in the year 1965 and a generalized model of a fuzzy set, known as the intuitionistic fuzzy set, was published by K. Atanassov[1] in 1986. Afterward, the notion of a neutrosophic set was developed and studied by Florentin Smarandache.[24–26] It had been established by Smarandache[26] that a neutrosophic set was a generalization of an intuitionistic fuzzy set. A neutrosophic set is entangled with three membership functions which are the truth-membership function, falsity-membership function, and indeterminacy-membership function. It is noteworthy that all these three neutrosophic components are impartial to one another. After Smarandache had brought the idea of neutrosophy to light, it was studied and taken in advance by many researchers[6,16,27,30,34] across the globe. There are some innate difficulties in the earlier methods (classical or fuzzy) because of the deficiency of parameterizing tools and so, those methods are insufficient to deal with several real-life problems. These problems can be handled in a more general and suitable way with the help of neutrosophic theory. Different types of practical-based works such as decision-making problems,[4,5] medical diagnosis,[14] image processing[18] had been carried out in a neutrosophic environment.

In 2012, Salama & Alblowi[28] revealed the idea of neutrosophic topological space as an extension of intuitionistic fuzzy topological space developed by D.Coker[12] in 1997. Later, a number of notions related to neutrosophic topological spaces had been developed by several researchers.[3,13,17,19,22,23,29,31] Rao & Rao[21] introduced the concepts of neutrosophic pre-open and pre-closed sets in 2017 and thereafter, Arokiarani *et al.*[2] developed the idea of neutrosophic pre-open, pre-closed, and pre-continuous functions. In recent years, separation properties were studied by some researchers.[8–11,15,20,32,33] In 2020, Ahu and Ferhat[7] introduced and studied the concept of pre-separation axioms in neutrosophic soft topological spaces. From the above discussion, it has been clear that the separation axioms in neutrosophic topological spaces via neutrosophic pre-open sets have not been studied so far.

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In this article, our primary motive is to define and learn about the separation axioms using neutrosophic pre-open sets which we shall call neutrosophic pre-separation axioms. But, before proceeding to that, we set up a few definitions and propositions based on neutrosophic interior, neutrosophic closure, neutrosophic pre-open sets, and neutrosophic pre-closed sets in section 3, which will be utilized in studying neutrosophic pre-separation axioms. In section 4, we define neutrosophic pre-T_0 space, neutrosophic pre-T_1 space, neutrosophic pre-T_2 space and study their various properties. We try to establish some relationships between neutrosophic separation axioms[15] and neutrosophic pre-separation axioms. We also study some hereditary properties. Apart from these, we establish some significant results implementing some functions such as neutrosophic continuous function, neutrosophic pre-irresolute function, neutrosophic pre*-continuous function. In section 5, we bestow a conclusion.

The article is organized by conferring some basic notions in section 2. In section 3, we establish some results in connection with single-valued neutrosophic sets. We then define neutrosophic subspace with example and investigate some properties. In section 4, we define neutrosophic T_0, T_1, T_2-spaces and study various properties. In section 5, we confer a conclusion.

2 Preliminaries:

2.1 Definition:[24]

Let X be the universe of discourse. A neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where the functions $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are real standard or non-standard subsets of $]^-0, 1^+[$, i.e., $\mathcal{T}_A : X \to]^-0, 1^+[$, $\mathcal{I}_A : X \to]^-0, 1^+[$, $\mathcal{F}_A : X \to]^-0, 1^+[$ and $^-0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3^+$.

The neutrosophic set A is characterized by the truth-membership function $\mathcal{T}_A$, indeterminacy-membership function $\mathcal{I}_A$, falsity-membership function $\mathcal{F}_A$.

2.2 Definition:[34]

Let X be the universe of discourse. A single-valued neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are functions from X to $[0, 1]$ and $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$.

The set of all single-valued neutrosophic sets over X is denoted by $\mathcal{N}(X)$.

Throughout this article, a neutrosophic set (NS, for short) will mean a single-valued neutrosophic set.

2.3 Definition:[19]

Let $A, B \in \mathcal{N}(X)$. Then

- (i) (Inclusion): If $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$ then A is said to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.
- (ii) (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.
- (iii) (Intersection): The intersection of A and B, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x)\rangle : x \in X\}$.
- (iv) (Union): The union of A and B, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x)\rangle : x \in X\}$.
- (v) (Complement): The complement of the NS A, denoted by A^c, is defined as $A^c = \{\langle x, \mathcal{F}_A(x), 1 - \mathcal{I}_A(x), \mathcal{T}_A(x)\rangle : x \in X\}$

(vi) (Universal Set): If $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

(vii) (Empty Set): If $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\tilde{\emptyset}$.

2.4 Definition:[28]

Let $\{A_i : i \in \triangle\} \subseteq \mathcal{N}(X)$, where $\triangle$ is an index set. Then

(i) $\cup_{i\in\triangle} A_i = \{\langle x, \vee_{i\in\triangle}\mathcal{T}_{A_i}(x), \wedge_{i\in\triangle}\mathcal{I}_{A_i}(x), \wedge_{i\in\triangle}\mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

(ii) $\cap_{i\in\triangle} A_i = \{\langle x, \wedge_{i\in\triangle}\mathcal{T}_{A_i}(x), \vee_{i\in\triangle}\mathcal{I}_{A_i}(x), \vee_{i\in\triangle}\mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

2.5 Definition:[22]

Let $\mathcal{N}(X)$ be the set of all neutrosophic sets over X. A NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic point (NP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = \alpha, \mathcal{I}_P(y) = \beta, \mathcal{F}_P(y) = \gamma$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$. A neutrosophic point $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ will be denoted by $P^x_{\alpha,\beta,\gamma}$ or $P < x, \alpha, \beta, \gamma >$ or simply by $x_{\alpha,\beta,\gamma}$. For the NP $x_{\alpha,\beta,\gamma}$, x will be called its support. The complement of the NP $P^x_{\alpha,\beta,\gamma}$ will be denoted by $(P^x_{\alpha,\beta,\gamma})^c$ or by $x^c_{\alpha,\beta,\gamma}$. A NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic crisp point (NCP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = 1, \mathcal{I}_P(y) = 0, \mathcal{F}_P(y) = 0$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$.

2.6 Proposition:[15]

Let X, Y, Z be three sets such that $\emptyset \neq Z \subseteq Y \subseteq X$. Let $A \in \mathcal{N}(X)$ and $\{A_\lambda : \lambda \in \triangle\} \subseteq \mathcal{N}(X)$, where $\triangle$ is an index set. Then

(i) $(\bigcup_{\lambda\in\triangle} A_\lambda) \mid_Y = \bigcup_{\lambda\in\triangle}(A_\lambda \mid_Y)$

(ii) $(\bigcap_{\lambda\in\triangle} A_\lambda) \mid_Y = \bigcap_{\lambda\in\triangle}(A_\lambda \mid_Y)$

(iii) $A^c \mid_Y = (A \mid_Y)^c$

(iv) $(A \mid_Y) \mid_Z = A \mid_Z$.

2.7 Definition:[29]

Let X and Y be two non-empty sets and $f : X \to Y$ be a function. Also let $A \in \mathcal{N}(X)$ and $B \in \mathcal{N}(Y)$. Then

(1) Image of A under f is defined by
$f(A) = \{\langle y, f(\mathcal{T}_A)(y), f(\mathcal{I}_A)(y), (1 - f(1 - \mathcal{F}_A))(y)\rangle : y \in Y\}$, where

$$f(\mathcal{T}_A)(y) = \begin{cases} \sup\{\mathcal{T}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

$$f(\mathcal{I}_A)(y) = \begin{cases} \inf\{\mathcal{I}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

$$(1 - f(1 - \mathcal{F}_A))(y) = \begin{cases} \inf\{\mathcal{F}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

(2) Pre-image of B under f is defined by
$f^{-1}(B) = \{\langle x, f^{-1}(\mathcal{T}_B)(x), f^{-1}(\mathcal{I}_B)(x), f^{-1}(\mathcal{F}_B)(x)\rangle : x \in X\}$

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2.8 Definition:[19]

Let $\tau \subseteq \mathcal{N}(X)$. Then τ is called a neutrosophic topology on X if

(i) $\tilde{\emptyset}$ and $\tilde{X}$ belong to τ.

(ii) Arbitrary union of neutrosophic sets in τ is in τ.

(iii) Intersection of any two neutrosophic sets in τ is in τ.

If τ is a neutrosophic topology on X then the pair (X, τ) is called a neutrosophic topological space (NTS, for short) over X. The members of τ are called neutrosophic τ-open sets (or neutrosophic open sets or open sets, for short) in X. If for a neutrosophic set A, $A^c \in \tau$ then A is said to be a neutrosophic τ-closed set (or neutrosophic closed set or closed set, for short) in X.

2.9 Definition:[19]

Let (X, τ) be a NTS and $A \in \mathcal{N}(X)$. Then the neutrosophic

(i) interior of A, denoted by $int(A)$, is defined as $int(A) = \cup\{G : G \in \tau$ and $G \subseteq A\}$.

(ii) closure of A, denoted by $cl(A)$, is defined as $cl(A) = \cap\{G : G^c \in \tau$ and $G \supseteq A\}$.

2.10 Theorem:[21]

Let (X, τ) be a NTS and $A \in \mathcal{N}(X)$. Then

(i) A ia a neutrosophic pre-open set (NPO, for short) in X if and only if $A \subseteq int(cl(A))$.

(ii) A ia a neutrosophic pre-closed (NPC, for short) set in X if and only if $cl(int(A)) \subseteq A$.

(iii) A ia an NPC set in X if and only if A^c is an NPO set in X.

(iv) Every neutrosophic open set in a NTS is an NPO set.

(v) Every neutrosophic closed set in a NTS is an NPC set.

If G is an NPO (resp. NPC) set in X then we may also say that G is a τ-NPO (resp. τ-NPC) set.

2.11 Theorem:[2]

Let $f : X \to Y$ be a function from a NTS (X, τ) to a NTS (Y, σ). If $f(G)$ is an NPO set in Y for every neutrosophic open set G in X then f is called a neutrosophic pre-open function.

2.12 Definition:[2]

Let $f : X \to Y$ be a function from a NTS (X, τ) to a NTS (Y, σ). If $f^{-1}(G)$ is an NPO set in X for every neutrosophic open set G in Y then f is called a neutrosophic pre-continuous function.

2.13 Definition:[17]

Let $f : X \to Y$ be a function from a NTS (X, τ) to a NTS (Y, σ). If $f^{-1}(G)$ is an NPO set in X for every NPO set G in Y then f is called a neutrosophic pre-irresolute function.

2.14 Definition:[15]

Let (X, τ) be a NTS. Let $\emptyset \neq Y \subseteq X$ and $\tau\mid_Y = \{G\mid_Y : G \in \tau\}$. Then $(Y, \tau\mid_Y)$ is a NTS. The topology $\tau\mid_Y$ is called the neutrosophic relative topology of τ on Y or the neutrosophic subspace topology of Y and the NTS $(Y, \tau\mid_Y)$ is called a neutrosophic subspace (or a subspace, for short) of the NTS (X, τ). Members of $\tau\mid_Y$ are called $\tau\mid_Y$-open sets in Y. A NS $A \in \mathcal{N}(Y)$ such that $A^c \in \tau\mid_Y$ is called a $\tau\mid_Y$-closed set in Y.

2.15 Definition:[15]

Let $(Y, \tau\mid_Y)$ be a neutrosophic subspace of a NTS (X, τ) and $A \in \mathcal{N}(Y)$. Then the neutrosophic interior of A, denoted by $int_Y(A)$, is defined as $int_Y(A) = \cup\{G : G \in \tau\mid_Y \text{ and } G \subseteq A\}$ and the neutrosophic closure of A, denoted by $cl_Y(A)$, is defined as $cl_Y(A) = \cap\{G : G^c \in \tau\mid_Y \text{ and } G \supseteq A\}$.

2.16 Definition:[15]

A property of a NTS (X, τ) is said to be hereditary if whenever the space X has that property, then so does every subspace of it.

2.17 Proposition:[15]

Let $(Y, \tau\mid_Y)$ be a neutrosophic subspace of a NTS (X, τ) and $A \in \mathcal{N}(Y)$. Then A is $\tau\mid_Y$-closed NS iff $A = F\mid_Y$ for some τ-closed NS F in X.

2.18 Definition:[15]

A NTS (X, τ) is called a neutrosophic

(i) T_0-space or (NT_0-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $U \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ or there exists a $V \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

(ii) T_1-space (NT_1-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $U \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a $V \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

(iii) T_2-space or neutrosophic Hausdorff space (NT_2-space or Hausdorff space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exist $U, V \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \tilde{\emptyset}$.

3 Some important results:

3.1 Proposition:

Let $(Y, \tau\mid_Y)$ be a neutrosophic subspace of a NTS (X, τ). Then

(i) $cl_X(G)\mid_Y = cl_Y(G\mid_Y)$ for every $G \in \mathcal{N}(X)$, where $cl_Y(G\mid_Y)$ is the $\tau\mid_Y$-closure of $G\mid_Y$ and $cl_X(G)$ is the τ-closure of G.

(ii) $int_X(G)\mid_Y = int_Y(G\mid_Y)$ for every $G \in \mathcal{N}(X)$, where $int_Y(G\mid_Y)$ is the $\tau\mid_Y$-interior of $G\mid_Y$ and $int_X(G)$ is the τ-interior of G.

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Proof: (i) $cl_X(G) \mid_Y = [\bigcap\{F : F$ is a τ-closed NS and $G \subseteq F\}] \mid_Y = \bigcap\{F \mid_Y : F$ is a τ-closed NS and $G \subseteq F\}$ [by 2.6(ii)] $= \bigcap\{F \mid_Y : F \mid_Y$ is a $\tau \mid_Y$-closed NS and $G \mid_Y \subseteq F \mid_Y\}$ [by 2.17] $= cl_Y(G \mid_Y)$.

(ii) $int_X(G) \mid_Y = [\bigcup\{A : A$ is a τ-open NS and $A \subseteq G\}] \mid_Y = \bigcup\{A \mid_Y : A$ is a τ-open NS and $A \subseteq G\}$ [by 2.6(i)] $= \bigcup\{A \mid_Y : A \mid_Y$ is a $\tau \mid_Y$-open NS and $A \mid_Y \subseteq G \mid_Y\}$ [by 2.14] $= int_Y(G \mid_Y)$.

3.2 Proposition:

Let $(Y, \tau \mid_Y)$ be a neutrosophic subspace of the NTS (X, τ) and $A \in \mathcal{N}(Y)$. Then A is a $\tau \mid_Y$-NPC set in Y iff A^c is a $\tau \mid_Y$-NPO set in Y.

Proof: A is a $\tau \mid_Y$-NPC set $\Leftrightarrow cl_Y(int_Y(A)) \subseteq A \Leftrightarrow A^c \subseteq [cl_Y(int_Y(A))]^c = int_Y((int_Y(A))^c) = int_Y(cl_Y(A^c)) \Leftrightarrow A^c \subseteq int_Y(cl_Y(A^c)) \Leftrightarrow A^c$ is a $\tau \mid_Y$-NPO set.

3.3 Proposition:

Let $(Y, \tau \mid_Y)$ be a neutrosophic subspace of the NTS (X, τ). Then

(i) $G \mid_Y$ is a $\tau \mid_Y$-NPO set in Y for every τ-NPO set G in X.

(ii) $G \mid_Y$ is a $\tau \mid_Y$-NPC set in Y for every τ-NPC set G in X.

Proof:

(i) G is a τ-NPO set in $X \Rightarrow G \subseteq int_X(cl_X(G)) \Rightarrow G \mid_Y \subseteq [int_X(cl_X(G))] \mid_Y \Rightarrow G \mid_Y \subseteq int_Y(cl_X(G) \mid_Y)$ [by 3.1(ii)] $\Rightarrow G \mid_Y \subseteq int_Y(cl_Y(G \mid_Y))$ [by 3.1(i)] $\Rightarrow G \mid_Y$ is a $\tau \mid_Y$-NPO set in Y.

(ii) G is a τ-NPC set $\Rightarrow G^c$ is a τ-NPO set $\Rightarrow G^c \mid_Y$ is a $\tau \mid_Y$-NPO set [by 3.3(i)] $\Rightarrow (G \mid_Y)^c$ is a $\tau \mid_Y$-NPO set [by 2.6(iii)] $\Rightarrow G \mid_Y$ is a $\tau \mid_Y$-NPC set in Y[by 3.2].

3.4 Definition:

Let (X, τ) be a NTS and $A \in \mathcal{N}(X)$. Then the neutrosophic

(i) pre-interior of A, denoted by $Pint(A)$, is defined as $Pint(A) = \cup\{G : G$ is an NPO set in X and $G \subseteq A\}$.

(ii) pre-closure of A, denoted by $Pcl(A)$, is defined as $Pcl(A) = \cap\{G : G$ is an NPC set in X and $G \supseteq A\}$.

3.5 Definition:

Let (X, τ) and (X, τ^*) be two NTSs. If every τ-NPO set in X is a τ^*-NPO set in X then τ is said to be pre-coarser than τ^* (denoted by $\tau < \tau^*$) or τ^* is said to be pre-finer than τ (denoted by $\tau^* > \tau$).

3.6 Example:

Let $X = \{a, b\}$, $\tau^* = \{\tilde{\emptyset}, \tilde{X}\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A\}$, where $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$. Obviously both (X, τ) and (X, τ^*) are NTSs. It is also clear that every τ-NPO set is a τ^*-NPO set. Therefore, by the definition 3.5, τ is pre-coarser than τ^*, i.e., τ^* is pre-finer than τ.

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3.7 Definition:

Let $f : X \to Y$ be a function from a NTS (X, τ) to a NTS (Y, σ). If $f^{-1}(G)$ is a neutrosophic open set set in X for every NPO set G in Y then f is called a neutrosophic pre*-continuous function.

4 Neutrosophic Pre-separation Axioms:

In this part of the article, we define neutrosophic pre-T_0, neutrosophic pre-T_1, and neutrosophic pre-T_2 spaces with examples and investigate various properties.

4.1 Definition:

A NTS (X, τ) is called a neutrosophic pre-T_0 space (NPT_0-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists an NPO set U in X such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ or there exists an NPO set V in X such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

4.2 Example:

Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$ and $B = \{\langle a, 0, 1, 1\rangle, \langle b, 1, 0, 0\rangle\}$. Clearly the NTS (X, τ) is a neutrosophic pre-T_0 space.

4.3 Proposition:

Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is pre-finer than τ. If (X, τ) is an NPT_0-space then (X, τ^*) is also an NPT_0-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X, τ) is an NPT_0-space, so there exists a τ-NPO set G in X such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-NPO set H in X such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since τ^* is pre-finer than τ, so every τ-NPO set in X is a τ^*-NPO set in X. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exists a τ^*-NPO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ^*-NPO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X, τ^*) is an NPT_0-space.

4.4 Proposition:

Let (X, τ) be a NTS. If X is NT_0-space then X is an NPT_0-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X, τ) is a NT_0-space, so there exists a τ-open NS G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-open NS H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since every neutrosophic open set is an NPO set [by 2.10(iv)], so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a τ-NPO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-NPO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X, τ) is an NPT_0-space.

4.5 Remark:

Converse of the proposition 4.4 is not true. We establish it by the following counter example.

Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}\}$ Clearly (X, τ) is not a NT_0-space.

We now show that (X, τ) is an NPT_0-space. Let $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$ be any two NPs in X such thar $a \neq b$. Also let $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$. Obviously $A \in \mathcal{N}(X)$, $a_{\alpha,\beta,\gamma} \in A$ but $b_{\alpha',\beta',\gamma'} \notin A$. Now $int(cl(A)) = int(\tilde{X}) = \tilde{X} \supseteq A$. Therefore A is an NPO set in X. Thus for any two NPs $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$, $a \neq b$, there exists an NPO set A in X such that $a_{\alpha,\beta,\gamma} \in A$ but $b_{\alpha',\beta',\gamma'} \notin A$. Therefore (X, τ) is an NPT_0-space.

Hence the NTS (X, τ) is an NPT_0-space but not a NT_0-space.

4.6 Proposition:

Let (X, τ) be an NPT_0-space. Then every neutrosophic subspace of X is an NPT_0-space and hence the property is hereditary.

Proof: Let $(Y, \tau\ |_Y)$ be a neutrosophic subspace of (X, τ), where $\tau\ |_Y = \{G\ |_Y : G \in \tau\}$. We want to show $(Y, \tau\ |_Y)$ is an NPT_0-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X, τ) is an NPT_0-space, so there exists a τ-NPO set U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ or there exists a τ-NPO set V such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Then $(x_{\alpha,\beta,\gamma} \in U\ |_Y,\ y_{\alpha',\beta',\gamma'} \notin U\ |_Y)$ or $(x_{\alpha,\beta,\gamma} \notin V\ |_Y,\ y_{\alpha',\beta',\gamma'} \in V\ |_Y)$. Also by the proposition 3.3(i), $U\ |_Y, V\ |_Y$ are $\tau\ |_Y$-NPO sets in Y as U, V are τ-NPO sets in X. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $\tau\ |_Y$-NPO set $U\ |_Y$ such that $x_{\alpha,\beta,\gamma} \in U\ |_Y$, $y_{\alpha',\beta',\gamma'} \notin U\ |_Y$ or there exists a $\tau\ |_Y$-NPO set $V\ |_Y$ such that $x_{\alpha,\beta,\gamma} \notin V\ |_Y$, $y_{\alpha',\beta',\gamma'} \in V\ |_Y$ Therefore $(Y, \tau\ |_Y)$ is an NPT_0-space and hence the property is hereditary.

4.7 Proposition:

Let (X, τ) be a NTS. Then X is an NPT_0-space iff for any two distinct neutrosophic crisp points $x_{1,0,0}$ and $y_{1,0,0}$, $(x_{1,0,0})\hat{q}[Pcl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[Pcl(x_{1,0,0})]$.

Proof: Necessary part: Suppose that both $(x_{1,0,0})\hat{q}[Pcl(y_{1,0,0})]$ and $(y_{1,0,0})\hat{q}[Pcl(x_{1,0,0})]$ are false. Then $(x_{1,0,0})q[Pcl(y_{1,0,0})]$ and $(y_{1,0,0})q[Pcl(x_{1,0,0})]$ are true. Now $(x_{1,0,0})q[Pcl(y_{1,0,0})] \Rightarrow x_{1,0,0} \notin [Pcl(y_{1,0,0})]^c$ $\Rightarrow x_{1,0,0} \notin [\cap\{G : G$ is an NPC set and $y_{1,0,0} \in G\}]^c \Rightarrow x_{1,0,0} \notin \cup\{G^c : G^c$ is an NPO set and $y_{1,0,0} \notin G^c\} \Rightarrow x_{1,0,0} \notin G^c$ for all NPO sets G^c in X such that $y_{1,0,0} \notin G^c$. This ascertains that if H is an NPO set such that $y_{1,0,0} \in H$ then $x_{1,0,0} \in H$. Similarly $(y_{1,0,0})q[Pcl(x_{1,0,0})]$ implies that if K is an NPO set such that $x_{1,0,0} \in K$ then $y_{1,0,0} \in K$. Thus every NPO set containing one of $x_{1,0,0}$ and $y_{1,0,0}$ must contain the other. But this is a contradiction to our assumption that X is an NPT_0-space. Therefore $(x_{1,0,0})\hat{q}[Pcl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[Pcl(x_{1,0,0})]$.

Converse part: $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ be any two NPs in X such that $x \neq y$. Now, if $(x_{1,0,0})\hat{q}[Pcl(y_{1,0,0})]$ then $x_{1,0,0} \in [Pcl(y_{1,0,0})]^c$. Obviously $x_{\alpha,\beta,\gamma} \in [Pcl(y_{1,0,0})]^c$ but $y_{p,q,r} \notin [Pcl(y_{1,0,0})]^c$. Since $Pcl(y_{1,0,0})$ is NPC set, so $[Pcl(y_{1,0,0})]^c$ is NPO set. Thus there exists an NPO set $[Pcl(y_{1,0,0})]^c$ in X such that $x_{\alpha,\beta,\gamma} \in [Pcl(y_{1,0,0})]^c$ but $y_{p,q,r} \notin [Pcl(y_{1,0,0})]^c$. Similarly if $(y_{1,0,0})\hat{q}[Pcl(x_{1,0,0})]$ then there exists a an NPO set $[Pcl(x_{1,0,0})]^c$ in X such that $x_{\alpha,\beta,\gamma} \notin [Pcl(x_{1,0,0})]^c$ and $y_{p,q,r} \in [Pcl(x_{1,0,0})]^c$. Therefore (X, τ) is an NPT_0-space.

Hence proved.

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4.8 Proposition:

Let f be a bijective neutrosophic pre-open function from a NTS (X,τ) to a NTS (Y,σ). If (X,τ) is NT_0 then (Y,σ) is an NPT_0-space.

Proof: Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ be two NPs in Y such that $y^1 \neq y^2$. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_0, so there exists a τ-open NS G such that $x^1_{\alpha,\beta,\gamma} \in G$ and $x^2_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-open NS H such that $x^1_{\alpha,\beta,\gamma} \notin H$ and $x^2_{\alpha',\beta',\gamma'} \in H$. Suppose G exists such that $x^1_{\alpha,\beta,\gamma} \in G$ and $x^2_{\alpha',\beta',\gamma'} \notin G$. Since f is a neutrosophic pre-open function, so $f(G)$ is a σ-NPO set such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly if H exists such that $x^1_{\alpha,\beta,\gamma} \notin H$ and $x^2_{\alpha',\beta',\gamma'} \in H$ then $f(H)$ is a σ-NPO set such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \notin f(H)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y there exists a σ-NPO set $f(G)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \notin f(G)$ or there exists a σ-NPO set $f(H)$ such that $y^1_{p,q,r} \notin f(H)$, $y^2_{p',q',r'} \in f(H)$. Therefore (Y,σ) is an NPT_0-space. Hence proved.

4.9 Proposition:

Let f be a one-one neutrosophic pre-continuous function from a NTS (X,τ) to a NTS (Y,σ). If (Y,σ) is NT_0 then (X,τ) is an NPT_0-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NT_0, so there exists a σ-open NS G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ or there exists a σ-open NS H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre-continuous function, so $f^{-1}(G)$ is a τ-NPO set in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-NPO set in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NPO set $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-NPO set $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X,τ) is an NPT_0-space. Hence proved.

4.10 Proposition:

Let f be a one-one neutrosophic pre-irresolute function from a NTS (X,τ) to a NTS (Y,σ). If (Y,σ) is NPT_0 then (X,τ) is an NPT_0-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NPT_0, so there exists a σ-NPO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ or there exists a σ-NPO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre-irresolute function, so $f^{-1}(G)$ is a τ-NPO set in X[by 2.13]. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-NPO set in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NPO set $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-NPO set $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X,τ) is an NPT_0-space. Hence proved.

4.11 Proposition:

Let f be a one-one neutrosophic pre*-continuous function from a NTS (X,τ) to a NTS (Y,σ). If (Y,σ) is NPT_0 then (X,τ) is a NT_0-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NPT_0, so there exists a σ-NPO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ or there exists a σ-NPO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre*-continuous function, so $f^{-1}(G)$ is a τ-open NS in X[by 3.7]. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-open NS in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-open NS $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-open NS $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X,τ) is a NT_0-space. Hence proved.

4.12 Definition:

A NTS (X,τ) is called a neutrosophic pre-T_1 space (NPT_1-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists an NPO set U in X such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists an NPO set V in X such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

4.13 Example:

Let $X = \{a,b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$ and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly the NTS (X,τ) is a neutrosophic pre-T_1 space.

4.14 Proposition:

Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is pre-finer than τ. If (X,τ) is an NPT_1-space then (X,τ^*) is also an NPT_1-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is an NPT_1-space, so there exists a τ-NPO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-NPO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since τ^* is pre-finer than τ, so every τ-NPO set is a τ^*-NPO set. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a τ^*-NPO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ^*-NPO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X,τ^*) is an NPT_1-space.

4.15 Proposition:

Let (X,τ) be a NTS. If X is NT_1-space then X is an NPT_1-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is a NT_1-space, so there exists a τ-open NS G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-open NS H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since every neutrosophic open set in X is an NPO set in X, so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a τ-NPO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-NPO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X,τ^*) is an NPT_1-space.

4.16 Remark:

Converse of the proposition 4.15 is not true. We establish it by the following counter example.

Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}\}$ Clearly (X, τ) is not a NT_1-space.

We now show that (X, τ) is an NPT_1-space. Let $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $a \neq b$. Also let $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$ and $B = \{\langle a, 0, 1, 1\rangle, \langle b, 1, 0, 0\rangle\}$. Clearly A and B are two NPO sets in X. Thus for any two NPs $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$, $a \neq b$, there exists an NPO set A in X such that $a_{\alpha,\beta,\gamma} \in A$, $b_{\alpha',\beta',\gamma'} \notin A$ and there exists an NPO set B in X such that $a_{\alpha,\beta,\gamma} \notin B$, $b_{\alpha',\beta',\gamma'} \in B$. Therefore (X, τ) is an NPT_1-space.

Hence the NTS (X, τ) is an NPT_1-space but not a NT_1-space.

4.17 Proposition:

Let (X, τ) be an NPT_1-space. Then every neutrosophic subspace of X is an NPT_1-space and hence the property is hereditary.

Proof: Let $(Y, \tau \mid_Y)$ be a neutrosophic subspace of (X, τ), where $\tau \mid_Y = \{G \mid_Y : G \in \tau\}$. We want to show $(Y, \tau \mid_Y)$ is an NPT_1-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X, τ) is an NPT_1-space, so there exists a τ-NPO set U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a τ-NPO set V such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Then $(x_{\alpha,\beta,\gamma} \in U \mid_Y, y_{\alpha',\beta',\gamma'} \notin U \mid_Y)$ and $(x_{\alpha,\beta,\gamma} \notin V \mid_Y, y_{\alpha',\beta',\gamma'} \in V \mid_Y)$. Also by the proposition 3.3(i), $U \mid_Y, V \mid_Y$ are $\tau \mid_Y$-NPO sets in Y as U, V are τ-NPO sets in X. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $\tau \mid_Y$-NPO set $U \mid_Y$ such that $x_{\alpha,\beta,\gamma} \in U \mid_Y$, $y_{\alpha',\beta',\gamma'} \notin U \mid_Y$ and there exists a $\tau \mid_Y$-NPO set $V \mid_Y$ such that $x_{\alpha,\beta,\gamma} \notin V \mid_Y$, $y_{\alpha',\beta',\gamma'} \in V \mid_Y$. Therefore $(Y, \tau \mid_Y)$ is an NPT_1-space and hence the property is hereditary.

4.18 Proposition:

Let (X, τ) be a NTS. If every neutrosophic point in X is an NPC set then X is an NPT_1-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in X such that $x \neq y$. Since every NP is an NPC set, so the neutrosophic crisp points $x_{1,0,0}$ and $y_{1,0,0}$ are NPC sets in X. Then $(x_{1,0,0})^c$ and $(y_{1,0,0})^c$ are NPO sets in X such that $x_{\alpha,\beta,\gamma} \in (y_{1,0,0})^c, y_{\alpha',\beta',\gamma'} \notin (y_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma} \notin (x_{1,0,0})^c, y_{\alpha',\beta',\gamma'} \in (x_{1,0,0})^c$. Therefore (X, τ) is an NPT_1-space.

4.19 Proposition:

Let (X, τ) be a NTS. Then every neutrosophic crisp point in X is an NPC set iff X is an NPT_1-space.

Proof: Necessary part: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in X such that $x \neq y$. By hypothesis, neutrosophic crisp points $x_{1,0,0}$ and $y_{1,0,0}$ are NPC sets in X. Then $(x_{1,0,0})^c$ and $(y_{1,0,0})^c$ are NPO sets in X such that $x_{\alpha,\beta,\gamma} \in (y_{1,0,0})^c, y_{\alpha',\beta',\gamma'} \notin (y_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma} \notin (x_{1,0,0})^c, y_{\alpha',\beta',\gamma'} \in (x_{1,0,0})^c$. Therefore (X, τ) is an NPT_1-space.

Sufficient part: Let $x_{1,0,0}$ be an NCP in X. Also let $y_{p,q,r} \in (x_{1,0,0})^c$ be any NP. Then obviously $x \neq y$. Let us consider an NP $x_{\alpha,\beta,\gamma}$ with support x. Since X is an NPT_1-space, so for $y_{p,q,r}$ and $x_{\alpha,\beta,\gamma}$, there exists a τ-NPO set G such that $y_{p,q,r} \in G$ and $x_{\alpha,\beta,\gamma} \notin G$. Since for all α, β, γ with $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$, one such G exists, therefore we must have a τ-NPO set H such that $y_{p,q,r} \in H$ and $x_{1,0,0} \cap H = \tilde{\emptyset}$. Therefore $y_{p,q,r} \in H \subseteq (x_{1,0,0})^c$. So, $(x_{1,0,0})^c$ is a τ-NPO set and consequently, $x_{1,0,0}$ is a τ-NPC set.

Hence proved.

4.20 Proposition:

Let (X, τ) be a NTS. If (X, τ) is an NPT_1-space then it is an NPT_0-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since X is NPT_1-space, so there exists a τ-NPO set U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists τ-NPO set V such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Hence (X, τ) is an NPT_0-space.

4.21 Remark:

Converse of the proposition 4.20 is not true. We establish it by the following counter example.

Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A\}$, where $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$. Clearly the NTS (X, τ) is an NPT_0-space.

We now show that (X, τ) is not an NPT_1-space.

We first establish that the NCP $a_{1,0,0}$ is a not an NPC set. We have $cl(int(a_{1,0,0})) = cl(A) = \tilde{X}$. Therefore $cl(int(a_{1,0,0})) \nsubseteq a_{1,0,0}$, i.e., $a_{1,0,0}$ is not an NPC set. Therefore by the proposition 4.19, (X, τ) is not an NPT_1-space.

Thus the NTS (X, τ) is an NPT_0-space but not an NPT_1-space.

4.22 Proposition:

Let f be a bijective neutrosophic pre-open function from a NTS (X, τ) to a NTS (Y, σ). If (X, τ) is NT_1 then (Y, σ) is an NPT_1-space.

Proof: Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, be two NPs in Y. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_1, so there exists a τ-open NS G such that $x^1_{\alpha,\beta,\gamma} \in G$, $x^2_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-open NS H such that $x^1_{\alpha,\beta,\gamma} \notin H$, $x^2_{\alpha',\beta',\gamma'} \in H$. Since f is a neutrosophic pre-open function, so $f(G)$ is a σ-NPO set such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly $f(H)$ is a σ-NPO set such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \notin f(H)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1 \neq y^2$, there exists a σ-NPO set $f(G)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \notin f(G)$ and there exists a σ-NPO set $f(H)$ such that $y^1_{p,q,r} \notin f(H)$, $y^2_{p',q',r'} \in f(H)$. Therefore (Y, σ) is an NPT_1-space. Hence proved.

4.23 Proposition:

Let f be a one-one neutrosophic pre-continuous function from a NTS (X, τ) to a NTS (Y, σ). If (Y, σ) is NT_1 then (X, τ) is an NPT_1-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NT_1, so there exists a σ-open NS G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ and there exists a σ-open NS H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre-continuous function, so $f^{-1}(G)$ and $f^{-1}(H)$ are τ-NPO sets in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$ and $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NPO set $f^{-1}(G)$ such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ and there exists a τ-NPO set $f^{-1}(H)$ such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X, τ) is an NPT_1-space. Hence proved.

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4.24 Proposition:

Let f be a one-one neutrosophic pre-irresolute function from a NTS (X,τ) to a NTS (Y,σ). If (Y,σ) is NPT_1 then (X,τ) is an NPT_1-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NPT_1, so there exists a σ-NPO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ and there exists a σ-NPO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre-irresolute function, so $f^{-1}(G)$ is a τ-NPO set in X[by 2.13]. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-NPO set in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NPO set $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-NPO set $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X,τ) is an NPT_1-space. Hence proved.

4.25 Proposition:

Let f be a one-one neutrosophic pre*-continuous function from a NTS (X,τ) to a NTS (Y,σ). If (Y,σ) is NPT_1 then (X,τ) is a NT_1-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NPT_1, so there exists a σ-NPO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ and there exists a σ-NPO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre*-continuous function, so $f^{-1}(G)$ is a τ-open NS in X[by 3.7]. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-open NS in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-open NS $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-open NS $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X,τ) is a NT_1-space. Hence proved.

4.26 Definition:

A NTS (X,τ) is called a neutrosophic pre-T_2 space or neutrosophic pre-Hausdorff space (NPT_2-space or NP-Hausdorff space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exist two neutrosophic pre-open sets U, V in X such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \tilde{\emptyset}$.

4.27 Example:

Let $X = \{a,b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$ and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly the NTS (X,τ) is a neutrosophic pre-T_2 space.

4.28 Proposition:

Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is pre-finer than τ. If (X,τ) is an NPT_2-space then (X,τ^*) is also an NPT_2-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is an NPT_2-space, so there exist τ-NPO sets G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$. Since τ^* is pre-finer than τ, so every τ-NPO set is a τ^*-NPO set. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exist τ^*-NPO sets G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$. Hence (X,τ^*) is an NPT_2-space.

4.29 Proposition:

Let (X,τ) be a NTS. If X is NT_2-space then X is an NPT_2-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is a NT_2-space, so there exist τ-open NSs G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$. Since every open NS is an NPO set, so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exist τ-NPO sets G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$.. Hence (X,τ) is an NPT_2-space.

4.30 Remark:

Converse of the proposition 4.29 is not true. We establish it by the following counter example.

Let $X = \{a,b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}\}$. Clearly (X,τ) is not a NT_2-space.

We now show that (X,τ) is an NPT_2-space. Let $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$ be any two NPs in X. Also let $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$ and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly A and B are two NPO sets in X such that $A \cap B = \tilde{\emptyset}$. Thus there exist NPO sets A and B such that $a_{\alpha,\beta,\gamma} \in A$, $b_{\alpha',\beta',\gamma'} \in B$ and $A \cap B = \tilde{\emptyset}$. Therefore (X,τ) is an NPT_2-space.

Thus the NTS (X,τ) is an NPT_2-space but not a NT_2-space.

4.31 Proposition:

Let (X,τ) be an NPT_2-space. Then every neutrosophic subspace of X is an NPT_2-space and hence the property is hereditary.

Proof: Let $(Y, \tau\mid_Y)$ be a neutrosophic subspace of (X,τ), where $\tau\mid_Y = \{G\mid_Y : G \in \tau\}$. We want to show $(Y,\tau\mid_Y)$ is an NPT_2-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X,τ) is an NPT_2-space, so there exist τ-NPO sets U, V such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \tilde{\emptyset}$. Then $x_{\alpha,\beta,\gamma} \in U\mid_Y$, $y_{\alpha',\beta',\gamma'} \in V\mid_Y$ and $(U\mid_Y) \cap (V\mid_Y) = (U \cap V)\mid_Y = \tilde{\emptyset}\mid_Y = \tilde{\emptyset}$. Also by the proposition 3.3(i), $U\mid_Y, V\mid_Y$ are $\tau\mid_Y$-NPO sets in Y as U, V are τ-NPO sets in X. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in Y such that $x \neq y$, there exist $\tau\mid_Y$-NPO sets $U\mid_Y, V\mid_Y$ such that $x_{\alpha,\beta,\gamma} \in U\mid_Y$, $y_{\alpha',\beta',\gamma'} \in V\mid_Y$ and $(U\mid_Y) \cap (V\mid_Y) = \tilde{\emptyset}$. Therefore $(Y,\tau\mid_Y)$ is an NPT_2-space and hence the property is hereditary.

4.32 Lemma:

In a co-finite NTS, every finite NS is an NPC set.

Proof: Let (X,τ) be a co-finite NTS and let U be a finite NS in X. As $U = (U^c)^c$ is a finite NS, so U^c is an open NS[as (X,τ) is a co-finite NTS], i.e., U is a closed NS. Since every closed NS is an NPC set, therefore U is an NPC set.

4.33 Remark:

From 4.32, it is clear that in a co-finite NTS, an NPO set is a NS whose complement is a finite NS. Therefore, in a co-finite NTS, an NPO set is a neutrosophic open set.

4.34 Lemma:

Let X be an infinite set. Then the co-finite NTS (X, τ) is not an NPT_2-space.

Proof: Suppose that (X, τ) is an NPT_2-space. Then for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exist τ-NPO sets G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$. Since (X, τ) is a co-finite NTS, so G, H are open NSs [by 4.33] and therefore, their complements, i.e., G^c, H^c are finite open NSs. Now $G \cap H = \tilde{\emptyset} \Rightarrow (G \cap H)^c = (\tilde{\emptyset})^c \Rightarrow G^c \cup H^c = \tilde{X}$, which is not possible as $\tilde{X}$ is an infinite neutrosophic set and $G^c \cup H^c$ is a finite neutrosophic set. Therefore (X, τ) is not an NPT_2-space.

4.35 Proposition:

Let (X, τ) be a NTS. If (X, τ) is an NPT_2-space then it is an NPT_1-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x \neq y$. Since (X, τ) is an NPT_2-space, so there exist τ-NPO sets H and K such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \in K$ and $H \cap K = \tilde{\emptyset}$. Since $x_{\alpha,\beta,\gamma} \in H$ and $H \cap K = \tilde{\emptyset}$, so $x_{\alpha,\beta,\gamma} \notin K$. Similarly $y_{\alpha',\beta',\gamma'} \notin H$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$ there exists a τ-NPO set H such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \notin H$ and there exists a τ-NPO set K such that $x_{\alpha,\beta,\gamma} \notin K$, $y_{\alpha',\beta',\gamma'} \in K$. Hence (X, τ) is an NPT_1-space.

4.36 Remark:

Converse of the proposition 4.35 is not true. We establish it by the following counter example.

Let $\mathbb{N}$ be the set of all natural numbers and $\mathcal{N}(\mathbb{N})$ be the set of all neutrosophic sets over $\mathbb{N}$. Also let $\tilde{\mathbb{N}} = \{\langle x, 1, 0, 0\rangle : x \in \mathbb{N}\}$ and $\tilde{\emptyset} = \{\langle x, 0, 1, 1\rangle : x \in \mathbb{N}\}$. Let τ be the set containing $\tilde{\emptyset}$ and all those neutrosophic sets over $\mathbb{N}$ whose complements are finite. Then $(\mathbb{N}, \tau)$ is a co-finite NTS.

Since $\mathbb{N}$ is an infinite set and since $(\mathbb{N}, \tau)$ is a co-finite NTS, so by the lemma 4.34, $(\mathbb{N}, \tau)$ is not an NPT_2-space.

We show that $(\mathbb{N}, \tau)$ is NPT_1-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in $\mathbb{N}$ such that $x \neq y$. Since $[(x_{1,0,0})^c]^c = x_{1,0,0}$, a finite NS, so $(x_{1,0,0})^c$ is a τ-open NS and therefore, a τ-NPO set. Clearly $y_{\alpha',\beta',\gamma'} \in (x_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma} \notin (x_{1,0,0})^c$. Therefore $(x_{1,0,0})^c$ is a τ-NPO set such that $y_{\alpha',\beta',\gamma'} \in (x_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma} \notin (x_{1,0,0})^c$. Similarly $(y_{1,0,0})^c$ is a τ-NPO set such that $y_{\alpha',\beta',\gamma'} \notin (y_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma} \in (y_{1,0,0})^c$. Therefore $(\mathbb{N}, \tau)$ is an NPT_1-space.

Thus $(\mathbb{N}, \tau)$ is an NPT_1-space but not an NPT_2-space.

4.37 Proposition:

Let (X, τ) be an NPT_2-space. Then for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ such that $x \neq y$, there exists an NPO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{p,q,r} \in G^c$ and $y_{p,q,r} \in [Pcl(G)]^c$.

Proof: Since X is NPT_2-space, so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ such that $x \neq y$, there exist two NPO sets G and H in X such that $x_{\alpha,\beta,\gamma} \in G$, $y_{p,q,r} \in H$ and $G \cap H = \tilde{\emptyset}$. Now $G \cap H = \tilde{\emptyset} \Rightarrow H \subseteq G^c \Rightarrow y_{p,q,r} \in G^c$. Since H^c is an NPC set and $G \subseteq H^c$, so $Pcl(G) \subseteq H^c \Rightarrow H \subseteq [Pcl(G)]^c \Rightarrow y_{p,q,r} \in [Pcl(G)]^c$. Hence proved.

4.38 Proposition:

Let (X,τ) be an NPT_2-space. Then for every NP $x_{\alpha,\beta,\gamma}$ in X, $x_{\alpha,\beta,\gamma}=\cap\{Pcl(G):x_{\alpha,\beta,\gamma}\in G$ and G is an NPO set $\}$.

Proof: Let $x_{\alpha,\beta,\gamma}$ be an NP in X. Also let $y_{1,0,0}$ be an NCP in X such that $x\neq y$. Since X is NPT_2-space, so there exist two NPO sets G and H in X such that $x_{\alpha,\beta,\gamma}\in G$, $y_{1,0,0}\in H$ and $G\cap H=\tilde{\emptyset}$. Now $G\cap H=\tilde{\emptyset}\Rightarrow H\subseteq G^c\Rightarrow y_{1,0,0}\in G^c$. Since H^c is an NPC set and $G\subseteq H^c$, so $Pcl(G)\subseteq H^c\Rightarrow H\subseteq[Pcl(G)]^c\Rightarrow y_{1,0,0}\in[Pcl(G)]^c\Rightarrow y_{0,1,1}\in Pcl(G)$, i.e., $y_{p,q,r}\cap Pcl(G)=\tilde{\emptyset}$ for every NP with support $y\neq x$. Thus for every NPO set G such that $x_{\alpha,\beta,\gamma}\in G$, we have $y_{p,q,r}\cap Pcl(G)=\tilde{\emptyset}$ for all NP such that $x\neq y$. Obviously $x_{\alpha,\beta,\gamma}\in Pcl(G)$ for every NPO G with $x_{\alpha,\beta,\gamma}\in G$. Therefore $x_{\alpha,\beta,\gamma}=\cap\{Pcl(G):x_{\alpha,\beta,\gamma}\in G$ and G is an NPO set $\}$. Hence proved.

4.39 Proposition:

Let f be a bijective neutrosophic pre-open function from a NTS (X,τ) to a NTS (Y,σ). If (X,τ) is NT_2 then (Y,σ) is an NPT_2-space.

Proof: Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1\neq y^2$, be two NPs in Y. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1\neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma})=y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'})=y^2_{p',q',r'}$. Since X is NT_2, so there exist τ-open NSs G,H such that $x^1_{\alpha,\beta,\gamma}\in G$, $x^2_{\alpha',\beta',\gamma'}\in H$ and $G\cap H=\tilde{\emptyset}$. Since f is a neutrosophic pre-open function, so $f(G),f(H)$ are σ-NPO sets such that $y^1_{p,q,r}=f(x^1_{\alpha,\beta,\gamma})\in f(G)$ and $y^2_{p',q',r'}=f(x^2_{\alpha',\beta',\gamma'})\in f(H)$. Again since f is bijective, so $f(G)\cap f(H)=f(G\cap H)=f(\tilde{\emptyset})=\tilde{\emptyset}$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1\neq y^2$, there exist σ-NPO sets $f(G),f(H)$ such that $y^1_{p,q,r}\in f(G)$, $y^2_{p',q',r'}\in f(H)$ and $f(G)\cap f(H)=\tilde{\emptyset}$. Therefore (Y,σ) is an NPT_2-space. Hence proved.

4.40 Proposition:

Let f be a one-one neutrosophic pre-continuous function from a NTS (X,τ) to a NTS (Y,σ). If (Y,σ) is NT_2 then (X,τ) is an NPT_2-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1\neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1\neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma})=y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'})=y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma}=f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'}=f^{-1}(y^2_{p',q',r'})$. Since (Y,σ) is NT_2, so there exist σ-open NSs H_1,H_2 such that $y^1_{p,q,r}\in H_1$, $y^2_{p',q',r'}\in H_2$ and $H_1\cap H_2=\tilde{\emptyset}$. Since f is a neutrosophic pre-continuous function, so $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are τ-NPO sets in X. Now $f^{-1}(H_1)\cap f^{-1}(H_2)=f^{-1}(H_1\cap H_2)=f^{-1}(\tilde{\emptyset})=\tilde{\emptyset}$. Also $y^1_{p,q,r}\in H_1\Rightarrow f^{-1}(y^1_{p,q,r})\in f^{-1}(H_1)\Rightarrow x^1_{\alpha,\beta,\gamma}\in f^{-1}(H_1)$. Similarly $x^2_{\alpha',\beta',\gamma'}\in f^{-1}(H_2)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1\neq x^2$, there exist τ-NPO sets $f^{-1}(H_1)$ and $f^{-1}(H_2)$ in X such that $x^1_{\alpha,\beta,\gamma}\in f^{-1}(H_1)$, $x^2_{\alpha',\beta',\gamma'}\in f^{-1}(H_2)$ and $f^{-1}(H_1)\cap f^{-1}(H_2)=\tilde{\emptyset}$. Therefore (X,τ) is an NPT_2-space. Hence proved.

4.41 Proposition:

Let f be a one-one neutrosophic pre-irresolute function from a NTS (X,τ) to a NTS (Y,σ). If (Y,σ) is NPT_2 then (X,τ) is an NPT_2-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1\neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1\neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma})=y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'})=y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma}=f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'}=f^{-1}(y^2_{p',q',r'})$. Since (Y,σ) is NPT_2, so there exist σ-NPO sets H_1,H_2 such that

$y^1_{p,q,r} \in H_1, y^2_{p',q',r'} \in H_2$ and $H_1 \cap H_2 = \tilde{\emptyset}$. Since f is a neutrosophic pre-irresolute function, so $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are τ-NPO sets in X[by 2.13]. Now $f^{-1}(H_1) \cap f^{-1}(H_2) = f^{-1}(H_1 \cap H_2) = f^{-1}(\tilde{\emptyset}) = \tilde{\emptyset}$. Also $y^1_{p,q,r} \in H_1 \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(H_1) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exist τ-NPO sets $f^{-1}(H_1)$ and $f^{-1}(H_2)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$ and $f^{-1}(H_1) \cap f^{-1}(H_2) = \tilde{\emptyset}$. Therefore (X, τ) is an NPT_2-space. Hence proved.

4.42 Proposition:

Let f be a one-one neutrosophic pre*-continuous function from a NTS (X, τ) to a NTS (Y, σ). If (Y, σ) is NPT_2 then (X, τ) is a NT_2-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since (Y, σ) is NPT_2, so there exist σ-NPO sets H_1, H_2 such that $y^1_{p,q,r} \in H_1, y^2_{p',q',r'} \in H_2$ and $H_1 \cap H_2 = \tilde{\emptyset}$. Since f is a neutrosophic pre*-continuous function, so $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are τ-open NSs in X[by 3.7]. Now $f^{-1}(H_1) \cap f^{-1}(H_2) = f^{-1}(H_1 \cap H_2) = f^{-1}(\tilde{\emptyset}) = \tilde{\emptyset}$. Also $y^1_{p,q,r} \in H_1 \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(H_1) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exist τ-open NSs $f^{-1}(H_1)$ and $f^{-1}(H_2)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$ and $f^{-1}(H_1) \cap f^{-1}(H_2) = \tilde{\emptyset}$. Therefore (X, τ) is a NT_2-space. Hence proved.

5 Conclusion:

In this article, we have defined neutrosophic pre-T_0, pre-T_1, and pre-T_2 spaces in connection with neutrosophic topological spaces which are based on single-valued neutrosophic sets and then studied their various properties with examples. We have shown that if X is a neutrosophic T_0-space (resp. T_1-space, T_2-space) then X is a neutrosophic pre-T_0 (resp. pre-T_1, pre-T_2) space but the converse is not true. We have established that a neutrosophic pre-T_2 (resp. pre-T_1) space is a neutrosophic pre-T_1 (resp. pre-T_0) space but the converse is not true. We have also proved that if Y is a neutrosophic subspace of X then for every neutrosophic pre-open (resp. pre-closed) set in X, there is a neutrosophic pre-open (resp. pre-closed) set in Y. We have proved that the property of a space of being NPT_0(resp. NPT_1, NPT_2) is a hereditary property. Lastly, we have tried to set up some results using various functions such as neutrosophic continuous function, neutrosophic pre-irresolute function, pre*-continuous function etc.

In the coming future, we shall try to develop and study some other types of separation properties. Hope that the findings of this article will assist the research fraternity to move forward with the development of different aspects of neutrosophic topological spaces.

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