



Multi Criteria Decision Making Algorithm Via Complex Neutrosophic Nano Topological Spaces

¹Mani Parimala, ¹Muthusamy Karthika, ²Sivaraman Murali, ³Florentin Smarandache, ⁴Muhammad Riaz, ⁵Saeid Jafari

¹Department of Mathematics, Bannari Amman Institute of Technology, Sathyamangalam-638401, Tamil Nadu, India.

Email: rishwanthpari@gmail.com

²Department of Mathematics, Coimbatore Institute of Technology, Coimbatore, Tamil Nadu, India. Email: muralisvino@gmail.com

³Mathematics & Science Department, University of New Mexico, 705 Gurley Ave., Gallup, NM 87301, USA

⁴Department of Mathematics, University of the Punjab Lahore, Pakistan

⁵College of Vestsjaelland South, Herrestraede 11, 4200 Slagelse, Denmark Email¹rishwanthpari@gmail.com, karthikamuthusamy1991@gmail.com, Email²muralisvino@gmail.com, Email³fsmarandache@gmail.com, Email⁴mriaz.math@pu.edu.pk, Email⁵jafaripersia@gmail.com

Abstract

The scope of this manuscript is to instigate the present-day perception of complex neutrosophic nano topological spaces and delve into a few of its spectacles. We also illustrate the spectacles with numerical quantities. Decision making plays an important role to diagnose a diseases in medical field. So a method is developed to achieve this under complex neutrosophic nano topological spaces (CNNTSs). A comparative assessment is provided to demonstrate the distinction between the unique concept and the existing approaches.

Keywords: complex neutrosophic topology, complex neutrosophic nano topological spaces, complex neutrosophic nano-closed sets, complex neutrosophic interior and closure.

1 Introduction

Multi-criteria decision-making (MCDM) is a process in decision-making that takes into account the best potential choices. Decisions have been made in medieval times without considering data ambiguities that may contribute to a prospective consequence. Insufficient consequences for real-world organizational circumstances. If we derive the consequence of collected data without hesitancy, the findings will be ambiguous, indeterminate, or incorrect. MCDM has played a critical part in real-world problems like as administration, illness diagnosis, finance, and industry. Each team leader makes dozens of decisions to carry out the majority of his or her task, but each decision should be based on logic. Medical diagnosis with MCDM gives clinicians with choices for recognizing disease symptoms and the degree of sickness. MCDM is used to tackle sophisticated and complex issues using a variety of characteristics. In MCDM, the challenge must be recognized by identifying viable alternatives, assessing each option against the criteria established by the decision-maker, and finally selecting the optimal option. To address the complications and complexity of multi-criteria decision-making situations, important mathematical approaches such as fuzzy sets and its generalized sets have been created.

Zadeh's²⁵ introduced fuzzy set theory. Zafer et al.²⁶ introduced and developed the MCDM method using rough fuzzy information. Among different generalised FFSs, the notion of neutrosophic logic, and NSs, was introduced by Smarandache^{22,23} that has considered as a generalization of fuzzy logic and IFSs, FSs, in light of Atanasov's³ IFSs lacks a realistic process with indeterminate and inconsistent knowledge involved in real time scenarios. In comparison to IFSs, NSs may successfully communicate the message of inconsistency, incompleteness, and indeterminacy by integrating an indeterminacy membership function that is focused on independently.

It has been applied many fields of science and engineering such as Algebra, topology,^{8-13,15-17} Graph theory and Image processing. Ali and Smarandache¹ developed novel complex neutrosophic sets(CNS) which consists of both amplitude values and phase values and it is applicable for the uncertainty and periodicity problems. Simultaneously, CNS has been applied in science and engineering field. Neutrosophic topology which is a generalization of general topology, FT, IFT and its applications^{2,8,10-12,14-16} in various fields gained attention in the literature. Lellis Thivagar et al.⁷ developed concept of NNT to the literature. It piqued the interest of scholars in understanding the development of nano topology and neutrosophic sets.^{8,10,14}

Numerous scientists have been working on topological spaces, aggregation operators, similarity measures, correlation coefficients, and decision-making applications in this age. These frameworks offer distinct formulas for various sets and provide superior decision-making solutions. It has several applicability in many domains such as medical science, artificial intelligence, object identification, social sciences.

1.1 Objective and Motivation

The expanded, goal work, and hybrid motivation of the manuscript is provided in the complete paper, step by step. CNS is a special case of fuzzy set. An algorithm is developed to study a problem in medical diagnosis. A medical diagnosis problem is assure the superiority, strength and ease of the proposed algorithm. In medical, engineering and artificial applications. Intelligence, forestry, and other issues of everyday life, this model is a common and applied to collect large data. This model motivates the researchers to develop a new model or study a various science and engineering problem using this model.

The structure of this document is as follows. Section 2 discusses the tentative concepts of the research in NS. A unique concept of CNNTSs, illustrated almost all of its processes such as interior and closure, and developed a scoring function are presented in section 3. We provided a technique and flowchart for the MCDM issue in Section 4. In Section 5, we built a strategy for solving the MCDM issue regarding medical diagnosis using CNNTS as an example. We also discussed the algorithms' benefit, efficiency, consistency, and validity. We presented a quick introduction and comparative analysis of our suggested strategy using several current approaches. In Section 6, the result of this work is fundamentally summarized, and the next field of research is indicated.

2 Preliminaries

The definitions from^{1,6,7} are used in sequel.

Definition 2.1.¹ An object \mathfrak{S} defined on a discourse universe \mathfrak{U} is called CNS, if it may be stated as $\mathfrak{S} = \{(\vartheta, \langle \mathfrak{M}(\vartheta), \mathfrak{I}(\vartheta), \mathfrak{F}(\vartheta) \rangle) \mid \vartheta \in \mathfrak{U}\}$. The values $\mathfrak{M}(\vartheta), \mathfrak{I}(\vartheta), \mathfrak{F}(\vartheta)$ and their number can be in the complex plane all inside the unit circle, and so is in the following form, $\mathfrak{M}(\vartheta) = p(\vartheta)e^{j\mu(\vartheta)}, \mathfrak{I}(\vartheta) = q(\vartheta)e^{j\nu(\vartheta)}, \mathfrak{F}(\vartheta) = r(\vartheta)e^{j\omega(\vartheta)}$ where $p(\vartheta), q(\vartheta), r(\vartheta)$ and $\mu(\vartheta), \nu(\vartheta), \omega(\vartheta)$ are respectively the amplitude terms and the phase terms, $\mu(\vartheta), \nu(\vartheta), \omega(\vartheta) \in [0, 1]$ such that $-0 \leq p(\vartheta) + q(\vartheta) + r(\vartheta) \leq 3^+$ and $\mu(\vartheta), \nu(\vartheta), \omega(\vartheta)$ are real valued with $j = \sqrt{-1}$. The scaling factors μ, ν and $\omega \in [0, 2\pi]$.

Definition 2.2.⁶ Let \mathfrak{C} be a folk of CNS on $U \neq \emptyset$. Then (X, \mathfrak{C}) is said to be a neutrosophic complex topological space if it fulfill the necessary criteria:

- $0_C, 1_C \in \mathfrak{C}$.
- Capricious union of complex neutrosophic set C in \mathfrak{C} if each C in \mathfrak{C}
- Restricted intersection of complex neutrosophic set C in \mathfrak{C} if each C in \mathfrak{C}

Definition 2.3.⁷ Let the equivalence relations be \mathfrak{R} and the neutrosophic set be \mathfrak{S} are defined on discourse of universe \mathfrak{U} . The satisfaction grade \mathfrak{M}_S , the indeterminacy grade \mathfrak{I}_S and dissatisfaction grade \mathfrak{F}_S are the elements of

\mathfrak{S} . The approximation space $(\mathfrak{U}, \mathfrak{R})$ has triplet elements such as, upper $\mathcal{E}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{S})$, lower $\mathcal{E}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{S})$, and boundary approximation $\mathcal{E}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{S})$ where

$$(i) \mathcal{E}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{S}) = \{ \langle \vartheta, \mathfrak{M}_{\overline{\mathfrak{R}}\mathfrak{S}}(\vartheta), \mathfrak{I}_{\overline{\mathfrak{R}}\mathfrak{S}}(\vartheta), \mathfrak{F}_{\overline{\mathfrak{R}}\mathfrak{S}}(\vartheta) \rangle \mid \xi \in [\vartheta]_{\mathfrak{R}}, \vartheta \in \mathfrak{U} \}$$

$$(ii) \mathcal{E}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{S}) = \{ \langle \vartheta, \mathfrak{M}_{\underline{\mathfrak{R}}\mathfrak{S}}(\vartheta), \mathfrak{I}_{\underline{\mathfrak{R}}\mathfrak{S}}(\vartheta), \mathfrak{F}_{\underline{\mathfrak{R}}\mathfrak{S}}(\vartheta) \rangle \mid \xi \in [\vartheta]_{\mathfrak{R}}, \vartheta \in \mathfrak{U} \}$$

$$(iii) \mathcal{E}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{S}) = \mathcal{E}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{S}) - \mathcal{E}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{S}).$$

where, $\mathfrak{M}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\vartheta) = \bigwedge_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{M}_{(\mathfrak{A})}(\xi)$, $\mathfrak{M}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\vartheta) = \bigvee_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{M}_{(\mathfrak{A})}(\xi)$

$\mathfrak{I}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\vartheta) = \bigvee_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{I}_{(\mathfrak{A})}(\xi)$, $\mathfrak{I}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\vartheta) = \bigwedge_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{I}_{(\mathfrak{A})}(\xi)$

$\mathfrak{F}_{\underline{\mathfrak{R}}(\mathfrak{A})}(\vartheta) = \bigvee_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{F}_{(\mathfrak{A})}(\xi)$, $\mathfrak{F}_{\overline{\mathfrak{R}}(\mathfrak{A})}(\vartheta) = \bigwedge_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{F}_{(\mathfrak{A})}(\xi)$.

3 Complex Neutrosophic Nano Topological Spaces

Definition 3.1. Two objects $\mathfrak{S}_1 = \{(\vartheta, \langle \mathfrak{M}_{\mathfrak{S}_1}(\vartheta), \mathfrak{I}_{\mathfrak{S}_1}(\vartheta), \mathfrak{F}_{\mathfrak{S}_1}(\vartheta) \rangle) : \vartheta \in \mathfrak{U}\}$ and $\mathfrak{S}_2 = \{(\vartheta, \langle \mathfrak{M}_{\mathfrak{S}_2}(\vartheta), \mathfrak{I}_{\mathfrak{S}_2}(\vartheta), \mathfrak{F}_{\mathfrak{S}_2}(\vartheta) \rangle) : \vartheta \in \mathfrak{U}\}$ are defined on \mathfrak{U} , the discourse of universe, and their intersection and union are denoted and defined as follows

1. The intersection of \mathfrak{S}_1 and \mathfrak{S}_2 is $\mathfrak{S}_1 \cap \mathfrak{S}_2 = \{(\vartheta, \langle \mathfrak{M}_{\mathfrak{S}_1 \cap \mathfrak{S}_2}(\vartheta), \mathfrak{I}_{\mathfrak{S}_1 \cap \mathfrak{S}_2}(\vartheta), \mathfrak{F}_{\mathfrak{S}_1 \cap \mathfrak{S}_2}(\vartheta) \rangle) : \vartheta \in \mathfrak{U}\}$, where

$$\mathfrak{M}_{\mathfrak{S}_1 \cap \mathfrak{S}_2}(\vartheta) = [p_{\mathfrak{S}_1}(\vartheta) \wedge p_{\mathfrak{S}_2}(\vartheta)] e^{j[\mu_{\mathfrak{S}_1}(\vartheta) \wedge \mu_{\mathfrak{S}_2}(\vartheta)]}$$

$$\mathfrak{I}_{\mathfrak{S}_1 \cap \mathfrak{S}_2}(\vartheta) = [q_{\mathfrak{S}_1}(\vartheta) \vee q_{\mathfrak{S}_2}(\vartheta)] e^{j[\nu_{\mathfrak{S}_1}(\vartheta) \vee \nu_{\mathfrak{S}_2}(\vartheta)]}$$

$$\mathfrak{F}_{\mathfrak{S}_1 \cap \mathfrak{S}_2}(\vartheta) = [r_{\mathfrak{S}_1}(\vartheta) \vee r_{\mathfrak{S}_2}(\vartheta)] e^{j[\omega_{\mathfrak{S}_1}(\vartheta) \vee \omega_{\mathfrak{S}_2}(\vartheta)]}$$

2. The union of \mathfrak{S}_1 and \mathfrak{S}_2 is $\mathfrak{S}_1 \cup \mathfrak{S}_2 = \{(\vartheta, \langle \mathfrak{M}_{\mathfrak{S}_1 \cup \mathfrak{S}_2}(\vartheta), \mathfrak{I}_{\mathfrak{S}_1 \cup \mathfrak{S}_2}(\vartheta), \mathfrak{F}_{\mathfrak{S}_1 \cup \mathfrak{S}_2}(\vartheta) \rangle) : \vartheta \in \mathfrak{U}\}$, where

$$\mathfrak{M}_{\mathfrak{S}_1 \cup \mathfrak{S}_2}(\vartheta) = [p_{\mathfrak{S}_1}(\vartheta) \vee p_{\mathfrak{S}_2}(\vartheta)] e^{j[\mu_{\mathfrak{S}_1}(\vartheta) \vee \mu_{\mathfrak{S}_2}(\vartheta)]}$$

$$\mathfrak{I}_{\mathfrak{S}_1 \cup \mathfrak{S}_2}(\vartheta) = [q_{\mathfrak{S}_1}(\vartheta) \wedge q_{\mathfrak{S}_2}(\vartheta)] e^{j[\nu_{\mathfrak{S}_1}(\vartheta) \wedge \nu_{\mathfrak{S}_2}(\vartheta)]}$$

$$\mathfrak{F}_{\mathfrak{S}_1 \cup \mathfrak{S}_2}(\vartheta) = [r_{\mathfrak{S}_1}(\vartheta) \wedge r_{\mathfrak{S}_2}(\vartheta)] e^{j[\omega_{\mathfrak{S}_1}(\vartheta) \wedge \omega_{\mathfrak{S}_2}(\vartheta)]}$$

3. The symmetric difference of \mathfrak{S}_1 and \mathfrak{S}_2 is $\mathfrak{S}_1 - \mathfrak{S}_2 = \{(\vartheta, \langle \mathfrak{M}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\vartheta), \mathfrak{I}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\vartheta), \mathfrak{F}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\vartheta) \rangle) : \vartheta \in \mathfrak{U}\}$, where

$$\mathfrak{M}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\vartheta) = [p_{\mathfrak{S}_1}(\vartheta) \wedge r_{\mathfrak{S}_2}(\vartheta)] e^{j[\mu_{\mathfrak{S}_1}(\vartheta) \wedge \omega_{\mathfrak{S}_2}(\vartheta)]}$$

$$\mathfrak{I}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\vartheta) = [q_{\mathfrak{S}_1}(\vartheta) \vee (1 - q_{\mathfrak{S}_2}(\vartheta))] e^{j[\nu_{\mathfrak{S}_1}(\vartheta) \vee (2\pi - \nu_{\mathfrak{S}_2}(\vartheta))]}$$

$$\mathfrak{F}_{\mathfrak{S}_1 - \mathfrak{S}_2}(\vartheta) = [r_{\mathfrak{S}_1}(\vartheta) \vee p_{\mathfrak{S}_2}(\vartheta)] e^{j[\omega_{\mathfrak{S}_1}(\vartheta) \vee \mu_{\mathfrak{S}_2}(\vartheta)]}$$

Definition 3.2. Let $\mathfrak{S}_1 = \{(\vartheta, \langle \mathfrak{M}_{\mathfrak{S}_1}(\vartheta), \mathfrak{I}_{\mathfrak{S}_1}(\vartheta), \mathfrak{F}_{\mathfrak{S}_1}(\vartheta) \rangle) : \vartheta \in \mathfrak{U}\}$ and $\mathfrak{S}_2 = \{(\vartheta, \langle \mathfrak{M}_{\mathfrak{S}_2}(\vartheta), \mathfrak{I}_{\mathfrak{S}_2}(\vartheta), \mathfrak{F}_{\mathfrak{S}_2}(\vartheta) \rangle) : \vartheta \in \mathfrak{U}\}$ are the two objects defined on a universe of discourse \mathfrak{U} , then

1. $\mathfrak{S}_1 \subseteq \mathfrak{S}_2$ if and only if $\mathfrak{M}_{\mathfrak{S}_1} \leq \mathfrak{M}_{\mathfrak{S}_2}$, $\mathfrak{I}_{\mathfrak{S}_1} \geq \mathfrak{I}_{\mathfrak{S}_2}$ and $\mathfrak{F}_{\mathfrak{S}_1} \geq \mathfrak{F}_{\mathfrak{S}_2}$ such that

$$\begin{aligned} \mathfrak{M}_{\mathfrak{S}_1} \leq \mathfrak{M}_{\mathfrak{S}_2} &= [p_{\mathfrak{S}_1}(\vartheta) \leq p_{\mathfrak{S}_2}(\vartheta)]e^{j[\mu_{\mathfrak{S}_1}(\vartheta) \leq \mu_{\mathfrak{S}_2}(\vartheta)]} \\ \mathfrak{I}_{\mathfrak{S}_1} \geq \mathfrak{I}_{\mathfrak{S}_2} &= [q_{\mathfrak{S}_1}(\vartheta) \geq q_{\mathfrak{S}_2}(\vartheta)]e^{j[\nu_{\mathfrak{S}_1}(\vartheta) \geq \nu_{\mathfrak{S}_2}(\vartheta)]} \\ \mathfrak{F}_{\mathfrak{S}_1} \geq \mathfrak{F}_{\mathfrak{S}_2} &= [r_{\mathfrak{S}_1}(\vartheta) \geq r_{\mathfrak{S}_2}(\vartheta)]e^{j[\omega_{\mathfrak{S}_1}(\vartheta) \geq \omega_{\mathfrak{S}_2}(\vartheta)]} \end{aligned}$$

Definition 3.3. Let \mathfrak{R} an equivalence relation on \mathfrak{S} , where $\mathfrak{S} = \{(\vartheta, \langle \mathfrak{M}(\vartheta), \mathfrak{I}(\vartheta), \mathfrak{F}(\vartheta) \rangle) : \vartheta \in \mathfrak{U}\}$ be a non-void set. Let \mathfrak{A} be a CNS in with satisfaction $\mathfrak{M}_{\mathfrak{A}}$, indeterminacy $\mathfrak{I}_{\mathfrak{A}}$ and dissatisfaction $\mathfrak{F}_{\mathfrak{A}}$. The complex neutrosophic nano minor approximation, complex neutrosophic nano major approximation and complex neutrosophic nano border of \mathfrak{A} in the approximation space $(\mathfrak{S}, \mathfrak{R})$ denoted by $\mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A})$, $\mathfrak{CNM}_{\mathfrak{R}}(\mathfrak{A})$ and $\mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A})$ are respectively defined as:

- (i) $\mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}) = \{(\vartheta, \langle \mathfrak{M}_{\mathfrak{R}(\mathfrak{A})}(\vartheta), \mathfrak{I}_{\mathfrak{R}(\mathfrak{A})}(\vartheta), \mathfrak{F}_{\mathfrak{R}(\mathfrak{A})}(\vartheta) \rangle) : \xi \in [\vartheta]_{\mathfrak{R}}, \vartheta \in \mathfrak{U}\}$
- (ii) $\mathfrak{CNM}_{\mathfrak{R}}(\mathfrak{A}) = \{(\vartheta, \langle \mathfrak{M}_{\overline{\mathfrak{R}(\mathfrak{A})}}(\vartheta), \mathfrak{I}_{\overline{\mathfrak{R}(\mathfrak{A})}}(\vartheta), \mathfrak{F}_{\overline{\mathfrak{R}(\mathfrak{A})}}(\vartheta) \rangle) : \xi \in [\vartheta]_{\mathfrak{R}}, \vartheta \in \mathfrak{U}\}$
- (iii) $\mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{CNM}_{\mathfrak{R}}(\mathfrak{A}) - \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A})$ where

$$\begin{aligned} \mathfrak{M}_{\mathfrak{R}(\mathfrak{A})}(\vartheta) &= \wedge_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{M}_{\mathfrak{A}}(\xi), \mathfrak{M}_{\overline{\mathfrak{R}(\mathfrak{A})}}(\vartheta) = \vee_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{M}_{\mathfrak{A}}(\xi) \\ \mathfrak{I}_{\mathfrak{R}(\mathfrak{A})}(\vartheta) &= \vee_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{I}_{\mathfrak{A}}(\xi), \mathfrak{I}_{\overline{\mathfrak{R}(\mathfrak{A})}}(\vartheta) = \wedge_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{I}_{\mathfrak{A}}(\xi) \\ \mathfrak{F}_{\mathfrak{R}(\mathfrak{A})}(\vartheta) &= \vee_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{F}_{\mathfrak{A}}(\xi), \mathfrak{F}_{\overline{\mathfrak{R}(\mathfrak{A})}}(\vartheta) = \wedge_{\xi \in [\vartheta]_{\mathfrak{R}}} \mathfrak{F}_{\mathfrak{A}}(\xi). \end{aligned}$$

The triplet $(\mathfrak{CNL}_{\mathfrak{R}}, \mathfrak{CNM}_{\mathfrak{R}}, \mathfrak{CNB}_{\mathfrak{R}})$ is said to be complex neutrosophic approximation space.

Definition 3.4. Let \mathfrak{R} be an equivalence relation on the non-empty set $\mathfrak{S} \subseteq \mathfrak{U}$ and \mathfrak{U} be the universe, if $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\sim}, 1_{\sim}, \mathfrak{CNL}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{CNM}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{CNB}_{\mathfrak{R}}(\mathfrak{A})\}$, where $\mathfrak{A} \subseteq \mathfrak{S}$ and $\tau_{\mathfrak{R}}$ that has the following forms:

- 1. $0_{\sim}, 1_{\sim} \in \tau_{\mathfrak{R}}$
- 2. If $\mathfrak{A}_i \in \tau_{\mathfrak{R}}(\mathfrak{A})$, for $i = 1, 2, 3, \dots$, then

$$\bigcup_{i=1}^{\infty} \mathfrak{A}_i \in \tau_{\mathfrak{R}}(\mathfrak{A})$$

- 3. If $\mathfrak{A}_i \in \tau_{\mathfrak{R}}(\mathfrak{A})$, for $i = 1, 2, 3, \dots, n$, then

$$\bigcap_{i=1}^n \mathfrak{A}_i \in \tau_{\mathfrak{R}}(\mathfrak{A})$$

then $\tau_{\mathfrak{R}}(\mathfrak{A})$ is termed as CNNTS on \mathfrak{S} with respect to \mathfrak{A} . where the neutrosophic complex sets $1_{\sim} = \{(\vartheta, \langle 1e^{j0}, 0e^{j1}, 0e^{j1} \rangle) : \vartheta \in \mathfrak{U}\}$ and $0_{\sim} = \{(\vartheta, \langle 0e^{j1}, 1e^{j0}, 1e^{j0} \rangle) : \vartheta \in \mathfrak{U}\}$. We call $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{A}))$ as CNNTS. The components of $\tau_{\mathfrak{R}}(\mathfrak{A})$ are said to be CNNOS.

The complement \mathfrak{A}^c of a CNNOS \mathfrak{A} in a CNNTS. $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{A}))$ is said to be a CNNCS in \mathfrak{S} .

Example 3.1. Let a factory that includes a car part. The factory has 3 workers in this section. Every worker in this plant gets 10 car components, to be polished every day. The quality assurance unit at the factory maintains that though three employees are polishing correctly / successfully. The car parts, some of the staff are doing a job higher output than the rest. The amplitude (number of jobs done) and phase (attribute) (quality of the job done) of CNS and their upper, lower and boundary approximations are given below:

Let $\mathfrak{S} = \{a_1, a_2, a_3\}$ be the discourse of universe. Let $\mathfrak{S}/\mathfrak{R} = \{\{a_1, a_2\}, \{a_3\}\}$ be an equivalence relation on \mathfrak{S} and $\mathfrak{A} = \{\langle a_1, (0.8e^{j\pi 0.7}, 0.5e^{j\pi 0.4}, 0.6e^{j\pi 0.2}) \rangle, \langle a_2, (0.3e^{j\pi 0.4}, 0.4e^{j\pi 0.3}, 0.1e^{j\pi 0.5}) \rangle,$

$\langle a_3, (0.1e^{j\pi 0.3}, 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6}) \rangle\}$ be a neutrosophic set on \mathfrak{S} , then

$\mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle a_1, (0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.4}, 0.6e^{j\pi 0.5}) \rangle, \langle a_2, (0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.4}, 0.6e^{j\pi 0.5}) \rangle,$

$\langle a_3, (0.1e^{j\pi 0.3}, 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6}) \rangle\}$,

$\mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle a_1, (0.8e^{j\pi 0.7}, 0.4e^{j\pi 0.3}, 0.1e^{j\pi 0.2}) \rangle, \langle a_2, (0.8e^{j\pi 0.7}, 0.4e^{j\pi 0.3}, 0.1e^{j\pi 0.2}) \rangle,$

$\langle a_3, (0.1e^{j\pi 0.3}, 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6}) \rangle\}$ and

$\mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle a_1, (0.1e^{j\pi 0.2}, 0.6e^{j\pi 0.7}, 0.8e^{j\pi 0.7}) \rangle, \langle a_2, (0.1e^{j\pi 0.2}, 0.6e^{j\pi 0.7}, 0.8e^{j\pi 0.7}) \rangle,$

$\langle a_3, (0.1e^{j\pi 0.3}, 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6}) \rangle\}$.

$\mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}) \cup \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle a_1, (0.8e^{j\pi 0.7}, 0.4e^{j\pi 0.3}, 0.1e^{j\pi 0.2}) \rangle, \langle a_2, (0.8e^{j\pi 0.7}, 0.4e^{j\pi 0.3}, 0.1e^{j\pi 0.2}) \rangle,$

$\langle a_3, (0.1e^{j\pi 0.3}, 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6}) \rangle\} = \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A})$

$\mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}) \cap \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A}) = \{\langle a_1, (0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.4}, 0.6e^{j\pi 0.5}) \rangle, \langle a_2, (0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.4}, 0.6e^{j\pi 0.5}) \rangle,$

$\langle a_3, (0.1e^{j\pi 0.3}, 0.7e^{j\pi 0.5}, 0.3e^{j\pi 0.6}) \rangle\} = \mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A})$

$0_{\sim} \cap \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A}) = 0_{\sim}, 0_{\sim} \cap \mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}) = 0_{\sim}, 0_{\sim} \cap \mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{A}) = 0_{\sim},$

$0_{\sim} \cup \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A}), 0_{\sim} \cup \mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}), 0_{\sim} \cup \mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{A}),$

$1_{\sim} \cap \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A}), 1_{\sim} \cap \mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}), 1_{\sim} \cap \mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{A}) = \mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{A}),$

$1_{\sim} \cup \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A}) = 1_{\sim}, 1_{\sim} \cup \mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}) = 1_{\sim}, 1_{\sim} \cup \mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{A}) = 1_{\sim},$

Therefore, $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\sim}, 1_{\sim}, \mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{A})\}$ forms a topology.

Proposition 3.1. Let \mathfrak{U} be a non-void universe and \mathfrak{A} be a complex neutrosophic set on \mathfrak{U} . Then the following statements hold:

1. The collection $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\sim}, 1_{\sim}\}$, is the in-discrete complex neutrosophic nano topology on \mathfrak{U} .
2. If $\mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}} = \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}} = \mathfrak{C}\mathfrak{N}_{\mathfrak{R}}$, then the complex neutrosophic nano topology is $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\sim}, 1_{\sim}, \mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{A})\}$.
3. If $\mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}} = \mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}$, then $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\sim}, 1_{\sim}, \mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}}(\mathfrak{A})\}$ is a complex neutrosophic nano topology.
4. If $\mathfrak{C}\mathfrak{N}\mathfrak{U}_{\mathfrak{R}} = \mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}$, then the complex neutrosophic nano topology is $\tau_{\mathfrak{R}}(\mathfrak{A}) = \{0_{\sim}, 1_{\sim}, \mathfrak{C}\mathfrak{N}\mathfrak{L}_{\mathfrak{R}}(\mathfrak{A}), \mathfrak{C}\mathfrak{N}\mathfrak{B}_{\mathfrak{R}}(\mathfrak{A})\}$

Definition 3.5. Let $(\mathfrak{U}; \tau_{\mathfrak{R}})$ be any CNNTS with respect to complex neutrosophic subset of \mathfrak{U} and let \mathfrak{A} be a complex neutrosophic nano set in \mathfrak{S} . Then the complex neutrosophic nano interior and complex neutrosophic nano closure of \mathfrak{A} are defined as follows:

1. $\mathfrak{A}^{\circ} = \cup\{\mathfrak{G} : \mathfrak{G} \text{ is a CNNOS in } \mathfrak{S} \text{ and } \mathfrak{G} \subseteq \mathfrak{A}\},$
2. $\mathfrak{A}^{-} = \cap\{\mathfrak{G} : \mathfrak{G} \text{ is a CNNCS in } \mathfrak{S} \text{ and } \mathfrak{G} \supseteq \mathfrak{A}\}.$

Remark 3.1. For any complex neutrosophic nano set \mathfrak{A} in $(\mathfrak{U}; \tau_{\mathfrak{N}})$, we have

1. $[\mathfrak{A}^c]^{-} = [\mathfrak{A}^{\circ}]^c.$
2. $[\mathfrak{A}^c]^{\circ} = [\mathfrak{A}^{-}]^c.$
3. \mathfrak{A} is a CNNCS if and only if $\mathfrak{A}^{-} = \mathfrak{A}.$
4. \mathfrak{A} is a CNNOS if and only if $\mathfrak{A}^{\circ} = \mathfrak{A}.$
5. \mathfrak{A}^{-} is a CNNCS in $\mathfrak{U}.$
6. \mathfrak{A}° is a CNNOS in $\mathfrak{U}.$

Theorem 3.1. Let $(\mathfrak{U}; \tau_{\mathfrak{N}})(\mathfrak{S})$ be a complex neutrosophic nano topological space with respect to \mathfrak{S} where \mathfrak{S} is a complex neutrosophic subset of \mathfrak{U} . Let \mathfrak{A}_1 and \mathfrak{A}_2 be complex neutrosophic subsets of \mathfrak{U} . Then the following statements hold:

1. $\mathfrak{A} \subseteq \mathfrak{A}^{-}.$
2. \mathfrak{A} is complex neutrosophic nano closed if and only if $\mathfrak{A}^{-} = \mathfrak{A}.$
3. $0_{\sim}^{-} = 0_{\sim}$ and $1_{\sim}^{-} = 1_{\sim}.$
4. $\mathfrak{A}_1 \subseteq \mathfrak{A}_2 \Rightarrow \mathfrak{A}_1^{-} \subseteq \mathfrak{A}_2^{-}.$
5. $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^{-} = \mathfrak{A}_1^{-} \cup \mathfrak{A}_2^{-}.$
6. $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^{-} = \mathfrak{A}_1^{-} \cap \mathfrak{A}_2^{-}.$
7. $(\mathfrak{A}^{-})^{-} = \mathfrak{A}^{-}.$

Proof.

1. By definition of complex neutrosophic nano closure, $\mathfrak{A} \subseteq \mathfrak{A}^{-}$
2. If \mathfrak{A} is a complex neutrosophic nano closed set, then \mathfrak{A} is the smallest complex neutrosophic nano closed set containing itself and hence $\mathfrak{A}^{-} = \mathfrak{A}$. Conversely, if $\mathfrak{A}^{-} = \mathfrak{A}$, then \mathfrak{A} is the smallest complex neutrosophic nano closed set containing itself and hence \mathfrak{A} is a complex neutrosophic nano closed set.
3. Since 0_{\sim} and 1_{\sim} are complex neutrosophic nano closed sets in $(\mathfrak{U}; \tau_{\mathfrak{N}})(\mathfrak{S})$, $0_{\sim}^{-} = 0_{\sim}$ and $1_{\sim}^{-} = 1_{\sim}.$

4. If CNN set \mathfrak{A}_1 is a subset of CNN set \mathfrak{A}_2 , since CNN set \mathfrak{A}_2 is a subset of \mathfrak{A}_2^- , then CNN set \mathfrak{A}_1 is a subset of \mathfrak{A}_2^- . That is, \mathfrak{A}_2^- is a CNNCS containing \mathfrak{A}_1 . But \mathfrak{A}_1^- is the smallest CNNCS containing \mathfrak{A}_1 . Therefore, $\mathfrak{A}_1^- \subseteq \mathfrak{A}_2^-$.
5. Since CNN set \mathfrak{A}_1 is a subset of union of two CNN sets \mathfrak{A}_1 and \mathfrak{A}_2 and CNN set \mathfrak{A}_2 is a subset of union of two CNN sets \mathfrak{A}_1 and \mathfrak{A}_2 , $\mathfrak{A}_1^- \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^-$. Then closure of CNN set \mathfrak{A}_1 is a subset of closure of union of two CNN sets \mathfrak{A}_1 and \mathfrak{A}_2 and closure of CNN set \mathfrak{A}_2 is a subset of closure of union of two CNN sets \mathfrak{A}_1 and \mathfrak{A}_2 . Therefore, union of closure of CNN sets \mathfrak{A}_1^- , \mathfrak{A}_2^- is a subset of closure of union of $(\mathfrak{A}_1^-, \mathfrak{A}_2)^-$. By the fact that $\mathfrak{A}_1 \cup \mathfrak{A}_2 \subseteq \mathfrak{A}_1^- \cup \mathfrak{A}_2^-$, and since $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^-$ is the smallest complex neutrosophic nano closed set containing $\mathfrak{A}_1 \cup \mathfrak{A}_2$, so $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^- \subseteq \mathfrak{A}_1^- \cup \mathfrak{A}_2^-$. Thus, $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^- = \mathfrak{A}_1^- \cup \mathfrak{A}_2^-$.
6. Since $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_1$ and $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_2$, $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^- \subseteq \mathfrak{A}_1^- \cap \mathfrak{A}_2^-$.
7. Since \mathfrak{A}^- is a complex neutrosophic nano closed set, then $(\mathfrak{A}^-)^- = \mathfrak{A}^-$.

Theorem 3.2. $(\mathfrak{U}; \tau_{\mathfrak{N}})(\mathfrak{S})$ be a complex neutrosophic nano topological space with respect to \mathfrak{S} where \mathfrak{S} is a complex neutrosophic subset of \mathfrak{U} . Let \mathfrak{A} be a complex neutrosophic subset of \mathfrak{U} . Then

1. $1_{\sim} - \mathfrak{A}^{\circ} = (1_{\sim} - \mathfrak{A})^-$.
2. $1_{\sim} - \mathfrak{A}^- = (1_{\sim} - \mathfrak{A})^{\circ}$.

Theorem 3.3. Let $(\mathfrak{U}; \tau_{\mathfrak{N}})(\mathfrak{S})$ be a complex neutrosophic nano topological space with respect to \mathfrak{S} where \mathfrak{S} is a complex neutrosophic subset of \mathfrak{U} . Let \mathfrak{A}_1 and \mathfrak{A}_2 be complex neutrosophic subsets of \mathfrak{U} . Then the following statements hold:

1. \mathfrak{A} is CNNOS $\Leftrightarrow \mathfrak{A}^{\circ} = \mathfrak{A}$.
2. $0_{\sim}^{\circ} = 0_{\sim}$ and $1_{\sim}^{\circ} = 1_{\sim}$.
3. $\mathfrak{A}_1 \subseteq \mathfrak{A}_2 \Rightarrow \mathfrak{A}_1^{\circ} \subseteq \mathfrak{A}_2^{\circ}$.
4. $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^{\circ} = \mathfrak{A}_1^{\circ} \cup \mathfrak{A}_2^{\circ}$.
5. $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^{\circ} = \mathfrak{A}_1^{\circ} \cap \mathfrak{A}_2^{\circ}$.
6. $(\mathfrak{A}^{\circ})^{\circ} = \mathfrak{A}^{\circ}$.

Proof.

1. \mathfrak{A} is a complex neutrosophic nano open set if and only if $1_{\sim} - \mathfrak{A}$ is a complex neutrosophic nano closed set, if and only if $(1_{\sim} - \mathfrak{A})^- = 1_{\sim} - \mathfrak{A}$, if and only if $1_{\sim} - (1_{\sim} - \mathfrak{A})^- = \mathfrak{A}$ if and only if $\mathfrak{A}^{\circ} = \mathfrak{A}$.
2. Since 0_{\sim} and 1_{\sim} are complex neutrosophic nano open sets in $(\mathfrak{U}; \tau_{\mathfrak{N}})(\mathfrak{S})$, $0_{\sim}^{\circ} = 0_{\sim}$ and $1_{\sim}^{\circ} = 1_{\sim}$.

3. If $\mathfrak{A}_1 \subseteq \mathfrak{A}_2$, since $\mathfrak{A}_2 \supseteq \mathfrak{A}_2^\circ$, then $\mathfrak{A}_1 \supseteq \mathfrak{A}_2^\circ$. That is, \mathfrak{A}_2° is a complex neutrosophic nano open set containing \mathfrak{A}_1 . But \mathfrak{A}_1° is the largest complex neutrosophic nano open set contained in \mathfrak{A}_1 . Therefore, $\mathfrak{A}_1^\circ \subseteq \mathfrak{A}_2^\circ$
4. Since $\mathfrak{A}_1 \subseteq \mathfrak{A}_1 \cup \mathfrak{A}_2$ and $\mathfrak{A}_2 \subseteq \mathfrak{A}_1 \cup \mathfrak{A}_2$, $\mathfrak{A}_1^\circ \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^\circ$ and $\mathfrak{A}_2^\circ \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^\circ$. Therefore, $\mathfrak{A}_1^\circ \cup \mathfrak{A}_2^\circ \subseteq (\mathfrak{A}_1 \cup \mathfrak{A}_2)^\circ$. By the fact that $\mathfrak{A}_1 \cup \mathfrak{A}_2 \subseteq \mathfrak{A}_1^\circ \cup \mathfrak{A}_2^\circ$, and since $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^\circ$ is the largest complex neutrosophic nano open set containing $\mathfrak{A}_1 \cup \mathfrak{A}_2$, so $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^\circ \subseteq \mathfrak{A}_1^\circ \cup \mathfrak{A}_2^\circ$. Thus, $(\mathfrak{A}_1 \cup \mathfrak{A}_2)^\circ = \mathfrak{A}_1^\circ \cup \mathfrak{A}_2^\circ$.
5. Since $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_1$ and $\mathfrak{A}_1 \cap \mathfrak{A}_2 \subseteq \mathfrak{A}_2$, $(\mathfrak{A}_1 \cap \mathfrak{A}_2)^\circ \subseteq \mathfrak{A}_1^\circ \cap \mathfrak{A}_2^\circ$.
6. Since \mathfrak{A}° is a complex neutrosophic nano open set, then $(\mathfrak{A}^\circ)^\circ = \mathfrak{A}^\circ$.

Definition 3.6. Let $\mathfrak{A} = \{\mathfrak{M}, \mathfrak{J}, \mathfrak{F}\}$ be a CNS, a score function $\mathfrak{S}_{cr}(\cdot)$, based on the satisfaction grade (\mathfrak{M}), abstinence grade (\mathfrak{J}), and dissatisfaction grade (\mathfrak{F}) which is defined as

$$\mathfrak{S}_{cr}(CNN) = \frac{1}{6m} \sum_{i=1}^m \{ [p_i(\vartheta) + (2 - (q_i(\vartheta) + r_i(\vartheta)))] + \frac{1}{2\pi} [\mu_i(\vartheta) + (4\pi - (\nu_i(\vartheta) + \omega_i(\vartheta)))] \}$$

The score value of a CNN measures the accuracy of the number CNN in access with satisfaction grades.

Clearly, if in some cases CNS has $\mathfrak{M} + \mathfrak{F} = 1$ then $\mathfrak{S}_{cr}(\cdot)$ reduces to $\mathfrak{K}_{cr}(\cdot)$. Based on it, a prioritized comparison method for any two CNSs \mathfrak{A}_1 and \mathfrak{A}_2 is defined as

1. if $\mathfrak{K}_{cr}(\mathfrak{A}_1) < \mathfrak{K}_{cr}(\mathfrak{A}_2)$, then $\mathfrak{A}_1 \prec \mathfrak{A}_2$
2. if $\mathfrak{K}_{cr}(\mathfrak{A}_1) = \mathfrak{K}_{cr}(\mathfrak{A}_2)$, then $\mathfrak{A}_1 \sim \mathfrak{A}_2$
3. if $\mathfrak{S}_{cr}(\mathfrak{A}_1) < \mathfrak{S}_{cr}(\mathfrak{A}_2)$, then $\mathfrak{A}_1 \prec \mathfrak{A}_2$
4. if $\mathfrak{S}_{cr}(\mathfrak{A}_1) > \mathfrak{S}_{cr}(\mathfrak{A}_2)$, then $\mathfrak{A}_1 \succ \mathfrak{A}_2$
5. if $\mathfrak{S}_{cr}(\mathfrak{A}_1) = \mathfrak{S}_{cr}(\mathfrak{A}_2)$, then $\mathfrak{A}_1 \sim \mathfrak{A}_2$

4 Complex neutrosophic nano topology in multi-criteria decision-making

MCDM is a process for selecting the optimal solution with the maximum degree of satisfaction from a group of available alternatives. These sorts of MCDM challenges emerge in a variety of real-time circumstances and are distinguished by a number of characteristics. This section presents a revolutionary complex neutrosophic nano topological technique for decision-making challenges using complex neutrosophic information. The following phases provide a systematic technique for picking the appropriate options and qualities in a decision-making environment.

4.1 Proposed Algorithm and Flowchart

Algorithm: (With CNNTSs, you can make the best decisions)

Input part:

Step-1: Examine the realm of conversation (set of objects) \mathfrak{A} , a collection of possibilities \mathfrak{E} , the collection of decision characteristics \mathfrak{D} . Consider an in-discernibility relation \mathfrak{R} on \mathfrak{A} . Frame a complex neutrosophic set in matrix representation related to the attributes. The objects and attributes are represented as columns and rows respectively and the table entries indicate the values of the attributes.

Computational part:

Step-2: Construct the ambiguity relation \mathfrak{R} .

Step-3: Frame the CNNTs $\mathfrak{C}_{\tau_1\eta}^*$ and $\mathfrak{C}_{\tau_2\xi}^*$.

Step-4: Concord the score function of every one of the entries of the CNNTSs

$$\mathfrak{S}_{cr}(CNN) = \frac{1}{6m} \sum_{i=1}^m \{ [p_i(\vartheta) + (2 - (q_i(\vartheta) + r_i(\vartheta)))] + \frac{1}{2\pi} [\mu_i(\vartheta) + (4\pi - (\nu_i(\vartheta) + \omega_i(\vartheta)))] \}$$

The accuracy of the CNN number measure provides a support the truth membership degree.

Ending Part:

Step-5: Final Decision

Regulate the complex neutrosophic score values of the alternatives $\mathfrak{G}_1 \leq \mathfrak{G}_2 \leq \dots \leq \mathfrak{G}_\beta$ and the attributes $\mathfrak{H}_1 \leq \mathfrak{H}_2 \leq \dots \leq \mathfrak{H}_\gamma$. Choose the attribute \mathfrak{H}_γ for the alternative \mathfrak{G}_1 and $\mathfrak{H}_{\gamma-1}$ for the alternative \mathfrak{G}_2 etc. If $\beta \leq \gamma$, then neglect \mathfrak{H}_ξ , where $\xi = 1, 2, \beta - \gamma$.

The flow chart of proposed Algorithm for MCDM is given in the Figure 1.

5 Numerical Example

For example of the proposed solution we find a medical diagnosis issue. Medicine Diagnosis entails uncertainty and an growing amount of knowledge available to doctors Fresh tools for pharmacy. Therefore, all the collected knowledge can be in a dynamic neutrosophic type. The triplet elements of a CNS are real-evaluated amalgamations Truth of matter amplitude term in combination with phase term, indeterminate amplitude term actual evaluated with phase term, and with real-evaluated, phase term false amplitude. And in a neutrosophical dynamic medical diagnosis environment, it is given to deal with further indeterminacy circumstances.

The method of classifying various sets of symptoms under a single disease name is very complicated and critical. In some real time scenarios, every dimension has the chance within a periodic type of the neutrosophic sets. So, further indeterminacy is involved in the medical diagnosis. Critical problems are superscribed by complex neutrosophic nano

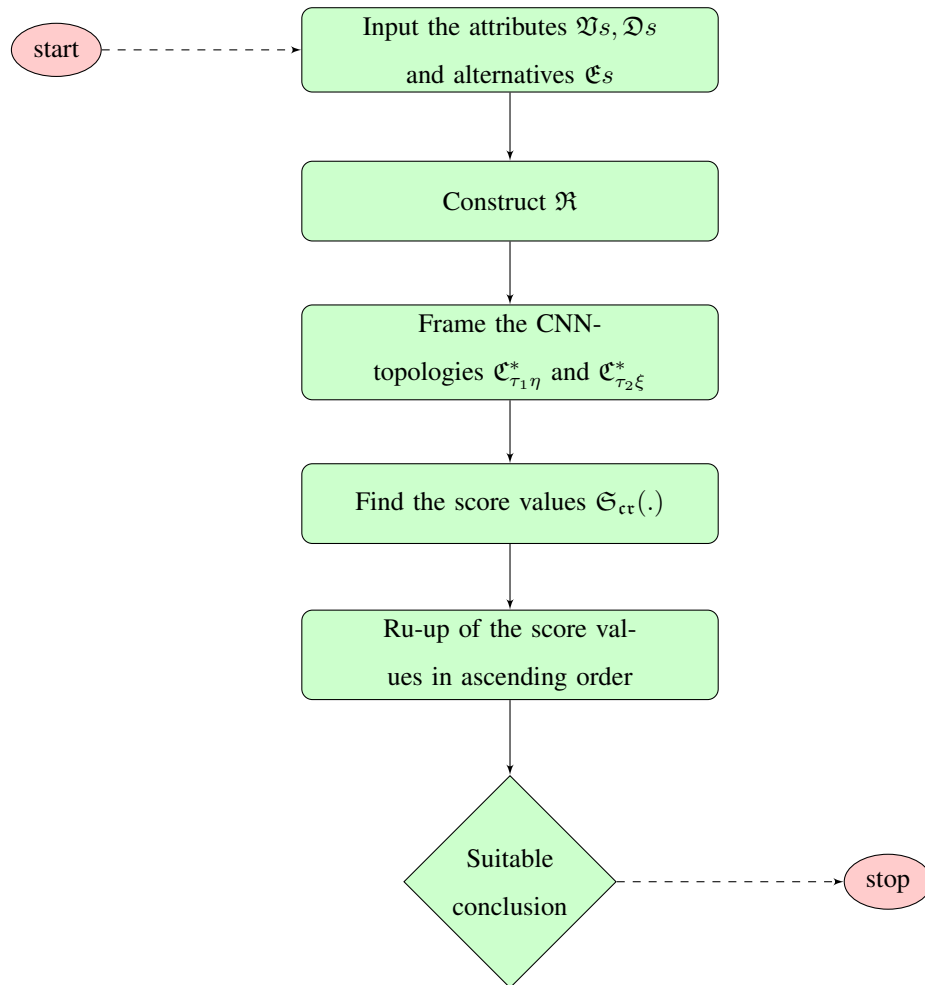


Figure 1: Flow diagram for the proposed algorithm

topologies. This plan of action is more generally versatile, when it comes to minimum places of indeterminacy, and simpler to apply. With a score function between patients versus symptoms and symptoms versus diseases, the put forward algorithm of complex neutrosophic nano topological spaces has the right medical diagnosis in complex neutrosophic milieu.

The main characteristic of the proposed technique is that it evaluates specific indeterminate, complex factual participation, and misrepresentation of every dimension taking periodic form in neutrosophic set.

Let $\mathfrak{P} = \{p_1, p_2, p_3, p_4\}$ be the set of patients, $\mathfrak{D} = \{d_1, d_2, d_3, d_4\}$ be the set of diseases and $\mathfrak{S} = \{s_1, s_2, s_3, s_4, s_5\}$ be the set of symptoms. The symptoms, practitioner decided to include, looked something like this: Chest pain, Cough, Headache, Stomach pain, Temperature. Normal representation showed following are the diseases to be the most indicated ones: Viral Fever, Malaria, Stomach problem, Chest problem.

The proposed work is to examine the patient and determine the patient’s illness in a complex neutrosophical environment.

Table 1: The complex neutrosophic system for Patients verses Symptoms

	p_1	p_2	p_3	p_4
s_1	$\langle 0.7e^{j\pi 0.9}, 0.4e^{j\pi 0.8}, 0.3e^{j\pi 0.7} \rangle$	$\langle 0.5e^{j\pi 0.6}, 0.5e^{j\pi 0.9}, 0.2e^{j\pi 0.8} \rangle$	$\langle 0.7e^{j\pi 0.7}, 0.4e^{j\pi 0.8}, 0.6e^{j\pi 0.9} \rangle$	$\langle 0.5e^{j\pi 0.9}, 0.5e^{j\pi 0.7}, 0.5e^{j\pi 0.6} \rangle$
s_2	$\langle 0.4e^{j\pi 0.2}, 0.4e^{j\pi 0.7}, 0.3e^{j\pi 0.3} \rangle$	$\langle 0.4e^{j\pi 0.5}, 0.6e^{j\pi 0.4}, 0.4e^{j\pi 0.6} \rangle$	$\langle 0.5e^{j\pi 0.1}, 0.4e^{j\pi 0.7}, 0.7e^{j\pi 0.9} \rangle$	$\langle 0.8e^{j\pi 0.7}, 0.0e^{j\pi 0.2}, 0.7e^{j\pi 0.5} \rangle$
s_3	$\langle 0.8e^{j\pi 0.7}, 0.1e^{j\pi 0.8}, 0.2e^{j\pi 0.2} \rangle$	$\langle 0.2e^{j\pi 0.7}, 0.1e^{j\pi 0.2}, 0.6e^{j\pi 0.7} \rangle$	$\langle 0.3e^{j\pi 0.7}, 0.1e^{j\pi 0.9}, 0.6e^{j\pi 0.2} \rangle$	$\langle 0.1e^{j\pi 0.5}, 0.2e^{j\pi 0.1}, 0.7e^{j\pi 0.6} \rangle$
s_4	$\langle 0.3e^{j\pi 0.4}, 0.2e^{j\pi 0.6}, 0.5e^{j\pi 0.6} \rangle$	$\langle 0.9e^{j\pi 0.6}, 0.3e^{j\pi 0.5}, 0.2e^{j\pi 0.9} \rangle$	$\langle 0.4e^{j\pi 0.2}, 0.3e^{j\pi 0.4}, 0.2e^{j\pi 0.4} \rangle$	$\langle 0.9e^{j\pi 0.2}, 0.1e^{j\pi 0.6}, 0.3e^{j\pi 0.8} \rangle$
s_5	$\langle 0.2e^{j\pi 0.4}, 0.1e^{j\pi 0.6}, 0.7e^{j\pi 0.9} \rangle$	$\langle 0.3e^{j\pi 0.4}, 0.4e^{j\pi 0.6}, 0.7e^{j\pi 0.9} \rangle$	$\langle 0.3e^{j\pi 0.6}, 0.1e^{j\pi 0.5}, 0.6e^{j\pi 0.9} \rangle$	$\langle 0.4e^{j\pi 0.6}, 0.2e^{j\pi 0.4}, 0.2e^{j\pi 0.8} \rangle$

Table 2: The complex neutrosophic system for Symptoms verses Diseases

	\mathfrak{s}_1	\mathfrak{s}_2	\mathfrak{s}_3	\mathfrak{s}_4	\mathfrak{s}_5
\mathfrak{d}_1	$\langle 0.4e^{j\pi 0.8}, 0.4e^{j\pi 0.6}, 0.3e^{j\pi 0.3} \rangle$	$\langle 0.7e^{j\pi 0.5}, 0.4e^{j\pi 0.7}, 0.3e^{j\pi 0.4} \rangle$	$\langle 0.6e^{j\pi 0.2}, 0.3e^{j\pi 0.7}, 0.7e^{j\pi 0.8} \rangle$	$\langle 0.6e^{j\pi 0.7}, 0.4e^{j\pi 0.8}, 0.6e^{j\pi 0.5} \rangle$	$\langle 0.7e^{j\pi 0.3}, 0.4e^{j\pi 0.7}, 0.3e^{j\pi 0.7} \rangle$
\mathfrak{d}_2	$\langle 0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.6}, 0.4e^{j\pi 0.5} \rangle$	$\langle 0.4e^{j\pi 0.4}, 0.5e^{j\pi 0.5}, 0.6e^{j\pi 0.7} \rangle$	$\langle 0.6e^{j\pi 0.2}, 0.5e^{j\pi 0.8}, 0.9e^{j\pi 0.5} \rangle$	$\langle 0.6e^{j\pi 0.8}, 0.2e^{j\pi 0.3}, 0.5e^{j\pi 0.6} \rangle$	$\langle 0.4e^{j\pi 0.7}, 0.6e^{j\pi 0.8}, 0.3e^{j\pi 0.6} \rangle$
\mathfrak{d}_3	$\langle 0.9e^{j\pi 0.6}, 0.3e^{j\pi 0.6}, 0.3e^{j\pi 0.4} \rangle$	$\langle 0.3e^{j\pi 0.6}, 0.2e^{j\pi 0.3}, 0.7e^{j\pi 0.6} \rangle$	$\langle 0.4e^{j\pi 0.6}, 0.2e^{j\pi 0.8}, 0.7e^{j\pi 0.5} \rangle$	$\langle 0.2e^{j\pi 0.6}, 0.1e^{j\pi 0.5}, 0.5e^{j\pi 0.7} \rangle$	$\langle 0.8e^{j\pi 0.4}, 0.6e^{j\pi 0.5}, 0.3e^{j\pi 0.9} \rangle$
\mathfrak{d}_4	$\langle 0.4e^{j\pi 0.5}, 0.3e^{j\pi 0.7}, 0.6e^{j\pi 0.4} \rangle$	$\langle 0.8e^{j\pi 0.7}, 0.2e^{j\pi 0.8}, 0.3e^{j\pi 0.6} \rangle$	$\langle 0.6e^{j\pi 0.3}, 0.4e^{j\pi 0.5}, 0.1e^{j\pi 0.5} \rangle$	$\langle 0.7e^{j\pi 0.4}, 0.2e^{j\pi 0.7}, 0.4e^{j\pi 0.6} \rangle$	$\langle 0.4e^{j\pi 0.2}, 0.3e^{j\pi 0.1}, 0.4e^{j\pi 0.7} \rangle$

Step-2: Frame the correlation of undetectable connection between the symptoms is given as $\mathfrak{R} = \{\{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_4\}, \{\mathfrak{s}_3, \mathfrak{s}_5\}\}$.

Step-3: Formulate the complex neutrosophic nano topological spaces for every patient and every disease with respect to the symptoms as follows:

CNNTSs for patients are $\mathfrak{C}_{\tau_1 \eta}^*$

- $\mathfrak{C}_{\tau_1}^*(\mathfrak{p}_1) = \{1_{\sim}, 0_{\sim}, \langle 0.7e^{j\pi 0.9}, 0.2e^{j\pi 0.6}, 0.3e^{j\pi 0.3} \rangle, \langle 0.8e^{j\pi 0.7}, 0.1e^{j\pi 0.6}, 0.2e^{j\pi 0.2} \rangle, \langle 0.3e^{j\pi 0.2}, 0.4e^{j\pi 0.8}, 0.5e^{j\pi 0.7} \rangle, \langle 0.2e^{j\pi 0.4}, 0.1e^{j\pi 0.8}, 0.7e^{j\pi 0.9} \rangle, \langle 0.3e^{j\pi 0.2}, 0.6e^{j\pi 1.2}, 0.7e^{j\pi 0.9} \rangle, \langle 0.2e^{j\pi 0.2}, 0.9e^{j\pi 1.2}, 0.8e^{j\pi 0.9} \rangle\}$
- $\mathfrak{C}_{\tau_1}^*(\mathfrak{p}_2) = \{1_{\sim}, 0_{\sim}, \langle 0.9e^{j\pi 0.6}, 0.3e^{j\pi 0.4}, 0.2e^{j\pi 0.6} \rangle, \langle 0.3e^{j\pi 0.7}, 0.1e^{j\pi 0.2}, 0.6e^{j\pi 0.7} \rangle, \langle 0.4e^{j\pi 0.5}, 0.6e^{j\pi 0.9}, 0.4e^{j\pi 0.9} \rangle, \langle 0.2e^{j\pi 0.4}, 0.4e^{j\pi 0.6}, 0.7e^{j\pi 0.9} \rangle, \langle 0.2e^{j\pi 0.5}, 0.4e^{j\pi 1.1}, 0.9e^{j\pi 0.9} \rangle, \langle 0.2e^{j\pi 0.4}, 0.6e^{j\pi 1.4}, 0.7e^{j\pi 0.9} \rangle\}$
- $\mathfrak{C}_{\tau_1}^*(\mathfrak{p}_3) = \{1_{\sim}, 0_{\sim}, \langle 0.7e^{j\pi 0.7}, 0.3e^{j\pi 0.4}, 0.2e^{j\pi 0.4} \rangle, \langle 0.3e^{j\pi 0.7}, 0.1e^{j\pi 0.5}, 0.6e^{j\pi 0.2} \rangle, \langle 0.4e^{j\pi 0.1}, 0.4e^{j\pi 0.8}, 0.7e^{j\pi 0.9} \rangle, \langle 0.3e^{j\pi 0.6}, 0.1e^{j\pi 0.9}, 0.6e^{j\pi 0.9} \rangle, \langle 0.2e^{j\pi 0.1}, 0.6e^{j\pi 1.2}, 0.7e^{j\pi 0.9} \rangle, \langle 0.3e^{j\pi 0.2}, 0.9e^{j\pi 1.1}, 0.6e^{j\pi 0.9} \rangle\}$
- $\mathfrak{C}_{\tau_1}^*(\mathfrak{p}_4) = \{1_{\sim}, 0_{\sim}, \langle 0.9e^{j\pi 0.9}, 0e^{j\pi 0.2}, 0.3e^{j\pi 0.5} \rangle, \langle 0.4e^{j\pi 0.6}, 0.2e^{j\pi 0.1}, 0.2e^{j\pi 0.6} \rangle, \langle 0.5e^{j\pi 0.2}, 0.5e^{j\pi 0.7}, 0.7e^{j\pi 0.8} \rangle, \langle 0.1e^{j\pi 0.5}, 0.4e^{j\pi 0.4}, 0.7e^{j\pi 0.8} \rangle, \langle 0.3e^{j\pi 0.2}, 0.5e^{j\pi 1.3}, 0.9e^{j\pi 0.9} \rangle, \langle 0.1e^{j\pi 0.5}, 0.6e^{j\pi 1.6}, 0.7e^{j\pi 0.8} \rangle\}$

Complex neutrosophic nano topologies for diseases are $\mathfrak{C}_{\tau_2 \xi}^*$

- $\mathfrak{C}_{\tau_2}^*(\mathfrak{d}_1) = \{1_{\sim}, 0_{\sim}, \langle 0.7e^{j\pi 0.8}, 0.4e^{j\pi 0.6}, 0.3e^{j\pi 0.3} \rangle, \langle 0.7e^{j\pi 0.3}, 0.3e^{j\pi 0.7}, 0.3e^{j\pi 0.7} \rangle,$

$$\langle 0.4e^{j\pi 0.5}, 0.4e^{j\pi 0.8}, 0.6e^{j\pi 0.5} \rangle, \langle 0.6e^{j\pi 0.2}, 0.4e^{j\pi 0.7}, 0.7e^{j\pi 0.8} \rangle, \langle 0.3e^{j\pi 0.3}, 0.6e^{j\pi 1.2}, 0.7e^{j\pi 0.8} \rangle, \\ \langle 0.3e^{j\pi 0.2}, 0.6e^{j\pi 1.3}, 0.7e^{j\pi 0.8} \rangle \}$$

$$2. \mathfrak{C}_{\tau_2}^*(\mathfrak{d}_2) = \{1_{\sim}, 0_{\sim}, \langle 0.6e^{j\pi 0.8}, 0.2e^{j\pi 0.3}, 0.4e^{j\pi 0.5} \rangle, \langle 0.6e^{j\pi 0.7}, 0.5e^{j\pi 0.8}, 0.3e^{j\pi 0.5} \rangle, \\ \langle 0.3e^{j\pi 0.4}, 0.5e^{j\pi 0.6}, 0.6e^{j\pi 0.7} \rangle, \langle 0.4e^{j\pi 0.2}, 0.6e^{j\pi 0.8}, 0.9e^{j\pi 0.6} \rangle, \langle 0.3e^{j\pi 0.4}, 0.5e^{j\pi 1.4}, 0.6e^{j\pi 0.8} \rangle, \\ \langle 0.3e^{j\pi 0.2}, 0.5e^{j\pi 1.2}, 0.9e^{j\pi 0.7} \rangle \}$$

$$3. \mathfrak{C}_{\tau_2}^*(\mathfrak{d}_3) = \{1_{\sim}, 0_{\sim}, \langle 0.9e^{j\pi 0.6}, 0.1e^{j\pi 0.3}, 0.3e^{j\pi 0.4} \rangle, \langle 0.8e^{j\pi 0.6}, 0.2e^{j\pi 0.5}, 0.3e^{j\pi 0.5} \rangle, \\ \langle 0.2e^{j\pi 0.6}, 0.3e^{j\pi 0.6}, 0.7e^{j\pi 0.7} \rangle, \langle 0.4e^{j\pi 0.4}, 0.6e^{j\pi 0.8}, 0.7e^{j\pi 0.9} \rangle, \langle 0.2e^{j\pi 0.4}, 0.7e^{j\pi 1.4}, 0.9e^{j\pi 0.7} \rangle, \\ \langle 0.3e^{j\pi 0.4}, 0.4e^{j\pi 1.2}, 0.8e^{j\pi 0.9} \rangle \}$$

$$4. \mathfrak{C}_{\tau_2}^*(\mathfrak{d}_4) = \{1_{\sim}, 0_{\sim}, \langle 0.8e^{j\pi 0.7}, 0.2e^{j\pi 0.7}, 0.3e^{j\pi 0.4} \rangle, \langle 0.6e^{j\pi 0.3}, 0.3e^{j\pi 0.1}, 0.1e^{j\pi 0.5} \rangle, \\ \langle 0.4e^{j\pi 0.4}, 0.3e^{j\pi 0.8}, 0.6e^{j\pi 0.6} \rangle, \langle 0.4e^{j\pi 0.2}, 0.4e^{j\pi 0.5}, 0.4e^{j\pi 0.7} \rangle, \langle 0.3e^{j\pi 0.4}, 0.7e^{j\pi 1.2}, 0.8e^{j\pi 0.7} \rangle, \\ \langle 0.1e^{j\pi 0.2}, 0.6e^{j\pi 1.5}, 0.6e^{j\pi 0.7} \rangle \}$$

Step-4: Computation of complex neutrosophic score functions for the patients and diseases are depleted as in step-4 for the technique are as given below

Score values for the patients are

$$\mathfrak{S}_{cr}(\mathfrak{p}_1) = 0.5052, \mathfrak{S}_{cr}(\mathfrak{p}_2) = 0.4917, \mathfrak{S}_{cr}(\mathfrak{p}_3) = 0.4906, \mathfrak{S}_{cr}(\mathfrak{p}_4) = 0.5042$$

Score values for the diseases are

$$\mathfrak{S}_{cr}(\mathfrak{d}_1) = 0.5010, \mathfrak{S}_{cr}(\mathfrak{d}_2) = 0.4875, \mathfrak{S}_{cr}(\mathfrak{d}_3) = 0.5073, \mathfrak{S}_{cr}(\mathfrak{d}_4) = 0.5146$$

Step-5: Arrange the complex neutrosophic score functions for the alternatives $\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3, \mathfrak{p}_4$ and the attributes $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4$ in lead-in structure. We contemplate the arrangement as follows $\mathfrak{p}_3 \leq \mathfrak{p}_2 \leq \mathfrak{p}_4 \leq \mathfrak{p}_1$ and $\mathfrak{d}_2 \leq \mathfrak{d}_1 \leq \mathfrak{d}_3 \leq \mathfrak{d}_4$. Thus the patient \mathfrak{p}_3 suffers with disease \mathfrak{d}_4 = chest problem, Thus the patient \mathfrak{p}_2 suffers with disease \mathfrak{d}_3 = stomach problem, Thus the patient \mathfrak{p}_4 suffers with disease \mathfrak{d}_1 = viral fever and Thus the patient \mathfrak{p}_1 suffers with disease \mathfrak{d}_2 = Malaria.

The comparison table show the characteristic between novel complex neutrosophic nano topological space with extant endeavor.

Sets	ambiguity	related to the knowledge about a component	fake valuation about a component	indeterminacy about a component	harshness&border of a set	unit complex plane
GT	-	-	-	-	-	-
FT	✓	✓	-	-	-	-
IFT	✓	✓	✓	-	-	-
NT	✓	✓	✓	✓	-	-
NNT	✓	✓	✓	✓	✓	-
CNNT	✓	✓	✓	✓	✓	✓

6 Conclusion

Indeterminate, contradictory, unclear, vague, and incomplete redundant / periodic information werte can be best dealt with it is the opinion that complex neutrosophic knowledge. This manuscript focused at conducting out the CNNTS which is suitable to a greater extent and changeable to practical problems. Novel notion CNNTS is established and essential operations such as interior and closure are developed. Also introduced and applied new algorithm for solving MCDM problem in medical science under CNS. A comparison was done between the suggested approach and the conventional models to demonstrate the benefits and usability. The findings are crucial in furthering the complicated neutrosophical awareness given for decision-making applications. Upcoming study will focus on applying the MCDM methodology to more potential implementation and developing the constructive interval valued CNNTS logic method for predicting challenges.

Abbreviation:

- CNN - Complex neutrosophic nano
- CNNCS - Complex neutrosophic nano closed set
- CNNOS - Complex neutrosophic nano open set
- CNS - Complex neutrosophic set
- CNNTS - Complex neutrosophic nano topological space
- IFS - Intuitionistic fuzzy sets
- MCDM - Multi-criteria Decision Making
- NNT - Neutrosophic nano Topological spaces
- NS - Neutrosophic set
- NNT - Neutrosophic nano topology

References

- [1] M. Ali, F. Smarandache. Complex neutrosophic set. *Neural Computing and Applications*, 12, 2015. DOI:10.1007/s00521-015-2154-y.
- [2] I. Arokiarani, R. Dhavaseelan, S. Jafari, M. Parimala. On some new notions and functions in neutrosophic topological spaces, *Neutrosophic Sets Syst.*, 16, pp. 16-19, 2017.
- [3] K. T. Atanassov. Intuitionistic fuzzy sets, *Fuzzy sets and systems*, 20, pp. 87-96, 1986.
- [4] D. Coker. An introduction to fuzzy topological spaces, *Fuzzy sets and systems*, 88, pp. 81-89, 1997.
- [5] K. Kuratowski. *Topology Vol. II (transl.)*, Academic Press, New York, 1966.
- [6] M. Karthika, M. Parimala, S. Jafari, F. Smarandache, Mohammad Alshumrani, Cenap Ozel, R. Udhayakumar. Neutrosophic complex $\alpha\psi$ connectedness in neutrosophic complex topological spaces. *Neutrosophic sets and systems*, 29, pp. 158-164, 2019.
- [7] M. Lellis Thivagar, S. Jafari, V. Sutha Devi, V. Antonysamy, A novel approach to nano topology via neutrosophic sets, *Neutrosophic sets and systems*, 20, pp. 86-94, 2018.
- [8] M. Parimala, S. Jafari, and S. Murali, Nano Ideal Generalized Closed Sets in Nano Ideal Topological Spaces, *Annales Univ. Sci. Budapest.*, 60, pp. 3-11, 2017.
- [9] M. Parimala, R. Jeevitha, S. Jafari, F. Smarandache and R. Udhayakumar. Neutrosophic $\alpha\psi$ -Homeomorphism in Neutrosophic Topological Spaces, *Information*, 9, 187, pp. 1-10, 2018. doi:10.3390/info9080187.
- [10] M. Parimala, R. Jeevitha, A. Selvakumar. A New Type of Weakly Closed Set in Ideal Topological Spaces, *International Journal of Mathematics and its Applications*, 5(4-C), pp. 301-312, 2017.
- [11] M. Parimala, M. Karthika, R. Dhavaseelan, S. Jafari. On neutrosophic supra pre-continuous functions in neutrosophic topological spaces, *New Trends in Neutrosophic Theory and Applications*, 2, pp. 371-383, 2018.
- [12] M. Parimala, M. Karthika, S. Jafari, F. Smarandache, R. Udhayakumar. Neutrosophic nano ideal topological structures, *Neutrosophic sets and systems*, 24, pp. 70-77, 2019.
- [13] M. Parimala, M. Karthika, S. Jafari, F. Smarandache, R. Udhayakumar. Decision-Making via Neutrosophic Support Soft Topological Spaces. *Symmetry*, 10, pp. 1-10, 2018. doi:10.3390/sym10060217
- [14] M. Parimala and R. Perumal, Weaker form of open sets in nano ideal topological spaces, *Global Journal of Pure and Applied Mathematics*, 12(1), pp. 302-305, 2016.
- [15] M. Parimala, F. Smarandache, S. Jafari, R. Udhayakumar, On Neutrosophic $\alpha\psi$ -Closed Sets, *Information*, 9, 103, pp. 1-7, 2018.

- [16] M. Riaz, F. Smarandache, F. Karaaslan, M. R. Hashmi, I. Nawaz. Neutrosophic Soft Topology and its Applications to Multi-Criteria Decision Making. *Neutrosophic sets and systems*, 35, pp. 198-219, 2020.
- [17] M. Riaz, Naeem, K.; Zareef, I. Afzal, D. Neutrosophic N-Soft Sets with TOPSIS method for Multiple Attribute Decision Making. *Neutrosophic sets and systems*, 32, pp. 1-24, 2020.
- [18] M. Riaz, B. Davvaz, A. Fakhar, A. Firdous. Hesitant fuzzy soft topology and its applications to multi-attribute group decision-making. *Soft Computing*, pp. 1-21, 2020. <https://doi.org/10.1007/s00500-020-04938-0>.
- [19] M. Riaz, S.T. Tehrim. Cubic bipolar fuzzy set with application to multi-criteria group decision making using geometric aggregation operators. *Soft Computing*, 2020, <https://doi.org/10.1007/s00500-020-04927-3>.
- [20] A. A. Salama, S. A. Alblowi. Neutrosophic Set and Neutrosophic Topological Spaces, *IOSR J. Math.*, 3, pp. 31-35, 2012.
- [21] A. A. Salama. Florentin Smarandache, Valeri Kromov. Neutrosophic Closed Set and Neutrosophic Continuous Functions, *Neutrosophic Sets and Systems*, 4, pp. 4-8, 2014.
- [22] F. Smarandache. *Neutrosophy and Neutrosophic Logic*, First International Conference on Neutrosophy, Neutrosophic Logic Set, Probability and Statistics; University of New Mexico, Gallup, NM, USA, 2002.
- [23] F. Smarandache. *A Unifying Field in Logics: Neutrosophic Logic*. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Research Press, Rehoboth, NM, USA, 1999.
- [24] F. Smarandache. Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra, *Neutrosophic Sets and Systems*, 33, pp. 290-296, 2020.
- [25] L. A. Zadeh. *Fuzzy Sets*, *Information and Control*, 18, pp. 338-353, 1965.
- [26] F. Zafer, M. Akram, A novel decision-making method based on rough fuzzy information. *Int. J. Fuzzy Syst.* 2017. doi:10.1007/s40815-017-0368-0.