A New Approach Of Supra-Neutrosophic Topological space

Riad K. Al-Hamido

Department of Mathematics, College of Science, Al-Furat University, Deir-ez-Zor, Syria.
riad-hamido1983@hotmail.com

Abstract

In this new paper, we introduced new and important concepts in neutrosophic topology for the first time. Where we introduced the interesting concept "a new approach of neutrosophic supra-topological space ($ŠN_{Å}$-NTS)", that we presented this neutrosophic supra-topological space without using neutrosophic sets, and we also studied the separation axioms in this new neutrosophic supra-space and studied the relationship between the new separation axioms in ($ŠN_{Å}$-NTS) and the separation axioms of the previously known supra-topological space. We also defined the neutrosophic supra-topological sup-space from this new space. We prove that $ŠN_{Å}$-NTS is not a classical supra-topological space. Also, $ŠN_{Å}$-NTS is neither neutrosophic supra-topological space nor neutrosophic crisp topological space. Many examples and theories are presented.

Keywords: New approach of neutrosophic supra-topological spaces, new approach of neutrosophic supra-open sets, new approach of neutrosophic supra-closed sets, sup-space,separation axioms.

1. Introduction

Recently, as a generalization of neutrosophic topological space, A new approach of neutrosophic supra-topological spaces was defined by Riad Al-Hamido [1]. The fantastic idea about this new neutrosophic topological space is that without the use of neutrosophic sets, although many researcheres built neutrosophic topology via neutrosophic sets as[2-5], and neutrosophic crisp sets as[6-7].

Separation axioms in neutrosophic crisp topological space were first introduced by A.Alnafee et al. in [8].

In 2019 neutrosophic supra-bitopological space and neutrosophic supra-topological space were built and presented by Riad Al-Hamido [9] via neutrosophic crisp sets, also G.Jayaparthasarathy, V.F.Little Flower, M.Arockia Dasan introduced the concept of neutrosophic supra topological space [10] in 2019, by using the neutrosophic sets.

In this paper, as a generalization of A new approach of neutrosophic topological spaces, A new approach of neutrosophic supra-topological spaces is defined. Also, we studied the separation axioms in this new $ŠN_{Å}$-NTS and studied the relationship between the new separation axioms in ($ŠN_{Å}$-NTS) and the separation axioms of the previously known supra-topological space. We also defined the neutrosophic supra-topological sup-space from this new space. We prove that ŠN$_{Å}$-NTS is not a classical supra-topological space. Also, ŠN$_{Å}$-NTS is neither neutrosophic supra-topological space nor neutrosophic crisp topological space. Many examples and theories are presented.

2. Definition

In this section, we give some basic definitions which are helpful in the paper.

Definition 2.1:[1]

Let Ӿ≠∅ be any set, $(Ӿ)_N$ ={ a⊕bI : a∈ Ӿ,b∈ Ӿ∪ {0 } }, also, bI is indeterminacy and bI = I.

Definition 2.2: [1]

1. If A ∈P(Ӿ) and A≠∅ then $(A)_N$ ∈ P[$(Ӿ)_N$]; $(Å)_N$ ={ a⊕bI : a∈ Ӿ,b∈ Ӿ∪ {0 } }, bI = I. But if Å=∅ then $(Å)_N$ =∅⊕I.
2. If $(Å)_N$ ∈ P[$(Ӿ)_N$], then (∅⊕I) ∪ $(Å)_N$ = $(Å)_N$ and (∅⊕I) ∩ $(Å)_N$ = ∅⊕I.

Definition 2.3: [1]

Let Ӿ≠∅, if T={$Å_i$}$_{i∈Δ}$ is a topology on Ӿ, then a new approach of neutrosophic topology (N$_{Å}$-NT) on Ӿ is a family Ť={$(Å_i)_N$}$_{i∈Δ}$ of $(Ӿ)_N$.

The pair (Ӿ,Ť) is called a new approach of neutrosophic topological space (N$_{Å}$-NTS) in Ӿ. Moreover, members of Ť are known as a new approach of neutrosophic open sets (N$_{Å}$-NOS), and their complements are a new approach of neutrosophic closed sets (N$_{Å}$-NCS), members of P[$(Ӿ)_N$] are known as a new approach of neutrosophic sets (N$_{Å}$-NS).

Remark 2.4: [1] Let(Ӿ,Ť)be a N$_{Å}$-NT and $(Å_i)_N$∈ Ť, for all i∈Δ, then:

1. $\cup_{i\in\Delta}(Å_i)_N = (\cup_{i\in\Delta}Å_i)_N$.
2. $(Å_1)_N \cap (Å_2)_N = (Å_1 \cap Å_2)_N$.

3.A New Approach Of Supra-Neutrosophic Topological space:

Definition 3.1:

Let Ӿ≠∅, if T={$Å_i$}$_{i∈Δ}$ is supra-topology on Ӿ, then family Ť={$(Å_i)_N$}$_{i∈Δ}$ of $(Ӿ)_N$ on Ӿ is called new approach of neutrosophic supra-topology (ŠN$_{Å}$-NT) .

The pair (Ӿ,Ť) is called a new approach of neutrosophic supra-topological space (ŠN$_{Å}$-NTS) in Ӿ Moreover, members of Ť are known as a new approach of neutrosophic supra-open sets and their complements are a new approach of neutrosophic supra-closed sets, members of P[$(Ӿ)_N$] are known as a new approach of neutrosophic supra-sets (ŠN$_{Å}$-NS).

Remark 3.2:

- ŠN$_{Å}$-NOS(Ӿ) = the family of the new approach of supra-neutrosophic open sets on Ӿ.
- ŠN$_{Å}$-NCS(Ӿ) = the family of the new approach of neutrosophic supra-closed sets on Ӿ.
- ŠN$_{Å}$-NOS= new approach of neutrosophic supra-open sets.

- $\check{S}N_{\mathring{A}}$-NCS= new approach of neutrosophic supra-closed sets.
- $\check{S}N_{\mathring{A}}$-NTS = new approach of neutrosophic supra-topological space.
- $\check{S}N_{\mathring{A}}$-NTS = new approach of neutrosophic supra-topology.
- $N_{\mathring{A}}$-NTS = new approach of neutrosophic topological space.

Example 3.3:

Let Ӿ={e , f , g}. $T = \{\emptyset, \mathring{A}, \beta, Ӿ\}$,

$\mathring{A} = \{e, f\}, \beta = \{e, g\}. \check{T} = \{\emptyset \oplus I, (\mathring{A})_N, (\beta)_N, (Ӿ)_N\}$

$(\mathring{A})_N = \{e, f, e\oplus I, f\oplus I\}, (\beta)_N = \{e, g, e\oplus I , g\oplus I\}$.

Then(Ӿ, Ť) is a new approach of neutrosophic supra-topological space ($\check{S}N_{\mathring{A}}$-NTS).

Remark 3.4:

1. $\check{S}N_{\mathring{A}}$-NTS is not a classical supra-topological space because $\emptyset \notin \check{T}$.
2. If I=0, then, $\check{T} = T$ therefore, $\check{S}N_{\mathring{A}}$-NTS is a classical supra-topological space.

Remark 3.5:

$N_{\mathring{A}}$-NTS is $\check{S}N_{\mathring{A}}$-NTS.

Proof:

Let (Ӿ,Ť) be a $N_{\mathring{A}}$-NTS; $\check{T}=\{(\mathring{A}_i)_N\}_{i\in\Delta}$ of $(Ӿ)_N$ on Ӿ , therefore $T=\{\mathring{A}_i\}_{i\in\Delta}$ is an topology on Ӿ, so $T=\{\mathring{A}_i\}_{i\in\Delta}$ is an supra topology. Therefore (Ӿ,Ť) be a $\check{S}N_{\mathring{A}}$-NTS.

Remark 3.6:

Let(Ӿ,Ť)be a $\check{S}N_{\mathring{A}}$-NTS and $(\mathring{A}_i)_N \in \check{T}$, for all $i\in\Delta$, then:

$\cup_{i\in\Delta}(\mathring{A}_i)_N = (\cup_{i\in\Delta}\mathring{A}_i)_N$.

Theorem 3.7: Let Ӿ≠∅ then if $\check{T}=\{(A_i)_N\}_{i\in\Delta}$ is a new approach of neutrosophic supra-topology, then:

$\tau=\check{T}\cup\{\emptyset, Ӿ\}$ is $\check{S}N_{\mathring{A}}$-NT on $(Ӿ)_N$, and (Ӿ, τ) is $\check{S}N_{\mathring{A}}$-NTS.

Proof:

- ∅,Ӿ ∈τ.
- For every $i\in\Delta$, let $(\mathring{A}_i)_N\in\tau$ then $\mathring{A}_i\in \check{T}$, but Ť is supra-topology, therefore, $\cup_{i\in\Delta}\mathring{A}_i \in \check{T}$, hence

  $(\cup_{i\in\Delta}\mathring{A}_i)_N \in \check{T}$.

  Therefore, $\cup_{i\in\Delta}(\mathring{A}_i)_N = (\cup_{i\in\Delta}\mathring{A}_i)_N \in\tau$ by remark 3.6.

Therefore (Ӿ, τ) is $\check{S}N_{\mathring{A}}$-NTS.

Theorem 3.8: Let (Ӿ, Ť) is a ($\check{S}N_{\mathring{A}}$-NTS), if $(\mathring{A}_1)_N, (\mathring{A}_2)_N \in \check{T}$ then $(\mathring{A}_1)_N \cup (\mathring{A}_2)_N \in \check{T}$.

Proof:

Let $(Ǻ_1)_N$, $(Ǻ_2)_N \in Ť$ then $Ǻ_1$, $Ǻ_2 \in$ T, but T is supra-topology, therefore, $Ǻ_1 \cup Ǻ_2 \in$T, hence $(Ǻ_1 \cup Ǻ_2)_N \in Ť$.

But $(Ǻ_1)_N \cup (Ǻ_2)_N = (Ǻ_1 \cup Ǻ_2)_N$ by remark 3.6,Therefore $(Ǻ_1)_N \cup (Ǻ_2)_N \in Ť$.

Remark 3.9: Let (Ӿ, Ť) is a ($N_{Ǻ}$-NTS), if $(Ǻ_1)_N$, $(Ǻ_2)_N \in Ť$ then $(Ǻ_1)_N \cup (Ǻ_2)_N \in Ť$.

Proof:

Since every $N_{Ǻ}$-NTS is Š$N_{Ǻ}$-NTS **by theorem 3.8,**Therefore $(Ǻ_1)_N \cup (Ǻ_2)_N$.

Definition 3.10:

Let Ӿ≠∅, if T={$Ǻ_i$}$_{i\in\Delta}$ is a topology on Ӿ, and (Ӿ,Ť) a new approach of neutrosophic topological space (Ť={$(Ǻ_i)_N$}$_{i\in\Delta}$), and τ={B_i}$_{i\in\Delta}$ is supra-topology on Ӿ, and (Ӿ, $\hat{\tau}$) a new approach of neutrosophic topological space ($\hat{\tau}$={$(B_i)_N$}$_{i\in\Delta}$), if T⊆τ then:

- Ť ⊆$\hat{\tau}$.
- (Ӿ, $\hat{\tau}$) A new approach of neutrosophic supra-topological space associated with (Ӿ,Ť).

Example 3.11:

Let Ӿ={e , f , g}. $T = \{\emptyset, \hat{E}, \grave{\grave{E}}, Ӿ\}$, $\tau = \{\emptyset, E, \hat{E}, \grave{\grave{E}}, Ӿ\}$
$E = \{e\}, \hat{E} = \{e,f\}, \grave{\grave{E}} = \{e,g\}$ $B.$ $Ť = \left\{\emptyset \oplus I, (\hat{E})_N, (\grave{\grave{E}})_N, (Ӿ)_N\right\}, \hat{\tau} = \left\{\emptyset \oplus I, (E)_N, (\hat{E})_N, (\grave{\grave{E}})_N, (Ӿ)_N\right\}$ *then*
$(\hat{E})_N = \{e, f, e\oplus I, f\oplus I\}, (\grave{\grave{E}})_N = \{e, g, e\oplus I , g\oplus I\}, (E)_N = \{e, e\oplus I\}$ since $T \subseteq \tau$.

Then (Ӿ, $\hat{\tau}$) A new approach of neutrosophic supra-topological space associated with (Ӿ,Ť).

Theorem 3.12:

Let (Ӿ, $\hat{\tau}$) a new approach of neutrosophic supra-topological space associated with a new approach of neutrosophic topological space (Ӿ,Ť) then:

If $(Ǻ_1)_N \in Ť$, $(Ǻ_2)_N \in \hat{\tau}$ then $(Ǻ_1)_N \cup (Ǻ_2)_N \in \hat{\tau}$.

Proof:

Since(Ӿ, $\hat{\tau}$) a new approach of neutrosophic supra-topological space associated with a new approach of neutrosophic topological space (Ꭓ,Ť), Ť ⊆$\hat{\tau}$. therefore $(Ǻ_1)_N$∈ Ť⊆$\hat{\tau}$ then $(Ǻ_1)_N$, $(Ǻ_2)_N$∈ $\hat{\tau}$, hence $(Ǻ_1 \cup Ǻ_2)_N$ ∈ Ť.

Theorem 3.13: Let (Ꭓ, Ť) is a ($N_Ǻ$-NTS), if $(Ǻ_1)_N$, $(Ǻ_2)_N$∈ Ť then $(Ǻ_1)_N \cap (Ǻ_2)_N$∈ Ť.

Proof:

Let $(Ǻ_1)_N$, $(Ǻ_2)_N$∈ Ť then $Ǻ_1$, $Ǻ_2$ ∈ T, but T is topology, therefore, $Ǻ_1 \cap Ǻ_2$ ∈T, hence $(Ǻ_1 \cap Ǻ_2)_N$ ∈ Ť.

But $(Ǻ_1)_N \cap (Ǻ_2)_N = (Ǻ_1 \cap Ǻ_2)_N$ ∈ Ť by remark 2.4,Therefore $(Ǻ_1)_N \cap (Ǻ_2)_N$ ∈ Ť.

Remark 3.14:

The above theorem is not true if we take a new approach of neutrosophic supra-topological space, see the example

Example 3.15:

In Example 3.11$(\hat{E})_N, (\grave{E})_N$ are new approach of neutrosophic supra-open set but $(\hat{E})_N \cap (\grave{E})_N = \{e, e\oplus I\}$ is new approach of neutrosophic supra-open set.

Remark 3.16:

Š$N_Ǻ$-NTS is not a neutrosophic supra-topological space.

Remark 3.17:

Š$N_Ǻ$-NTS is not a neutrosophic crisp supra-topological space.

4. Separation axioms in a new approach of neutrosophic supra-topological space:

Definition 4.1:

Let (Ꭓ, Ť) is a Š$N_Ǻ$-NTS), x,y ∈ Ꭓ then the point x,y from the same kind if both x,y without I or x,y with I.

Example 4.2:

Let Ꭓ={a , b , c}. T = {∅, A, B, C, Ꭓ},
A = $\{a\}, B = \{a,b\}, C = \{a,c\}$. Ť = {$\emptyset\oplus I$, $(A)_N, (B)_N, (C)_N$, $(Ꭓ)_N$}

$$(A)_N = \{a, a\oplus I\}, (B)_N = \{a, b, a\oplus I, b\oplus I\}, (C)_N = \{a, c, a\oplus I, c\oplus I\}.$$

a,b are points from the same kind, $a\oplus I$, $b\oplus I$ are points from the same kind, but a , $b\oplus I$ are points not from the same kind.

Definition 4.3:

Let (Ꭓ, Ť) is a (Š$N_Ǻ$-NTS), Then Ꭓ is called:

a. $Ť_o$-space if for every point from the same kind x≠ y∈ $(Ꭓ)_N$ There exists Š$N_Ǻ$-NOS containing one of them but not the other.

b. $Ť_1$-space if for every point from the same kind x≠ y∈$(Ӿ)_N$ There exist ŠN$_{Å}$-NOS, D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2.

c. $Ť_2$-space if for every point from the same kind x≠ y∈$(Ӿ)_N$ There exist ŠN$_{Å}$-NOS, D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2 with $D_1 \cap D_2 = \emptyset$.

Theorem 4.4:

Let $(Ӿ, Ť)$ is a (ŠN$_{Å}$-NTS), then :

1. **1.** $Ť_2$-space ⟹ $Ť_1$-space.
2. **2.** $Ť_1$-space ⟹ $Ť_0$-space.

Proof:

1. Suppose that $(Ӿ, Ť)$ is an $Ť_2$-space, therefore for every two points from the same kind x≠ y∈$(Ӿ)_N$ there exist ŠN$_{Å}$-NOS, D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2 with $D_1 \cap D_2 = \emptyset$. So there exists an ŠN$_{Å}$-NOS, D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2, therefore X is $Ť_1$-space.
2. Suppose that $(Ӿ, Ť)$is an $Ť_1$-space, therefore for every two points from the same kind x≠ y∈$(Ӿ)_N$ there exist ŠN$_{Å}$-NOS, D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2.So there exists an ŠN$_{Å}$-NOS, D_1, such that x∈ D_1, y ∉ D_1, therefore X is $Ť_0$-space.

Remark 4.5:

The converse of theorem 4.4 is not true., as it is shown in the following examples.

Example 4.6:

Let Ӿ={a , b , c}. $T = \{\emptyset, A, B, C, Ӿ\}$,
$A = \{a\}, B = \{a, b\}, C = \{a, c\}$. $Ť = \{\emptyset \oplus I, (A)_N, (B)_N, (C)_N, (Ӿ)_N\}$
$(A)_N = \{a, a\oplus I\}, (B)_N = \{a, b, a\oplus I , b\oplus I\}, (C)_N = \{a, c, a\oplus I , c\oplus I\}$.))
Then Ӿ is $Ť_o$-space, Ӿ is not $Ť_1$-space .

Remark 4.7:

Let $(Ӿ, Ť)$ is a (ŠN$_{Å}$-NTS), then :

$Ť_2$-space ⟹ $Ť_1$-space ⟹ $Ť_0$-space.
The converse of the Theorem 4.7 is not true.

Remark 4.8:

Relations among the different types of neutrosophic separation axioms which were studied in this paper, appear in the following diagram.

$Ť_2$-space	⟹ ⇍	$Ť_1$-space	⟹ ⇍	$Ť_0$-space.

Theorem 4.9:

Let Ӿ≠∅, if T={$Å_i$}$_{i∈Δ}$ is topology on Ӿ, and (Ӿ,Ť) a new approach of neutrosophic supra-topological space (

Ť={$(Å_i)_N$}$_{i∈Δ}$), then:

1. (Ӿ,Ť) $Ť_0$-space ⟹ (Ӿ,T) is T_0-space.
2. (Ӿ,Ť) $Ť_1$-space ⟹ (Ӿ,T) is T_1-space.

3. (Ӿ,Ť) $Ť_2$-space ⇒ (Ӿ,T) is T_2-space.

Proof:

1. Let (Ӿ,Ť) $Ť_0$-space then, by definition for every point from the same kind x≠ y∈ $(Ӿ)_N$ There exists $ŠN_{Å}$-NOS containing one of them but not the other, therefore in a special case for every points x≠ y without I there exists $ŠN_{Å}$-NOS containing one of them but not the other, that mean (Ӿ,T) is T_0-space.
2. Let (Ӿ,Ť) $Ť_1$-space then, by definition for every point from the same kind x≠ y∈$(Ӿ)_N$ There exist $ŠN_{Å}$-NOS, D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2, therefore in a special case for every points x≠ y without I there exists $ŠN_{Å}$-NOS, D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2, that mean (Ӿ,T) is T_1-space.

Let (Ӿ,Ť) $Ť_2$-space then, by definition for every point from the same kind x≠ y∈$(Ӿ)_N$ There exist $ŠN_{Å}$-NOS, D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2, with $D_1 \cap D_2 = \emptyset$. Therefore in a special case for every point x≠ y without I there exists $ŠN_{Å}$-NOS, D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2,

-now we are thinking, are the converse of Theorem 4.9 true? the answer in the following theorem.

Theorem 4.10:

Let Ӿ≠∅, if T={$\acute{Å}_i$}$_{i\in\Delta}$ is topology on Ӿ, and (Ӿ,Ť) a new approach of neutrosophic supra-topological space (Ť={$(\acute{Å}_i)_N$}$_{i\in\Delta}$), then:

1. (Ӿ,T) is T_0-space ⇒(Ӿ,Ť) $Ť_0$-space.
2. (Ӿ,T) is T_0-space ⇒(Ӿ,Ť) $Ť_0$-space.
3. (Ӿ,T) is T_0-space ⇒(Ӿ,Ť) $Ť_0$-space.

Proof:

1. Let (Ӿ,T) T_0-space then, by definition for every point x≠ y∈ Ӿ there exists supra-open set H containing one of them but not the other. Now there is two cases for any points x≠ y :
   a) x , y without I, since(Ӿ,T) T_0-space then, there exists supra-open set H containing one of them but not the other.

b) x , y with I, there exists ŠN$_{Å}$-NOS $(H)_N$ (H as in case a) containing one of them but not the other.

Therefore (Ӿ,Ť) is T_0-space.

2. Let (Ӿ,T) T_1-space then, by definition for every point x≠ y∈ Ӿ there exists two supra-open sets D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2.

   Now there is two cases for any points x≠ y :

   a) x , y without I, since(Ӿ,T) T_1-space then, there exists two supra-open sets D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2.

   b) x , y with I, there exists two ŠN$_{Å}$-NOS $(D_1)_N$, $(D_2)_N$ (as in case a) such that x∈ $(D_1)_N$, y ∉ $(D_1)_N$ and x ∉ $(D_2)_N$, y ∈ $(D_2)_N$,therefore (Ӿ,Ť) is Ť$_1$-space.

3. Let (Ӿ,T) T_2-space then, by definition for every point x≠ y∈ Ӿ there exists two supra-open sets D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2 with $D_1 \cap D_2 = \emptyset$.

   Now there is two case for any points x≠ y :

   a) x , y without I, since(Ӿ,T) T_1-space then, there exists two supra-open sets D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2 with $D_1 \cap D_2 = \emptyset$.

   b) x , y with I, there exists two ŠN$_{Å}$-NOS $(D_1)_N$, $(D_2)_N$ (as in case a) such that x∈ $(D_1)_N$, y ∉ $(D_1)_N$ and x ∉ $(D_2)_N$, y ∈ $(D_2)_N$ with $(D_1)_N \cap (D_2)_N = \emptyset$,therefore (Ӿ,Ť) is Ť$_2$-space.

Remark 4.11:

The relationship between the old and new separation axioms that we studied in this paper, appears in this diagram.

Ť$_2$-space	⇒ ⇍	Ť$_1$-space	⇒ ⇍	Ť$_0$-space.
⇓ ⇑		⇓ ⇑		⇓ ⇑
T_2-space	⇒ ⇍	T_1-space	⇒ ⇍	T_o-space.

5. A new approach of neutrosophic sub-space:

Definition5.1: If $(Ӿ, Ť)$ is a new approach of neutrosophic supra-topological space , $\Phi \neq \hat{G} \subseteq Ӿ$,

$\hat{Y}_{\hat{G}} = \{(\hat{G} \cap Ǻ)_N : Ǻ \in T\} \cup \{\varnothing \oplus I\}$, then ($\hat{G}, \hat{Y}_{\hat{G}}$) is called a new approach of neutrosophic supra-topological subspace of $(Ӿ, Ť)$.

Example 5.2:

Let Ӿ={e , f , g}. $T = \{\varnothing, Ǻ, \beta, Ӿ\}$,

$Ǻ = \{e, f\}, \beta = \{e, g\}. Ť = \{\varnothing \oplus I, (Ǻ)_N, (\beta)_N, (Ӿ)_N\}, \hat{G} = \{f, g\} then$

$\hat{Y}_{\hat{G}} = \left\{\varnothing \oplus I, \{f, f \oplus I\}, \{g, g \oplus I\}, \left\{(\hat{G})_N = \{f, g, g \oplus I, f \oplus I\}\right\}\right\}.$

($\hat{G}, \hat{Y}_{\hat{G}}$) a new approach of neutrosophic supra-topological subspace of $(Ӿ, Ť)$.

Definition 5.3: If $(Ӿ, Ť)$ is a new approach of neutrosophic topological space , $\Phi \neq \hat{G} \subseteq Ӿ$, $\hat{Y}_{\hat{G}} = \{(\hat{G} \cap Ǻ)_N :$ $Ǻ \in T\} \cup \{\varnothing \oplus I\}$, then ($\hat{G}, \hat{Y}_{\hat{G}}$) is called a new approach of neutrosophic topological subspace of $(Ӿ, Ť)$.

Theorem 5.4:

Let (Ӿ,Ť) an new approach of neutrosophic supra-topological space, ($\hat{G}, \hat{Y}_{\hat{G}}$) is an new approach of neutrosophic supra-topological subspace of $(Ӿ, Ť)$then:

1. (Ӿ,Ť) $Ť_0$-space ⇒ ($\hat{G}, \hat{Y}_{\hat{G}}$) is $Ť_0$-space.
2. (Ӿ,Ť) $Ť_1$-space ⇒ ($\hat{G}, \hat{Y}_{\hat{G}}$) is $Ť_1$-space.
3. (Ӿ,Ť) $Ť_2$-space ⇒ ($\hat{G}, \hat{Y}_{\hat{G}}$) is $Ť_2$-space.

Proof:

1. Let (Ӿ,Ť) $Ť_0$-space then, by definition for every points from the same kind x≠ y∈ $(Ӿ)_N$ there exists $ŠN_{Ǻ}$-NOS H containing one of them but not the other. But H∩ $\hat{G} \in \hat{Y}_{\hat{G}}$ therefore for every points x≠ y there exists $ŠN_{Ǻ}$-NOS($\hat{G}$) containing one of them but not the other, that mean ($\hat{G}, \hat{Y}_{\hat{G}}$) is $Ť_0$-space.
2. Let (Ӿ,Ť) $Ť_1$-space then, by definition for every points from the same kind x≠ y∈$(Ӿ)_N$ there exist $ŠN_{Ǻ}$-NOS, D_1, D_2 such that x∈ D_1, y ∉ D_1 and x ∉ D_2 , y ∈ D_2, therefore for every points x≠ y there exists $ŠN_{Ǻ}$-NOS($\hat{G}$), $D_1 \cap \hat{G}$, $D_2 \cap \hat{G}$ such that x∈ $D_1 \cap \hat{G}$, y ∉ $D_1 \cap \hat{G}$ and x ∉ $D_2 \cap \hat{G}$, y ∈ $D_2 \cap \hat{G}$, that mean ($\hat{G}, \hat{Y}_{\hat{G}}$) is $Ť_1$-space.
3. Let (Ӿ,Ť) $Ť_2$-space then, by definition for every points from the same kind x≠ y∈$(Ӿ)_N$ there exist $ŠN_{Ǻ}$-

NOS, D_1, D_2 such that $x \in D_1$, $y \notin D_1$ and $x \notin D_2$, $y \in D_2$, with $D_1 \cap D_2 = \emptyset$, therefore for every points $x \neq y$ there exists $\check{S}N_{\mathring{A}}$-NOS($\hat{G}$), $D_1 \cap \hat{G}$, $D_2 \cap \hat{G}$ such that $x \in D_1 \cap \hat{G}$, $y \notin D_1 \cap \hat{G}$ and $x \notin D_2 \cap \hat{G}$, $y \in D_2 \cap \hat{G}$, with $(D_1 \cap \hat{G}) \cap (D_2 \cap \hat{G}) = \emptyset$that mean $(\hat{G}, \hat{Y}_{\hat{G}})$ is $\check{T}_2$-space.

Conclusion

In this work, we have introduced $\check{S}N_{\mathring{A}}$-NT and $\check{S}N_{\mathring{A}}$-NTS. Also, we studied some of their basic properties. Then, we have introduced neutrosophic separation axioms in $\check{S}N_{\mathring{A}}$-NTS. Finally, This paper is just a beginning.

More theoretical research will be necessary to establish a general framework for practical application. In the future, using these new space" $\check{S}N_{\mathring{A}}$-NT", various classes of supra-mappings and supra-weak open sets as supra-alpha open sets,…ets, on the new approach of neutrosophic topological space can be studied.

References

[1] Al-Hamido, R. K, " A New Approach Of Neutrosophic Topological space ", International Journal of Neutrosophic Science, Vol 1, pp66-73, 2018.

[2] Salma, A. A. Alblowi, S.A. "Neutrosophic set and neutrosophic topological spaces", IOSR J. Math., Vol 3, pp31–35. 2012.

[3] Al-Hamido, R. K, "A study of multi-Topological Spaces", PhD Theses, AlBaath university , Syria, 2019.

[4] Al-Omeri, W. Jafari, S. "On Generalized Closed Sets and Generalized Pre-Closed Sets in Neutrosophic Topological Spaces", Mathematics, Vol 7, pp1-12, 2019. doi:doi.org/10.3390/math7010001.

[5] Smarandache, F. Pramanik, S."New Neutrosophic Sets via Neutrosophic Topological Spaces", In Neutrosophic Operational Research; Pons Editions: Brussels, Belgium, Vol 1, pp189–209, 2017.

[6] Al-Omeri, W."Neutrosophic crisp sets via neutrosophic crisp topological spaces", Neutrosophic Sets Syst, Vol 13, pp96–104, 2016.

[7] R.K. Al-Hamido, "Neutrosophic Crisp Bi-Topological Spaces", Neutrosophic Sets and Systems, Vol 21, pp66-73, 2018.

[8] AL-Nafee, A. B. Al-Hamido, R.K. Smarandache, F."Separation Axioms In Neutrosophic Crisp Topological Spaces", Neutrosophic Sets and Systems, Vol 25, pp25-32, 2019.

[9] Al-Hamido, R. K, "Neutrosophic Crisp Supra Bi-Topological Spaces", International Journal of Neutrosophic Science, Vol 1, pp66-73, 2018.

[10] Jayaparthasarathy, G.; Little Flower, V.F.; Arockia Dasan M.; Neutrosophic Supra Topological Applications in Data Mining Process, Neutrosophic Sets and Systems, ,(2019) ,vol. 27, pp. 80-97.