n-Valued Refined Neutrosophic Crisp Sets

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Abstract

The main purpose of this manuscript is to expand the notion of neutrosophic crisp set (NCS) by presenting the notion of n-valued refined neutrosophic crisp set with some illustration examples. We also establish some of its set-theoretical operations.

Keyword: Neutrosophic Crisp Set; n-Valued Refined Neutrosophic Crisp Set; Quadripartitioned Neutrosophic Crisp Set; Pentapartitioned Neutrosophic Crisp Set.

1. Introduction

Smarandache presented in [1, 2] the concept of "neutrosophic set" which is a generalization of the concept of "fuzzy set" and the concept "intuitionistic fuzzy" set to handle with uncertainty and imprecision by incurporating degrees of non-membership and indeterminacy as independent components. Salama and Smarandache presented in [3, 4] the concept of neutrosophic crisp set (NCS) which is a generalization of the concept of crisp set and the concept of "intuitionistic set" [5]. Thereafter, many researchers have applied the notion of NCSs in topology, image processing and decision-making problems (for example see [6, 7, 8, 9, 10], [11], [12-21]).

Rajashi et al introduced in [22] the notion of (QSVNS) that involves "truth", "falsity", "unknown" and "contradiction" depending on four-valued logics [23,24]. Shawkat introduced in [25] the notion of "n-values refined neutrosophic soft set" which is a generalization of the notion of "neutrosophic soft set". Thereafter, Shawkat and Ayman used this concept to solve decision-making problems [26]. Rama and Surapati introduced in [27] the notion of "pentapartitioned neutrosophic set" by splitting indeterminacy to independent components namely "contradiction", "ignorance, and "unknown"-membership based on [24].

In [24], Smarandache split truth into many types of truths, indeterminacy into several types of indeterminacies and falsehood into several types of falsities and proposed "n-symbol-valued refined neutrosophic logic" (NSVRNL). In

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this manuscript, we use the NSVRNL and propose n-valued refined neutrosophic crisp set. We also establish some of its set-theoretical operations. The proposed set is generalization of existent set NCS.

2. Preliminaries

In this section, we recall some basic concepts relevant to the upcoming sections. For more details on the concepts presented in the section (return to the refs, [4], [24]).

I. Neutrosophic Crisp Set (NCS) [4].

Let G be a non-empty fixed set. The form $\mathrm{Q} = < \mathrm{A}, \mathrm{B}, \mathrm{C} >$ is called (NCS). Hear A, B and C, represent the set of "memberships","indeterminacies" and "non-memberships" respectively of elements of G to Q, where $\mathrm{A}, \mathrm{B}, \mathrm{C}$ are subsets of G.

II. n-Valued Refined Neutrosophic Logic [24].

The neutrosophic logic value of a given proposition has the values, A =(truth), B =(indeterminacy), and C = (falsehood). Smarandache have defined two types of n-valued logic, symbolic and numerical.

1 - The n-symbol-valued refined neutrosophic logic (NSVRNL).

In general: C can be split into many types of falsities $C_1; C_2; \dots C_s$, B into several types of indeterminacies: $B_1; B_2; \dots B_r$, and A into several types of truths: $A_1; A_2; \dots A_P$, where all $p,r,s \geq 1$ are integers, and $n = p+r+s$. All subcomponents $A_i; B_j; C_k$ are symbols for all $i \in \{1,2,\dots p\}$, for all $j \in \{1,2,.., r\}$, and for all $k \in \{1,2,.., s\}$.

2 - The n- numerical-valued refined neutrosophic logic (NNVRNL).

In same way, but all subcomponents $A_i; B_j; C_k$ are not symbols, but subset of [0,1], for all $i \in \{1,2,..,p\}$, for all $j \in \{1,2,..,r\}$ and for all $k \in \{1,2,.., s\}$. In this manuscript, we use the NSVRNL and propose n-valued refined neutrosophic crisp set as follows:

3. n-Valued Refined Neutrosophic Crisp Set ($\mathrm{vnc}^{\mathrm{n}}$-set).

In this section, we define the notion of $\mathrm{vnc}^{\mathrm{n}}$-set and give some illustration examples.

3.1. Definition

Let G be a nonempty fixed set. The form $\mathrm{Q} = < A_1,..., A_p; B_1,..., B_r; C_1,..., C_s >$ is called an n-valued refined neutrosophic crisp set (for short, $\mathrm{vnc}^{\mathrm{n}}$-set), where $A_1, ..., A_p; B_1, ..., B_r$ and $C_1, ..., C_s$ are subsets of G. Here, $A_1,..., A_p; B_1,..., B_r$ and $C_1,..., C_s$, are called membership sets; indeterminacy sets and non-membership sets, respectively of $\mathrm{vnc}^{\mathrm{n}}$-set Q, where all $p, r, s \geq 1$ are integers, and $n= p+ r+s$. The set of all $\mathrm{vnc}^{\mathrm{n}}$-sets over G will be denoted by $\mathrm{VNCS}^{\mathrm{n}}_{\mathrm{G}}$. The following **special cases** can be obtained from the above definition.

Case (I): Quadripartitioned neutrosophic crisp set or (vnc^4-set).

3.2. Example

Let $\mathrm{G} = \{h_{1,} h_{1,} h_{3,} h_4, h_{5,}\}$ be a nonempty fixed set. Let the indeterminacy B be refined (split) as B_1 = unknown and B_2 = contradiction. A_1, B_1, B_2 and C_1 are subsets of G,

$n = 4 = 1+ 2+1$. Then, we can write the vnc^4-set Q as follows: $\mathrm{Q} = < \{h_2\}; \{h_5\}, \{h_{3,} h_4\}; \{h_1\} >$.

Case (II): **Pentapartitioned neutrosophic crisp set or (vnc^5-set).**

3.3. Example

Let $\mathrm{G} = \{h_{1,} h_{1,} h_{3,} h_4, h_{5,}\}$ be a nonempty fixed set. Let the indeterminacy B be refined (split) as B_1 = unknown, B_2 = ignorance and B_3 = contradiction. A_1, B_1, B_2, B_3 and C_1 are subsets of G,

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n = 5 = 1 + 3 + 1. Then, we can write the vnc^5-set Q as follows: $Q = < \{h_1\}; \{h_2\}, \{h_4\}, \{h_5\}; \{h_3, h_1\} >$.

Case (III): Hexapartitioned neutrosophic crisp set or (vnc^6-set).

3.4. Example

Let $G = \{h_{1,}\ h_{1,}\ h_{3,}\ h_4, h_5\}$ be a nonempty fixed set. Let the truth A be refined (split) as A_1= absolute truth and A_2= relative truth, the indeterminacy B be refined (split) as B_1 = absolute indeterminacy and B_2 = relative indeterminacy and the falsity C be refined (split) as C_1 = absolute falsity and C_2 = relative falsity. A_1, A_2, B_1, B_2, C_1 and C_2 are subsets of G, n = 6 = 2 + 2 + 2. Then, we can write the vnc^6-set Q as follows:

$$Q = < \{h_1\}, \{h_3\}; \{h_4\}, \{_{h5}\}; \{h_2\}, \{h_{5,}\ h_3\} >.$$

3.5. Definition

Let $Q = < A_1,..., A_p; B_1,..., B_r; C_1,..., C_s > \in VNCS_G^n$. If $A_i \cap B_j$ = empty, $A_i \cap C_k$ = empty and $B_j \cap C_k$ = empty, where $i \in \{1,2,,,,p\}$, $j \in \{1,2,,,,r\}$ and $k \in \{1,2,,,,s\}$, then the vnc^n-set Q is called star vnc^n-set (for short, $svnc^n$-set).

3.6. Definition

Let $Q = < A_1,..., A_p; B_1,..., B_r; C_1,..., C_s > \in VNCS_G^n$. If the union of all subcomponents, $A_1,..., A_p; B_1,..., B_r; C_1,..., C_s$ equals G and $A_i \cap B_j$ = empty, $A_i \cap C_k$ = empty and $B_j \cap C_k$ = empty and, where $i \in \{1,2,,,,p\}$, $j \in \{1,2,,,,r\}$ and $k \in \{1,2,,,,s\}$, then the vnc^n-set Q is called an 2star-vnc^n-set (for short, $2svnc^n$-set).

3.7. Definition

Let $Q = < A_1,..., A_p; B_1,..., B_r; C_1,..., C_s > \in VNCS_G^n$. If the union of all subcomponents, $A_1,..., A_p; B_1,..., B_r; C_1,..., C_s$ equals G and $A_i \cap B_j \cap C_k$ = empty, where $i \in \{1,2,,,,p\}$, $j \in \{1,2,,,,r\}$ and $k \in \{1,2,,,,s\}$, then the vnc^n-set Q is called an 3star vnc^n-set (for short, $3svnc^n$-set).

The set of all $svnc^n$-sets, $2svnc^n$-sets and $3svnc^n$-sets over G will be denoted by $SVNCS_G^n$, $2SVNCS_G^n$ and $3SVNCS_G^n$, respectively.

4. Operations of n-Valued Refined Neutrosophic Crisp Set.

Since our goal is to build the tools to develop n-valued refined neutrosophic crisp set. we will organize the existing definitions into three types in each type these operations will be consistent and functional . As follows:

((Operations of $svnc^n$-set, Type I))

4.1. Definition

Let $Q = < A_1,..., A_p; B_1,..., B_r; C_1,..., C_s >$ and $W = < X_1,..., X_p; Y_1,..., Y_r; Z_1,..., Z_s > \in VNCS_G^n$. Then,

- The vnc^n-empty set, denoted by $\hat{\hat{\hat{\emptyset}}}$ is defined as follows:

$$\hat{\hat{\hat{\emptyset}}} = < \emptyset_1,..., \emptyset_p; G_1,..., G_r; G_1,..., G_s >.$$

- The vnc^n- absolute set, denoted by $\hat{\hat{\hat{G}}}$ is defined as follows:

$$\hat{\hat{\hat{G}}} = < G_1,..., G_p; \emptyset_1,..., \emptyset_r; \emptyset_1,..., \emptyset_s >.$$

- The inclusion between Q and W, denoted by $Q \mathrel{\hat{\hat{\subseteq}}} W$ is defined as follows:

$$Q \mathrel{\hat{\hat{\subseteq}}} W = < A_1 \subseteq X_1,...,A_P \subseteq X_P; Y_1 \subseteq B_1,...,Y_r \subseteq B_r; Z_1 \subseteq C_1,...,Z_s \subseteq C_s >.$$

- The union between Q and W, denoted by $Q \mathbin{\hat{\cup}} W$ is defined as follows:

$$Q \mathbin{\hat{\hat{\cup}}} W = < A_1 \cup X_1,...,A_P \cup X_P; B_1 \cap Y_1,...,B_r \cap Y_r; C_1 \cap Z_1,...,C_s \cap Z_s >.$$

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- The intersection between Q and W, denoted by $Q \hat{\hat{\cap}} W$ is defined as follows:

$Q \hat{\hat{\cap}} W = < A_1 \cap X_1,\ldots,A_P \cap X_P; B_1 \cup Y_1,\ldots,B_r \cup Y_r; C_1 \cup Z_1,\ldots,C_s \cup Z_s >$.

- The complement of Q, denoted by Q^C is defined as follows:

$$Q^C = < C_1,\ldots, C_s; {B^c}_1,\ldots, {B^c}_r; A_1,\ldots, A_p >.$$

- Q and W are equal, denoted by $Q \hat{\hat{=}} W$ if $Q \hat{\hat{\subseteq}} W$ and $W \hat{\hat{\subseteq}} Q$.

((Operations of svnc^n-set, Type II))

4.2. Definition

Let $Q = < A_1,\ldots, A_p; B_1,\ldots, B_r; C_1,\ldots, C_s >$ and $W = < X_1,\ldots, X_p; Y_1,\ldots, Y_r; Z_1,\ldots, Z_s > \in \mathrm{VNCS}_G^n$. Then,

- The vnc^n-empty set, denoted by $\hat{\hat{\emptyset}}$ is defined as follows:

$$\hat{\hat{\emptyset}} = < \emptyset_1,\ldots, \emptyset_p; \emptyset_1,\ldots, \emptyset_r; G_1,\ldots, G_s >.$$

- The vnc^n- absolute set, denoted by $\hat{\hat{G}}$ is defined as follows:

$$\hat{\hat{G}} = < G_1,\ldots, G_p; G_1,\ldots, G_r; \emptyset_1,\ldots, \emptyset_s >.$$

- The inclusion between Q and W, denoted by $Q \hat{\hat{\subseteq}} W$ is defined as follows:

$Q \hat{\hat{\subseteq}} W = < A_1 \subseteq X_1,\ldots,A_P \subseteq X_P; B_1 \subseteq Y_1,\ldots,B_r \subseteq Y_r; Z_1 \subseteq C_1,\ldots,Z_s \subseteq C_s >$.

- The union between Q and W, denoted by $Q \hat{\cup} W$ is defined as follows:

$Q \hat{\cup} W = < A_1 \cup X_1,\ldots,A_P \cup X_P; B_1 \cup Y_1,\ldots,B_r \cup Y_r; C_1 \cap Z_1,\ldots,C_s \cap Z_s >$.

- The intersection between Q and W, denoted by $Q \hat{\hat{\cap}} W$ is defined as follows:

$Q \hat{\hat{\cap}} W = < A_1 \cap X_1,\ldots,A_P \cap X_P; B_1 \cap Y_1,\ldots,B_r \cap Y_r; C_1 \cup Z_1,\ldots,C_s \cup Z_s >$.

- The complement of Q, denoted by Q^C is defined as follows:

$$Q^C = < {A^c}_1,\ldots, {A^c}_p; {B^c}_1,\ldots, {B^c}_r; {C^c}_1,\ldots, {C^c}_s >.$$

- Q and W are equal, denoted by $Q \hat{\hat{=}} W$ if $Q \hat{\hat{\subseteq}} W$ and $W \hat{\hat{\subseteq}} Q$.

((Operations of svnc^n-set, Type III))

4.3. Definition

Let $Q = < A_1,\ldots, A_p; B_1,\ldots, B_r; C_1,\ldots, C_s >$ and $W = < X_1,\ldots, X_p; Y_1,\ldots, Y_r; Z_1,\ldots, Z_s > \in \mathrm{VNCS}_G^n$. Then,

- The vnc^n-empty set, denoted by $\hat{\hat{\emptyset}}$ is defined as follows:

$$\hat{\hat{\emptyset}} = < \emptyset_1,\ldots, \emptyset_p; \emptyset_1,\ldots, \emptyset_r; \emptyset_1,\ldots, \emptyset_s>.$$

- The vnc^n- absolute set, denoted by $\hat{\hat{G}}$ is defined as follows:

$$\hat{\hat{G}} = < G_1,\ldots, G_p; G_1,\ldots, G_r; G_1,\ldots, G_s >.$$

- The inclusion between Q and W, denoted by $Q \hat{\hat{\subseteq}} W$ is defined as follows:

$Q \hat{\hat{\subseteq}} W = < A_1 \subseteq X_1,\ldots,A_P \subseteq X_P; B_1 \subseteq Y_1,\ldots,B_r \subseteq Y_r; C_1 \subseteq Z_1,\ldots,C_s \subseteq Z_s >$.

- The union between Q and W, denoted by $Q \hat{\cup} W$ is defined as follows:

$Q \hat{\cup} W = < A_1 \cup X_1,\ldots,A_P \cup X_P; B_1 \cup Y_1,\ldots,B_r \cup Y_r; C_1 \cup Z_1,\ldots,C_s \cup Z_s >$.

- The intersection between Q and W, denoted by $Q \hat{\hat{\cap}} W$ is defined as follows:

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$Q \hat{\hat{\cap}} W = < A_1 \cap X_1,...,A_P \cap X_P; B_1 \cap Y_1,...,B_r \cap Y_r; C_1 \cap Z_1,...,C_s \cap Z_s >$.

- The complement of Q, denoted by Q^C is defined as follows: $Q^C = < A^c{}_1,..., A^c{}_p; B^c{}_1,..., B^c{}_r; C^c{}_1,..., C^c{}_s >$.
- Q and W are equal, denoted by $Q \hat{\hat{=}} W$ if $Q \hat{\hat{\subseteq}} W$ and $W \hat{\hat{\subseteq}} Q$.

4.4. Example

Consider example 1 in case 1.

Let $Q = < \{h_2\}; \{h_5\}, \{h_3, h_4\}; \{h_1, h_4\} >$ and $W = < \{h_2, h_1\}; \{h_5\}, \{h_3\}; \{h_1\} > \in VNCS^4_G$. Then,

- Type I.
  - $Q \hat{\hat{\cup}} W = < \{h_2\} \cup \{h_2, h_1\}; \{h_5\} \cap \{h_5\}, \{h_3\} \cap \{h_3, h_4\}; \{h_1\} \cap \{h_1, h_4\} >$
    $= < \{h_2, h_1\}; \{h_5\}, \{h_3\}; \{h_1\} >$.
  - $Q \hat{\hat{\cap}} W = < \{h_2\} \cap \{h_2, h_1\}; \{h_5\} \cup \{h_5\}, \{h_3\} \cup \{h_3, h_4\}; \{h_1\} \cup \{h_1, h_4\} >$
    $= < \{h_2\}; \{h_5\}, \{h_3, h_4\}; \{h_1, h_4\} >$.
  - $Q^C = < \{h_1, h_4\}; \{h_1, h_2, h_3, h_4, h_6\}, \{h_1, h_2, h_5, h_6\}; \{h_2\} >$.
- Type II.
  - $Q \hat{\hat{\cup}} W = < \{h_2\} \cup \{h_2, h_1\}; \{h_5\} \cup \{h_5\}, \{h_3\} \cup \{h_3, h_4\}; \{h_1\} \cap \{h_1, h_4\} >$
    $= < \{h_2, h_1\}; \{h_5\}, \{h_3, h_4\}; \{h_1\} >$.
  - $Q \hat{\hat{\cap}} W = < \{h_2\} \cap \{h_2, h_1\}; \{h_5\} \cap \{h_5\}, \{h_3\} \cap \{h_3, h_4\}; \{h_1\} \cup \{h_1, h_4\} >$
    $= < \{h_2\}; \{h_5\}, \{h_3\}; \{h_1, h_4\} >$.
  - $Q^C = < \{h_1, h_3, h_4, h_5, h_6\}; \{h_1, h_2, h_3, h_4, h_6\}, \{h_1, h_2, h_5, h_6\}; \{h_2, h_3, h_5, h_6\} >$.
- Type III.
  - $Q \hat{\hat{\cup}} W = < \{h_2\} \cup \{h_2, h_1\}; \{h_5\} \cup \{h_5\}, \{h_3\} \cup \{h_3, h_4\}; \{h_1\} \cup \{h_1, h_4\} >$
    $= < \{h_2, h_1\}; \{h_5\}, \{h_3, h_4\}; \{h_1, h_4\} >$.
  - $Q \hat{\hat{\cap}} W = < \{h_2\} \cap \{h_2, h_1\}; \{h_5\} \cap \{h_5\}, \{h_3\} \cap \{h_3, h_4\}; \{h_1\} \cap \{h_1, h_4\} >$
    $= < \{h_2\}; \{h_5\}, \{h_3\}; \{h_1\} >$.
  - $Q^C = < \{h_1, h_3, h_4, h_5, h_6\}; \{h_1, h_2, h_3, h_4, h_6\}, \{h_1, h_2, h_5, h_6\}; \{h_2, h_3, h_5, h_6\} >$.

4.5. Corollary

- For, Type I and Type II. If Q and W are two $2svnc^n$-sets, then $Q \hat{\hat{\cap}} W$ does not have to be an $2svnc^n$-set.
- For, Type I and Type II. If Q and W are two $2svnc^n$-sets, then $Q \hat{\hat{\cup}} W$ does not have to be an $2svnc^n$-set.
- For, Type I and Type II. If Q and W are two $3svnc^n$-sets, then $Q \hat{\hat{\cap}} W$ does not have to be an $3svnc^n$-set.
- For, Type I and Type II. If Q are two $3svnc^n$-sets, then $Q \hat{\hat{\cup}} W$ does not have to be an $3svnc^n$-set.
- For, Type I. If Q is an $3svnc^n$-set, then $(Q)^C$ does not have to be an $3svnc^n$-set.

Note that:

For, Type I and Type II. $Q = < \{h_1, h_2\}; \{h_3, h_4\}, \emptyset ; \{h_5\}>$ and $W = < \{h_1\}; \{h_2, h_3\}, \emptyset ; \{h_4, h_5\} >$ are two $2svnc^4$-sets. But, $Q \hat{\hat{\cap}} W$ is not an $2svnc^4$-set**.**

For, Type I and Type II. $Q = < \{h_1, h_2, h_6\}; \{h_3, h_4\}, \emptyset ; \{h_4, h_5, h_6\}>$ and $W = < \{h_1, h_6\}; \{h_2, h_6\}, \emptyset ; \{h_3, h_4, h_5\} >$ are two $3svnc^4$-sets. But, $Q \hat{\hat{\cap}} W$ **is not** an $3svnc^4$-set**.**

For, Type I and Type II. $Q = < \{h_1, h_2\}; \{h_3, h_4\}, \emptyset ; \{h_5\}>$ and $W = < \{h_1\}; \{h_2, h_3\}, \emptyset ; \{h_4, h_5\} >$ are two 2svnc^4-sets. But, $Q \hat{\cup} W$ **is not** an 2svnc^4-set**.**

For, Type I and Type II. $Q = < \{h_1, h_2, h_6\}; \{h_3, h_4\}, \emptyset ; \{h_2, h_5, h_6\}>$ and $W = < \{h_1, h_6\}; \{h_2, h_6\}, \emptyset ; \{ h_2, h_3, h_4, h_5\} >$ are two 3svnc^4-sets. But, $Q \hat{\cup} W$ **is not** an 3svnc^4-set**.**

4.6. Proposition

Let Q, £ and $W \in VNCS_G^n$. Then,

- $Q \hat{\hat{\subseteq}} \hat{\hat{G}}$.
- $\hat{\hat{\emptyset}} \hat{\hat{\subseteq}} Q$.
- $Q \hat{\hat{\subseteq}} Q$.
- If $Q \hat{\hat{\subseteq}} W$ and $W \hat{\hat{\subseteq}}$ £, then $Q \hat{\hat{\subseteq}}$ £.

Proof. It is clear.

4.7. Proposition

Let Q, £ and $W \in VNCS_G^n$. Then,

- $Q \hat{\cup} W = W \hat{\cup} Q$.
- $Q \hat{\hat{\cap}} W = W \hat{\hat{\cap}} Q$.
- $Q \hat{\cup} Q = Q$, $Q \hat{\hat{\cap}} Q = Q$.
- $Q \hat{\cup} \hat{\hat{\emptyset}} = Q$, $Q \hat{\hat{\cap}} \hat{\emptyset} = \hat{\hat{\emptyset}}$.
- $Q \hat{\cup} \hat{\hat{G}} = \hat{\hat{G}}$, $Q \hat{\hat{\cap}} \hat{\hat{G}} = Q$.
- $(Q \hat{\hat{\cap}} W) \hat{\hat{\cap}}$ £ $= Q \hat{\hat{\cap}} (W \hat{\hat{\cap}}$ £$)$.
- $(Q \hat{\cup} W) \hat{\cup}$ £ $= Q \hat{\cup} (W \hat{\cup}$ £$)$.
- $(Q^C)^C = Q$.
- For, Type I and Type II, the equalities $Q \hat{\cup} Q^C \neq \hat{\hat{G}}$ and $Q \hat{\hat{\cap}} Q^C \neq \hat{\hat{\emptyset}}$ are in general not true.
- For, Type III, $Q \hat{\cup} Q^C = \hat{\hat{G}}$, $Q \hat{\hat{\cap}} Q^C = \hat{\hat{\emptyset}}$, (in general).

Proof. It is clear.

4.8. Proposition

Let Q and $W \in VNCS_G^n$. Then,

- $(Q \hat{\hat{\cap}} W)^C = Q^C \hat{\cup} W^C$.
- $(Q \hat{\cup} W)^C = Q^C \hat{\hat{\cap}} W^C$.

Proof.

For, Type I.

$(Q \hat{\hat{\cap}} W)^C = (< A_1 \cap X_1,\ldots, A_P \cap X_P; B_1 \cup Y_1,\ldots, B_r \cup Y_r; C_1 \cup Z_1,\ldots,C_s \cup Z_s >)^C$

$= < C_1 \cup Z_1,\ldots,C_s \cup Z_s; (B_1 \cup Y_1)^C,\ldots,(B_r \cup Y_r)^C; A_1 \cap X_1,\ldots, A_P \cap X_P >$

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$= < C_1 \cup Z_1,\dots,C_s \cup Z_s; (B_1)^C \cap (Y_1)^C,\dots, (B_r)^C \cap (Y_r)^C ; A_1 \cap X_1,\dots, A_P \cap X_P >$

$= < C_1,\dots, C_s; (B_1)^C,\dots,(B_r)^C; A_1,\dots, A_P > \widehat{\cup} < Z_1,\dots, Z_s; (Y_1)^C,\dots,(Y_r)^C; X_1,\dots, X_P >$

$= (< A_1,\dots, A_p; B_1,\dots, B_r; C_1,\dots, C_s >)^C \,\widehat{\cup}\, (< X_1,\dots, X_p; Y_1,\dots, Y_r; Z_1,\dots, Z_s >)^C = Q^C \,\widehat{\cup}\, W^C.$

Similarly, we can prove (2).

For, Type II.

$(Q \,\widehat{\cap}\, W)^C = (< A_1 \cap X_1,\dots,A_P \cap X_P; B_1 \cap Y_1,\dots,B_r \cap Y_r; C_1 \cup Z_1,\dots,C_s \cup Z_s >)^C$

$= < (A_1 \cap X_1)^C,\dots,(A_P \cap X_P)^C; (B_1 \cap Y_1)^C,\dots,(B_r \cap Y_r)^C; (C_1 \cup Z_1)^C,\dots,(C_s \cup Z_s)^C >$

$= < (A_1)^C \cup (X_1)^C,\dots,(A_P)^C \cup (X_P)^C; (B_1)^C \cup (Y_1)^C,\dots,(B_r)^C \cup (Y_r)^C; (C_1)^C \cap (Z_1)^C,\dots, (C_s)^C \cap (Z_s)^C >$

$= < (A_1)^C,\dots,(A_P)^C; (B_1)^C,\dots,(B_r)^C; (C_1)^C,\dots, (C_s)^C > \widehat{\cup} < (X_1)^C,\dots,(X_P)^C; (Y_1)^C,\dots,(Y_r)^C; (Z_1)^C,\dots, (Z_s)^C >$

$= (< A_1,\dots, A_p; B_1,\dots, B_r; C_1,\dots, C_s >)^C \,\widehat{\cup}\, (< X_1,\dots, X_p; Y_1,\dots, Y_r; Z_1,\dots, Z_s >)^C = Q^C \,\widehat{\cup}\, W^C.$

Similarly, we can prove (2).

For, Type III.

$(Q \,\widehat{\cap}\, W)^C = (< A_1 \cap X_1,\dots,A_P \cap X_P; B_1 \cap Y_1,\dots,B_r \cap Y_r; C_1 \cap Z_1,\dots,C_s \cap Z_s >)^C$

$= < (A_1 \cap X_1)^C,\dots,(A_P \cap X_P)^C; (B_1 \cap Y_1)^C,\dots,(B_r \cap Y_r)^C; (C_1 \cap Z_1)^C,\dots,(C_s \cap Z_s)^C >$

$= < (A_1)^C \cup (X_1)^C,\dots,(A_P)^C \cup (X_P)^C; (B_1)^C \cup (Y_1)^C,\dots,(B_r)^C \cup (Y_r)^C; (C_1)^C \cup (Z_1)^C,\dots, (C_s)^C \cup (Z_s)^C >$

$= < (A_1)^C,\dots,(A_P)^C; (B_1)^C,\dots,(B_r)^C; (C_1)^C,\dots, (C_s)^C > \widehat{\cup} < (X_1)^C,\dots,(X_P)^C; (Y_1)^C,\dots,(Y_r)^C; (Z_1)^C,\dots, (Z_s)^C >$

$= (< A_1,\dots, A_p; B_1,\dots, B_r; C_1,\dots, C_s >)^C \,\widehat{\cup}\, (< X_1,\dots, X_p; Y_1,\dots, Y_r; Z_1,\dots, Z_s >)^C = Q^C \,\widehat{\cup}\, W^C.$

Similarly, we can prove (2).

4.9. Definition

Let $f: G \to K$ be a mapping, $Q = < A_1,\dots, A_p; B_1,\dots, B_r; C_1,\dots, C_s > \in VNCS^n_G$,

$W = < X_1,\dots, X_p; Y_1,\dots, Y_r; Z_1,\dots, Z_s > \in VNCS^n_K$. Then,

- The image of Q under f, denoted by $f(Q) \in VNCS^n_K$, is defined as:

$f(Q) = < f(A_1),\dots, f(A_p); f(B_1),\dots, f(B_r); f(C_1),\dots, f(C_s) >$.

- The preimage of W under f, denoted by $f^{-1}(W) \in VNCS^n_G$, is defined as:

$f^{-1}(W) = < f^{-1}(X_1),\dots, f^{-1}(X_p); f^{-1}(Y_1),\dots, f^{-1}(Y_r); f^{-1}(Z_1),\dots, f^{-1}(Z_s) >$.

4.10. Corollary

Let $f: G \to K$ be a mapping,

$Q = < A_1,\dots, A_p; B_1,\dots, B_r; C_1,\dots, C_s > \in VNCS^n_G$,

$Q_1 = < A_{11},\dots, A_{1p}; B_{11},\dots, B_{1r}; C_{11},\dots, C_{1s} > \in VNCS^n_G$,

$W = < X_1,\dots, X_p; Y_1,\dots, Y_r; Z_1,\dots, Z_s > \in VNCS^n_K$,

$W_1 = < X_{11},\dots, X_{1p}; Y_{11},\dots, Y_{1r}; Z_{11},\dots, Z_{1s} > \in VNCS^n_K$. Then,

- $W_1 \,\widehat{\subseteq}\, W \leftrightarrow f^{-1}(W_1) \,\widehat{\subseteq}\, f^{-1}(W)$.
- $Q_1 \,\widehat{\subseteq}\, Q \leftrightarrow f(Q_1) \,\widehat{\subseteq}\, f(Q)$.
- $Q \,\widehat{\subseteq}\, f^{-1}(f(Q))$, and if f is an injective, then $Q = f^{-1}(f(Q))$.
- $f(f^{-1}(W)) \,\widehat{\subseteq}\, W$, and if f is a surjective, then $f(f^{-1}(W)) = W$.

- $f^{-1}(W_1 \widehat{\widehat{\cup}} W) = f^{-1}(W_1) \widehat{\widehat{\cup}} f^{-1}(W)$.
- $f^{-1}(W_1 \widehat{\widehat{\widehat{\cap}}} W) = f^{-1}(W_1) \widehat{\widehat{\widehat{\cap}}} f^{-1}(W)$.
- $f(Q_1 \widehat{\widehat{\cup}} Q) = f(Q_1) \widehat{\widehat{\cup}} f(Q)$.
- $f(Q_1 \widehat{\widehat{\widehat{\cap}}} Q) \widehat{\widehat{\widehat{\subseteq}}} f(Q_1) \widehat{\widehat{\widehat{\cap}}} f(Q)$ and, if f is an injective, then $f(Q_1 \widehat{\widehat{\widehat{\cap}}} Q) = f(Q_1) \widehat{\widehat{\widehat{\cap}}} f(Q)$.
- $f^{-1}(\widehat{\widehat{\widehat{\emptyset}}}) = \widehat{\widehat{\widehat{\emptyset}}}$, $f^{-1}(\widehat{\widehat{K}}) = \widehat{\widehat{\widehat{G}}}$.
- $f(\widehat{\widehat{\widehat{\emptyset}}}) = \widehat{\widehat{\widehat{\emptyset}}}$, and $f(\widehat{\widehat{\widehat{G}}}) = \widehat{\widehat{K}}$, if f is a surjective.

Proof. Straightforward.

6. Conclusion

We used the NSVRNL and proposed n-valued refined neutrosophic crisp set with some of its set-theoretical operations. We can propose a group decision making method based on vnc^n-sets (In particular, n=4 or 5 or 6), and give algorithm of proposed method. We hope that the results of this study will be useful for resear-chers to present additional new studies on the n-valued refined neutrosophic crisp sets .

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