



# On Some Properties of Interior, Closure, Boundary and Exterior of Neutrosophic Multi Topological Space

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## Abstract

This paper introduced the concept of neutrosophic multi topological space. The definitions of the interior, closure, exterior, and boundary of neutrosophic multiset is defined and also, its propositions will be studied with examples.

**Keywords:** Neutrosophic Multiset, Neutrosophic Multi Topological Space, Neutrosophic Multi Exterior, Neutrosophic Multi Boundary

## 1. Introduction

Zadeh [1] discovered the fuzzy set (FS) and proposed the concept of membership value in 1965 and explained the concept of uncertainty using a FS. Atanassov introduced [2] the intuitionistic fuzzy (IF) set in 1989, which generalised FS concepts and suggested the degree of non-membership (DNM) as an independent component. Following the introduction of FSs, various types of research on extensions of FS notions were done. Considering the generalisation of FSs, several researchers applied FS theory to a variety of fields in science & technology. Chang [3] established the development of fuzzy topological space (FTS). Intuitionistic FTS was defined by Coker [4]. Kelly proposed [5] the concept of bitopological spaces in 1963 and also, Mwachary et al. [6] investigated neutrosophic bitopological spaces.

Smarandache [7-10] proposed the (NS) neutrosophic set and after that many researchers applied it in science & technology. Broumi et al. [11, 12] investigated relations on interval-valued neutrosophic soft sets, as well as Neutrosophic parametrized soft set theory and decision-making. Sumathi et al. [13, 14] discussed the fuzzy neutrosophic groups and later studied the topological group structure of the NS. Many researchers [15-18] have

investigated topological spaces based on NSs. Basumatary et al. [19] studied the neutrosophic bitopological group and also, Basumatary et al. [20] discussed the properties of plithogenic neutrosophic hypersoft. Blizard [21] defined the idea of multisets. Kazanc [22] studied how multi-fuzzy soft sets can be used in hyper modules. Many researchers [23, 24] discussed multiset topology, and also many authors [25-27] investigated fuzzy multisets. Bakbak [28] studied neutrosophic multigroup and its applications. Basumatary et al. [29] investigated the neutrosophic multi topological group and its properties and also Basumatary [30] investigated fuzzy closure regarding to the fuzzy boundary.

In this paper, the definitions of the neutrosophic multi interior (NMI), neutrosophic multi closure (NMC), the neutrosophic multi exterior (NME), and neutrosophic multi boundary (NMB) and their propositions will be studied with examples.

**2. Preliminaires**

**Definition 2.1:** [10]

Let  $\mathbb{A}$  be the NS on  $X$  is expressed as  $\mathbb{A} = \{ \langle x, \chi_{\mathbb{A}}, \psi_{\mathbb{A}}, \xi_{\mathbb{A}} \rangle : x \in X \}$ , where  $\tau, \mathcal{J}, \mathcal{F} : X \rightarrow [0, 1]$ ;  $0 \leq \chi_{\mathbb{A}} + \psi_{\mathbb{A}} + \xi_{\mathbb{A}} \leq 3$ , and  $\chi_{\mathbb{A}}(x)$  represents the degree of membership value,  $\psi_{\mathbb{A}}(x)$  the degree of indeterminacy, and  $\xi_{\mathbb{A}}(x)$  is the degree of non- membership value respectively.

**Definition 2.2:** [9]

A neutrosophic multiset (NMS) is a type of NS in which one or more elements with the same or different neutrosophic components are repeated several times.

**Definition 2.3:** [29]

The Empty NMS can be expressed as  $0_N = \{ n^{NM} \in X; \langle n^{NM}_{(0,1,1)} \rangle \}$ .

**Definition 2.4:** [29]

The Whole NMS can be expressed as  $1_N = \{ n^{NM} \in X; \langle n^{NM}_{(1,0,0)} \rangle \}$ .

**Definition 2.5:** [29]

Let  $\mathbb{A}$  be a NMS on  $X$  and

$$\mathbb{A} = \left\{ n^{NM} \in X; \left( n^{NM} \langle T_{\mathbb{A}(n^{NM})}, I_{\mathbb{A}(n^{NM})}, F_{\mathbb{A}(n^{NM})} \rangle \right) \right\},$$

then  $\mathbb{A}^C = \left\{ n^{NM} \in X; \left( n^{NM} \langle F_{\mathbb{A}(n^{NM})}, 1 - I_{\mathbb{A}(n^{NM})}, T_{\mathbb{A}(n^{NM})} \rangle \right) \right\}$  is the the complement of  $\mathbb{A}$ .

**Definition 2.6:** [29]

Let  $\mathbb{A} = \left\{ n^{NM} \in X; \left( n^{NM} \langle T_{\mathbb{A}(n^{NM})}, I_{\mathbb{A}(n^{NM})}, F_{\mathbb{A}(n^{NM})} \rangle \right) \right\}$  and

$\mathbb{B} = \left\{ n^{NM} \in X; \left( n^{NM} \langle T_{\mathbb{B}(n^{NM})}, I_{\mathbb{B}(n^{NM})}, F_{\mathbb{B}(n^{NM})} \rangle \right) \right\}$  be neutrosophic multisets on  $X$ . Then

- (i)  $\mathbb{A} \cap \mathbb{B} = \left\{ n^{NM} \in X; n^{NM} \langle \min(T_{\mathbb{A}(n^{NM})}, T_{\mathbb{B}(n^{NM})}), \max(I_{\mathbb{A}(n^{NM})}, I_{\mathbb{B}(n^{NM})}), \max(F_{\mathbb{A}(n^{NM})}, F_{\mathbb{B}(n^{NM})}) \rangle \right\}$
- (ii)  $\mathbb{A} \cup \mathbb{B} = \left\{ n^{NM} \in X; n^{NM} \langle \max(T_{\mathbb{A}(n^{NM})}, T_{\mathbb{B}(n^{NM})}), \min(I_{\mathbb{A}(n^{NM})}, I_{\mathbb{B}(n^{NM})}), \min(F_{\mathbb{A}(n^{NM})}, F_{\mathbb{B}(n^{NM})}) \rangle \right\}$ .

**Definition 2.7:** [10]

Let  $X \neq \emptyset$  and  $\mathfrak{S}_{NM}$  be a family of neutrosophic multi subsets of  $X$  if it satisfies the following:

- (i)  $0_N, 1_N \in \mathfrak{S}_{NM}$
- (ii)  $G_1^{NM} \cap G_2^{NM} \in \mathfrak{S}_{NM}$  for  $G_1^{NM}, G_2^{NM} \in \mathfrak{S}_{NM}$
- (iii)  $\cup G_i^{NM} \in \mathfrak{S}_{NM}, \forall \{G_i^{NM} : i \in J\} \subseteq \mathfrak{S}_{NM}$ .

Then  $(X, \mathfrak{S}_{NM})$  is neutrosophic multi topological space (NMTS).

### 3. Results

#### 3.1. Definition

Let  $(X, \mathfrak{S}_{NM})$  be a NMTS and let  $\mathcal{A} = \{ \langle x, \mu_{NM_i}, \sigma_{NM_i}, \delta_{NM_i} \rangle : x \in X \}$  be NMS, then the NMI of  $\mathcal{A}$  is the union of all neutrosophic multi open sets (NMOS) of  $X$  contained in  $\mathcal{A}$  and is defined as

$$int_{NM}(\mathcal{A}) = \{ \langle x, \uplus \mu_{NM_i}, \upmho \sigma_{NM_i}, \upmho \delta_{NM_i} \rangle : x \in X \}.$$

#### 3.1. Proposition

Let  $(X, \mathfrak{S}_{NM})$  be a NMTS, then

- (i)  $int_{NM}(0_N) = 0_N, int_{NM}(1_N) = 1_N$ ;
- (ii)  $int_{NM}(\mathcal{A}) \subseteq \mathcal{A}$ ;
- (iii)  $\mathcal{A} \subseteq \mathcal{B} \Rightarrow int_{NM}(\mathcal{A}) \subseteq int_{NM}(\mathcal{B})$ .

Proof:

- (i) Since  $0_N$  and  $1_N$  are open sets, we have  
 $int_{NM}(0_N) = 0_N, int_{NM}(1_N) = 1_N$ .
- (ii) Let  $x \in int_{NM}(\mathcal{A})$ .  
 $\Rightarrow x$  is an interior point of  $\mathcal{A}$ .  
 $\Rightarrow \mathcal{A}$  is a neighbourhood of  $x$ .  
 $\Rightarrow x \in \mathcal{A}$ .

Hence,  $int_{NM}(\mathcal{A}) \subseteq \mathcal{A}$ .

- (iii) Let  $x \in int_{NM}(\mathcal{A})$ .

Then  $x$  is an interior point of  $\mathcal{A}$ , so  $\mathcal{A}$  is a neighbourhood of  $x$ .

Since  $\mathcal{A} \subseteq \mathcal{B}$ , so  $\mathcal{B}$  is also a neighbourhood of  $x$ .

$$\Rightarrow x \in int_{NM}(\mathcal{B}).$$

Thus,  $x \in int_{NM}(\mathcal{A}) \Rightarrow x \in int_{NM}(\mathcal{B})$ .

Hence,  $\mathcal{A} \subseteq \mathcal{B} \Rightarrow int_{NM}(\mathcal{A}) \subseteq int_{NM}(\mathcal{B})$ .

#### 3.1. Example

Let  $X = \{a\}$  and  $\mathcal{D} = \left\{ \frac{a}{\langle .7, .8, .6, .5 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .1, .4, .3, .2 \rangle} \right\}$

Then  $\mathfrak{S}_{NM} = \{0_N, 1_N, \mathcal{D}\}$  is a NMTS.

Let  $\mathcal{E} = \left\{ \frac{a}{\langle .2, .3, .3, .2 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .7, .8, .6, .5 \rangle} \right\}$  and

$$\mathcal{F} = \left\{ \frac{a}{\langle .8, .8, .9, .6 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .1, .4, .3, .1 \rangle} \right\}.$$

Since  $\mathcal{E} \subseteq \mathcal{F}$ .

Then  $int_{NM}(\mathcal{E}) = 0_N$  and  $int_{NM}(\mathcal{F}) = \mathcal{D}$

Hence,  $\mathcal{E} \subseteq \mathcal{F} \Rightarrow int_{NM}(\mathcal{E}) \subseteq int_{NM}(\mathcal{F})$ .

#### 3.2. Proposition

Let  $(X, \mathfrak{S}_{NM})$  be a NMTS, then

- (a)  $int_{NM}(\mathcal{A} \upmho \mathcal{B}) = int_{NM}(\mathcal{A}) \upmho int_{NM}(\mathcal{B})$ ;
- (b)  $int_{NM}(\mathcal{A}) \uplus int_{NM}(\mathcal{B}) \subseteq int_{NM}(\mathcal{A} \uplus \mathcal{B})$ .

Proof:

- (a) Since  $\mathcal{A} \upmho \mathcal{B} \subseteq \mathcal{A}$  and  $\mathcal{A} \upmho \mathcal{B} \subseteq \mathcal{B}$ , then we have from (iii),  
 $int_{NM}(\mathcal{A} \upmho \mathcal{B}) \subseteq int_{NM}(\mathcal{A})$  and  $int_{NM}(\mathcal{A} \upmho \mathcal{B}) \subseteq int_{NM}(\mathcal{B})$   
 $\Rightarrow int_{NM}(\mathcal{A} \upmho \mathcal{B}) \subseteq int_{NM}(\mathcal{A}) \upmho int_{NM}(\mathcal{B}) \quad \dots (1)$

Again, let  $x \in int_{NM}(\mathcal{A}) \upmho int_{NM}(\mathcal{B})$ .

Then  $x \in int_{NM}(\mathcal{A})$  and  $x \in int_{NM}(\mathcal{B})$ .

Hence,  $x$  is an interior point of each of the sets  $\mathcal{A}$  and  $\mathcal{B}$ .

It gives that  $\mathcal{A}$  and  $\mathcal{B}$  are neighbourhoods of  $x$ , so their intersection  $\mathcal{A} \upmho \mathcal{B}$  is also a neighbourhood of  $x$ .

Therefore,  $x \in int_{NM}(\mathcal{A} \upmho \mathcal{B})$ .

$$\begin{aligned} \text{Thus, } & x \in \text{int}_{NM}(\mathcal{A}) \cap \text{int}_{NM}(\mathcal{B}) \\ & \Rightarrow x \in \text{int}_{NM}(\mathcal{A} \cap \mathcal{B}) \\ & \Rightarrow \text{int}_{NM}(\mathcal{A}) \cap \text{int}_{NM}(\mathcal{B}) \subseteq \text{int}_{NM}(\mathcal{A} \cap \mathcal{B}) \quad \dots (2) \end{aligned}$$

From (1) and (2), we have

$$\text{int}_{NM}(\mathcal{A} \cap \mathcal{B}) = \text{int}_{NM}(\mathcal{A}) \cap \text{int}_{NM}(\mathcal{B}).$$

(b) From **Proposition 3.1** (iii), we have

$$\mathcal{A} \subseteq (\mathcal{A} \cup \mathcal{B}) \Rightarrow \text{int}_{NM}(\mathcal{A}) \subseteq \text{int}_{NM}(\mathcal{A} \cup \mathcal{B}) \text{ and}$$

$$\mathcal{B} \subseteq (\mathcal{A} \cup \mathcal{B}) \Rightarrow \text{int}_{NM}(\mathcal{B}) \subseteq \text{int}_{NM}(\mathcal{A} \cup \mathcal{B}).$$

Hence,  $\text{int}_{NM}(\mathcal{A}) \cup \text{int}_{NM}(\mathcal{B}) \subseteq \text{int}_{NM}(\mathcal{A} \cup \mathcal{B})$ .

### 3.2. Definition

Let  $(X, \mathfrak{S}_{NM})$  be a NMTS and let  $\mathcal{A} = \{ \langle x, \mu_{NM_i}, \sigma_{NM_i}, \delta_{NM_i} \rangle : x \in X \}$  be NMS, then the NMC of  $\mathcal{A}$  is the intersection of all neutrosophic multi closed sets (NMCoS) of  $X$  containing in  $\mathcal{A}$  and is defined as

$$cl_{NM}(\mathcal{A}) = \{ \langle x, \cap \mu_{NM_i}, \cup \sigma_{NM_i}, \cup \delta_{NM_i} \rangle : x \in X \}.$$

### 3.3. Proposition

Let  $(X, \mathfrak{S}_{NM})$  be a NMTS, then

$$(a) \quad cl_{NM}(0_N) = 0_N, cl_{NM}(1_N) = 1_N;$$

$$(b) \quad \mathcal{U} \subseteq cl_{NM}(\mathcal{U});$$

$$(c) \quad \mathcal{U} \text{ is NMCoS iff } \mathcal{U} = cl_{NM}(\mathcal{U});$$

$$(d) \quad \mathcal{U} \subseteq \mathcal{V} \Rightarrow cl_{NM}(\mathcal{U}) \subseteq cl_{NM}(\mathcal{V}).$$

**Proof:**

(a) Since  $0_N$  and  $1_N$  are closed sets, we have

$$cl_{NM}(0_N) = 0_N, cl_{NM}(1_N) = 1_N.$$

(b) Since  $cl_{NM}(\mathcal{U})$  is the smallest NMCoS containing  $\mathcal{U}$ , so  $\mathcal{U} \subseteq cl_{NM}(\mathcal{U})$ .

(c) Since  $\mathcal{U}$  is NMCoS, then  $\mathcal{U}$  itself is the smallest NMCoS containing  $\mathcal{U}$  and so,  $cl_{NM}(\mathcal{U}) = \mathcal{U}$ .

Conversely, let  $cl_{NM}(\mathcal{U}) = \mathcal{U}$ . Then  $cl_{NM}(\mathcal{U})$  is NMCoS and hence  $\mathcal{U}$  is also NMCoS.

(d) From (b), we have  $\mathcal{V} \subseteq cl_{NM}(\mathcal{V})$ . Since  $\mathcal{U} \subseteq \mathcal{V}$ , so we have  $\mathcal{U} \subseteq cl_{NM}(\mathcal{V})$ .

But  $cl_{NM}(\mathcal{V})$  is a NMCoS. So,  $cl_{NM}(\mathcal{V})$  is a NMCoS containing  $\mathcal{U}$ . Since  $cl_{NM}(\mathcal{U})$  is the smallest NMCoS containing  $\mathcal{U}$ , so we have  $cl_{NM}(\mathcal{U}) \subseteq cl_{NM}(\mathcal{V})$ .

Hence,  $\mathcal{U} \subseteq \mathcal{V} \Rightarrow cl_{NM}(\mathcal{U}) \subseteq cl_{NM}(\mathcal{V})$ .

### 3.4. Proposition

Let  $(X, \mathfrak{S}_{NM})$  be a NMTS, then

$$(a) \quad cl_{NM}(\mathcal{U} \cup \mathcal{V}) = cl_{NM}(\mathcal{U}) \cup cl_{NM}(\mathcal{V});$$

$$(b) \quad cl_{NM}(\mathcal{U} \cap \mathcal{V}) \subseteq cl_{NM}(\mathcal{U}) \cap cl_{NM}(\mathcal{V}).$$

**Proof of (a):**

Since  $\mathcal{U} \subseteq \mathcal{U} \cup \mathcal{V}$  and  $\mathcal{V} \subseteq \mathcal{U} \cup \mathcal{V}$ .

From **Proposition 3.3** (d), we have  $cl_{NM}(\mathcal{U}) \subseteq cl_{NM}(\mathcal{U} \cup \mathcal{V})$  and  $cl_{NM}(\mathcal{V}) \subseteq cl_{NM}(\mathcal{U} \cup \mathcal{V})$ .

Hence,  $cl_{NM}(\mathcal{U}) \cup cl_{NM}(\mathcal{V}) \subseteq cl_{NM}(\mathcal{U} \cup \mathcal{V}) \dots (1)$

Since  $cl_{NM}(\mathcal{U})$ ,  $cl_{NM}(\mathcal{V})$  are NMCoSs, so  $cl_{NM}(\mathcal{U}) \cup cl_{NM}(\mathcal{V})$  is also NMCoS.

Also,  $\mathcal{U} \subseteq cl_{NM}(\mathcal{U})$  and  $\mathcal{V} \subseteq cl_{NM}(\mathcal{V})$

$$\Rightarrow \mathcal{U} \cup \mathcal{V} \subseteq cl_{NM}(\mathcal{U}) \cup cl_{NM}(\mathcal{V}).$$

Thus,  $cl_{NM}(\mathcal{U}) \cup cl_{NM}(\mathcal{V})$  is a NMCoS containing  $\mathcal{U} \cup \mathcal{V}$ .

Since  $cl_{NM}(\mathcal{U} \cup \mathcal{V})$  is the smallest NMCoS containing  $\mathcal{U} \cup \mathcal{V}$ .

Therefore,  $cl_{NM}(\mathcal{U} \cup \mathcal{V}) \subseteq cl_{NM}(\mathcal{U}) \cup cl_{NM}(\mathcal{V}) \dots (2)$

From (1) and (2), we have

$$cl_{NM}(\mathcal{U} \cup \mathcal{V}) = cl_{NM}(\mathcal{U}) \cup cl_{NM}(\mathcal{V}).$$

### 3.2. Example

Let  $X = \{a\}$  and  $\mathcal{A} = \left\{ \frac{a}{\langle .7, .8, .6, .9 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .2, .4, .3, .2 \rangle} \right\}$  and

$$\mathcal{A}^c = \left\{ \frac{a}{\langle .2, .4, .3, .2 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .7, .8, .6, .9 \rangle} \right\}.$$

Then  $\mathfrak{S}_{NM} = \{0_N, 1_N, \mathcal{A}\}$  is a NMTS.

Let  $\mathcal{B} = \left\{ \frac{a}{\langle .7, .8, .6, .8 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .3, .5, .4, .3 \rangle} \right\}$  and

$$\mathcal{C} = \left\{ \frac{a}{\langle .8, .8, .7, .9 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .2, .3, .1, .2 \rangle} \right\}.$$

Then  $\mathcal{B} \cup \mathcal{C} = \left\{ \frac{a}{\langle .7, .8, .7, .9 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .2, .3, .1, .2 \rangle} \right\}$

So,  $cl_{NM}(\mathcal{B} \cup \mathcal{C}) = 1_N$

$$cl_{NM}(\mathcal{B}) = 1_N \text{ and } cl_{NM}(\mathcal{C}) = 1_N$$

Then  $cl_{NM}(\mathcal{B}) \cup cl_{NM}(\mathcal{C}) = 1_N \cup 1_N$

$$\Rightarrow cl_{NM}(\mathcal{B}) \cup cl_{NM}(\mathcal{C}) = 1_N.$$

Hence,  $cl_{NM}(\mathcal{B} \cup \mathcal{C}) = cl_{NM}(\mathcal{B}) \cup cl_{NM}(\mathcal{C})$ .

**Proof of (b):**

Since  $\mathcal{U} \cap \mathcal{V} \subseteq \mathcal{U} \Rightarrow cl_{NM}(\mathcal{U} \cap \mathcal{V}) \subseteq cl_{NM}(\mathcal{U})$  and

$$\mathcal{U} \cap \mathcal{V} \subseteq \mathcal{V} \Rightarrow cl_{NM}(\mathcal{U} \cap \mathcal{V}) \subseteq cl_{NM}(\mathcal{V}).$$

Hence,  $cl_{NM}(\mathcal{U} \cap \mathcal{V}) \subseteq cl_{NM}(\mathcal{U}) \cap cl_{NM}(\mathcal{V})$ .

**3.3. Example**

Let  $X = \{a\}$  and  $\mathcal{D} = \left\{ \frac{a}{\langle .7, .8, .6, .5 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .1, .4, .3, .2 \rangle} \right\}$  and

$$\mathcal{D}^c = \left\{ \frac{a}{\langle .1, .4, .3, .2 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .7, .8, .6, .5 \rangle} \right\}.$$

Then  $\mathfrak{S}_{NM} = \{0_N, 1_N, \mathcal{D}\}$  is a NMTS.

Let  $\mathcal{U} = \left\{ \frac{a}{\langle .1, .5, .3, .2 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .5, .8, .6, .5 \rangle} \right\}$  and

$$\mathcal{V} = \left\{ \frac{a}{\langle .2, .4, .3, .2 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .7, .7, .6, .5 \rangle} \right\}.$$

Then  $\mathcal{U} \cap \mathcal{V} = \left\{ \frac{a}{\langle .1, .4, .3, .2 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .7, .8, .6, .5 \rangle} \right\}$

So,  $cl_{NM}(\mathcal{U} \cap \mathcal{V}) = \mathcal{D}^c$

$$cl_{NM}(\mathcal{U}) = 1_N \text{ and } cl_{NM}(\mathcal{V}) = 1_N$$

Therefore,  $cl_{NM}(\mathcal{U}) \cap cl_{NM}(\mathcal{V}) = 1_N \cap 1_N$ .

$$\Rightarrow cl_{NM}(\mathcal{U}) \cap cl_{NM}(\mathcal{V}) = 1_N.$$

Hence,  $cl_{NM}(\mathcal{U} \cap \mathcal{V}) = cl_{NM}(\mathcal{U}) \cap cl_{NM}(\mathcal{V})$ .

**3.3. Definition**

Let  $(X, \mathfrak{S}_{NM})$  be a NMTS and let  $\mathcal{A} = \{ \langle x, \mu_{NM_i}, \sigma_{NM_i}, \delta_{NM_i} \rangle : x \in X \}$  be NMS, then the NMB of  $\mathcal{A}$  is defined as

$$Bd_{NM}(\mathcal{A}) = cl_{NM}(\mathcal{A}) \cap cl_{NM}(\mathcal{A}^c).$$

$$\text{i.e., } Bd_{NM}(\mathcal{A}) = 1_N - int_{NM}(\mathcal{A}) \cap [int_{NM}(1_N - \mathcal{A})].$$

**3.5. Proposition**

Let  $(X, \mathfrak{S}_{NM})$  be a NMTS and  $A$  be NMCoS then  $Bd_{NM}(A) \subseteq A$ .

Proof: Since  $A$  is NMCoS.

$$\Rightarrow cl_{NM}(A) = A.$$

By definition,  $Bd_{NM}(A) = cl_{NM}(A) \cap cl_{NM}(A^c)$

$$= cl_{NM}(A) \cap cl_{NM}(1_N - A)$$

$$= A \cap cl_{NM}(1_N - A)$$

$$\subseteq A$$

$$\Rightarrow Bd_{NM}(A) \subseteq A.$$

But the converse may not be true.

For this we cite an example-

**3.4. Example**

Let  $X = \{a\}$  and  $\mathcal{A} = \left\{ \frac{a}{\langle .8, .6, .7, .7 \rangle, \langle .7, .6, .7, .5 \rangle, \langle .8, .6, .7, .6 \rangle} \right\}$  and

$$\mathcal{A}^c = \left\{ \frac{a}{\langle .8, .6, .7, .6 \rangle, \langle .3, .4, .3, .5 \rangle, \langle .8, .6, .7, .7 \rangle} \right\}.$$

Then  $\mathfrak{S}_{NM} = \{0_N, 1_N, \mathcal{A}\}$  is a NMFS.

Let  $\mathcal{B} = \left\{ \frac{a}{\langle .9, .7, .8, .7 \rangle, \langle .7, .6, .7, .5 \rangle, \langle .8, .5, .6, .5 \rangle} \right\}$ ,

$$\mathcal{B}^c = \left\{ \frac{a}{\langle .8, .5, .6, .5 \rangle, \langle .3, .4, .3, .5 \rangle, \langle .9, .7, .8, .7 \rangle} \right\}$$

Then  $cl_{NM}(\mathcal{B}) = 1_N$  and  $cl_{NM}(\mathcal{B}^c) = \mathcal{A}^c$

So,  $Bd_{NM}(\mathcal{B}) = 1_N \cap \mathcal{A}^c = \mathcal{A}^c$

Thus,  $Bd_{NM}(\mathcal{B}) = \mathcal{A}^c \subseteq \mathcal{B}$ .

Hence,  $Bd_{NM}(\mathcal{B}) \subseteq \mathcal{B}$ .

But  $\mathcal{B}$  is not NMCoS.

### 3.6. Proposition

Let  $(X, \mathfrak{S}_{NM})$  be a NMFS and  $P, Q \subseteq X$ , then

- (a)  $Bd_{NM}(P) = Bd_{NM}(1_N - P) = cl_{NM}(P) - int_{NM}(P) = cl_{NM}(P) \cap cl_{NM}(1_N - P)$ .
- (b)  $cl_{NM}(P) = int_{NM}(P) \cup Bd_{NM}(P)$ ;
- (c)  $int_{NM}(P) = P - Bd_{NM}(P)$ ;
- (d)  $Bd_{NM}(cl_{NM}(P)) \subseteq Bd_{NM}(P)$ ;
- (e)  $Bd_{NM}(int_{NM}(P)) \subseteq Bd_{NM}(P)$ ;

**Proof:**

(a) We know that  $Bd_{NM}(P) = 1_N - int_{NM}(P) \cup int_{NM}(1_N - P)$  ... (1)

Replacing  $P$  by  $1_N - P$  in (1), we get

$$\begin{aligned} Bd_{NM}(1_N - P) &= 1_N - int_{NM}(1_N - P) \cup int_{NM}[1_N - (1_N - P)] \\ &= 1_N - int_{NM}(1_N - P) \cup int_{NM}(P) \\ &= Bd_{NM}(P) \quad [\text{from (1)}] \end{aligned}$$

Therefore,  $Bd_{NM}(1_N - P) = Bd_{NM}(P)$  ... (2)

Since  $1_N - int_{NM}(P) = cl_{NM}(1_N - P)$  ... (3)

and  $int_{NM}(1_N - P) = 1_N - cl_{NM}(P)$  ... (4)

Using (4) in (1), we have

$$Bd_{NM}(P) = 1_N - int_{NM}(P) \cup (1_N - cl_{NM}(P)).$$

Then  $x \in Bd_{NM}(P) \Leftrightarrow x \notin int_{NM}(P)$  and  $x \notin 1_N - cl_{NM}(P)$

$$\Leftrightarrow x \in 1_N - int_{NM}(P) \text{ and } x \in cl_{NM}(P)$$

Therefore,  $Bd_{NM}(P) = (1_N - int_{NM}(P)) \cap cl_{NM}(P)$

$$= cl_{NM}(1_N - P) \cap cl_{NM}(P) \quad [\text{from (3)}] \quad \dots (5)$$

From (2) and (5), we have

$$Bd_{NM}(P) = Bd_{NM}(1_N - P) = cl_{NM}(P) \cap cl_{NM}(1_N - P) \quad \dots (6)$$

Now,  $P \cap Q = P - (1_N - Q)$ .

$$\begin{aligned} cl_{NM}(P) \cap cl_{NM}(1_N - P) &= cl_{NM}(P) - [1_N - cl_{NM}(1_N - P)] \\ &= cl_{NM}(P) - [1_N - (1_N - int_{NM}(P))] \\ &= cl_{NM}(P) - int_{NM}(P) \end{aligned}$$

$$\Rightarrow cl_{NM}(P) \cap cl_{NM}(1_N - P) = cl_{NM}(P) - int_{NM}(P) \quad \dots (7)$$

Using (7) in (6), we have

$$\begin{aligned} Bd_{NM}(P) &= Bd_{NM}(1_N - P) \\ &= cl_{NM}(P) \cap cl_{NM}(1_N - P) \\ &= cl_{NM}(P) - int_{NM}(P). \end{aligned}$$

(b) From (5), we have

$$Bd_{NM}(P) = cl_{NM}(P) \cap cl_{NM}(1_N - P)$$

Now,

$$\begin{aligned} int_{NM}(P) \cup Bd_{NM}(P) &= int_{NM}(P) \cup [cl_{NM}(P) \cap cl_{NM}(1_N - P)] \\ &= (int_{NM}(P) \cup cl_{NM}(P)) \cap [cl_{NM}(P) \cup cl_{NM}(1_N - P)] \\ &= cl_{NM}(P) \cap [int_{NM}(P) \cup cl_{NM}(1_N - P)] \quad [\text{as } int_{NM}(P) \subseteq P \subseteq cl_{NM}(P)] \\ &= cl_{NM}(P) \cap [int_{NM}(P) \cup (1_N - int_{NM}(P))] \\ &= cl_{NM}(P) \cap 1_N \\ &= cl_{NM}(P). \end{aligned}$$

Hence,  $int_{NM}(P) \cup Bd_{NM}(P) = cl_{NM}(P)$ .

(c) From (5), we have

$$Bd_{NM}(P) = cl_{NM}(P) \cap cl_{NM}(1_N - P)$$

Now,

$$\begin{aligned} P - Bd_{NM}(P) &= P - cl_{NM}(P) \cap cl_{NM}(1_N - P) \\ &= (P - cl_{NM}(P)) \cup [P - cl_{NM}(1_N - P)] \\ &= 0_N \cup [P - cl_{NM}(1_N - P)] \\ &= 0_N \cup [P - (1_N - int_{NM}(P))] \\ &= P - (1_N - int_{NM}(P)) \\ &= P - int_{NM}(P) \\ &= int_{NM}(P). \end{aligned}$$

Hence,  $P - Bd_{NM}(P) = int_{NM}(P)$ .

(d) Since  $Bd_{NM}(P) = cl_{NM}(P) \cap cl_{NM}(1_N - P)$

We have

$$\begin{aligned} Bd_{NM}(cl_{NM}(P)) &= cl_{NM}(cl_{NM}(P)) \cap cl_{NM}[1_N - cl_{NM}(P)] \\ &= cl_{NM}(P) \cap cl_{NM}[1_N - cl_{NM}(P)] \quad [\text{as } cl_{NM}(cl_{NM}(A)) = cl_{NM}(A)] \\ &= cl_{NM}(P) \cap cl_{NM}[int_{NM}(1_N - P)] \\ &\subseteq cl_{NM}(P) \cap cl_{NM}(1_N - P) \\ &= Bd_{NM}(P) \end{aligned}$$

Hence,  $Bd_{NM}(cl_{NM}(P)) \subseteq Bd_{NM}(P)$ .

(e) Since  $Bd_{NM}(P) = cl_{NM}(P) \cap cl_{NM}(1_N - P)$

Then  $Bd_{NM}(int_{NM}(P)) = cl_{NM}(int_{NM}(P)) \cap cl_{NM}[1_N - int_{NM}(P)]$

But  $1_N - int_{NM}(P) = cl_{NM}(1_N - P)$

$$\begin{aligned} \text{So, } cl_{NM}(1_N - int_{NM}(P)) &= cl_{NM}[cl_{NM}(1_N - P)] \\ &= cl_{NM}(1_N - P) \quad [\text{as } cl_{NM}(cl_{NM}(A)) = cl_{NM}(A)] \\ &\subseteq cl_{NM}(P) \cap cl_{NM}(1_N - P) \\ &= Bd_{NM}(P) \end{aligned}$$

Hence,  $Bd_{NM}(int_{NM}(P)) \subseteq Bd_{NM}(P)$ .

### 3.7. Proposition

Let  $(X, \mathfrak{N}_{NM})$  be a NMTS and  $\mathcal{U}, \mathcal{V} \subseteq X$ , then

$$(a) \quad Bd_{NM}(\mathcal{U} \cup \mathcal{V}) \subseteq Bd_{NM}(\mathcal{U}) \cup Bd_{NM}(\mathcal{V});$$

$$(b) \quad Bd_{NM}(\mathcal{U} \cap \mathcal{V}) \subseteq Bd_{NM}(\mathcal{U}) \cup Bd_{NM}(\mathcal{V}).$$

Proof:

$$\begin{aligned} (a) \quad Bd_{NM}(\mathcal{U} \cap \mathcal{V}) &= cl_{NM}(\mathcal{U} \cup \mathcal{V}) \cap cl_{NM}(\mathcal{U} \cup \mathcal{V})^c \\ &= (cl_{NM}(\mathcal{U}) \cup cl_{NM}(\mathcal{V})) \cap (cl_{NM}(\mathcal{U}^c) \cap cl_{NM}(\mathcal{V}^c)) \\ &\subseteq (\mathcal{U} \cup \mathcal{V}) \cap (cl_{NM}(\mathcal{U}^c) \cap cl_{NM}(\mathcal{V}^c)) \\ &= [cl_{NM}(\mathcal{U}) \cap (cl_{NM}(\mathcal{U}^c) \cap cl_{NM}(\mathcal{V}^c))] \cup [cl_{NM}(\mathcal{V}) \cap (cl_{NM}(\mathcal{U}^c) \cap cl_{NM}(\mathcal{V}^c))] \\ &= [(cl_{NM}(\mathcal{U}) \cap cl_{NM}(\mathcal{U}^c)) \cap cl_{NM}(\mathcal{V}^c)] \cup [(cl_{NM}(\mathcal{V}) \cap cl_{NM}(\mathcal{V}^c)) \cap cl_{NM}(\mathcal{U}^c)] \\ &= [Bd_{NM}(\mathcal{U}) \cap cl_{NM}(\mathcal{V}^c)] \cup [Bd_{NM}(\mathcal{V}) \cap \mathcal{U}^c] \\ &\subseteq Bd_{NM}(\mathcal{U}) \cup Bd_{NM}(\mathcal{V}) \end{aligned}$$

Hence,  $Bd_{NM}(\mathcal{U} \cup \mathcal{V}) \subseteq Bd_{NM}(\mathcal{U}) \cup Bd_{NM}(\mathcal{V})$ .

$$\begin{aligned}
 (b) \quad Bd_{NM}(U \cap V) &= cl_{NM}(U \cap V) \cap cl_{NM}(U \cap V)^c \\
 &\subseteq (cl_{NM}(U) \cap cl_{NM}(V)) \cap cl_{NM}(U^c \cap V^c) \\
 &= (cl_{NM}(U) \cap cl_{NM}(V)) \cap (cl_{NM}(U^c) \cap cl_{NM}(V^c)) \\
 &= [(cl_{NM}(U) \cap cl_{NM}(V)) \cup cl_{NM}(U^c)] \cup [(cl_{NM}(U) \cap cl_{NM}(V)) \cap cl_{NM}(U^c)] \\
 &= [(cl_{NM}(U) \cap cl_{NM}(U^c)) \cap cl_{NM}(V)] \cup [cl_{NM}(U) \cap (cl_{NM}(V) \cap cl_{NM}(V^c))] \\
 &= [Bd_{NM}(U) \cap cl_{NM}(V)] \cup [cl_{NM}(U) \cap Bd_{NM}(V)] \\
 &\subseteq Bd_{NM}(U) \cup Bd_{NM}(V)
 \end{aligned}$$

Hence,  $Bd_{NM}(U \cap V) \subseteq Bd_{NM}(U) \cup Bd_{NM}(V)$ .

**3.4. Definition**

Let  $(X, \mathfrak{S}_{NM})$  be a NMTS and let  $\mathcal{A} = \{ \langle x, \mu_{NM_i}, \sigma_{NM_i}, \delta_{NM_i} \rangle : x \in X \}$  be NMS, then the NME of  $\mathcal{A}$  is defined as  $ext_{NM}(\mathcal{A}) = \cup \{ G \in \mathfrak{S}_{NM} : G \subseteq \mathcal{A}^c \}$ .

**3.8. Proposition**

Let  $(X, \mathfrak{S}_{NM})$  be a NMTS and  $P, Q \subseteq X$ , then

- (a)  $ext_{NM}(1_N) = 0_N, ext_{NM}(0_N) = 1_N;$
- (b)  $ext_{NM}(P) \subseteq P^c;$
- (c)  $ext_{NM}(P) = ext_{NM}[(ext_{NM}(P))^c].$

Proof:

- (a) Since  $ext_{NM}(1_N) = int_{NM}(1_N - 1_N) = int_{NM}(0_N) = 0_N$   
and  $ext_{NM}(0_N) = int_{NM}(1_N - 0_N) = int_{NM}(1_N) = 1_N.$

- (b) Since  $ext_{NM}(P) = int_{NM}(1_N - P) \subseteq 1_N - P = P^c.$

Hence,  $ext_{NM}(P) \subseteq P^c.$

$$\begin{aligned}
 (c) \quad \text{Since } [ext_{NM}(P)]^c &= [int_{NM}(1_N - P)]^c \\
 &= 1_N - (int_{NM}(1_N - P)) \\
 \Rightarrow ext_{NM}[(ext_{NM}(P))^c] &= ext_{NM}[1_N - (int_{NM}(1_N - P))] \\
 &= int_{NM}[1_N - \{1_N - (int_{NM}(1_N - P))\}] \\
 &= int_{NM}[int_{NM}(1_N - P)] \\
 &= int_{NM} int_{NM}(1_N - P) \\
 &= int_{NM}(1_N - P) \quad [as \ int_{NM} \ int_{NM}(A) = int_{NM}(A)] \\
 &= ext_{NM}(P)
 \end{aligned}$$

Hence,  $ext_{NM}[(ext_{NM}(P))^c] = ext_{NM}(P).$

**3.5. Example**

Let  $X = \{a\}$  and  $\mathcal{A} = \left\{ \frac{a}{\langle (8,8,7,7), (5,5,5,5), (2,3,2,3) \rangle} \right\}$

Then  $\mathfrak{S}_{NM} = \{0_N, 1_N, \mathcal{A}\}$  is a NMTS.

Let  $\mathcal{B} = \left\{ \frac{a}{\langle (2,7,6,5), (5,5,5,5), (8,6,4,6) \rangle} \right\}$  and

$$\mathcal{B}^c = \left\{ \frac{a}{\langle (8,6,4,6), (5,5,5,5), (2,7,6,5) \rangle} \right\}.$$

Then  $ext_{NM}(\mathcal{B}) = 0_N$  and

$$\begin{aligned}
 [ext_{NM}(\mathcal{B})]^c &= 1_N \\
 \Rightarrow ext_{NM}[(ext_{NM}(\mathcal{B}))^c] &= ext_{NM}(1_N) = 0_N.
 \end{aligned}$$

Hence,  $ext_{NM}[(ext_{NM}(P))^c] = ext_{NM}(P).$

**3.9. Proposition**

Let  $(X, \mathfrak{S}_{NM})$  be a NMTS and  $P, Q \subseteq X$ , then

- (i)  $P \subseteq Q \Rightarrow ext_{NM}(Q) \subseteq ext_{NM}(P);$
- (ii)  $int_{NM}(P) = ext_{NM}[ext_{NM}(P)];$

$$(iii) \text{ ext}_{NM}(P \Psi Q) = \text{ ext}_{NM}(P) \cap \text{ ext}_{NM}(Q).$$

**Proof of (i)**

$$\begin{aligned} \text{Since } P \subseteq Q &\Rightarrow 1_N - Q \subseteq 1_N - P \\ &\Rightarrow \text{ int}_{NM}(1_N - Q) \subseteq \text{ int}_{NM}(1_N - P) \\ &\Rightarrow \text{ ext}_{NM}(Q) \subseteq \text{ ext}_{NM}(P). \end{aligned}$$

$$\text{Hence, } P \subseteq Q \Rightarrow \text{ ext}_{NM}(Q) \subseteq \text{ ext}_{NM}(P).$$

### 3.6. Example

Let  $X = \{a\}$  and

$$\mathcal{E} = \left\{ \frac{a}{\langle .8, .8, .7, .7 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .2, .3, .2, .3 \rangle} \right\}$$

Then  $\mathfrak{S}_{NM} = \{0_N, 1_N, \mathcal{E}\}$  is a NMTS.

$$\text{Let } \mathcal{R} = \left\{ \frac{a}{\langle .2, .3, .3, .4 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .8, .8, .9, .9 \rangle} \right\} \text{ and}$$

$$\mathcal{S} = \left\{ \frac{a}{\langle .8, .9, .7, .8 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .2, .2, .3, .3 \rangle} \right\}.$$

$$\text{Then } \mathcal{R}^c = \left\{ \frac{a}{\langle .8, .8, .9, .9 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .2, .3, .3, .4 \rangle} \right\} \text{ and}$$

$$\mathcal{S}^c = \left\{ \frac{a}{\langle .2, .2, .3, .3 \rangle, \langle .5, .5, .5, .5 \rangle, \langle .8, .9, .7, .8 \rangle} \right\}.$$

Since  $\mathcal{R} \subseteq \mathcal{S}$ .

$$\text{Then } \text{ ext}_{NM}(\mathcal{R}) = \mathcal{E} \text{ and } \text{ ext}_{NM}(\mathcal{S}) = 0_N$$

$$\text{Hence, } \mathcal{R} \subseteq \mathcal{S} \Rightarrow \text{ ext}_{NM}(\mathcal{S}) \subseteq \text{ ext}_{NM}(\mathcal{R}).$$

**Proof of (ii)**

$$\begin{aligned} \text{Since } \text{ ext}_{NM}(P) &= \text{ int}_{NM}(1_N - P) \subseteq 1_N - P \\ &\Rightarrow \text{ ext}_{NM}(P) \subseteq 1_N - P. \end{aligned}$$

$$\begin{aligned} \text{Now, } P \subseteq Q &\Rightarrow \text{ ext}_{NM}(Q) \subseteq \text{ ext}_{NM}(P), \text{ we have} \\ &\Rightarrow \text{ ext}_{NM}(1_N - P) \subseteq \text{ ext}_{NM}[\text{ ext}_{NM}(P)] \quad \dots (1) \end{aligned}$$

$$\begin{aligned} \text{But } \text{ ext}_{NM}(1_N - P) &= \text{ ext}_{NM}(P^c) \\ &= \text{ int}_{NM}(1_N - P^c) \\ &= \text{ int}_{NM}[1_N - (1_N - P)] \\ &= \text{ int}_{NM}(P). \end{aligned}$$

$$\text{From (1), we have } \text{ int}_{NM}(P) \subseteq \text{ ext}_{NM}[\text{ ext}_{NM}(P)].$$

**Proof of (iii)**

$$\begin{aligned} \text{Since } \text{ ext}_{NM}(P \Psi Q) &= \text{ int}_{NM}[1_X - (P \Psi Q)] \\ &= \text{ int}_{NM}[(1_N - P) \cap (1_N - Q)] \\ &= \text{ int}_{NM}[P^c \cap Q^c] \\ &= \text{ int}_{NM}(P^c) \cap \text{ int}_{NM}(Q^c) \\ &= \text{ ext}_{NM}(P) \cap \text{ ext}_{NM}(Q). \end{aligned}$$

$$\text{Hence, } \text{ ext}_{NM}(P \Psi Q) = \text{ ext}_{NM}(P) \cap \text{ ext}_{NM}(Q).$$

## 5. Conclusions

In this study, topological space is studied based on NMS. In this work, the definitions of NMI, NMC, NME, and NMB were observed. With examples, the various NMTS propositions are examined. Hope this study will help in the further development of NMTS.

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## References

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- [1] Zadeh, L.A. "Fuzzy sets". *Inf. Control*, 8, pp.338–353, 1965.
- [2] Atanassov, K.T. "Intuitionistic fuzzy sets". *Fuzzy Sets and Systems*, 20, pp.87–96, 1986.
- [3] Chang, C.L. "Fuzzy topological spaces". *J. Math. Anal. Appl.*, 24(1), pp.182–190, 1968.
- [4] Çoker, D. "An introduction to intuitionistic fuzzy topological spaces". *Fuzzy Sets and Systems*, 88(1), pp.81–89, 1997.
- [5] Kelly, J. "Bitopological spaces". *Proc. Lond. Math. Soc.*, 3(1), pp.71–89, 1963.
- [6] Mwachary, D.D. and Basumatary, B. "A note on neutrosophic bitopological spaces". *Neutrosophic Sets and Systems*, 33, pp.134–144, 2020.
- [7] Smarandache, F. "Neutrosophy / Neutrosophic probability, set, and logic". American Research Press, 1998. <http://gallup.unm.edu/~smarandache/NeutLog.txt>
- [8] Smarandache, F. "Neutrosophic set-a generalization of the intuitionistic fuzzy set". *Int. J. Pure Appl. Math.*, 24(3), pp.287–297, 2005.
- [9] Smarandache, F. "Neutrosophic perspectives: triplets, duplets, multisets, hybrid operators, modal logic, hedge algebras and applications". Pons Editions, Bruxelles, 323, pp.115–119, 2017.
- [10] Smarandache, F. "Neutrosophy and neutrosophic logic, first international conference on neutrosophy, neutrosophic logic, set, probability, and statistics". University of New Mexico, Gallup, NM 87301, USA, 2002.
- [11] Broumi, S., Deli, I. and Smarandache, F. "Relations on interval valued neutrosophic soft sets". *J. New Results Sci.*, 3(5), pp.1–20, 2014.
- [12] Broumi, S., Deli, I. and Smarandache, F. "Neutrosophic parametrized soft set theory and its decision making". *International Frontier Science Letters*, 1(1), pp.1–11, 2014.
- [13] Sumathi, I.R. and Arockiarani, I. "Fuzzy neutrosophic groups". *Adv. Fuzzy Math*, 10(2), pp.117–122, 2015.
- [14] Sumathi, I.R. and Arockiarani, I. Topological group structure of neutrosophic set. *Journal of Advanced Studies in Topology*, 7(1), pp.12–20, 2016.
- [15] Salama, A.A. and Alblowi, S.A. "Neutrosophic set and neutrosophic topological spaces". *IOSR Journal of Mathematics (IOSR-JM)*, 3(4), pp.31–35, 2012.
- [16] Al-Hamido, R.K. "Neutrosophic crisp bi-topological spaces". *Neutrosophic Sets and Systems*, 21, pp.66–73, 2018.
- [17] Hamido, R.K. and Gharibah, T. "Neutrosophic crisp tri-topological spaces". *Journal of New Theory*, (23), pp.13–21, 2018.
- [18] Devi, R., Dhavaseelan, R. and Jafari, S. "On separation axioms in an ordered neutrosophic bitopological space". *Neutrosophic Sets and Systems*, 18(1), pp.27–36, 2017.
- [19] Basumatary, B. and Wary, N. "A study on neutrosophic bitopological group". *Neutrosophic Sets and Systems*, (38), pp.164–178, 2020.
- [20] Basumatary, B., Wary, N., Saeed, M. and Saqlain, M. "On some properties of plithogenic neutrosophic hypersoft almost topological group". *Neutrosophic Sets and Systems*, 43, pp.169–179, 2020.
- [21] Kazancı, O., Hoskova-Mayerova, S. and Davvaz, B. "Application multi-fuzzy soft sets in hypermodules". *Mathematics*, 9(18), pp.2182, 2021.
- [22] Blizard, W. D. "Multiset theory". *Notre Dame Journal of formal logic*, 30(1), pp.36–66, 1989.
- [23] Tella, Y. and Daniel, S. "A note on power whole multiset of a multiset and multiset topologies". *Journal of Computer and Mathematical Sciences*, 5(1), pp.1–122, 2014.
- [24] Onasanya, B. O. and Hoskova-Mayerova, S. "Multi-fuzzy group induced by multisets". *Ital. J. Pure Appl. Math.*, 41, pp.597–604, 2019.
- [25] Al Tahan, M., Hoskova-Mayerova, S. and Davvaz, B. "Fuzzy multi-polygroups". *Journal of Intelligent & Fuzzy Systems*, 38(2), pp.2337–2345, 2020.
- [26] Sebastian, S. and Ramakrishnan, T. V. "Multi-fuzzy sets: An extension of fuzzy sets". *Fuzzy Information and Engineering*, 3(1), pp.35–43, 2011.
- [27] Dey, A. and Pal, M. "Multi-fuzzy complex numbers and multi-fuzzy complex sets". *International Journal of*

- Fuzzy System Applications (IJFSA), 4(2), pp.15–27, 2015.
- [28] Uluçay, V. and Şahin, M. Neutrosophic multigroups and applications. *Mathematics*, 7(1), pp.95, 2019.
- [29] Basumatary, B., Wary, N., Mwchahary, D.D., Brahma, A.K., Moshahary, J., Basumatary, U.R., and Basumatary, J. “A study on some properties of neutrosophic multi topological group”. *Symmetry*, 13(9), pp.1689, 2021.
- [30] Basumatary, B. “Towards forming the field of fuzzy closure with reference to fuzzy boundary”. *Journal of Process Management. New Technologies*, 4(1), pp.30–40, 2016.