Bipolar neutrosophic soft generalized pre-continuous mappings

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Abstract

In this study, new classes of continuous mappings in bipolar neutrosophic soft topological space, namely bipolar neutrosophic soft continuous mappings and bipolar neutrosophic soft generalized pre-continuous mappings has been introduced. Continuity mappings preserves topological structures such as closeness, openness, compactness and so on. Here, we have proposed and investigated various continuous mappings based on bipolar neutrosophic soft sets. Further, we investigated some of their properties and relations with other mappings with examples.
Keywords: Bipolar neutrosophic soft set; BNSGP-continuous; BNSG-closed set; Neutrosophic set; BNSS-topology.

1 Introduction

Not all real world problems has exact solutions; in that case, we have to explore most suitable solutions according to the problem nature. Fuzzy sets are one of the kind to compute the solution for the given problem which contains inconsistent information. The fuzzy sets was originally introduced by Zadeh.[9] Fuzzy set has numerous applications in almost all mathematical modeling situations. Cheng[6] introduced the study of fuzzy topological spaces in 1968. Atanassov[8] developed some concepts in fuzzy sets and proposed intiutionistic fuzzy sets which is the extension of conventional Fuzzy sets. Smarandache[14,15] developed a new concept called neutrosophic sets which deals with uncertain information in scientific problems efficiently. Also Smarandache[16] extend the soft set to hyper soft set by generalizing existing soft set in 2018. Since the introduction of neutrosophic sets, many researchers were proposed various kind of neutrosophic sets by extending/ altering the existing sets.[3–5,13]

On the other side, topology is the mathematical area which is used in many scientific and engineering problems. A.A.Salama et al.[1] developed a new concept neutrosophic topology, i.e. the topology concepts were introduced for neutrosophic sets. Molodtsov[12] introduced soft set theory in 1999. Many neutrosophic sets were introduced by researchers.[2,7,18] Norman Levine[10,11] introduced generalized closed set and some continuity mappings in point set topology in 1970. In this study, bipolar neutrosophic soft generalized pre-continuous mappings are proposed.

This paper is organized as follows: Section 2 consists of the required results and definitions for the proposed mappings. Section 3 has the proposed mapping namely, bipolar neutrosophic soft continuous mapping and the relationship between the proposed mapping with the conventional mappings. In section 4, bipolar neutrosophic soft generalized pre-continuous mapping have been proposed and some of the relation between the proposed mapping and other existing mappings are discussed. Section 5 concludes the proposed works and outline of the future work.

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2 Preliminaries

Definition 2.1. [14] Let X be a universe set. For every $x \in X$, the components $u(x), v(x)$ and $w(x)$ are truth, indeterminate and false degrees of x. Then the Neutrosophic set (NS) over X be defined as follows.
$N = \{u(x), v(x), w(x) : x \in X\}$
Here, $u(x), v(x), w(x)$ ranges in the non-standard interval $]^{-}0, 1^{+}[$ and their sum $^{-}0 \leq u+v+w \leq 3^{+}$. For scientific problems, we prefer standard interval $[0, 1]$ instead of non-standard interval and it is called single-valued neutrosophic set.

Definition 2.2. [2] For the universe set X and positive member values $u^{+}, v^{+}, w^{+} : E \to [0, 1]$, negative member values $u^{-}, v^{-}, w^{-} : E \to [-1, 0]$, A bipolar neutrosophic set (BNS) is defined by//

$$B = \left\{ \langle x, u^{+}(x), v^{+}(x), w^{+}(x), u^{-}(x), v^{-}(x), w^{-}(x) \rangle : x \in X \right\}$$

Definition 2.3. [12] A soft set is a function which maps a parameter set to the power set of X. It is denoted by (f, E) and is defined by
$f : E \to P(x)$
Each member of X is parameterized with the parameter set E by the function f.

Definition 2.4. [2] A bipolar neutrosophic soft set (BNSS) is the fusion of soft set and bipolar neutrosophic set and is defined as follws.

$$BNS = (f_A, E) = \{\langle e, f_A(x) \rangle : e \in A \subset E, f_A(x) \in BNS(X)\}$$

Here $f_A(x) = \left\{ \left\langle x, u^{+}_{f_{A(e)}}(x), v^{+}_{f_{A(e)}}(x), w^{+}_{f_{A(e)}}(x), u^{-}_{f_{A(e)}}(x), v^{-}_{f_{A(e)}}(x), w^{-}_{f_{A(e)}}(x) \right\rangle : x \in X \right\}$.

Definition 2.5. [2] Let B be a $BNSS$. Then the complement of B is defined as

$$B^c = \left\{ \left\langle e, w^{+}_{f}(e), 1 - v^{+}_{f}(e), u^{+}_{f}(e), w^{-}_{f}(e), -1 - v^{-}_{f}(e), u^{-}_{f}(e) \right\rangle \right\}.$$

Definition 2.6. [2] Let $\phi_{\mathbb{B}}$ be a null $BNSS$ and is defined as

$$\phi_{\mathbb{B}} = \left\{ \langle e_i, \{x_i, 0, 1, 1, 0, -1, -1\} \rangle : x \in X, e \in E \right\}$$

Definition 2.7. [2] Let $1_{\mathbb{B}}$ be a complete $BNSS$ and is defined as

$$1_{\mathbb{B}} = \left\{ \langle e_i, \{x_i, 1, 0, 0, -1, 0, 0\} \rangle : x \in X, e \in E \right\}$$

Definition 2.8. [2] Let B_1 and B_2 be two $BNSS$s. Then their union $B_1 \cup B_2$ is defined as

$$B_1 \cup B_2 = \left\{ \left\langle e, \cup_i f^{(i)}(e) \right\rangle \right\}.$$

Here,

$$\bigcup_i f^{(i)}(e) = \Big\{ \langle x, max \left[u^{+}_{fi}(e)(x) \right], min \left[v^{+}_{fi}(e)(x) \right], min \left[w^{+}_{fi}(e)(x) \right], min \left[u^{-}_{fi}(e)(x) \right], max \left[v^{-}_{fi}(e)(x) \right], max \left[w^{-}_{fi}(e)(x) \right] \rangle \Big\}$$

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Definition 2.9. [2] Let B_1 and B_2 be two $BNSS$s. Then their intersection $B_1 \cap B_2$ is defined as

$$B_1 \cap B_2 = \left\{ \left\langle e, \cap_i f^{(i)}(e) \right\rangle \right\}.$$

Here,

$$\bigcap_i f^{(i)}(e) = \left\{ \langle x, min\left[u^+_{fi}(e)(x)\right], max\left[v^+_{fi}(e)(x)\right], max\left[w^+_{fi}(e)(x)\right], \right.$$
$$\left. max\left[u^-_{fi}(e)(x)\right], min\left[v^-_{fi}(e)(x)\right], min\left[w^-_{fi}(e)(x)\right] \rangle \right\}$$

Definition 2.10. [2] Let B_1 and B_2 be two $BNSS$s. Then B_1 is called subset of B_2 (i.e. $B_1 \subseteq B_2$) only if the following condition hold.
For every $x \in X$ and $e \in E$,

$$\{B_1 \subseteq B_2\} = \left\{ \begin{array}{c} \left[u^+_{B_1}(x) \leq u^+_{B_2}(x)\right], \left[v^+_{B_1}(x) \geq v^+_{B_2}(x)\right], \left[w^+_{B_1}(x) \geq w^+_{B_2}(x)\right] \\ \left[u^-_{B_1}(x) \geq u^-_{B_2}(x)\right], \left[v^-_{B_1}(x) \leq v^-_{B_2}(x)\right], \left[w^-_{B_1}(x) \leq w^-_{B_2}(x)\right] \end{array} \right\}$$

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Definition 2.11. [10] Let B be a subset of a topological space (X, τ).

1. If $int(cl(B)) \subseteq B$, then B is semi-closed set (SCS)
2. If $cl(int(B)) \subseteq B$, then B is pre-closed set (PCS)
3. If $int(cl(int(B))) \subseteq B$, then B is semi pre-closed set (SPCS)
4. If $cl(int(cl(B))) \subseteq B$, then B is α-closed set (αCS)
5. If $B = cl(int(B))$, then B is regular closed set (RCS).

Definition 2.12. [6,10,11] Let B be a subset of a topological space (X, τ).

1. if $cl(B) \subseteq U$, then B is generalized closed set (g-closed) whenever $B \subseteq U$ and U is open in X.
2. if $scl(B) \subseteq U$, then B is generalized semi closed set (gs-closed) whenever $B \subseteq U$ and U is open in X.
3. if $\alpha cl(B) \subseteq U$, then B is generalized α-closed set (gα-closed) whenever $B \subseteq U$ and U is open in X.
4. if $pcl(B) \subseteq U$, then B is generalized pre-closed set (gp-closed) whenever $B \subseteq U$ and U is open in X.
5. if $spcl(B) \subseteq U$, then B is generalized semi pre-closed set (gsp-closed) whenever $B \subseteq U$ and U is open in X.

Definition 2.13. [11] Let (X, τ) and (Y, σ) be any two topological spaces. For every closed set V of (Y, σ), a map $f : (X, \tau) \to (Y, \sigma)$ is said to be

1. semi-continuous if $f^{-1}(V)$ is semi-closed set in (X, τ).
2. pre-continuous if $f^{-1}(V)$ is pre-closed set in (X, τ).

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3. semi pre-continuous if $f^{-1}(V)$ is semi pre-closed set in (X, τ).
4. α-continuous if $f^{-1}(V)$ is α-closed set in (X, τ).
5. generalized continuous if $f^{-1}(V)$ is generalized closed set in (X, τ).
6. generalized semi-continuous if $f^{-1}(V)$ is generalized semi-closed set in (X, τ).
7. generalized pre-continuous if $f^{-1}(V)$ is generalized pre-closed set in (X, τ).
8. generalized semi pre-continuous if $f^{-1}(V)$ is generalized semi pre-closed set in (X, τ).
9. α-generalized continuous if $f^{-1}(V)$ is α-generalized closed set in (X, τ).

Definition 2.14. [17] A bipolar neutrosophic soft topology (BNST) on X is a collection τ of bipolar neutrosophic soft sets (BNSS) in X satisfying the following conditions:

1). $\phi_{\mathbb{B}}, 1_{\mathbb{B}} \in \tau_{\mathbb{B}}$

2). $\bigcup_{i \in n} \mathbb{B}_i \in \tau_{\mathbb{B}}$ for each $\mathbb{B}_i \in \tau_{\mathbb{B}}$

3). $\mathbb{B}_i \cap \mathbb{B}_j \in \tau_{\mathbb{B}}$ for any $\mathbb{B}_i, \mathbb{B}_j \in \tau_{\mathbb{B}}$

The pair $(X, \tau_{\mathbb{B}})$ is called $\mathbb{BNSS}$-topological space. The members of $\tau_{\mathbb{B}}$ are called bipoar neutrosophic soft open sets (BNOS) and their complements are called bipoar neutrosophic soft closed sets (BNCS).

3 Bipolar neutrosophic soft continuous mapping

Definition 3.1. Let $(X, \tau^1_{\mathbb{B}}, E)$ and $(Y, \tau^2_{\mathbb{B}}, E')$ be two bipolar neutrosophic soft topological spaces. For every bipolar neutrosophic closed set B of $(Y, \tau^2_{\mathbb{B}}, E')$, a map $\psi : ((X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ is said to be,

1. Bipolar neutrosophic soft continuous (BNS-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft closed set in $(X, \tau^1_{\mathbb{B}}, E)$.
2. Bipolar neutrosophic soft semi-continuous (BNSS-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft semi-closed set in $(X, \tau^1_{\mathbb{B}}, E)$.
3. Bipolar neutrosophic soft pre-continuous (BNSP-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft pre-closed set in $(X, \tau^1_{\mathbb{B}}, E)$.
4. Bipolar neutrosophic soft semi pre-continuous (BNSSP-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft semi pre-closed set in $(X, \tau^1_{\mathbb{B}}, E)$.
5. Bipolar neutrosophic soft α-continuous (BNSα-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft α-closed set in $(X, \tau^1_{\mathbb{B}}, E)$.
6. Bipolar neutrosophic soft regular continuous (BNSR-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft regular closed set in $(X, \tau^1_{\mathbb{B}}, E)$.

Definition 3.2. Let $(X, \tau^1_{\mathbb{B}}, E)$ and $(Y, \tau^2_{\mathbb{B}}, E')$ be two bipolar neutrosophic soft topological spaces. For every bipolar neutrosophic closed set B of $(Y, \tau^2_{\mathbb{B}}, E')$, a map $\psi : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ is said to be,

1. Bipolar neutrosophic soft generalized continuous (BNSG-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft generalized closed set in $(X, \tau^1_{\mathbb{B}}, E)$.
2. Bipolar neutrosophic soft generalized semi-continuous (BNSGS-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft generalized semi-closed set in $(X, \tau^1_{\mathbb{B}}, E)$.
3. Bipolar neutrosophic soft generalized α-continuous (BNSGα-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft generalized α-closed set in $(X, \tau^1_{\mathbb{B}}, E)$.

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4 Bipolar neutrosophic soft generalized pre-continuous mapping

Definition 4.1. A map $\psi : (X, \tau^1_{\mathbb{B}}, E) \rightarrow (Y, \tau^2_{\mathbb{B}}, E')$ is said to be bipolar neutrosophic soft generalized pre-continuous (BNSGP-continuous) mapping if $\psi^{-1}(B)$ is BNSGPCS in $(X, \tau^1_{\mathbb{B}}, E)$ for every bipolar neutrosophic soft closed set B of $(Y, \tau^2_{\mathbb{B}}, E')$.

Example 4.2. Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}$ and $E = \{e_1, e_2\}, E' = \{e'_1, e'_2\}$. Also the $BNSS$s are

$$B_1 = \left\{ \begin{array}{l} \langle e_1, \{x_1, 0.5, 0.4, 0.2, -0.5, -0.5, -0.8\}, \{x_2, 0.3, 0.7, 0.4, -0.6, -0.3, -0.5\}\rangle, \\ \langle e_2, \{x_1, 0.4, 0.6, 0.3, -0.2, -0.3, -0.5\}, \{x_2, 0.7, 0.3, 0.2, -0.2, -0.5, -0.5\}\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \langle e_1, \{x_1, 0.6, 0.3, 0.1, -0.7, -0.3, -0.5\}, \{x_2, 0.7, 0.4, 0.2, -0.7, -0.2, -0.2\}\rangle, \\ \langle e_2, \{x_1, 0.6, 0.3, 0.3, -0.4, -0.2, -0.3\}, \{x_2, 0.8, 0.2, 0.2, -0.4, -0.2, -0.4\}\rangle \end{array} \right\}$$

$$B_3 = \left\{ \begin{array}{l} \langle e'_1, \{y_1, 0.4, 0.6, 0.5, -0.3, -0.6, -0.9\}, \{y_2, 0.2, 0.8, 0.5, -0.4, -0.4, -0.6\}\rangle, \\ \langle e'_2, \{y_1, 0.3, 0.7, 0.5, -0.1, -0.5, -0.5\}, \{y_2, 0.5, 0.4, 0.3, -0.2, -0.6, -0.6\}\rangle \end{array} \right\}$$

Then, $\tau^1_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_1, B_2\}$ and $\tau^2_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_3\}$ are $BNST$s on X and Y respectively. Now define a mapping $\psi : (X, \tau^1_{\mathbb{B}}, E) \rightarrow (Y, \tau^2_{\mathbb{B}}, E')$ such that $\psi(x_1) = y_1,\ \psi(x_2) = y_2$ and $\psi(e) = e'$. Hence ψ is a $BNSGP - continuous$ mapping.

Theorem 4.3. *Let $(X, \tau^1_{\mathbb{B}}, E)$ and $(Y, \tau^2_{\mathbb{B}}, E')$ be any two bipolar neutrosophic soft topological sapces. For a bipolar neutrosophic soft continuous mapping $\psi : (X, \tau^1_{\mathbb{B}}, E) \rightarrow (Y, \tau^2_{\mathbb{B}}, E')$, we obtain the following results.*

1. *Every BNS-continuous mapping is a BNSG-continuous mapping.*
2. *Every BNS-continuous mapping is a BNSα-continuous mapping.*
3. *Every BNS-continuous mapping is a BNSP-continuous mapping.*
4. *Every BNSα-continuous mapping is a BNSP-continuous mapping.*
5. *Every BNSR-continuous mapping is a BNS-continuous mapping.*
6. *Every BNSP-continuous mapping is a BNSSP-continuous mapping.*
7. *Every BNS-continuous mapping is a BNSGP-continuous mapping.*
8. *Every BNSG-continuous mapping is a BNSGP-continuous mapping.*
9. *Every BNSP-continuous mapping is a BNSGP-continuous mapping.*
10. *Every BNSα-continuous mapping is a BNSGP-continuous mapping.*
11. *Every BNSGα-continuous mapping is a BNSGP-continuous mapping.*
12. *Every BNSGP-continuous mapping is a BNSSP-continuous mapping.*
13. *Every BNSGP-continuous mapping is a BNSGSP-continuous mapping.*

Proof. 1. Let $\psi : (X, \tau^1_{\mathbb{B}}, E) \rightarrow (Y, \tau^2_{\mathbb{B}}, E')$ be BNS-continuous mapping. Suppose B be BNSCS in Y. Then $\psi^{-1}(B)$ is BNSCS in X. Since every BNSCS is BNSGCS, $\psi^{-1}(B)$ is BNSGCS in X. Hence ψ is BNSG-continuous mapping.

2. Let $\psi : (X, \tau^1_{\mathbb{B}}, E) \rightarrow (Y, \tau^2_{\mathbb{B}}, E')$ be BNS-continuous mapping. Suppose B be BNSCS in Y. Then $\psi^{-1}(B)$ is BNSCS in X. Since every BNSCS is BNSαCS, $\psi^{-1}(B)$ is BNSαCS in X. Hence ψ is BNSα-continuous mapping.

Remaining proofs are similar. □

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Remark 4.4. The converse of theorem 4.3 not necessarily be true as shown in the following examples.

Example 4.5. Let $X = \{x_1, x_2, x_3\}, Y = \{y_1, y_2, y_3\}$ and $E = \{e_1, e_2\}, E' = \{e'_1, e'_2\}$.

$$B_1 = \left\{ \begin{array}{c} \langle e_1, \{x_1, 0.6, 0.3, 0.1, -0.7, -0.3, -0.5\}, \{x_2, 0.7, 0.4, 0.2, -0.7, -0.2, -0.2\}, \\ \{x_3, 0.5, 0.4, 0.4, -0.6, -0.3, -0.6\}\rangle, \langle e_2, \{x_1, 0.6, 0.3, 0.3, -0.4, -0.2, -0.3\}, \\ \{x_2, 0.8, 0.2, 0.2, -0.4, -0.2, -0.4\}, \{x_3, 0.3, 0.5, 0.7, -0.3, -0.5, -0.6\}\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} \langle e_1, \{x_1, 0.5, 0.4, 0.2, -0.5, -0.5, -0.8\}, \{x_2, 0.3, 0.7, 0.4, -0.6, -0.3, -0.5\}, \\ \{x_3, 0.5, 0.4, 0.4, -0.6, -0.3, -0.6\}\rangle, \langle e_2, \{x_1, 0.4, 0.6, 0.3, -0.2, -0.3, -0.5\}, \\ \{x_2, 0.7, 0.3, 0.2, -0.2, -0.5, -0.5\}, \{x_3, 0.6, 0.3, 0.2, -0.4, -0.5, -0.7\}\rangle \end{array} \right\}$$

$$B_3 = \left\{ \begin{array}{c} \langle e'_1, \{y_1, 0.4, 0.6, 0.5, -0.3, -0.6, -0.9\}, \{y_2, 0.2, 0.8, 0.5, -0.4, -0.4, -0.6\}, \\ \{y_3, 0.6, 0.2, 0.4, -0.2, -0.2, -0.8\}\rangle, \langle e'_2, \{y_1, 0.3, 0.7, 0.5, -0.1, -0.5, -0.5\}, \\ \{y_2, 0.5, 0.4, 0.3, -0.2, -0.6, -0.6\}, \{y_3, 0.5, 0.4, 0.4, -0.6, -0.3, -0.6\}\rangle \end{array} \right\}$$

Then, $\tau^1_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_1, B_2\}$ and $\tau^2_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_3\}$ are $BNST$s on X and Y respectively. Now define a mapping $\psi : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ such that $\psi(x_1) = y_1,\ \psi(x_2) = y_2,\ \psi(x_3) = y_3$ and $\psi(e) = e'$. Then ψ is a BNSG-continuous mapping, but not BNS-continuous mapping. Because,

$$B^c_3 = \left\{ \begin{array}{c} \langle e'_1, \{y_1, 0.5, 0.4, 0.4, -0.9, -0.4, -0.3\}, \{y_2, 0.5, 0.2, 0.2, -0.6, -0.6, -0.4\}, \\ \{y_3, 0.4, 0.8, 0.6, -0.8, -0.8, -0.2\}\rangle, \langle e'_2, \{y_1, 0.5, 0.3, 0.3, -0.5, -0.5, -0.5\}, \\ \{y_2, 0.3, 0.6, 0.5, -0.6, -0.4, -0.2\}, \{y_3, 0.4, 0.6, 0.5, -0.6, -0.7, -0.6\}\rangle \end{array} \right\}$$

is BNSCS in Y, but $\psi^{-1}(B^c_3)$ is not BNSCS in X.

Example 4.6. Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}$ and $E = \{e_1, e_2\}, E' = \{e'_1, e'_2\}$.

$$B_1 = \left\{ \begin{array}{c} \langle e_1, \{x_1, 0.6, 0.3, 0.1, -0.7, -0.3, -0.5\}, \{x_2, 0.7, 0.4, 0.2, -0.7, -0.2, -0.2\}, \rangle, \\ \langle e_2, \{x_1, 0.6, 0.3, 0.3, -0.4, -0.2, -0.3\}, \{x_2, 0.8, 0.2, 0.2, -0.4, -0.2, -0.4\}, \rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} \langle e_1, \{x_1, 0.5, 0.4, 0.2, -0.5, -0.5, -0.8\}, \{x_2, 0.3, 0.7, 0.4, -0.6, -0.3, -0.5\}, \rangle, \\ \langle e_2, \{x_1, 0.4, 0.6, 0.3, -0.2, -0.3, -0.5\}, \{x_2, 0.7, 0.3, 0.2, -0.2, -0.5, -0.5\}, \rangle \end{array} \right\}$$

$$B_3 = \left\{ \begin{array}{c} \langle e'_1, \{y_1, 0.4, 0.6, 0.5, -0.3, -0.6, -0.9\}, \{y_2, 0.2, 0.8, 0.5, -0.4, -0.4, -0.6\}, \rangle, \\ \langle e'_2, \{y_1, 0.3, 0.7, 0.5, -0.1, -0.5, -0.5\}, \{y_2, 0.5, 0.4, 0.3, -0.2, -0.6, -0.6\}, \rangle \end{array} \right\}$$

Then, $\tau^1_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_1, B_2\}$ and $\tau^2_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_3\}$ are $BNST$s on X and Y respectively. Now define a mapping $\psi : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ such that $\psi(x_1) = y_1,\ \psi(x_2) = y_2$ and $\psi(e) = e'$. Then ψ is a BNSα-continuous mapping, but not BNS-continuous mapping. Because,

$$B^c_3 = \left\{ \begin{array}{c} \langle e'_1, \{y_1, 0.5, 0.4, 0.4, -0.9, -0.4, -0.3\}, \{y_2, 0.5, 0.2, 0.2, -0.6, -0.6, -0.4\}, \rangle, \\ \langle e'_2, \{y_1, 0.5, 0.3, 0.3, -0.5, -0.5, -0.5\}, \{y_2, 0.3, 0.6, 0.5, -0.6, -0.4, -0.2\}\rangle \end{array} \right\}$$

is BNSCS in Y, but $\psi^{-1}(B^c_3)$ is not BNSCS in X.

Example 4.7. Let $X = \{x_1, x_2, x_3\}, Y = \{y_1, y_2, y_3\}$ and $E = \{e_1, e_2\}, E' = \{e'_1, e'_2\}$.

$$B_1 = \left\{ \begin{array}{c} \langle e_1, \{x_1, 0.5, 0.4, 0.2, -0.5, -0.5, -0.8\}, \{x_2, 0.3, 0.7, 0.4, -0.6, -0.3, -0.5\}, \\ \{x_3, 0.7, 0.1, 0.3, -0.6, -0.2, -0.6\}\rangle, \langle e_2, \{x_1, 0.4, 0.6, 0.3, -0.2, -0.3, -0.5\}, \\ \{x_2, 0.7, 0.3, 0.2, -0.2, -0.5, -0.5\}, \{x_3, 0.6, 0.3, 0.2, -0.7, -0.2, -0.5\}\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} \langle e'_1, \{y_1, 0.4, 0.6, 0.5, -0.3, -0.6, -0.9\}, \{y_2, 0.2, 0.8, 0.5, -0.4, -0.4, -0.6\}, \\ \{y_3, 0.6, 0.2, 0.4, -0.2, -0.2, -0.8\}\rangle, \langle e'_2, \{y_1, 0.3, 0.7, 0.5, -0.1, -0.5, -0.5\}, \\ \{y_2, 0.5, 0.4, 0.3, -0.2, -0.6, -0.6\}, \{y_3, 0.5, 0.4, 0.4, -0.6, -0.3, -0.6\}\rangle \end{array} \right\}$$

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Then, $\tau^1_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_1\}$ and $\tau^2_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_2\}$ are $BNST$s on X and Y respectively. Now define a mapping $\psi : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ such that $\psi(x_1) = y_1,\ \psi(x_2) = y_2,\ \psi(x_3) = y_3$ and $\psi(e) = e'$. Then ψ is a BNSP-continuous mapping, but not BNSα-continuous mapping. Because,

$$B_2^c = \left\{ \begin{array}{c} \langle e_1', \{y_1, 0.5, 0.4, 0.4, -0.9, -0.4, -0.3\}, \{y_2, 0.5, 0.2, 0.2, -0.6, -0.6, -0.4\}, \\ \{y_3, 0.4, 0.8, 0.6, -0.8, -0.8, -0.2\}\rangle, \langle e_2', \{y_1, 0.5, 0.3, 0.3, -0.5, -0.5, -0.5\}, \\ \{y_2, 0.3, 0.6, 0.5, -0.6, -0.4, -0.2\}, \{y_3, 0.4, 0.6, 0.5, -0.6, -0.7, -0.6\}\rangle \end{array} \right\}$$

is BNSCS in Y, but $\psi^{-1}(B_3^c)$ is not BNSCS and BNSαCS in X.

Example 4.8. Let $X = \{x_1, x_2, x_3\}, Y = \{y_1, y_2, y_3\}$ and $E = \{e_1, e_2\}, E' = \{e_1', e_2'\}$.

$$B_1 = \left\{ \begin{array}{c} \langle e_1, \{x_1, 0.5, 0.4, 0.2, -0.5, -0.5, -0.8\}, \{x_2, 0.3, 0.7, 0.4, -0.6, -0.3, -0.5\}, \\ \{x_3, 0.7, 0.1, 0.3, -0.6, -0.2, -0.6\}\rangle, \langle e_2, \{x_1, 0.4, 0.6, 0.3, -0.2, -0.3, -0.5\}, \\ \{x_2, 0.7, 0.3, 0.2, -0.2, -0.5, -0.5\}, \{x_3, 0.6, 0.3, 0.2, -0.7, -0.2, -0.5\}\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} \langle e_1', \{y_1, 0.5, 0.4, 0.2, -0.5, -0.5, -0.8\}, \{y_2, 0.3, 0.7, 0.4, -0.6, -0.3, -0.5\}, \\ \{y_3, 0.7, 0.1, 0.3, -0.6, -0.2, -0.6\}\rangle, \langle e_2', \{y_1, 0.4, 0.6, 0.3, -0.2, -0.3, -0.5\}, \\ \{y_2, 0.7, 0.3, 0.2, -0.2, -0.5, -0.5\}, \{y_3, 0.6, 0.3, 0.2, -0.7, -0.2, -0.5\}\rangle \end{array} \right\}$$

Then, $\tau^1_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_1\}$ and $\tau^2_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_2\}$ are $BNST$s on X and Y respectively. Now define a mapping $\psi : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ such that $\psi(x_1) = y_1,\ \psi(x_2) = y_2,\ \psi(x_3) = y_3$ and $\psi(e) = e'$. Then ψ is a BNS-continuous mapping, but not BNSR-continuous mapping. Because,

$$B_2^c = \left\{ \begin{array}{c} \langle e_1', \{y_1, 0.2, 0.6, 0.5, -0.8, -0.5, -0.5\}, \{y_2, 0.4, 0.3, 0.3, -0.5, -0.7, -0.6\}, \\ \{y_3, 0.3, 0.9, 0.7, -0.6, -0.8, -0.6\}\rangle, \langle e_2', \{y_1, 0.3, 0.4, 0.4, -0.5, -0.7, -0.2\}, \\ \{y_2, 0.2, 0.7, 0.3, -0.5, -0.5, -0.8\}, \{y_3, 0.2, 0.7, 0.6, -0.5, -0.8, -0.7\}\rangle \end{array} \right\}$$

is BNSCS in Y, but $\psi^{-1}(B_2^c)$ is not BNSRCS and in X.

The remaining Examples would be similar.

Theorem 4.9. *A mapping $\psi : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ is BNSGP-continuous if and only if the inverse image of each BNSOS(Bipolar neutrosophic soft open set) in Y is BNSGPOS(Bipolar neutrosophic soft generalized pre-open set) in X.*

Proof. Let B be BNSOS in Y. So B^c is BNSCS in Y. Since ψ is BNSGP-continuous mapping, $\psi^{-1}(B^c)$ is BNSGPCS in X. Also since, $\psi^{-1}(B^c) = (\psi^{-1}(B))^c$, $\psi^{-1}(B)$ is BNSGPOS in X. Converse part is straight forward from the definition of BNSGP-continuous map. □

Theorem 4.10. *Let $\psi_1 : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ be BNSGP-continuous mapping and $\psi_2 : (Y, \tau^2_{\mathbb{B}}, E') \to (Z, \tau^3_{\mathbb{B}}, E'')$ be bipolar neutrosophic soft continuous mapping, then $\psi_1 \circ \psi_2 : (X, \tau^1_{\mathbb{B}}, E) \to (Z, \tau^3_{\mathbb{B}}, E'')$ is a BNSGP-continuous mapping.*

Proof. Let B be BNSCS in Z. Then $\psi_2^{-1}(B)$ is BNSCS, by the hypothesis. Since ψ_1 is BNSGP-continuous mapping, $\psi_1^{-1}(\psi_2^{-1}(B))$ is BNSGPCS in X. Hence $\psi_1 \circ \psi_2$ is BNSGP-continuous mapping. □

Definition 4.11. Let $(X, \tau_{\mathbb{B}}, E)$ be a BNST. The bipolar neutrosophic soft generalized pre-closure (BNSgpcl) and bipolar neutrosophic soft generalized pre-interior (BNSgpint) for any $BNSS\ B$ is defined by,

$$BNSgpint(B) = \cup\{U |\ U\ is\ a\ BNSGPOS\ in\ X\ and\ U \subseteq B\}$$
$$BNSgpcl(B) = \cap\{V |\ V\ is\ a\ BNSGPCS\ in\ X\ and\ B \subseteq V\}$$

If B is $BNSGPCS$, then $BNSgpcl(B) = B$.

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Theorem 4.12. *Let $\psi : (X, \tau_1, E) \to (Y, \tau_2, E')$ be BNSGP-continuous mapping. Then the following conditions are hold.*

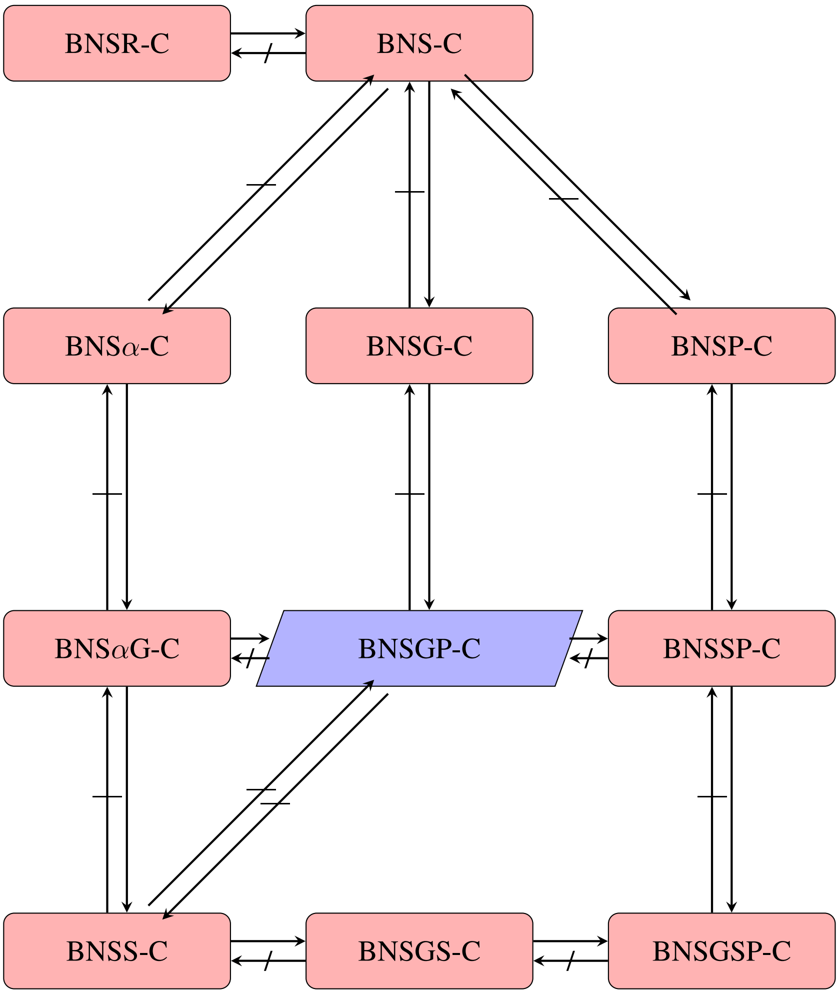
i). $\psi(BNSgpcl(B)) \subseteq BNScl(\psi(B))$*, for every BNSS B in* X*.*

ii). $BNSgpcl(\psi^{-1}(B)) \subseteq \psi^{-1}(BNScl(B))$*, for every BNSS B in* Y*.*

Proof. i) Since $BNScl(\psi(B))$ is BNSCS in Y and ψ is BNSGP-continuous mapping, then $\psi^{-1}(BNScl(\psi(B)))$ is BNSGPCS in X. So $BNSgpcl(B) \subseteq \psi^{-1}(BNScl(\psi(B)))$. Therefore $\psi(BNSgpcl(B)) \subseteq BNScl(\psi(B))$ for every BNSS B in X.

ii) Replace B by $\psi^{-1}(B)$ in i), we have $\psi(BNSgpcl(\psi^{-1}(B))) \subseteq BNScl(\psi(\psi^{-1}(B))) \subseteq BNScl(B)$. Therefore $BNSgpcl(\psi^{-1}(B)) \subseteq \psi^{-1}(BNScl(B))$, for every BNSS B in Y. □

The relation between the proposed bipolar neutrosophic soft generalized pre continuity with various continuity mappings are expressed as a chart below.



Here, the arrow ”⟶” between A and B means, A implies B but converse not true.

Definition 4.13. A map $\psi : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ is said to be bipolar neutrosophic soft generalized pre-irresolute (BNSGP-irresolute) mapping if $\psi^{-1}(B)$ is BNSGPCS in $(X, \tau^1_{\mathbb{B}}, E)$ for every bipolar neutrosophic soft generalized pre closed set B of $(Y, \tau^2_{\mathbb{B}}, E')$.

Example 4.14. Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}$ and $E = \{e_1, e_2\}, E' = \{e'_1, e'_2\}$. Also the $BNSS$s are

$$B_1 = \left\{ \begin{array}{l} \langle e_1, \{x_1, 0.5, 0.4, 0.2, -0.5, -0.5, -0.8\}, \{x_2, 0.3, 0.7, 0.4, -0.6, -0.3, -0.5\}\rangle, \\ \langle e_2, \{x_1, 0.4, 0.6, 0.3, -0.2, -0.3, -0.5\}, \{x_2, 0.7, 0.3, 0.2, -0.2, -0.5, -0.5\}\rangle \end{array} \right\}$$

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$$B_2 = \left\{ \begin{array}{l} \langle e_1, \{x_1, 0.6, 0.3, 0.1, -0.7, -0.3, -0.5\}, \{x_2, 0.7, 0.4, 0.2, -0.7, -0.2, -0.2\}\rangle, \\ \langle e_2, \{x_1, 0.6, 0.3, 0.3, -0.4, -0.2, -0.3\}, \{x_2, 0.8, 0.2, 0.2, -0.4, -0.2, -0.4\}\rangle \end{array} \right\}$$

$$B_3 = \left\{ \begin{array}{l} \langle e'_1, \{y_1, 0.4, 0.6, 0.5, -0.3, -0.6, -0.9\}, \{y_2, 0.2, 0.8, 0.5, -0.4, -0.4, -0.6\}\rangle, \\ \langle e'_2, \{y_1, 0.3, 0.7, 0.5, -0.1, -0.5, -0.5\}, \{y_2, 0.5, 0.4, 0.3, -0.2, -0.6, -0.6\}\rangle \end{array} \right\}$$

Then, $\tau^1_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_1, B_2\}$ and $\tau^2_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}, B_3\}$ are $BNST$s on X and Y respectively. Now define a mapping $\psi : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ such that $\psi(x_1) = y_1,\ \psi(x_2) = y_2$ and $\psi(e) = e'$. Hence ψ is a $BNSGP - irresolute$ mapping.

Theorem 4.15. *Let $\psi : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ be a BNSGP-irresolute mapping, then ψ is BNSGP-continuous mapping but not conversely.*

Proof. Let ψ be BNSGP-irresolute mapping and let B ba a BNSCS in Y. Since every BNSCS is BNSGPCS, B is BNSGPCS in Y. Also, ψ is BNSGP-irresolute mapping. Therefore, by definition, $\psi^{-1}(B)$ is BNSGPCS in X. So ψ is BNSGP-continuous mapping. □

Theorem 4.16. *A mapping $\psi : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ is BNSGP-irresolute mapping if and only if the inverse image of every BNSGPOS in Y is BNSGPOS in X.*

Proof. Let B be BNSGPOS in Y. So B^c is BNSGPCS in Y. Since ψ is BNSGP-irresolute mapping, $\psi^{-1}(B^c)$ is BNSGPCS in X. Since $\psi^{-1}(B^c) = (\psi^{-1}(B))^c$, $\psi^{-1}(B)$ is BNSGPOS in X. The converse proof is straight forward. □

Theorem 4.17. *Let $\psi_1 : (X, \tau^1_{\mathbb{B}}, E) \to (Y, \tau^2_{\mathbb{B}}, E')$ and $\psi_2 : (Y, \tau^2_{\mathbb{B}}, E') \to (Z, \tau^3_{\mathbb{B}}, E'')$ be BNSGP-irresolute mapping, where X, Y, Z are BNSGTS, then $\psi_1 \circ \psi_2$ is a BNSGP-irresolute mapping.*

Proof. Let B be BNSGPCS in Z. Since ψ_1 is BNSGP-irresolute mapping, $\psi_2^{-1}(B)$ is BNSGPCS in Y. Since ψ_1 is BNSGP-irresolute mapping, $\psi_1^{-1}(\psi_2^{-1}(B))$ is BNSGPCS in X. Hence $(\psi_1 \circ \psi_2)^{-1}(B)$ is BNSGPCS in X. Hence $\psi_1 \circ \psi_2$ is BNSGP-irresolute mapping. □

5 Conclusion

In this paper, we have introduced bipolar neutrosophic soft generalized pre-continuous mapping. Many results in this paper shows that how the various type of continuities preserve the topological structures such as closeness and openness of sets. Our future work will include some applications of the proposed mapping in topology.

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