



MBJ-neutrosophic T-ideal on B-algebra

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Abstract

In this paper we define and study the MBJ-neutrosophic T-ideal through different concept like union, intersection. further we use the important properties to investigate the MBJ-neutrosophic T-ideal under cartesian product and homomorphic results.

Keywords: B-algebra, MBJ-neutrosophic set, MBJ-neutrosophic T-ideal, Cartesian product, Homomorphism.

1. Introduction

[1] presented the idea of fuzzy set. [2] gave the idea of BCK-algebra. [3] introduced the BCI-algebra and it is obvious that BCK-algebra is a proper sub class of BCI-algebra. [4] wrote on intuitionistic fuzzy G-algebra. [5] introduced the intuitionistic fuzzy sets. [6] provided the idea of interval-valued intuitionistic fuzzy sets. [7] studied the neutrosophic soft subalgebra and investigated many results. [8] initiated the intuitionistic fuzzy structure of B-algebra. [9] discussed the fuzzy dot subalgebra and fuzzy dot ideal of B-algebras. [10] introduced the B-algebra in 2002. [11,12] worked on t-intuitionistic fuzzy sets in fuzzy subgroups and subrings. Barbhuiya [13] studied the t-intuitionistic fuzzy BG-subalgebra. [14] studied ideals with its level subsets. [15] introduced the T-neutrosophic cubic set and deeply investigated this set with significant characteristics. [16] defined and studied the MBJ-neutrosophic set through subalgebra and S-extension. [17,18] The concept of Neutrosophic set is given by Smarandache with truth, indeterminate and false membership function.

This paper is presented to study the idea of MBJ-neutrosophic set through the concept of T-ideal, homomorphic characteristics and cartesian product.

2. Preliminaries

First we cite some definitions which are used to present this paper. [3] An algebra $(Y, *, 0)$ of type $(2, 0)$ is called a BCI-algebra if it satisfies the following conditions:

$$i) (t_1 * t_2) * (t_1 * t_3) \leq (t_3 * t_2),$$

$$\text{ii) } t_1 * (t_1 * t_2) \leq t_2,$$

$$\text{iii) } t_1 \leq t_1,$$

$$\text{iv) } t_1 \leq t_2 \text{ and } t_2 \leq t_1 \Rightarrow t_1 = t_2,$$

$$\text{v) } t_1 \leq 0 \Rightarrow t_1 = 0, \text{ where } t_1 \leq t_2 \text{ is defined by } t_1 * t_2 = 0, \text{ for all } t_1, t_2, t_3 \in Y.$$

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$$\text{v) } 0 \leq t_1 \Rightarrow t_1 = 0, \text{ where } t_1 \leq t_2 \text{ is defined by } t_1 * t_2 = 0, \text{ for all } t_1, t_2, t_3 \in Y.$$

[10] A nonempty set Y with a constant 0 and having binary operation $*$ is called B-algebra if it satisfies the following conditions:

$$\text{i) } t_1 * t_1 = 0$$

$$\text{ii) } t_1 * 0 = x$$

$$\text{iii) } (t_1 * t_2) * t_3 = (t_1 * (t_3 * (0 * t_2))) \text{ for all } t_1, t_2 \in Y.$$

[10] Let K be a nonempty subset of B-algebra Y , then K is called a subalgebra of Y if $t_1 * t_2 \in K$, for all $t_1, t_2 \in K$.

[14] Let Y be a PS-algebra. A fuzzy set B of Y is called a fuzzy PS ideal of Y if it satisfies the following conditions:

$$\text{i) } \vartheta(0) \geq \vartheta(t_1),$$

$$\text{ii) } \vartheta(t_1) \geq \min\{\vartheta(t_2 * t_1), \vartheta(t_2)\}, \text{ for all } t_1, t_2 \in Y.$$

Let Y be a group of objects denoted generally by t_1 . Then a fuzzy set B of Y is defined as $B = \{ \langle t_1, \vartheta_B(t_1) \rangle \mid t_1 \in Y \}$, where $\vartheta_B(t_1)$ is called the existence ship value of t_1 in B and $\vartheta_B(t_1) \in [0,1]$.

A fuzzy set B [14] of PS-algebra Y is called a fuzzy PS subalgebra of Y if $\vartheta(t_1 * t_2) \geq \min\{\vartheta(t_1), \vartheta(t_2)\}$, for all $t_1, t_2 \in Y$.

An intuitionistic fuzzy set (IFS) [5] B over Y is an object having the form $B = \{ \langle t_1, \vartheta_B(t_1), \phi_B(t_1) \rangle \mid t_1 \in Y \}$, where $\vartheta_B(t_1) \mid Y \rightarrow [0,1]$ and $\phi_B(t_1) \mid Y \rightarrow [0,1]$, with the condition $0 \leq \vartheta_B(t_1) + \phi_B(t_1) \leq 1$, for all $t_1 \in Y$. $\vartheta_B(t_1)$ and $\phi_B(t_1)$ represent the degree of existence and the degree of non-existence of the element t_1 in the set B respectively.

An IFS $B = \{ \langle t_1, \vartheta_B(t_1), \phi_B(t_1) \rangle \mid t_1 \in Y \}$ of Y is said to be an IFID of Y if it satisfies these three conditions:

$$\text{(i) } \vartheta_B(0) \geq \vartheta_B(t_1), \phi_B(0) \leq \phi_B(t_1),$$

$$\text{(ii) } \vartheta_B(t_1) \geq \min\{\vartheta_B(t_1 * t_2), \vartheta_B(t_2)\},$$

$$\text{(iii) } \phi_B(t_1) \leq \max\{\phi_B(t_1 * t_2), \phi_B(t_2)\}, \text{ for all } t_1, t_2 \in Y.$$

An IVFS [1] B over Y is of the form $B = \{(t_1, \hat{\vartheta}_B(t_1)) | t_1 \in Y\}$, where $\hat{\vartheta}_B(t_1) | Y \rightarrow D[0,1]$, where $D[0,1]$ is the set of all sub intervals of $[0,1]$. $\hat{\vartheta}_B(t_1)$ is the interval of the existence value of the element t_1 in B , where $\hat{\vartheta}_B(t_1) = [\hat{\vartheta}_{B_L}(t_1), \hat{\vartheta}_{B_U}(t_1)] \forall t_1 \in Y$.

An IVIFS [6] B over Y is an object of the form $B = \{(t_1, \hat{\vartheta}_B(t_1), \hat{\phi}_B(t_1)) | t_1 \in Y\}$ where $\hat{\vartheta}_B(t_1) | Y \rightarrow [0,1]$ and $\hat{\phi}_B(t_1) | Y \rightarrow [0,1]$, $D[0,1]$ is the set of all subintervals of $[0,1]$. $\hat{\vartheta}_B(t_1)$ and $\hat{\phi}_B(t_1)$ are the intervals of existence value and non-existence value of element t_1 to B , $\hat{\vartheta}_B(t_1) = [\hat{\vartheta}_{B_L}(t_1), \hat{\vartheta}_{B_U}(t_1)]$ and $\hat{\phi}_B(t_1) = [\hat{\phi}_{B_L}(t_1), \hat{\phi}_{B_U}(t_1)]$, $\forall t_1 \in Y$, with the condition $0 \leq \hat{\vartheta}_{B_U}(t_1) + \hat{\phi}_{B_U}(t_1) \leq 1$.

An IVIFS $\hat{B} = (\hat{\vartheta}_B, \hat{\phi}_B)$ in Y is called an IVIF-ideal [9] of Y when it fulfills these axioms.

$$\text{i) } \hat{\vartheta}_B(0) \geq \hat{\vartheta}_B(t_1) \text{ and } \hat{\phi}_B(0) \leq \hat{\phi}_B(t_1),$$

$$\text{ii) } \hat{\vartheta}_B(t_1) \geq rmin\{\hat{\vartheta}_B(t_1 * t_2), \hat{\vartheta}_B(t_2)\},$$

$$\text{iii) } \hat{\phi}_B(t_1) \leq rmax\{\hat{\phi}_B(t_1 * t_2), \hat{\phi}_B(t_2)\}.$$

Let $B = (\vartheta_B, \phi_B)$ be an IFS of Y . Let $t \in [0,1]$, then the IFS B^t is called the t -intuitionistic fuzzy subset [11, 12] of Y w.r.t B and is defined as $B^t = \{ \langle t_1, \vartheta_{B^t}(t_1), \phi_{B^t}(t_1) \rangle | t_1 \in Y \} = \langle \vartheta_{B^t}, \phi_{B^t} \rangle$, where $\vartheta_{B^t}(t_1) = \min\{\vartheta_B(t_1), t\}$ and $\phi_{B^t}(t_1) = \max\{\phi_B(t_1), 1 - t\}$ for all $t_1 \in Y$.

Let IFS $B = \{(t_1, \vartheta_B(t_1), \phi_B(t_1)) | t_1 \in Y\}$ of Y is said to be an **IFTID** of Y if it satisfies these three conditions:

$$\text{(i) } \vartheta_B(0) \geq \vartheta_B(t_1), \phi_B(0) \leq \phi_B(t_1),$$

$$\text{(ii) } \vartheta_B(t_1 * t_3) \geq \min\{\vartheta_B((t_1 * t_2) * t_3), \vartheta_B(t_2)\},$$

$$\text{(iii) } \phi_B(t_1 * t_3) \leq \max\{\phi_B((t_1 * t_2) * t_3), \phi_B(t_2)\}, \text{ for all } t_1, t_2 \in Y.$$

An Neutrosophic fuzzy set [17, 18] on Y is defined by $B = \langle B_T, B_I, B_F \rangle | x \in Y$, where $B_T \rightarrow [0,1]$ is a truth membership function, $B_I \rightarrow [0,1]$, is an indeterminate membership function and $B_F \rightarrow [0,1]$ is a false membership function.

Let Y be a non empty set. MBJ-neutrosophic set [16] in Y , is a structure of the form $C = \{(M_C t_1, \hat{B}_C t_1, J_C t_1) | t_1 \in Y\}$ where M_C and J_C are fuzzy sets in Y and M_C is a truth membership function, J_C is a false membership function and \hat{B} is interval valued fuzzy set in Y and is an Indeterminate Interval Valued membership function.

3. MBJ-neutrosophic T-ideal of B-algebra

Definition 3.1. Let $C = (M_C, \hat{B}_C, J_C)$ is a MBJ-neutrosophic set of B-algebra Y . Let $t \in [0,1]$ then C is called MBJ-neutrosophic T-ideal (MBJNTID) of Y if it fulfills these assertions:

$$\text{i) } M_C(0) \geq M_C(t_1), \hat{B}_C(0) \geq \hat{B}_C(t_1) \text{ and } J_C(0) \leq J_C(t_1).$$

$$\text{ii) } M_C(t_1 * t_3) \geq \min\{M_C((t_1 * t_2) * t_3), M_C(t_2)\}.$$

$$\text{iii) } \hat{B}_C(t_1 * t_3) \geq rmin\{\hat{B}_C((t_1 * t_2) * t_3), \hat{B}_C(t_2)\}.$$

$$\text{iv) } J_C(t_1 * t_3) \leq \max\{J_C((t_1 * t_2) * t_3), J_C(t_2)\}.$$

Theorem 3.1. Let $C = (M_C, \hat{B}_C, J_C)$ be a MBJNID of a B-algebra Y . If $t_1 * t_2 \leq t_3$, then $M_C(t_1 * x) \geq \min\{M_C(t_3), \hat{B}_C(t_2)\}$, $\hat{B}_C(t_1 * x) \geq rmin\{\hat{B}_C(t_3), \hat{B}_C(t_2)\}$ and $J_C(t_1 * x) \leq \max\{J_C(t_3), J_C(t_2)\}$ for all $t_1, t_2, t_3 \in Y$.

Proof. Let $t_1, t_2, t_3 \in Y$ such that $t_1 * t_2 = t_3$, now

$$\begin{aligned} M_C(t_1 * x) &\geq \{\min\{M_C((t_1 * t_2) * x), M_C(t_2)\}\} \\ &\geq \min\{\min\{M_C(((t_1 * t_2) * t_3) * x), M_C(t_3)\}, M_C(t_2)\} \\ &\geq \min\{\min\{M_C(0), M_C(t_3)\}, M_C(t_2)\} \\ &\geq \min\{M_C(t_3), M_C(t_2)\} \\ M_C(t_1 * x) &= \min\{M_C(t_3), M_C(t_2)\} \end{aligned}$$

and

$$\begin{aligned} \hat{B}_C(t_1 * x) &\geq \{rmin\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}\} \\ &\geq rmin\{rmin\{\hat{B}_C(((t_1 * t_2) * t_3) * x), \hat{B}_C(t_3)\}, \hat{B}_C(t_2)\} \\ &\geq rmin\{rmin\{\hat{B}_C(0), \hat{B}_C(t_3)\}, \hat{B}_C(t_2)\} \\ &\geq rmin\{\hat{B}_C(t_3), \hat{B}_C(t_2)\} \\ \hat{B}_C(t_1 * x) &= rmin\{\hat{B}_C(t_3), \hat{B}_C(t_2)\} \end{aligned}$$

and

$$\begin{aligned} J_C(t_1 * x) &\leq \{\max\{J_C((t_1 * t_2) * x), J_C(t_2)\}\} \\ &\leq \max\{\max\{J_C(((t_1 * t_2) * t_3) * x), J_C(t_3)\}, J_C(t_2)\} \\ &\leq \max\{\max\{J_C(0), J_C(t_3)\}, J_C(t_2)\} \\ &\leq \max\{J_C(t_3), J_C(t_2)\} \\ J_C(t_1 * x) &= \max\{J_C(t_3), J_C(t_2)\}. \end{aligned}$$

Theorem 3.2. Let $C = (M_C, J_C)$ be a MBJNTID of a B-algebra Y . If $t_1 \leq t_2$ then $M_C(t_1 * x) \geq M_C(t_2)$, $\hat{B}_C(t_1 * x) \geq \hat{B}_C(t_2)$ and $J_C(t_1 * x) \leq J_C(t_2)$ for all $t_1, t_2, t_3 \in Y$.

Proof. Let $t_1, t_2 \in Y$ such that $t_1 \leq t_2$. Then $t_1 * t_2 = 0$, now

$$\begin{aligned} M_C(t_1 * x) &\geq \{\min\{M_C((t_1 * t_2) * x), M_C(t_2)\}\} \\ &\geq \{\min\{M_C(0), M_C(t_2)\}\} \\ M_C(t_1 * x) &= M_C(t_2) \end{aligned}$$

and

$$\hat{B}_C(t_1 * x) \geq \{rmin\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}\}$$

$$\geq \{rmin\{\hat{B}_C(0), \hat{B}_C(t_2)\}$$

$$\hat{B}_C(t_1 * x) = \hat{B}_C(t_2)$$

and

$$J_C(t_1 * x) \leq \{\max\{J_C((t_1 * t_2) * x), J_C(t_2)\}$$

$$\leq \{\max\{J_C(0), J_C(t_2)\}$$

$$J_C(t_1 * x) = J_C(t_2).$$

Hence $M_C(t_1 * x) \geq M_C(t_2)$, $\hat{B}_C(t_1 * x) \geq \hat{B}_C(t_2)$ and $J_C(t_1 * x) \leq J_C(t_2)$ for all $t_1, t_2, t_3 \in Y$.

Theorem 3.3. If C is a MBJNTID, then it fulfills the condition $(M_C((t_1 * (t_2 * t_1)) * x) \geq (M_C(t_2)$, $(\hat{B}_C((t_1 * (t_2 * t_1)) * x) \geq (\hat{B}_C(t_2)$ and $(J_C((t_1 * (t_2 * t_1)) * x) \leq (J_C(t_2)$.

Proof. Let C is a MBJNTID, so

$$M_C((t_1 * (t_2 * t_1)) * x) \geq \min\{M_C(((t_1 * (t_2 * t_1)) * t_2) * x), M_C(t_2)\}$$

$$= \min\{M_C(0), M_C(t_2)\}$$

$$M_C((t_1 * (t_2 * t_1)) * x) = M_C(t_2)$$

and

$$\hat{B}_C((t_1 * (t_2 * t_1)) * x) \geq rmin\{\hat{B}_C(((t_1 * (t_2 * t_1)) * t_2) * x), \hat{B}_C(t_2)\}$$

$$= rmin\{\hat{B}_C(0), \hat{B}_C(t_2)\}$$

$$\hat{B}_C((t_1 * (t_2 * t_1)) * x) = \hat{B}_C(t_2)$$

and

$$J_C((t_1 * (t_2 * t_1)) * x) \leq \max\{J_C(((t_1 * (t_2 * t_1)) * t_2) * x), J_C(t_2)\}$$

$$= \max\{J_C(0), J_C(t_2)\}$$

$$J_C((t_1 * (t_2 * t_1)) * x) = J_C(t_2).$$

Hence $(M_C((t_1 * (t_2 * t_1)) * x) \geq (M_C(t_2)$, $(\hat{B}_C((t_1 * (t_2 * t_1)) * x) \geq (\hat{B}_C(t_2)$ and $(J_C((t_1 * (t_2 * t_1)) * x) \leq (J_C(t_2)$.

Theorem 3.4. Let $A = (M_A, \hat{B}_A, J_A)$ and $C = (M_C, \hat{B}_C, J_C)$ are two MBJNTIDs of a B-algebra Y . Then the intersection $A \cap C$ is also MBJNTID of Y .

Proof. Let $t_1, t_2 \in A \cap C$, then $t_1, t_2 \in A$ and $t_1, t_2 \in C$.

$$M_{A \cap C}(0) = M_{A \cap C}(t_1 * t_1)$$

$$= \min\{M_A(t_1 * t_1), M_C(t_1 * t_1)\}$$

$$\geq \min\{\min\{M_A(t_1), M_A(t_1)\}, \min\{M_C(t_1), M_C(t_1)\}\}$$

$$= \min\{M_A(t_1), M_C(t_1)\}$$

$$M_{A \cap C}(0) = M_{A \cap C}(t_1)$$

and

$$\hat{B}_{A \cap C}(0) = \hat{B}_{A \cap C}(t_1 * t_1)$$

$$= r\min\{\hat{B}_A(t_1 * t_1), \hat{B}_C(t_1 * t_1)\}$$

$$\geq r\min\{r\min\{\hat{B}_A(t_1), \hat{B}_A(t_1)\}, r\min\{\hat{B}_C(t_1), \hat{B}_C(t_1)\}\}$$

$$= r\min\{\hat{B}_A(t_1), \hat{B}_C(t_1)\}$$

$$\hat{B}_{A \cap C}(0) = \hat{B}_{A \cap C}(t_1)$$

and

$$J_{A \cap C}(0) = J_{A \cap C}(t_1 * t_1)$$

$$= \max\{J_C(t_1 * t_1), J_C(t_1 * t_1)\}$$

$$\leq \max\{\max\{J_C(t_1), J_C(t_1)\}, \max\{J_C(t_1), J_C(t_1)\}\}$$

$$= \max\{J_C(t_1), J_C(t_1)\}$$

$$J_{A \cap C}(0) = J_{A \cap C}(t_1),$$

now

$$M_{A \cap C}(t_1 * x) = \min\{M_A(t_1 * x), M_C(t_1 * x)\}$$

$$\geq \min\{\min\{M_A((t_1 * t_2) * x), M_A(t_2)\}, \min\{M_C((t_1 * t_2) * x), M_C(t_2)\}\}$$

$$= \min\{\min\{M_A((t_1 * t_2) * x), M_C((t_1 * t_2) * x)\}, \min\{M_A(t_2), M_C(t_2)\}\}$$

$$= \{\min\{M_{A \cap C}((t_1 * t_2) * x), M_{A \cap C}(t_2)\}$$

and

$$\hat{B}_{A \cap C}(t_1 * x) = r\min\{\hat{B}_A(t_1 * x), \hat{B}_C(t_1 * x)\}$$

$$\geq r\min\{r\min\{\hat{B}_A((t_1 * t_2) * x), \hat{B}_A(t_2)\}, r\min\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}\}$$

$$= r\min\{r\min\{\hat{B}_A((t_1 * t_2) * x), \hat{B}_C((t_1 * t_2) * x)\}, r\min\{\hat{B}_A(t_2), \hat{B}_C(t_2)\}\}$$

$$= \{r\min\{\hat{B}_{A \cap C}((t_1 * t_2) * x), \hat{B}_{A \cap C}(t_2)\}$$

and

$$J_{A \cap C}(t_1 * x) = \max\{J_A(t_1 * x), J_C(t_1 * x)\}$$

$$\leq \max\{\max\{J_A((t_1 * t_2) * x), J_A(t_2)\}, \max\{J_C((t_1 * t_2) * x), J_C(t_2)\}\}$$

$$\begin{aligned}
&= \max\{\max\{J_A((t_1 * t_2) * x), J_C((t_1 * t_2) * x)\}, \max\{J_A(t_2), J_C(t_2)\}\} \\
&= \max\{J_{A \cap C}((t_1 * t_2) * x), J_{A \cap C}(t_2)\}.
\end{aligned}$$

Hence the intersection $A \cap C$ is MBJNTID of Y .

Theorem 3.5. Let $A = (M_A, \hat{B}_A, J_A)$ and $C = (M_C, \hat{B}_C, J_C)$ are two MBJNTIDs of a B-algebra Y . Then the union $A \cup C$ is also MBJNTID of Y .

Proof. Let $t_1, t_2 \in A \cup C$, then $t_1, t_2 \in A$ and $t_1, t_2 \in C$.

$$\begin{aligned}
M_{A \cup C}(0) &= M_{A \cup C}(t_1 * t_1) \\
&= \max\{M_A(t_1 * t_1), M_C(t_1 * t_1)\} \\
&\geq \max\{\max\{M_A(t_1), M_A(t_1)\}, \max\{M_C(t_1), M_C(t_1)\}\} \\
&= \max\{M_A(t_1), M_C(t_1)\} \\
M_{A \cup C}(0) &= M_{A \cup C}(t_1)
\end{aligned}$$

and

$$\begin{aligned}
\hat{B}_{A \cup C}(0) &= \hat{B}_{A \cup C}(t_1 * t_1) \\
&= r\max\{\hat{B}_A(t_1 * t_1), \hat{B}_C(t_1 * t_1)\} \\
&\geq r\max\{r\max\{\hat{B}_A(t_1), \hat{B}_A(t_1)\}, r\max\{\hat{B}_C(t_1), \hat{B}_C(t_1)\}\} \\
&= r\max\{\hat{B}_A(t_1), \hat{B}_C(t_1)\} \\
\hat{B}_{A \cup C}(0) &= \hat{B}_{A \cup C}(t_1)
\end{aligned}$$

and

$$\begin{aligned}
J_{A \cup C}(0) &= J_{A \cup C}(t_1 * t_1) \\
&= \min\{J_A(t_1 * t_1), J_C(t_1 * t_1)\} \\
&\leq \min\{\min\{J_A(t_1), J_A(t_1)\}, \min\{J_C(t_1), J_C(t_1)\}\} \\
&= \min\{J_A(t_1), J_C(t_1)\} \\
J_{A \cup C}(0) &= J_{A \cup C}(t_1)
\end{aligned}$$

now

$$\begin{aligned}
M_{A \cup C}(t_1 * x) &= \max\{M_A(t_1 * x), M_C(t_1 * x)\} \\
&\geq \max\{\max\{M_A((t_1 * t_2) * x), M_A(t_2)\}, \max\{M_C((t_1 * t_2) * x), M_C(t_2)\}\} \\
&= \max\{\max\{M_A((t_1 * t_2) * x), M_C((t_1 * t_2) * x)\}, \max\{M_A(t_2), M_C(t_2)\}\} \\
&= \max\{M_{A \cup C}((t_1 * t_2) * x), M_{A \cup C}(t_2)\}
\end{aligned}$$

and

$$\begin{aligned} \hat{B}_{A \cup C}(t_1 * x) &= rmax\{\hat{B}_A(t_1 * x), \hat{B}_C(t_1 * x)\} \\ &\geq rmax\{rmax\{\hat{B}_A((t_1 * t_2) * x), \hat{B}_A(t_2)\}, rmax\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}\} \\ &= rmax\{rmax\{\hat{B}_A((t_1 * t_2) * x), \hat{B}_C((t_1 * t_2) * x)\}, rmax\{\hat{B}_A(t_2), \hat{B}_C(t_2)\}\} \\ &= \{rmax\{\hat{B}_{A \cup B}((t_1 * t_2) * x), \hat{B}_{A \cup B}(t_2)\} \} \end{aligned}$$

and

$$\begin{aligned} J_{A \cup C}(t_1 * x) &= \min\{J_A(t_1 * x), J_C(t_1 * x)\} \\ &\leq \min\{\min\{J_A((t_1 * t_2) * x), J_A(t_2)\}, \min\{J_C((t_1 * t_2) * x), J_C(t_2)\}\} \\ &= \min\{\min\{J_A((t_1 * t_2) * x), J_C((t_1 * t_2) * x)\}, \min\{J_A(t_2), J_C(t_2)\}\} \\ &= \{\min\{J_{A \cup C}((t_1 * t_2) * x), J_{A \cup C}(t_2)\} \}. \end{aligned}$$

Hence the union $A \cup C$ is also MBJNTID of Y .

Theorem 3.6. Let A and C are the MBJ-neutrosophic sets in Y . The cartesian product $A \times C: Y \times Y \rightarrow [0,1]$ is defined by $M_A \times M_C(t_1, t_2) = \max\{M_A(t_1), M_C(t_2)\}$, $\hat{B}_A \times \hat{B}_C(t_1, t_2) = rmax\{\hat{B}_A(t_1), \hat{B}_C(t_2)\}$ and $J_A \times J_C(t_1, t_2) = \min\{J_A(t_1), J_C(t_2)\}$ for all $t_1, t_2 \in Y$. Let A and C are two MBJNTIDs of Y , then $A \times C$ is a MBJNTID of $Y \times Y$.

Proof. For any $t_1, t_2 \in Y \times Y$, we have $M_A \times M_C(0,0) = \min\{M_A(0), M_C(0)\} \geq \min\{M_A(t_1), M_C(t_2)\} = M_A \times M_C(t_1, t_2)$, $\hat{B}_A \times \hat{B}_C(0,0) = rmin\{\hat{B}_A(0), \hat{B}_C(0)\} \geq rmin\{\hat{B}_A(t_1), \hat{B}_C(t_2)\} = \hat{B}_A \times \hat{B}_C(t_1, t_2)$ and $J_A \times J_C(0,0) = \max\{J_A(0), J_C(0)\} \leq \max\{J_A(t_1), J_C(t_2)\} = J_A \times J_C(t_1, t_2)$. That is $M_A \times M_C(0,0) \geq M_A \times M_C(t_1, t_2)$, $\hat{B}_A \times \hat{B}_C(0,0) \geq \hat{B}_A \times \hat{B}_C(t_1, t_2)$ and $J_A \times J_C(0,0) \leq J_A \times J_C(t_1, t_2)$.

Now let $x, (t_1, t_2)$ and $(y_1, y_2) \in Y \times Y$. Then, $(M_A \times M_C)(t_1 * x, t_2 * x) = \min\{M_A(t_1 * x), M_C(t_2 * x)\} \geq \min\{\min\{M_A((t_1 * y_1) * x), M_A(y_1)\}, \min\{M_C((t_2 * y_2) * x), M_C(y_2)\}\} = \min\{\min\{M_A((t_1 * y_1) * x), M_C((t_2 * y_2) * x)\}, \min\{M_A(y_1), M_C(y_2)\}\} = \min\{(M_A \times M_C)((t_1 * y_1) * x), ((t_2 * y_2) * x)\}, (M_A \times M_C)(y_1, y_2)\} = \min\{(M_A \times M_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (M_A \times M_C)(y_1, y_2)\}$. That is $(M_A \times M_C)(t_1 * x, t_2 * x) \geq \min\{(M_A \times M_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (M_A \times M_C)(y_1, y_2)\}$ and $(\hat{B}_A \times \hat{B}_C)(t_1 * x, t_2 * x) = rmin\{\hat{B}_A(t_1 * x), \hat{B}_C(t_2 * x)\} \geq rmin\{rmin\{\hat{B}_A((t_1 * y_1) * x), \hat{B}_A(y_1)\}, rmin\{\hat{B}_C((t_2 * y_2) * x), \hat{B}_C(y_2)\}\} = rmin\{rmin\{\hat{B}_A((t_1 * y_1) * x), \hat{B}_C((t_2 * y_2) * x)\}, rmin\{\hat{B}_A(y_1), \hat{B}_C(y_2)\}\} = rmin\{(\hat{B}_A \times \hat{B}_C)((t_1 * y_1) * x), ((t_2 * y_2) * x)\}, (\hat{B}_A \times \hat{B}_C)(y_1, y_2)\} = rmin\{(\hat{B}_A \times \hat{B}_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (\hat{B}_A \times \hat{B}_C)(y_1, y_2)\}$. That is $(\hat{B}_A \times \hat{B}_C)(t_1 * x, t_2 * x) \geq rmin\{(\hat{B}_A \times \hat{B}_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (\hat{B}_A \times \hat{B}_C)(y_1, y_2)\}$ and $(J_A \times J_C)(t_1 * x, t_2 * x) = \max\{J_A(t_1 * x), J_C(t_2 * x)\} \leq \max\{\max\{J_A((t_1 * y_1) * x), J_A(y_1)\}, \max\{J_C((t_2 * y_2) * x), J_C(y_2)\}\} = \max\{\max\{J_A((t_1 * y_1) * x), J_C((t_2 * y_2) * x)\}, \max\{J_A(y_1), J_C(y_2)\}\} = \max\{(J_A \times J_C)((t_1 * y_1) * x), ((t_2 * y_2) * x)\}, (J_A \times J_C)(y_1, y_2)\} = \max\{(J_A \times J_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (J_A \times J_C)(y_1, y_2)\}$.

That is $(J_A \times J_C)(t_1 * x, t_2 * x) \leq \max\{(J_A \times J_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (J_A \times J_C)(y_1, y_2)\}$.

Theorem 3.7. Let A and C are two MBJNs in Y such that $A \times C$ is an MBJNTID of $Y \times Y$, then

1. $M_A(0) \geq M_A(t_1)$, $\hat{B}_A(0) \geq \hat{B}_A(t_1)$, $J_A(0) \leq J_A(t_1)$ and $M_C(0) \geq M_C(t_1)$, $\hat{B}_C(0) \geq \hat{B}_C(t_1)$, $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

2. If $M_A(0) \geq M_A(t_1)$ then $M_C(0) \geq M_A(t_1)$ and $M_C(0) \geq M_C(t_1)$ for all $t_1 \in Y$ also if $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ then $\hat{B}_C(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$ also if $J_A(0) \leq J_A(t_1)$ then $J_C(0) \leq J_A(t_1)$ and $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

3. If $M_C(0) \geq M_C(t_1)$ then $M_A(0) \geq M_A(t_1)$ and $M_A(0) \geq M_C(t_1)$ for all $t_1 \in Y$ also if $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ then $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_A(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$ also if $J_C(0) \leq J_C(t_1)$ then $J_A(0) \leq J_A(t_1)$ and $J_A(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

Proof. 1. Suppose $M_A(t_1) > M_A(0)$ or $M_C(t_1) > M_C(0)$ for all $t_1 \in Y$ $(M_A \times M_C)(t_1, t_1) = \min\{M_A(t_1), M_C(t_1)\} > \min\{M_A(0), M_C(0)\} = (M_A \times M_C)(0,0)$. Thus $(M_A \times M_C)(t_1, t_1) > (M_A \times M_C)(0,0)$ for all $t_1 \in Y$, which is the contradiction to $(M_A \times M_C)$ is a MBJNTID of $Y \times Y$. Therefore $M_A(0) \geq M_A(t_1)$ and $M_C(0) \geq M_C(t_1)$ for all $t_1 \in Y$, also $\hat{B}_A(t_1) > \hat{B}_A(0)$ or $\hat{B}_C(t_1) > \hat{B}_C(0)$ for all $t_1 \in Y$, $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) = rmin\{\hat{B}_A(t_1), \hat{B}_C(t_1)\} > rmin\{\hat{B}_A(0), \hat{B}_C(0)\} = (\hat{B}_A \times \hat{B}_C)(0,0)$. Thus $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) > (\hat{B}_A \times \hat{B}_C)(0,0)$ for all $t_1 \in Y$, which is the contradiction to $(\hat{B}_A \times \hat{B}_C)$ is a MBJNTID of $Y \times Y$. Therefore $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$ and also $J_A(t_1) < J_A(0)$ or $J_C(t_1) < J_C(0)$ for all $t_1 \in Y$. $(J_A \times J_C)(t_1, t_1) = \max\{J_A(t_1), J_C(t_1)\} > \max\{J_A(0), J_C(0)\} = (J_A \times J_C)(0,0)$. Thus $(J_A \times J_C)(t_1, t_1) > (J_A \times J_C)(0,0)$ for all $t_1 \in Y$, which is the contradiction to $(J_A \times J_C)$ is a MBJNTID of $Y \times Y$. Therefore $J_A(0) \leq J_A(t_1)$ and $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

2. Suppose $M_C(0) < M_A(t_1)$ or $M_A(0) < M_A(t_1)$ for all $t_1 \in Y$. Then $(M_A \times M_C)(0,0) = \min\{M_A(0), M_C(0)\} = M_C(0)$ and $(M_A \times M_C)(t_1, t_1) = \min\{M_A(t_1), M_C(t_1)\} > M_C(0) = (M_A \times M_C)(0,0)$. This implies $(M_A \times M_C)(t_1, t_1) > (M_A \times M_C)(0,0)$. Which is the contradiction to $(M_A \times M_C)$ is a MBJNTID of $Y \times Y$. Hence if $M_A(0) \geq M_A(t_1)$ then $M_C(0) \geq M_A(t_1)$ and $M_C(0) \geq M_C(t_1)$ for all $t_1 \in Y$. Now Suppose $\hat{B}_C(0) < \hat{B}_A(t_1)$ or $\hat{B}_A(0) < \hat{B}_A(t_1)$ for all $t_1 \in Y$. Then $(\hat{B}_A \times \hat{B}_C)(0,0) = rmin\{\hat{B}_A(0), \hat{B}_C(0)\} = \hat{B}_C(0)$ and $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) = rmin\{\hat{B}_A(t_1), \hat{B}_C(t_1)\} > \hat{B}_C(0) = (\hat{B}_A \times \hat{B}_C)(0,0)$. This implies $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) > (\hat{B}_A \times \hat{B}_C)(0,0)$. Which is the contradiction to $(\hat{B}_A \times \hat{B}_C)$ is a MBJNTID of $Y \times Y$. Hence if $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ then $\hat{B}_C(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$. Now suppose $J_C(0) > J_A(t_1)$ or $J_A(0) > J_A(t_1)$ for all $t_1 \in Y$. Then $(J_A \times J_C)(0,0) = \max\{J_A(0), J_C(0)\} = J_C(0)$ and $(J_A \times J_C)(t_1, t_1) = \max\{J_A(t_1), J_C(t_1)\} > J_C(0) = (J_A \times J_C)(0,0)$. This implies $(J_A \times J_C)(t_1, t_1) > (J_A \times J_C)(0,0)$. Which is the contradiction to $(J_A \times J_C)$ is a MBJNTID of $Y \times Y$. Hence if $J_A(0) \leq J_A(t_1)$ then $J_C(0) \leq J_A(t_1)$ and $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

3. The proof is quit same to 2.

Theorem 3.8. Let $\Gamma: Y \rightarrow X$ is a B-homomorphism of B-algebra. If $C = (M_C, \hat{B}_C, J_C)$ is a MBJNTID of X then the pre-image $\Gamma^{-1}(C) = (\Gamma^{-1}(M_C), \Gamma^{-1}(\hat{B}_C), \Gamma^{-1}(J_C))$ of B under Γ is a MBJNTID in Y.

Proof. For any $t_1 \in Y$,

1. $\Gamma^{-1}(M_C)(t_1) = M_C(\Gamma(t_1)) \geq M_C(0) = M_C(\Gamma(0)) = \Gamma^{-1}(M_C)(0)$, $\Gamma^{-1}(\hat{B}_C)(t_1) = \hat{B}_C(\Gamma(t_1)) \geq \hat{B}_C(0) = \hat{B}_C(\Gamma(0)) = \Gamma^{-1}(\hat{B}_C)(0)$ and $\Gamma^{-1}(J_C)(t_1) = J_C(\Gamma(t_1)) \leq J_C(0) = J_C(\Gamma(0)) = \Gamma^{-1}(J_C)(0)$.

2. $\Gamma^{-1}(M_C)(t_1 * x) = M_C(\Gamma(t_1 * x)) \geq \min\{M_C(\Gamma(t_1) * \Gamma(t_2)), M_C(\Gamma(t_2))\} = \min\{M_C(\Gamma((t_1 * t_2) * x)), M_C(\Gamma(t_2))\} = \min\{\Gamma^{-1}(M_C)((t_1 * t_2) * x), \Gamma^{-1}(M_C)(t_2)\}$, also $\Gamma^{-1}(\hat{B}_C)(t_1 * x) = \hat{B}_C(\Gamma(t_1 * x)) \geq rmin\{\hat{B}_C(\Gamma(t_1) * \Gamma(t_2)), \hat{B}_C(\Gamma(t_2))\} = rmin\{\hat{B}_C(\Gamma((t_1 * t_2) * x)), \hat{B}_C(\Gamma(t_2))\} = rmin\{\Gamma^{-1}(\hat{B}_C)((t_1 * t_2) * x), \Gamma^{-1}(\hat{B}_C)(t_2)\}$ and $\Gamma^{-1}(J_C)(t_1 * x) = J_C(\Gamma(t_1 * x)) \leq \max\{J_C(\Gamma((t_1 * t_2) * x)), J_C(\Gamma(t_2))\} = \max\{J_C(\Gamma((t_1 * t_2) * x)), J_C(\Gamma(t_2))\} = \max\{\Gamma^{-1}(J_C)((t_1 * t_2) * x), \Gamma^{-1}(J_C)(t_2)\}$.

Theorem 3.9. Let $\Gamma: Y \rightarrow X$ be an epimorphism of B-algebra. Then $C = (M_C, \hat{B}_C, J_C)$ is a MBJNTID of X, if $\Gamma^{-1}(C) = (\Gamma^{-1}(M_C), \Gamma^{-1}(\hat{B}_C), \Gamma^{-1}(J_C))$ of B under Γ is a MBJNTID in Y.

Proof. For any $t_1 \in X$, there exist $a \in Y$ such that $\Gamma(a) = t_1$. Then $M_C(t_1) = M_C(\Gamma(a)) = \Gamma^{-1}(M_C)(a) \geq \Gamma^{-1}(M_C)(0) = M_C(\Gamma(0)) = M_C(0)$, $\hat{B}_C(t_1) = \hat{B}_C(\Gamma(a)) = \Gamma^{-1}(\hat{B}_C)(a) \geq \Gamma^{-1}(\hat{B}_C)(0) = \hat{B}_C(\Gamma(0)) = \hat{B}_C(0)$ and $J_C(t_1) = J_C(\Gamma(a)) = \Gamma^{-1}(J_C)(a) \leq \Gamma^{-1}(J_C)(0) = J_C(\Gamma(0)) = J_C(0)$. Now let $t_1, t_2 \in X$ then $\Gamma(a) = t_1$ and $\Gamma(b) = t_2$ for some $a, b \in Y$, now $M_C(t_1 * x) = M_C(\Gamma(a * x)) = \Gamma^{-1}(M_C)(a * x) \geq \min\{\Gamma^{-1}(M_C)((a * b) * x), \Gamma^{-1}(M_C(b))\} = \min\{M_C(\Gamma((a * b) * x)), M_C(\Gamma(b))\} = \min\{M_C(\Gamma(a) * \Gamma(b) * x), M_C(\Gamma(b))\} = \min\{M_C((t_1 * t_2) * x), M_C(t_2)\}$ and $\hat{B}_C(t_1 * x) = \hat{B}_C(\Gamma(a * x)) = \Gamma^{-1}(\hat{B}_C)(a * x) \geq r\min\{\Gamma^{-1}(\hat{B}_C)((a * b) * x), \Gamma^{-1}(\hat{B}_C(b))\} = r\min\{\hat{B}_C(\Gamma((a * b) * x)), \hat{B}_C(\Gamma(b))\} = r\min\{\hat{B}_C(\Gamma(a) * \Gamma(b) * x), \hat{B}_C(\Gamma(b))\} = r\min\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}$ and $J_C(t_1 * x) = J_C(\Gamma(a * x)) = \Gamma^{-1}(J_C)(a * x) \leq \max\{\Gamma^{-1}(J_C)((a * b) * x), \Gamma^{-1}(J_C(b))\} = \max\{J_C(\Gamma((a * b) * x)), J_C(\Gamma(b))\} = \max\{J_C(\Gamma(a) * \Gamma(b) * x), J_C(\Gamma(b))\} = \max\{J_C((t_1 * t_2) * x), J_C(t_2)\}$. Hence $C = (M_C, \hat{B}_C, J_C)$ is MBJ-neutrosophic T-ideal of Y .

7. Conclusion

In this paper, we studied the MBJ neutrosophic T-ideal, cartesian product of MBJ neutrosophic T-ideals and homomorphism of MBJ neutrosophic T-ideal through significant properties and results. This paper will give us new direction to use MBJ neutrosophic set in different atmosphere. In future, work can be done on MBJ-neutrosophic T-normal ideal, MBJ-neutrosophic cubic T-BMBJ ideal.

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