Continuity and Compactness on Neutrosophic Soft Bitopological Spaces

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Abstract:

In this manuscript, continuity, compactness concepts in neutrosophic soft bitopologi-cal space have been defined using star bineutrosophic soft open notion. Theorems and properties concerning to these two notions have been investigated here.

Keyword: Neutrosophic Soft Set, Fuzzy Set, Star Bineutrosophic Soft Open, Neutrosophic Soft Bitopological Spaces.

1. Introduction

The notion of soft set as a general mathematical tool for coping with objects involving vagueness and uncertainty has been introduced by Molodtsov [2]. F. Smarandache [3] introduced the concept of neutrosophic sets which is a generalization of Zadeh's fuzzy set, and Atanassov's intuitionistic fuzzy set, as a new mathematical tool for dealing with problems involving indeterminacy, inconsistent knowledge, incompleteness.

In 2013, the notion of neutrosophic soft set was introduced by merging soft set concept and neutrosophic set concept, and later this concept and its operations have been redefined (see, [5,6,7]). The notion of neutrosophic soft topological spaces was presented by [8], and later this concept has been redefined by [9] differently from the study [8]. In 2021, Al-Nafee, et al. [1] generalized the concept of neutrosophic soft topological spaces to the concept of neutrosophic soft bitopological spaces and they investigated several related properties and theories. The concepts of neutrosophic soft continuous mapping and compactness with their properties and some theorems were presented by many authors (see [8,10,11,12]). For more details on these concepts, you can see [13-20].

In this manuscript, the authors introduced the concepts of continuity, compactness and Hausdorff in neutrosophic soft bitopological spaces, through presenting the concepts of $N_3(bi)^*$-continuous mapping, NSbi-open mapping, NSbi-closed mapping, $N_3(bi)^*$-compact and $N_3(bi)^*$-Hausdorff based on the definition of $N_3(bi)^*$-open, some of related theorems and properties also have investigated.

2. Basic Concepts

In this section, some basic and related definitions are recalled as background and to give the reader a deep insight on the basic tools of the upcoming section. For the purpose of abbreviation, the authors will denote to the neutrosophic soft set by (NSS). Also the soft set is denoted by (SS).

Definition 2.1 [3]

The neutrosophic'set S over G is defined as follows: $S = \{< g, I_S(g), B_S(g), F_S(g) >) : g \in G\}$,

where the functions, $I,B,F : G \to]-0,+1[$ and $-0 \le I_S(g) + B_S(g) + F_S(g) \le +3$.

"((From philosophical point of view the neutrosophic set takes the value from real standard or non-standard subsets of $]-0,+1[$. But in real life application in scientific and engineering problems it is difficult to use a neutrosophic set with value from real standard or non-standard subset of $]-0,+1[$. Hence we consider the neutrosophic set which takes the value from the subset of [0, 1]))".

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Firstly, the concept of (SS) defined by Molodtsov in [2], and later this concept has been redefined by [20], as in definition 2.2 below.

Definition 2.2 [20]

Consider P(G) the set of all subsets of G. A soft set (SS) H on G is a set valued function from E to P(G). we can rewrite it as a set of ordered pairs, H = {(e, H(e)), e ∈ E}, where E is a set of parameters.

Firstly, the concept of (NSS) defined by Maji in [4], and later this concept has been redefined by Deli, and Broumi in [5], as in definition 2.3 below.

Definition 2.3 [5]

Consider G an initial universe set and E a set of parameters. P(G) denotes the set of all the neutrosophic subsets from G. An (NSS) H_E on the initial universe set G is a set defined by a set valued function H representing a mapping from E to P(G), where H is called approximate function of the (NSS) H_E. that's mean, H_E is a parameterized family of some elements of P(G) which implies to it can be rewritten as a set of ordered pairs,

$H_E = \{(e, \{< g^{(I_{H(e)}(g), B_{H(e)}(g), F_{H(e)}(g))} >: g \in G\}), e \in E\}$. $I_{H(e)}(g), B_{H(e)}(g), F_{H(e)}(g) \in [0,1]$, respectively known as Truth-Membership, Indeterminacy-Membership and Falsity-Membership function of H(e). It is well known that the supremum of each I, B, F equal 1 , so the inequality, $0 \leq I_{H(e)}(g), B_{H(e)}(g), F_{H(e)}(g) \leq 3$ is apparent.

Note:

1) In this work, we will use definition of (NSS) given by Deli, and Broumi in [5].

2) From the def. 2.3, and up to the rest of this paper, the notion $N_3(G)$ will be represent to the set of all (NSSs) over G**.**

Definition 2.4 [4,9]

Consider H_E and $k_E \in N_3(G)$, where

$H_E = \{(e, \{< g^{(I_{H(e)}(g), B_{H(e)}(g), F_{H(e)}(g))} >: g \in G\}), e \in E\}$.

$k_E = H_E = \{(e, \{< g^{(I_{K(e)}(g), B_{K(e)}(g), F_{K(e)}(g))} >: g \in G\}), e \in E\}$. Then:

- $\widetilde{G}_E = \{(e, \{< g^{(1,1,0)} > : g \in G\}), e \in E\}$.
- $\widetilde{\emptyset}_E = \{(e, \{< g^{(0,0,1)} > : g \in G\}), e \in E\}$.
- $H_E \sqsubseteq k_E \leftrightarrow \{(e, \{< g^{(I_{H(e)}(g) \leq I_{K(e)}(g), B_{H(e)}(g) \leq B_{K(e)}(g), F_{H(e)}(g) \geq F_{K(e)}(g))} > : g \in G\}), e \in E\}$.
- $H_E \sqcup k_E = \{(e, \{< g^{(I_{H(e)}(g) \vee I_{K(e)}(g), B_{H(e)}(g) \vee B_{K(e)}(g), F_{H(e)}(g) \wedge F_{K(e)}(g))} > : g \in G\}), e \in E\}$.
- $H_E \sqcap k_E = \{(e, \{< g^{(I_{H(e)}(g) \wedge I_{K(e)}(g), B_{H(e)}(g) \wedge B_{K(e)}(g), F_{H(e)}(g) \vee F_{K(e)}(g))} > : g \in G\}), e \in E\}$.

Definition 2.5 [1]

Let $H_E = \{(e, \{< g^{(I_{H(e)}(g), B_{H(e)}(g), F_{H(e)}(g))} >: g \in G\}), e \in E\} \in N_3(G)$. Then, the complement of H_E ($(H_E)^C$) is defined as:

$(H_E)^C = \{(e, \{< g^{(1-I_{H(e)}(g), 1-B_{H(e)}(g), 1-F_{H(e)}(g))} > : g \in G\}), e \in E\}$.

Definition 2.6 [13]

Consider G an initial universe set, and E a set of parameters. Then the neutrosophic soft set "${x^e}_{(\alpha,\beta,\gamma)}$" is called a "neutrosophic soft point", for every $x \in G$, $0 < \alpha, \beta, \gamma \leq 1, e \in E$, and is defined as follows:

$${x^e}_{(\alpha,\beta,\gamma)}(e')(y) = \begin{cases} (\alpha, \beta, \gamma) \text{ if } e = e' \text{and } x = y \\ (0,0,1) \text{ if } e \neq e' \text{and } x \neq y \end{cases}.$$

Definition 2.7 [9]

Consider $\sigma \subseteq N_3(G)$. Then σ is called a neutrosophic soft topology on G if,

1) $\widetilde{G}_E, \widetilde{\emptyset}_E \in \sigma$.
2) If $U_E, V_E \in \sigma$ then $U_E \sqcap V_E \in \sigma$.

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3) If $V_{i_E} \in \sigma$, for each $i \in I$, then $\sqcup_{i\in I} V_{i_E} \in \sigma$.

The triplet (G,E,σ) is called a neutrosophic soft topological space (in abbrev, Nst-space).

Each member of σ is named as a neutrosophic soft open set and their complement is called a neutrosophic soft closed set.

The neutrosophic soft interior of $\beta_B \in N_3(G)$"($(V_E)^0$) is defended as:

$(V_E)^0 = \sqcup\{ U_E : U_E$ is a neutrosophic soft open set, $U_E \sqsubseteq V_E\}$.

The neutrosophic soft closure of $\beta_B \in N_3(G)$ ($\overline{(V_E)}$) is defended as:

$\overline{(V_E)} = \sqcap\{ U_E : U_E$ is a neutrosophic soft closed set, $V_E \sqsubseteq U_E\}$.

Definition 2.8 [1]

Let (G,E,τ) and (G,E,σ) be two Nst-spaces defined on G. Then (G,E,τ,σ) or (G, for abbreviation purposes) is called a neutrosophic soft bitopological space or (in abbrev, BIN-space).

From this definition up to the rest of the paper and for abbreviation purpose, we give an attention to the readers that (G, E, τ, σ) will sometimes be represented as G.

Definition 2.9 [1]

A subset $V_E \in N_3(G)$ of BIN-space (G, E, τ, σ) is called a star bineutrosophic soft open (abbreviation, $N_3(bi)^*$-open) over G iff $V_E \sqsubseteq \overline{(V_E)^{o\sigma}}^{(\tau)^{o\sigma}}$ and their complement is a star bineutrosophic soft closed (in abbrev, $N_3(bi)^*$-closed). The set of all $N_3(bi)^*$-open ($N_3(bi)^*$-closed) sets over G is denoted by $G^{(Bi)*-NSO}$ ($G^{(Bi)*-NSC}$), respectively.

Definition 2.10 [1]

Let (G, E, τ, σ) be an BIN-space and $V_E \in N_3(G)$. Then,

- $(bi)^*$-neutrosophic.soft interior of V_E ($(V_E)^{o(bi)*}$) is defined as:

$(V_E)^{o(bi)*} = \sqcup\{ U_E : U_E$ is an $N_3(bi)^*$-open set, $U_E \sqsubseteq V_B\}$.

- $(bi)^*$-neutrosophic-soft closure of V_E ($\overline{(V_E)}^{(bi)*}$) is defined as:

$\overline{(V_E)}^{(bi)*} = \sqcap\{ U_E : U_E$ is an $N_3(bi)^*$-closed set, $V_E \sqsubseteq U_E\}$.

Remark 2.11

In ref. [1], the theorem 4.9. and theorem 4.12., the equalities $((V_E \sqcap U_E)^{o(bi)*} = (V_E)^{o(bi)*} \sqcap (U_E)^{o(bi)*})$, $\overline{V_E \sqcup U_E)}^{(bi)*} = \overline{(V_E)}^{(bi)*} \sqcup \overline{(U_E)}^{(bi)*}$ are in general not true.

i.e. $((V_E \sqcap U_E)^{o(bi)*} \sqsubset (V_E)^{o(bi)*} \sqcap (U_E)^{o(bi)*})$, $\overline{(V_E)}^{(bi)*} \sqcup \overline{(U_E)}^{(bi)*} \sqsubset \overline{V_E \sqcup U_E)}^{(bi)*}$, the following example has been originated by the authors to demonstrate this remark:

Example 2.12

Let G = $\{g_1, g_2, g_3, g_4\}$ and E = {e}. And let $H_E, K_E, R_E, S_E, L_E, B_E, A_E, F_E \in N_3(G)$ such that,

$$H_E = \{ (e, \{< g_1{}^{(1,1,0)} >, < g_2{}^{(0,0,1)} >, < g_3{}^{(0,0,1)} >, < g_4{}^{(0,0,1)} >\}) \}.$$

$$K_E = \{ (e, \{< g_1{}^{(0,0,1)} >, < g_2{}^{(0,0,1)} >, < g_3{}^{(0,0,1)} >, < g_4{}^{(1,1,0)} >\}) \}.$$

$$R_E = \{ (e, \{< g_1{}^{(1,1,0)} >, < g_2{}^{(0,0,1)} >, < g_3{}^{(0,0,1)} >, < g_4{}^{(1,1,0)} >\}) \}.$$

$$S_E = \{ (e, \{< g_1{}^{(0,0,1)} >, < g_2{}^{(1,1,0)} >, < g_3{}^{(1,1,0)} >, < g_4{}^{(0,0,1)} >\}) \}.$$

$$L_E = \{ (e, \{< g_1{}^{(1,1,0)} >, < g_2{}^{(1,1,0)} >, < g_3{}^{(1,1,0)} >, < g_4{}^{(0,0,1)} >\}) \}.$$

$$B_E = \{ (e, \{< g_1{}^{(0,0,1)} >, < g_2{}^{(1,1,0)} >, < g_3{}^{(1,1,0)} >, < g_4{}^{(1,1,0)} >\}) \}.$$

$$A_E = \{ (e, \{< g_1{}^{(1,1,0)} >, < g_2{}^{(0,0,1)} >, < g_3{}^{(1,1,0)} >, < g_4{}^{(0,0,1)} >\}) \}.$$

$$F_E = \{ (e, \{< g_1{}^{(0,0,1)} >, < g_2{}^{(1,1,0)} >, < g_3{}^{(0,0,1)} >, < g_4{}^{(1,1,0)} >\}) \}.$$

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$\tau = \{\widetilde{\emptyset}_E, \widetilde{G}_E, H_E, K_E, R_E, S_E, L_E, B_E\}$ is an Nst-space on G, $\sigma = \{\widetilde{\emptyset}_E, \widetilde{G}_E, H_E, K_E, R_E\}$ is an Nst-space on G. Then $((L_E \sqcap S_E)^{0(bi)*} \neq (L_E)^{0(bi)*} \sqcap (S_E)^{0(bi)*})$. Also, $\overline{F_E \sqcup R_E}^{(bi)*} \neq \overline{(F_E)}^{(bi)*} \sqcup \overline{(R_E)}^{(bi)*}$.

For more details and background on these concepts (return to the ref. [1]).

3. Continuity on neutrosophic soft Bitopological Spaces

The authors have dedicated this portion of the manuscript to introducing the concept of N3(bi)* -continuous mapping, NSbi-open mapping, NSbi-closed mapping in neutrosophic soft bitopological spaces, they also present a deep investigation into the related theorems and properties.

Definition 3.1

Let (G, E, τ, σ) and (G', E', γ, β) be two BIN-spaces. A mapping $f_\mu:(G, E, \tau, \sigma) \to (G', E', \gamma, \beta)$ is said to be $N_3(bi)^*$-continuous at $x^e_{(\alpha,\beta,\gamma)}$ iff every $N_3(bi)^*$-open set $V_{E'}$ over G' containing $f_\mu(x^e_{(\alpha,\beta,\gamma)})$, there exists $N_3(bi)^*$-open set U_E over G containing $x^e_{(\alpha,\beta,\gamma)}$ such that $f_\mu(U_E) \sqsubseteq V_{E'}$. If f_μ is an $N_3(bi)^*$-continuous for all $x^e_{(\alpha,\beta,\gamma)}$, then f_μ is called $N_3(bi)^*$-continuous over G.

Theorem 3.2

If (G, E, τ, σ), and (G', E', γ, β) are two BIN-spaces and $f_\mu: (G, E, \tau, \sigma) \to (G', E', \gamma, \beta)$ is a mapping. Then, the upcoming conditions, are identical:

(1) $f_\mu: (G, E, \tau, \sigma) \to (G', E', \gamma, \beta)$ is an $N_3(bi)^*$-continuous.

(2) For each $N_3(bi)^*$-open set $V_{E'}$ over G', $f_\mu^{-1}(V_{E'})$ is an $N_3(bi)^*$-open set over G.

(3) For each $N_3(bi)^*$-closed set $V_{E'}$ over G', $f_\mu^{-1}(V_{E'})$ is an $N_3(bi)^*$-closed set over G.

(4) For each $U_E \in N_3(G)$, $f_\mu\overline{(U_E)}^{(bi)*} \sqsubset \overline{(f_\mu(U_E))}^{(bi)*}$.

(5) For each $V_{E'} \in N_3(G')$, $\overline{(f_\mu^{-1}(V_{E'}))}^{(bi)*} \sqsubset f_\mu^{-1}\overline{(V_{E'})}^{(bi)*}$.

(6) For each $V_{E'} \in N_3(G')$, $f_\mu^{-1}\left((V_{E'})^{o\,(bi)*}\right) \sqsubset (f_\mu^{-1}(V_{E'}))^{o(bi)*}$.

Proof: (1) → (2)

Let $V_{E'}$ be an $N_3(bi)^*$-open set over $G'$and $x^e_{(\alpha,\beta,\gamma)} \in f_\mu^{-1}(V_{E'})$ be an arbitrary neutrosophic soft point. Then $f_\mu\left(x^e_{(\alpha,\beta,\gamma)}\right) = (f_\mu(x))^{\mu(e)}_{(\alpha,\beta,\gamma)} \in V_{E'}$.

Since f_μ is an $N_3(bi)^*$-continuous mapping, there exists $N_3(bi)^*$-open set U_E over G containing $x^e_{(\alpha,\beta,\gamma)}$ such that $f_\mu(U_E) \sqsubset V_{E'}$.

This implies that $x^e_{(\alpha,\beta,\gamma)} \in U_E \sqsubset f_\mu^{-1}(V_{E'})$, $f_\mu^{-1}(V_{E'})$ is an $N_3(bi)^*$-open set over G.

(2) → (1)

Let $x^e_{(\alpha,\beta,\gamma)}$ be a neutrosophic soft point and $V_{E'}$ be an $N_3(bi)^*$-open set over G'containing $f_\mu\left(x^e_{(\alpha,\beta,\gamma)}\right)$.

So $f_\mu^{-1}(V_{E'})$ is an $N_3(bi)^*$-open set over G containing $\left(x^e_{(\alpha,\beta,\gamma)}\right)$ and $f_\mu(f_\mu^{-1}(V_{E'})) \sqsubset V_{E'}$.

(2) → (3) (obvious).

(3) → (4)

Let $U_E \in N_3(G)$. Since $\overline{(f_\mu(U_E))}^{(bi)*}$ is an $N_3(bi)^*$-closed set over G'.

$\therefore f_\mu^{-1}\overline{(f_\mu(U_E))}^{(bi)*}$ is an $N_3(bi)^*$-closed set over G, $f_\mu^{-1}\overline{(f_\mu(U_E))}^{(bi)*} = \overline{f_\mu^{-1}\overline{(f_\mu(U_E))}^{(bi)*}}^{(bi)*}$

Now:

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Since $U_E \sqsubset f_\mu^{-1}(f_\mu(U_E))$, $U_E \sqsubset f_\mu^{-1}(f_\mu(U_E)) \sqsubset f_\mu^{-1}\overline{(f_\mu(U_E))}^{(bi)*}$.

This implies that, $\overline{U_E}^{(bi)*} \sqsubset f_\mu^{-1}\overline{(f_\mu(U_E))}^{(bi)*}$, $f_\mu(\overline{U_E}^{(bi)*}) \sqsubset f_\mu(f_\mu^{-1}\overline{(f_\mu(U_E))}^{(bi)*}) \sqsubset \overline{(f_\mu(U_E))}^{(bi)*}$.

(4) → (5)

Let $V_{E'} \in N_3(G')$ and $f_\mu^{-1}(V_{E'}) = U_E$.

From (4), we have $f_\mu\left(\overline{U_E}^{(bi)*}\right) = f_\mu\left(\overline{\left(f_\mu^{-1}(V_{E'})\right)}^{(bi)*}\right) \sqsubset \overline{f_\mu\left(f_\mu^{-1}(V_{E'})\right)}^{(bi)*} \sqsubset \overline{V_{E'}}^{(bi)*}$.

Then $\overline{\left(f_\mu^{-1}(V_{E'})\right)}^{(bi)*} = \overline{U_E}^{(bi)*} \sqsubset f_\mu^{-1}\left(f_\mu\left(\overline{U_E}^{(bi)*}\right)\right) \sqsubset f_\mu^{-1}\overline{(V_{E'})}^{(bi)*}$.

(5) → (6)

Let $V_{E'} \in N_3(G')$. Substituting $(V_{E'})^c$ for condition in (5).

Then $\overline{f_\mu^{-1}(V_{E'})^c}^{(bi)*} \sqsubset f_\mu^{-1}\overline{(V_{E'})^c}^{(bi)*}$.

It is clear that $(V_{E'})^{o(bi)*} = \left(\overline{(V_{E'})^c}^{(bi)*}\right)^c$. Then we have,

$$f_\mu^{-1}\left((V_E)^{o(bi)*}\right) = f_\mu^{-1}\left(\left(\overline{(V_{E'})^c}^{(bi)*}\right)^c\right) = \left(f_\mu^{-1}\overline{(V_{E'})^c}^{(bi)*}\right)^c \sqsubset \left(\overline{f_\mu^{-1}(V_{E'})^c}^{(bi)*}\right)^c = \left(f_\mu^{-1}(V_{E'})\right)^{o(bi)*}.$$

(6) → (2)

Let $V_{E'}$ be an $N_3(bi)^*$-open set over G'.

Since $\left(f_\mu^{-1}(V_{E'})\right)^{o\,(bi)*} \sqsubset f_\mu^{-1}(V_{E'}) = f_\mu^{-1}\left((V_{E'})^{o(bi)*}\right) \sqsubset \left(f_\mu^{-1}(V_{E'})\right)^{o(bi)*}$,

then $\left(f_\mu^{-1}(V_{E'})\right)^{o\,(bi)*} = f_\mu^{-1}(V_{E'})$ is obtained.

This implies that $f_\mu^{-1}(V_E)$ is an $N_3(bi)^*$-open set over G.

Definition 3.3

Let (G, E, τ, σ) and (G', E', γ, β) be an BIN-spaces and $f_\mu: (G, E, \tau, \sigma) \to (G', E', \gamma, \beta)$ be a mapping. Then,

1) A mapping f_μ is called an NSbi-open, if the image $f_\mu(U_E)$ of each $N_3(bi)^*$-open set U_E over G is an $N_3(bi)^*$-open set over G'.
2) A mapping f_μ is called an NSbi-closed if the image $f_\mu(U_E)$ of each $N_3(bi)^*$-closed set U_E over G is an $N_3(bi)^*$- closed set over G'.

Theorem 3.4

If (G, E, τ, σ), and (G', E', γ, β) are two BIN-spaces and $f_\mu: (G, E, \tau, \sigma) \to (G', E', \gamma, \beta)$ is a mapping. Then, f_μ is an NSbi-open mapping iff for each $U_E \in N_3(G)$, $f_\mu\left((U_E)^{o(bi)*}\right) \sqsubset \left(f_\mu(U_E)\right)^{o(bi)*}$ is satisfied.

Proof

Let f_μ be an NSbi-open mapping and $U_E \in N_3(G)$.

Then $(U_E)^{o(bi)*}$ is an $N_3(bi)^*$-open set and $(U_E)^{o(bi)*} \sqsubset U_E$.

Since f_μ is an NSbi-open mapping, $f_\mu\left((U_E)^{o(bi)*}\right)$ is an $N_3(bi)^*$-open set over G' and

$f_\mu\left((U_E)^{o(bi)*}\right) \sqsubset f_\mu(U_E)$. Thus $f_\mu\left((U_E)^{o(bi)*}\right) \sqsubset (f_\mu(U_E))^{o(bi)*}$ is obtained.

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Conversely

Let U_E be any $N_3(bi)^*$-open set over G. Then $U_E = (U_E)^{o(bi)*}$.

From the condition of theorem, we have $f_\mu\big((U_E)^{o\,(bi)*}\big) \sqsubset (f_\mu(U_E))^{o\,(bi)*}$.

Then $f_\mu(U_E) = f_\mu\big((U_E)^{o\,(bi)*}\big) \sqsubset (f_\mu(U_E))^{o(bi)*} \sqsubset f_\mu(U_E)$.

This implies that $f_\mu(U_E) = f\big((U_E)^{o\,(bi)*}\big)$.That is, $f_\mu$is an NSbi-open mapping.

Theorem 3.5

If (G, E, τ, σ), and (G', E', γ, β) are two BIN-spaces and $f_\mu: (G, E, \tau, \sigma) \to (G', E', \gamma, \beta)$ is a mapping. Then, f_μ is an NSbi-closed mapping iff for each $U_E \in N_3(G)$, $\overline{(f_\mu(U_E))}^{(bi)*} \sqsubset f_\mu\left(\overline{U_E}^{(bi)*}\right)$ is satisfied.

Proof

Let f_μ be an $N_3(bi)^*$- closed mapping and $U_E \in N_3(G)$.

Since f_μ is an NSbi-closed mapping, $f_\mu\left(\overline{U_E}^{(bi)*}\right)$ is an $N_3(bi)^*$-closed set over G' and

$f(U_E) \sqsubset f_\mu\left(\overline{U_E}^{(bi)*}\right)$. Thus $\overline{(f_\mu(U_E))}^{(bi)*} \sqsubset f_\mu\left(\overline{U_E}^{(bi)*}\right)$ is obtained.

Conversely

Let U_E be any $N_3(bi)^*$-closed set over G.

From the condition of the theorem $\overline{(f_\mu(U_E))}^{(bi)*} \sqsubset f_\mu\left(\overline{U_E}^{(bi)*}\right) = f_\mu(U_E) \sqsubset \overline{(f_\mu(U_E))}^{(bi)*}$.

This means that $\overline{(f_\mu(U_E))}^{(bi)*} = f_\mu(U_E)$. That is, f_μ is an NSbi-closed mapping.

Example 3.6

Let $G = \{g_1, g_2, g_3\}$, $G' = \{y_1, y_2, y_3\}$ and $E = \{e\}$.

And let $H_E, K_E, R_E \in N_3(G)$ such that:

$$H_E = \{(e, \{<g_1{}^{(1,1,0)}>, <g_2{}^{(0,0,1)}>, <g_3{}^{(0,0,1)}>\})\}.$$

$$K_E = \{(e, \{<g_1{}^{(1,1,0)}>, <g_2{}^{(1,1,0)}>, <g_3{}^{(0,0,1)}>\})\}.$$

$$R_E = \{(e, \{<g_1{}^{(1,1,0)}>, <g_2{}^{(0,0,1)}>, <g_3{}^{(1,1,0)}>\})\}.$$

$\sigma = \{\widetilde{\emptyset}_E, \widetilde{G}_E, H_E\}$ is an Nst-space on G.

$\tau = \{\widetilde{\emptyset}_E, \widetilde{G}_E\}$ is an Nst-space on G.

Then,

(G,E,τ,σ) is an BIN-space,

$G^{(Bi)*-NSO} = \{\widetilde{\emptyset}_E, \widetilde{G}_E, H_E, K_E, R_E\}$.

And let $H'_E, K'_E, R'_E \in N_3(G')$ such that:

$$H'_E = \{(e, \{<y_1{}^{(1,1,0)}>, <y_2{}^{(0,0,1)}>, <y_3{}^{(0,0,1)}>\})\}.$$

$$K'_E = \{(e, \{<y_1{}^{(1,1,0)}>, <y_2{}^{(1,1,0)}>, <y_3{}^{(0,0,1)}>\})\}.$$

$$R'_E = \{(e, \{<y_1{}^{(1,1,0)}>, <y_2{}^{(0,0,1)}>, <y_3{}^{(1,1,0)}>\})\}.$$

$\beta = \{\widetilde{\emptyset}_E, \widetilde{G'}_E, H'_E\}$ is an Nst-space on G'.

$\gamma = \{\widetilde{\emptyset}_E, \widetilde{G'}_E\}$ is an Nst-space on G'.

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Then (G',E,γ,β) is an BIN-space and $G'^{(Bi)*-NSO} = \{\widetilde{\emptyset}_E, \widetilde{G'}_E, H'_E, K'_E, R'_E\}$.

Now, if f_μ is a mapping from (G,E,τ,σ) to (G',E,γ,β), defined as follows:

$f_\mu(g_1) = y_1$, $f_\mu(g_2) = y_2$, $f_\mu(g_3) = y_3$, $\mu: E \to E$, $\mu(e) = e$.

Then it is easy to verify,

$f_\mu^{-1}(D_E)$ is an N3(bi)*-open set over G, for all N3(bi)*-open set D_E over G'.

$f_\mu(D_E)$ is an N3 (bi)*-open set over G', for all N3 (bi)*-open set D_E over G.

Therefore,

f_μ is an N3(bi)*-continuous mapping from (G,E,τ,σ) to (G',E,γ,β),

f_μ is an NSbi-open mapping from (G,E,τ,σ) to (G',E,γ,β).

Example 3.7

Let $G = \{g_1,g_2,g_3\}$, $G' = \{y_1,y_2\}$ and $E = \{e\}$. And let $H_E, K_E, R_E \in N_3(G)$ such that:

$$H_E = \{ (e, \{< g_1{}^{(1,1,0)} >, < g_2{}^{(0,0,1)} >, < g_3{}^{(0,0,1)} >\}) \}.$$

$$K_E = \{ (e, \{< g_1{}^{(0,0,1)} >, < g_2{}^{(1,1,0)} >, < g_3{}^{(0,0,1)} >\}) \}.$$

$$R_E = \{ (e, \{< g_1{}^{(1,1,0)} >, < g_2{}^{(1,1,0)} >, < g_3{}^{(0,0,1)} >\}) \}.$$

$\sigma = \{\widetilde{\emptyset}_E, \widetilde{G}_E, H_E\}$ is an Nst-space on G.

$\tau = \{\widetilde{\emptyset}_E, \widetilde{G}_E, K_E, R_E\}$ is an Nst-space on G.

Then,

(G,E,τ,σ) is an BIN-space,

$G^{(Bi)*-NSO} = \{\widetilde{\emptyset}_E, \widetilde{G}_E, H_E\}$.

And let $H'_E, K'_E, \in N_3(G')$ such that:

$$H'_E = \{ (e, \{< y_1{}^{(1,1,0)} >, < y_2{}^{(0,0,1)} >\}) \}.\ K'_E = \{ (e, \{< y_1{}^{(0,0,1)} >, < y_2{}^{(1,1,0)} >\}) \}.$$

$\beta = \{\widetilde{\emptyset}_B, \widetilde{G'}_E, H'_E, K'_E \}$ is an Nst-space on G'.

$\gamma = \{\widetilde{\emptyset}_E, \widetilde{G'}_E, H'_E\}$ is an Nst-space on G'.

Then (G',E,γ,β) is an BIN-space and $G'^{(Bi)*-NSO} = \{\widetilde{\emptyset}_B, \widetilde{G'}_E, H'_E, K'_E\}$.

Now, if f_μ is a mapping from (G,E,τ,σ) to (G',E,γ,β), defined as follows:

$f_\mu(g_1) = y_1$, $f_\mu(g_2) = f_\mu(g_3) = y_2$, $\mu: E \to E$, $\mu(e) = e$. Note that:

$f_\mu{}^{-1}(\widetilde{\emptyset}_E) = \widetilde{\emptyset}_E$ is an N3 (bi)*-open set over G,

$f_\mu{}^{-1}(\widetilde{G'}_E) = \widetilde{G}_E$ is an N3 (bi)*-open set over G,

$f_\mu{}^{-1}(K'_E) = \{ (e, \{< g_1{}^{(0,0,1)} >, < g_2{}^{(1,1,0)} >, < g_3{}^{(1,1,0)} >\})\}$ is not an N3 (bi)*-open set over G.

And

$f_\mu(\widetilde{\emptyset}_E) = \widetilde{\emptyset}_E$ is an N3 (bi)*-open set over G',

$f_\mu(\widetilde{G}_E) = \widetilde{G'}_E$ is an N3 (bi)*-open set over G',

$f_\mu(H_E) = H'_E$ is an N3 (bi)*-open set over G',

Therefore, f_μ is an NSbi-open mapping from (G,E,τ,σ) to (G',E,γ,β), but not N3(bi)*-continuous.

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Theorem 3.8

Let(G,E,τ,σ), (G',E',γ,β) and $(G'',E'',\pi,\varepsilon)$be an BIN-spaces. If $f_{\mu}:(G,E,\tau,\sigma)\rightarrow(G',E',\gamma,\beta)$ and $g_{\varphi}:(G',E',\gamma,\beta)\rightarrow(G'',E'',\pi,\varepsilon)$ are an $N_3(bi)^*$-continuous mappings, then $(gof)_{(\varphi o\mu)}:(G,E,\tau,\sigma)\rightarrow(G'',E'',\pi,\varepsilon)$ is an $N_3(bi)^*$-continuous mapping.

Proof

Let $U_{E''}$ be any $N_3(bi)^*$-open set over G''.

Since $((gof)_{(\varphi o\mu)})^{-1}(U_{E''})=((f^{-1}og^{-1})_{(\mu^{-1}o\varphi^{-1})})^{\cdot}(U_{E''})$ and

$g_{\varphi}:(G',E',\gamma,\beta)\rightarrow(G'',E'',\pi,\varepsilon)$ is an $N_3(bi)^*$-continuous mapping.

Then $(g_{\varphi})^{-1}(U_{E''})$ is an $N_3(bi)^*$-open set over G'.

On the other hand, since $f_{\mu}:(G,E,\tau,\sigma)\rightarrow(G',E',\gamma,\beta)$ is an $N_3(bi)^*$-continuous mapping.

Then $((f^{-1}og^{-1})_{(\mu^{-1}o\varphi^{-1})})^{\cdot}(U_{E''})$ is an $N_3(bi)^*$-open set over G.

That is, $((gof)_{(\varphi o\mu)})^{-1}(U_{E''})$ is an $N_3(bi)^*$-open set over G and

$(gof)_{(\varphi o\mu)}:(G,E,\tau,\sigma)\rightarrow(G'',E'',\pi,\varepsilon)$ is an $N_3(bi)^*$-continuous mapping.

Theorem 3.9

Let(G,E,τ,σ) and (G',E',γ,β) be an BIN-spaces, $f_{\mu}:(G,E,\tau,\sigma)\rightarrow(G',E',\gamma,\beta)$ be a bijective mapping. Then the following conditions are equivalent:

(i) f_{μ} is an $N_3(bi)^*$- homeomorphism,

(ii) f_{μ} is an $N_3(bi)^*$- continuous and NSbi-closed mapping,

(iii) f_{μ} is an $N_3(bi)^*$- continuous and NSbi-open mapping.

4. Compactness on neutrosophic soft Bitopological Spaces

This part of the manuscript has been devoted to introduce the notion of N3(bi)* -compact, N3(bi)* - Hausdorff in neutrosophic soft bitopological spaces, we also investigated their related theorems and properties.

Definition 4.1

A family NS= $\{B_{j_E}\}_{j\in J}$ of $N_3(bi)^*$-open subsets of BIN-space (G,E,τ,σ) is called an $N_3(bi)^*$-open cover of $B_E\in N_3(G)$ iff $B_E\sqsubseteq\sqcup_{j\in J}B_{j_E}$holds. If $\widetilde{G}_E=\sqcup_{j\in J}B_{j_E}$, then NS $=\{B_{j_E}\}_{j\in J}$ is said to be an $N_3(bi)^*$-open cover of (G,E,τ,σ). If NS is a finite, then NS is called a finite $N_3(bi)^*$-open cover of $\widetilde{G}_E$.

Definition 4.2

A finite subfamily of an $N_3(bi)^*$-open cover $\{B_{j_E}\}_{j\in J}$ of (G,E,τ,σ) is called a finite $N_3(bi)^*$-subcover of $\{B_{j_E}\}_{j\in J}$, if it is also an $N_3(bi)^*$-open cover of (G,E,τ,σ).

Definition 4.3

A BIN-space (G,E,τ,σ) is said to be an $N_3(bi)^*$-compact iff every an $N_3(bi)^*$-open cover of (G,E,τ,σ) has a finite $N_3(bi)^*$-subcover .

Definition 4.4

A subset B_E of an BIN-space (G,E,τ,σ) is called an $N_3(bi)^*$-compact provided for every family $\{B_{j_E}\}_{j\in J}$ of $N_3(bi)^*$-open subsets of(G,E,τ,σ)such that$B_E\sqsubseteq\sqcup_{j\in J}B_{j_E}$, there exists $j_1,j_2,\dots,j_n$, such that $B_E\sqsubseteq\sqcup_{i=1}^{n}B_{j_{i_E}}$.

Theorem 4.5

If f_{μ} is an $N_3(bi)^*$-continuous mapping from an $N_3(bi)^*$-compact space (G,E,τ,σ) onto an BIN-space (G',E',γ,β). Then (G',E',γ,β) is an $N_3(bi)^*$-compact.

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Theorem 4.6

An BIN-space is an $N_3(bi)^*$-compact iff given any family $\{B_{j_E}\}_{j\in J}$ of $N_3(bi)^*$-closed subsets of (G, E, τ, σ) such that the intersection of any finite number of the B_{j_E} is nonempty.

Theorem 4.7

Every $N_3(bi)^*$-closed subset of $N_3(bi)^*$-compact space is an $N_3(bi)^*$-compact**.**

Note : The proofs of the theorems (4.5, 4.6, 4.7) are similar to the corresponding theorems in the neutrosophic soft compact topological spaces (For more details the reader can return to the ref. [12]).

Definition 4.8

An BIN-space (G,E,τ,σ) is called an $N_3(bi)^*$-Hausdorff if and only if for each pair of distinct points ${g_1}^e_{(\alpha,\beta,\gamma)}$, ${g_2}^e_{(\alpha,\beta,\gamma)}$ of (G,E,τ,σ), there exists two $N_3(bi)^*$-open sets L_E, K_E Such that ${g_1}^e_{(\alpha,\beta,\gamma)} \in H_E$, ${g_2}^e_{(\alpha,\beta,\gamma)} \in L_E$, $H_E \sqcap L_E = \widetilde{\emptyset}_E$. (See Example 4.11, (G,E,τ,σ) is an $N_3(bi)^*$-Hausdorff).

Theorem 4.9 [12]

Every neutrosophic soft compact subset of a neutrosophic soft Hausdorff topological space is a neutrosophic soft closed .

Remark 4.10

The above theorem in neutrosophic soft bitopological spaces (BIN-spaces) is not true, the authors have originated the upcoming example to demonstrate this claim.

Example 4.11

Let $G = \{g_1, g_2, g_3, g_4\}$ and $E = \{e\}$. And let $H_E, K_E, R_E, S_E, L_E, B_E, A_E, C_E, D_E, F_E, W_E, M_E \in N_3(G)$ such that:

$$H_E = \{ (e, \{< {g_1}^{(1,1,0)} >, < {g_2}^{(0,0,1)} >, < {g_3}^{(0,0,1)} >, < {g_4}^{(0,0,1)} >\}) \}.$$

$$K_E = \{ (e, \{< {g_1}^{(0,0,1)} >, < {g_2}^{(1,1,0)} >, < {g_3}^{(0,0,1)} >, < {g_4}^{(0,0,1)} >\}) \}.$$

$$R_E = \{ (e, \{< {g_1}^{(1,1,0)} >, < {g_2}^{(1,1,0)} >, < {g_3}^{(0,0,1)} >, < {g_4}^{(0,0,1)} >\}) \}.$$

$$S_E = \{ (e, \{< {g_1}^{(0,0,1)} >, < {g_2}^{(0,0,1)} >, < {g_3}^{(1,1,0)} >, < {g_4}^{(1,1,0)} >\}) \}.$$

$$L_E = \{ (e, \{< {g_1}^{(1,1,0)} >, < {g_2}^{(0,0,1)} >, < {g_3}^{(1,1,0)} >, < {g_4}^{(1,1,0)} >\}) \}.$$

$$B_E = \{ (e, \{< {g_1}^{(0,0,1)} >, < {g_2}^{(1,1,0)} >, < {g_3}^{(1,1,0)} >, < {g_4}^{(1,1,0)} >\}) \}.$$

$$A_E = \{ (e, \{< {g_1}^{(1,1,0)} >, < {g_2}^{(1,1,0)} >, < {g_3}^{(1,1,0)} >, < {g_4}^{(0,0,1)} >\}) \}.$$

$$C_E = \{ (e, \{< {g_1}^{(1,1,0)} >, < {g_2}^{(0,0,1)} >, < {g_3}^{(1,1,0)} >, < {g_4}^{(0,0,1)} >\}) \}.$$

$$D_E = \{ (e, \{< {g_1}^{(1,1,0)} >, < {g_2}^{(0,0,1)} >, < {g_3}^{(0,0,1)} >, < {g_4}^{(1,1,0)} >\}) \}.$$

$$F_E = \{ (e, \{< {g_1}^{(0,0,1)} >, < {g_2}^{(1,1,0)} >, < {g_3}^{(1,1,0)} >, < {g_4}^{(0,0,1)} >\}) \}.$$

$$W_E = \{ (e, \{< {g_1}^{(0,0,1)} >, < {g_2}^{(1,1,0)} >, < {g_3}^{(0,0,1)} >, < {g_4}^{(1,1,0)} >\}) \}.$$

$$M_E = \{ (e, \{< {g_1}^{(1,1,0)} >, < {g_2}^{(1,1,0)} >, < {g_3}^{(0,0,1)} >, < {g_4}^{(1,1,0)} >\}) \}.$$

$\sigma = \{\widetilde{\emptyset}_E, \widetilde{G}_E, H_E, K_E, R_E, S_E, L_E, B_E\}$ is an Nst-space on G. $\tau = \{\widetilde{\emptyset}_E, \widetilde{G}_E, H_E\}$ is an Nst-space on G.

Then, $G^{(Bi)*-NSO} = \{\widetilde{\emptyset}_E, \widetilde{G}_E, H_E, K_E, R_E, S_E, L_E, B_E, A_E, C_E, D_E, F_E, W_E, M_E\}$, (G,E,τ,σ) is an BIN-space.

Note that: (G,E,τ,σ) is an $N_3(bi)^*$-Hausdorff is an $N_3(bi)^*$-compact, also it is an $N_3(bi)^*$-compact and so any subset of (G,E,τ,σ). But $M_E = \{ (e, \{< {g_1}^{(1,1,0)} >, < {g_2}^{(1,1,0)} >, < {g_3}^{(0,0,1)} >, < {g_4}^{(1,1,0)} >\}) \}$ is not $N_3(bi)^*$-closed set over G.

5.Conclusion

Topological concepts are used in building important mathematical concepts in different fields as well as their applications in other sciences. The motive of this research is to expand the topological concepts based on the (NSS). In this manuscript, the authors introduced continuity, compactness and Hausdorff concepts in neutrosophic soft bitopological spaces by introducing the concept of $N_3(bi)^*$-continuous mapping, NSbi-open mapping, NSbi-closed mapping, $N_3(bi)^*$-compact and $N_3(bi)^*$-Hausdorff based on the definition of $N_3(bi)^*$-open, we also investigated the related theorems and properties of these concepts.

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We hope that the results of this study will be useful for researchers to present additional new studies on the neutrosophic soft sets.

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